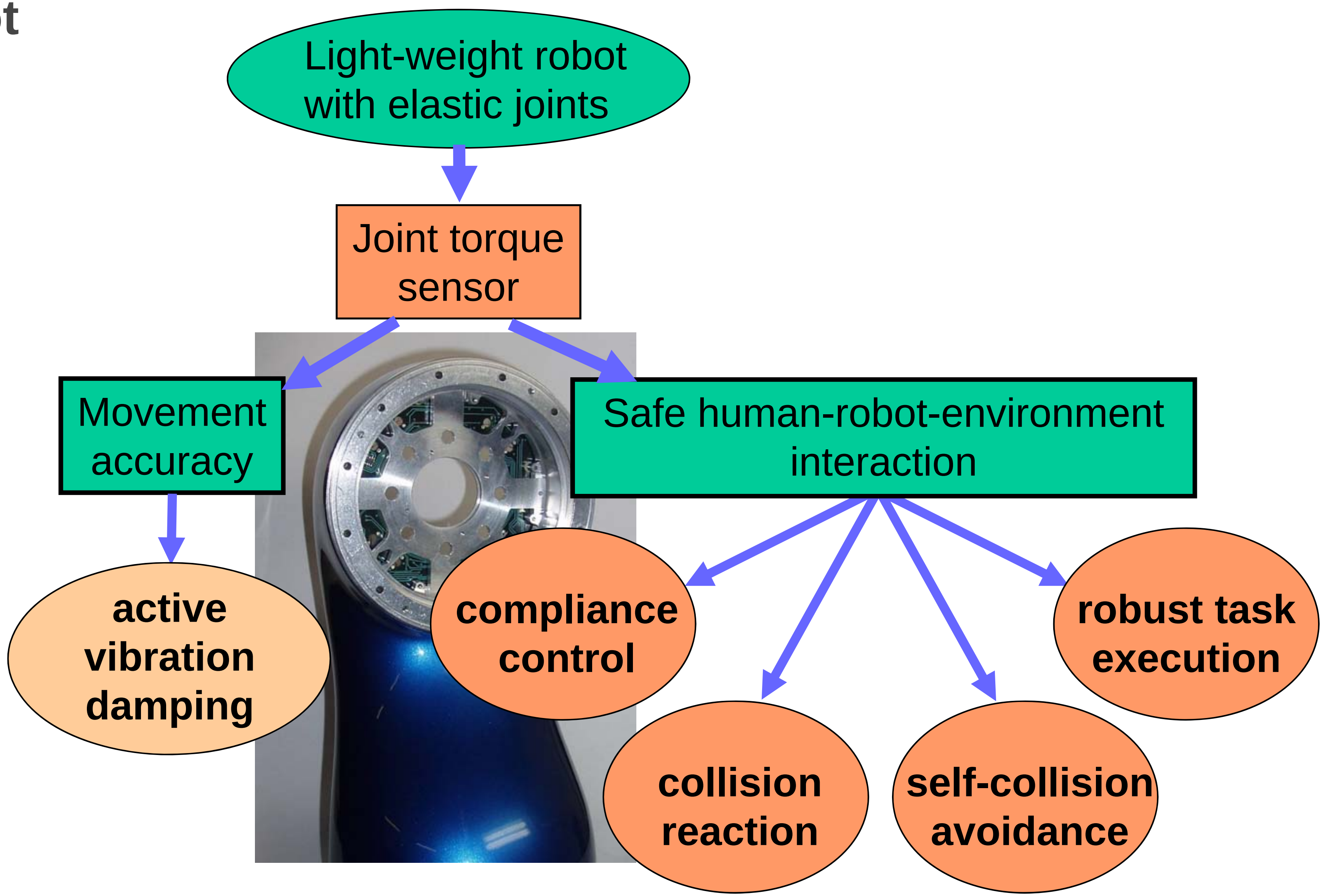
Interaction Control

Control Components for the Torque Controlled Light-weight Robot



Cartesian Impedance Control:

- vertical
  - stiffness=500 N/m
  - damping factor 0.001
- horizontal
  - stiffness=500 N/m
  - damping factor 0.7

Joint Position Control

Simplest Controller in Joint Space

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau$$

PD-Controller with gravity compensation:

$$\tau = k_P(q_d - q) - k_D\dot{q} + g(q)$$

closed loop system

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + k_D\dot{q} + k_P(q - q_d) = 0$$

Damping Design

- the mass matrix is position dependent
- the variation is assumed to be quasi-stationary
- constant stiffness is asumed

How should the damping matrix be chosen for a well damped behaviour?

