- Robot Parameters Identification

Problem Statement

$$\tau = M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q)$$

The model depends on following groups of parameters:

- joint variables $(q, \dot{q}, \ddot{q})$ - measured
- kinematics parameters (such as the constant DH-Parameter p_{DH}) – from CAD
- dynamics parameters p (masses, centers of mass, momentum of inertia) – often unprecisely known from CAD or variable (variable loads on the tip or along the structure).

τ is known, because it is the controller output. For the light-weight robot, it can be directly measured

We will present an off-line procedure for identification of the dynamics parameters

(for unknown kinematics parameters, similar identification methods exist.)

Linear Parameterization of the Dynamics Model

the dynamics model can be rewritten as follows

$$\tau = Y(q, \dot{q}, \ddot{q}, p_{DH})\, p$$

Regressor

unknown vector of dynamics parameters

$\tau \in \mathbb{R}^n$

$Y \in \mathbb{R}^{n \times k}$

$p \in \mathbb{R}^k, \quad typically: k > n$

Basic algorithm for identification:
perform several measurements in different configuration:

$$\begin{aligned} \tau_1 &= Y_1 p \\ \tau_2 &= Y_2 p \\ &\vdots \\ \tau_l &= Y_l p \end{aligned} \quad \Rightarrow \quad \tau_{tot} = Y_{tot}\, p$$

$\tau_{tot} \in \mathbb{R}^{l \cdot n}$

$Y_{tot} \in \mathbb{R}^{l \cdot n \times k}$

$p \in \mathbb{R}^k, \quad k \le l \cdot n$

If $Rank(Y) = k$, then the system can be solved for *p*.

$$p = Y_{tot}^{-1} \tau_{tot}$$ for $k = l \cdot n$ or $$p = Y_{tot}^{\#} \tau_{tot}$$ for $k \le l \cdot n$

(right pseudoinverse: $Y_{tot}^{\#} Y_{tot} = I_{k \times k}$)

Background:Linear Parametrization of the Dynamics Model

(Which are the components of the parameter vector p?)

The dynamics equations of the robot can be obtained by the dynamics formalism:

$$\frac{d}{dt}\left(\frac{dL(q,\dot{q})}{d\dot{q}}\right) - \frac{dL(q,\dot{q})}{dq} = \tau \quad \text{with} \quad L(q,\dot{q}) = T(q,\dot{q}) - U(q)$$

kinetics energy   potential energy

Remark: the Lagrangian-Formalism is used here for didactical purposes. For a practical implementation of dynamics computation, the Newton-Euler Algorithm is much more efficient (O(n) vrs. O(n^3)). If M, C, and g are required independently, the Featherstone algorithm is currently the most efficient method for dynamics computation (O(n)).

The kinetic energy of robot segment j is (without proof):

$$T = \frac{1}{2}\begin{bmatrix} v^T & \omega^T \end{bmatrix}\begin{bmatrix} mI_{3\times 3} & -m\hat{l}_c \\ m\hat{l}_c & J_I \end{bmatrix}\begin{bmatrix} v \\ \omega \end{bmatrix}$$

(matrix notation for vector product)

m - mass

l_c – cener of mass (3 parameters)

J_I –momentum of inertia (6 parameters))

all quantities w.r.t. reference frame of joint j (whiteboard)

$$J_I = I_c - m\hat{l}_c\hat{l}_c$$

I_c – inertia w.r.t. center of mass

The kinetic energy is linear in the parameters $\{m, ml_c, J_I\}$

Determining a minimal parameter set

One can analogously show that the potential energy is linear in $\{m, ml_c\}$

One therefore has

$$p_0^T = \left[m_1, m_1 l