Example: one-dimensional, linear case (constant mass)):

$$m\ddot{q} + d\dot{q} + kq = 0 \qquad d = 2\xi\sqrt{mk}, \quad 0 \leq \xi \leq 1$$

two real poles

Generalization to matrices:

A-p.d., sym. $\exists A_1$ sym, p.d, such that $A_1 A_1 = A$

Square root of a p.d., symmetric matrix

$$K_{P1} K_{P1} = K_P$$

$$M_1 M_1 = M \qquad K_D = \xi\left(M_1 K_{P1} + K_{P1} M_1\right)$$

Damping Design

For the linearized system

$$M\ddot{e} + K_D \dot{e} + K_P e = 0$$

$$\xi = 1$$

$$K_D = \left(M_1 K_{P1} + K_{P1} M_1 \right)$$

It follows

$$M_1 M_1 \ddot{e} + M_1 K_{P1} \dot{e} + K_{P1} M_1 \dot{e} + K_{P1} K_{P1} e = 0$$

Or, equivalently

$$M_1 \dot{e} + K_{P1} e = w \quad \longleftarrow \text{ intermediate variable}$$

$$M_1 \dot{w} + K_{P1} w = 0$$

Two first order differential equations and thus real eigenvalues

Remark: The quasi-static approximation is needed only for damping design.
The stability analysis is still valid for the non-linear system.

Double Diagonalization Damping Design

(alternative)

For the linearized system

$$M\ddot{e} + K_D\dot{e} + K_P e = 0$$

$$\xi = 1$$

$$K_D = \left(M_1 K_{P1} + K_{P1} M_1 \right)$$

Through an appropriate coordinate transformation, two p.d. symmetric matrices (K_P,M) can be simultaneously diagonalized.

$\exists Q$ such that $K = Q^T Q, \quad M = Q^T \Lambda Q$

such that Λ is diagonal.

With $\dot{w} = Q\dot{e}$ and $K_D = Q^T D Q$ the dynamics is decoupled

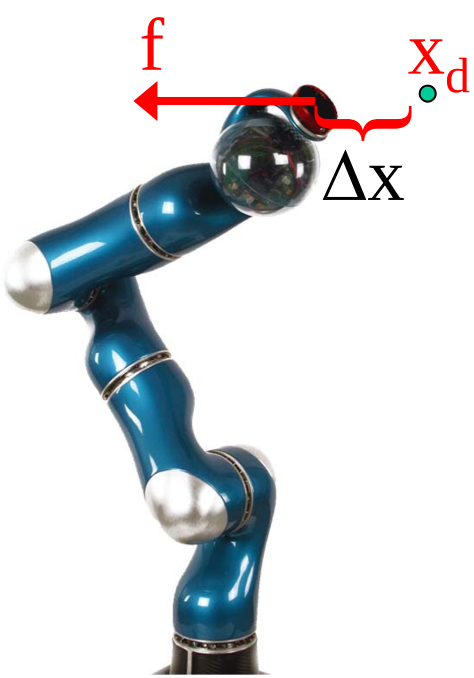
$$\Lambda \ddot{w} + D\dot{w} + w = 0$$

by choosing component-wise

$$D_i = 2\xi\sqrt{\Lambda_i}$$

Cartesian Impedance Control

Cartesian Impedance Control



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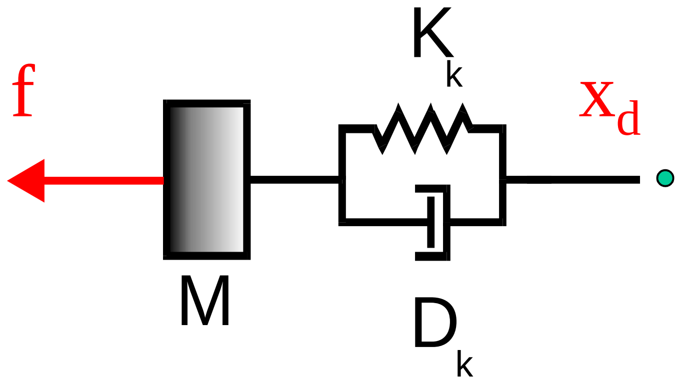
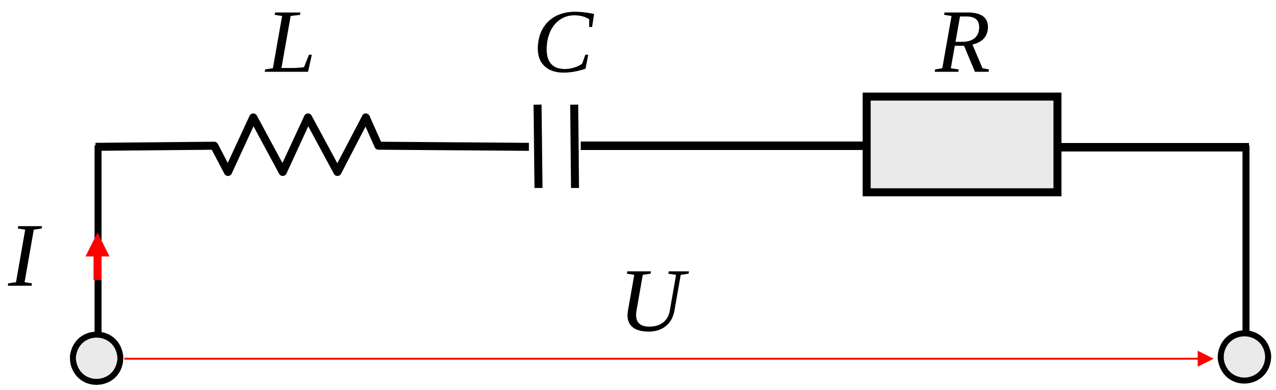


$$f = M\Delta\ddot{x} + D_k\Delta\dot{x} + K_k\Delta x$$

Notion of Impedance

Known from electrical circuits:



$$U = RI + L\dot{I} + \int \frac{1}{C} I \, dt$$

Impedance: $Z(s) = \frac{U(s)}{I(s)}$ ← Output ← Input

z.B. $Z(s) = \frac{1}{R + Ls + \frac{1}{Cs}}$

Admittance: $Y(s) = \frac{I(s)}{U(s)}$ ← Output ← Input

z.B. $Y(s) = R + Ls + \frac{1}{Cs}$

Notion of Impedance

Analogy electrical - mechanical

$$U = L\ddot{Q} + R\dot{Q} + \frac{1}{C}Q$$

$$f = M\ddot{x} + D\dot{x} + Kx$$

$U \rightarrow f$

$I \rightarrow \dot{x}$

$Q \rightarrow x$

$R \rightarrow D$ Damping

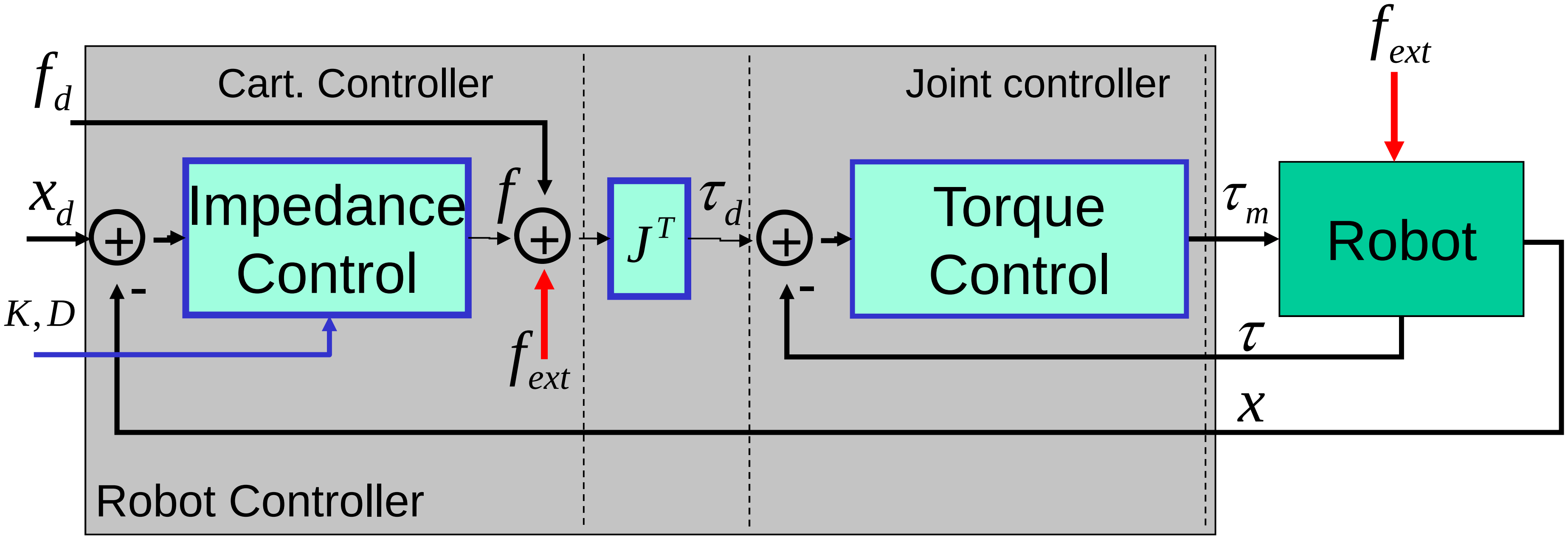
$L \rightarrow M$ Mass

$\frac{1}{C} \rightarrow K$ Ariffness

Impedance: $Z(s) = \frac{f(s)}{x(s)}$ ← Output ← Input — Impedance causality

Admittance: $Y(s) = \frac{x(s)}{f(s)}$ ← Output ← Input — Admittance causality

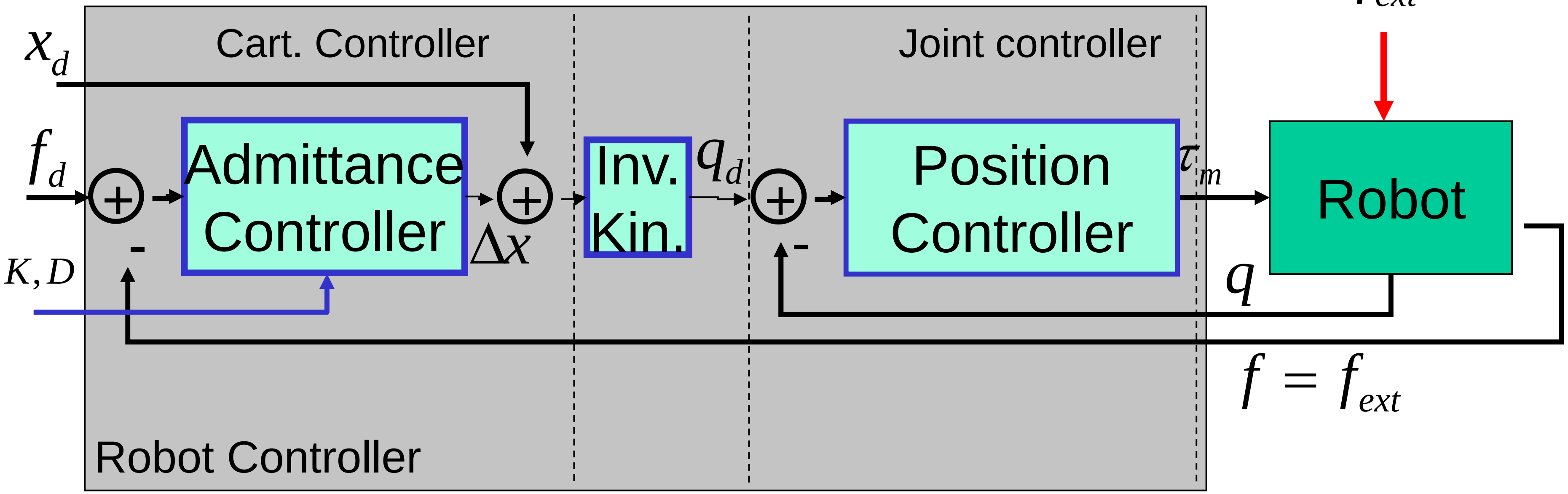
Principle of Impedance Control



- Inner loop: compliant
- outer loop: increases stiffness

Principle Admittance Control

Cartesian Controller with Admittance Causality



- Inner loop: high stiffness
- outer loop: compliant

=>
- Good behaviour for large and medium stiffness
- Good positioning accuracy
- Limitations for small stiffness (large compliance)
- Stability problems in contact

Cartesian Impedance Control

For a non-redundant, non singular robot with torque interface a simple PD controller in Cartesian coordinates can do the job.

Cartesian PD-Controller with gravity compensation

$$F = K_P(x_d - x) - K_D\dot{x}$$

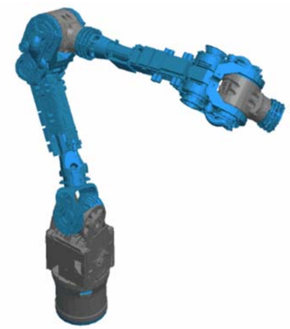
Transformation of the desired Cartesian Forces to desired joint torques

$$\tau = J^T(q)F + g(q) \ + \tau_N$$

Adding a null-space torque component τ_N for a redundant robot

$$\tau_N = (I - J^T J^{\#T})\tau_0$$

with τ_0 being an arbitrary joint space torque



the pseudoinverse has the property

$J^{\#T} J^T = I$ for example $J^{\#T} = (JJ^T)^{-1} J$

Usage of of Potentials

A quite general approach for generating robot controllers based on the passivity formalism

1. Define a task coordinate p, describing the control goal

$$p = h(q)$$

2. Define a p.d. (e.g. quadratical) potential as a function of p:

$$U(p) = \frac{1}{2} p^T K_p p$$

3. Differentiate the potential in order to obtain generalized forces dual to $\dot{p}$

$$F_p^T = -\frac{\partial U_p(p)}{\partial p} \quad \Rightarrow F_p = -K_p p$$

4. Apply the chain rule to get „elastic“ joint torques

$$\tau^T = -\underbrace{\frac{\partial U_p(p)}{\partial p}}_{F_p^T} \underbrace{\frac{\partial p(q)}{\partial q}}_{J_{pq}} \quad \Rightarrow \quad \tau = J_{pq}^T F_p$$

5. Add a dissipative term

$$F_{\dot{p}} = -D_p \dot{p} \qquad \Rightarrow \; \tau_{\dot{p}} = J_{pq}^T F_{\dot{p}}$$

Thus, one obtains an PD controller in the coordinate p

$$F = F_p + F_{\dot{p}} = -K_p p - D_p \dot{p}$$

Usage of Potentials

Remarks:

- For $p = x - x_d$ one obtains the Cartesian controller
- For the damping matrix one needs an appropriate damping design
- Further applications
  - collision avoidance, avoidance of joint limits
  - singularity avoidance
  - nullspace stiffness …

Collision Avoidance

Control of Flexible Joint Robots

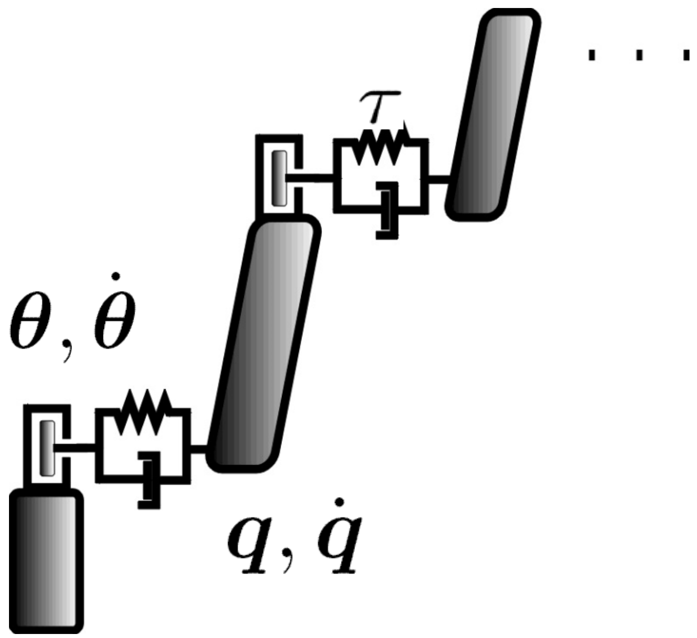
Torque Control with Gravity Compensation

Impedance control assumes an ideal torque source at joint level

In the rigid robot model, joint torque is not a state

- a torque controller with a P-term is not causal!

Model of the Flexible Joint Robot



possible state vector:

$$x_1^T = \{\theta, \dot{\theta}, q, \dot{q}\}$$

used state vector:

$$x^T = \{\theta, \dot{\theta}, \tau, \dot{\tau}\}$$

$$M(q)\ddot{q} + c(q, \dot{q}) + g(q) = \tau + \tau_{ext}$$

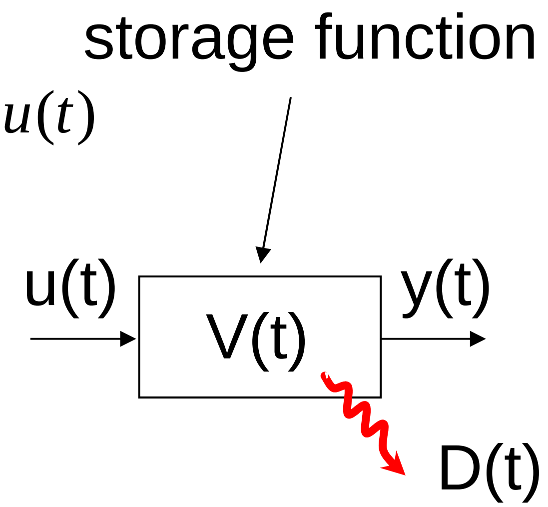
$$B\ddot{\theta} + \tau = \tau_m$$

$$\tau = K(\theta - q)$$

Definition of Passivity

A system is passive, if

$$\exists \alpha > -\infty, \qquad \int_0^t u^T(t) y(t) \mathrm{d}t > \alpha \qquad \forall u(t)$$



alternative (differential) formulation

energy variation
input power
dissipated power

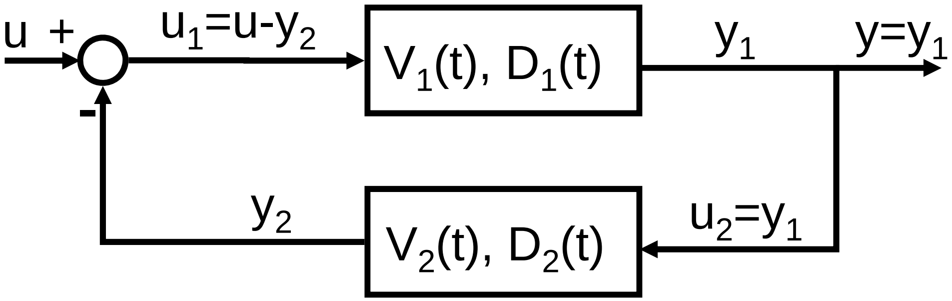
$$\dot{V}(t) = u^T(t) y(t) - D(t)$$

with $D(t) \geq 0$ and $V(t)$ lower bounded

Interconnection of Passive Systems

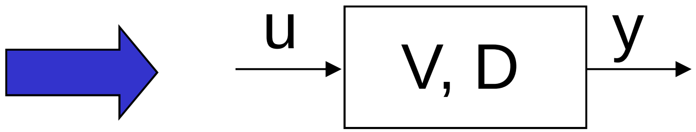
$$\dot{V}(t) = u^T(t)y(t) - D(t)$$

feedback coupling

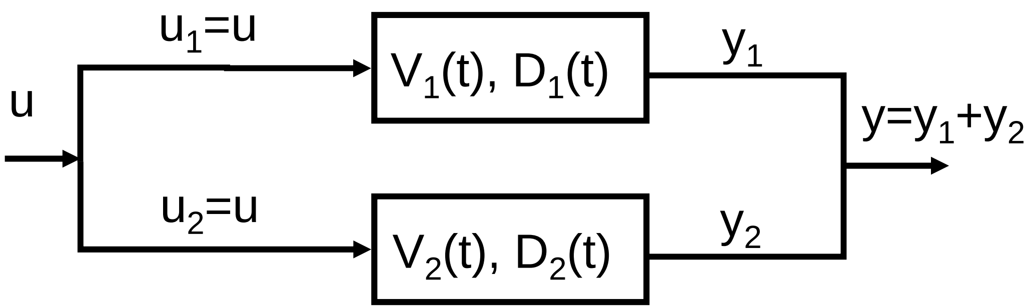


$$u = 0 \quad => \quad u_1^T y_1 = -y_2^T u_2$$

$$\dot{V}_1(t) + \dot{V}_2(t) = -D_1(t) - D_2(t) \leq 0$$



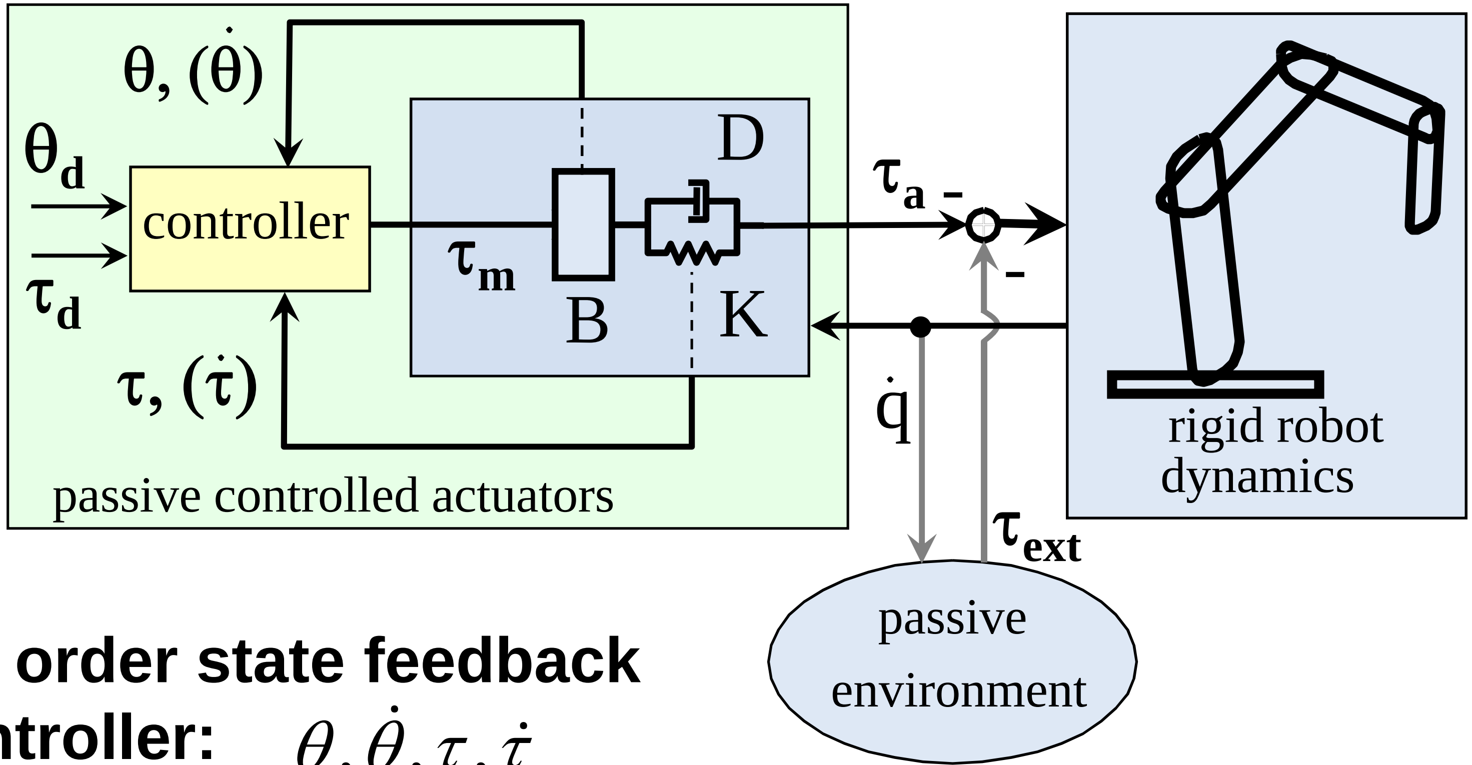
parallel coupling



$$V = \sum_i V_i$$

$$D = \sum_i D_i$$

Full state feedback for flexible joint robots



4th order state feedback controller: $\theta, \dot{\theta}, \tau, \dot{\tau}$

same structure used for

- **toque control**
- **position control**
- **impedance control**

Vibration Damping

Vibration Damping ON

Vibration OFF

Robot reaches the dynamics and accuracy of an industrial arm (according to KUKA ISO-Tests)

Cartesian Impedance Controller and Position Controller

Generalization of approaches from rigid robots to the flexible case

- Shaping the potential energy - collocated feedback
  - Asymptotic stabilization around $\boldsymbol{x}_{\mathbf{d}}$ ($\boldsymbol{\tau}_{ext} = \mathbf{0}$)
  - Implementation of the desired compliance relationship ($\boldsymbol{\tau}_{ext} \neq \mathbf{0}$)
  - Feedback of $\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}$ (rigid robot impedance controller!)
- Shaping of the kinetic energy - noncollocated feedback
  - Damping of vibrations => increased performance
  - Feedback o $\boldsymbol{\tau}, \dot{\boldsymbol{\tau}}$ (torque controller)

=> Full state feedback

Torque controller

Motor dynamics:

$$B\ddot{\theta} + \tau = \tau_m$$

Torque controller:

$$\tau_m = BB_\theta^{-1}u - (I - BB_\theta^{-1})\tau$$

Closed loop system:

$$B_\theta\ddot{\theta} + \tau = u$$

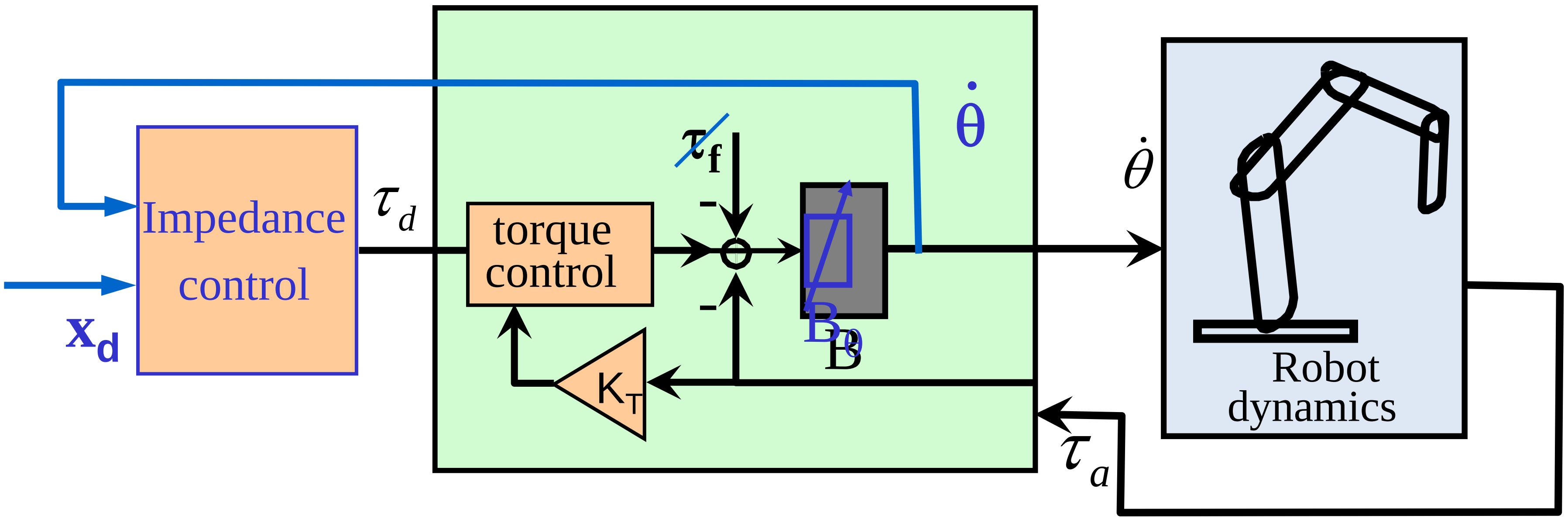
New input

Modified actuator inertia

Physical interpretation of torque feedback

Cartesian Impedance Control

Unified approach for torque, position and impedance control on Cartesian and joint level



$$\tau_F \to (1+K_T)^{-1}\tau_F \qquad B \to B_\theta = (1+K_T)^{-1}B$$

Passivity ⟶ Robustness in contact with the environment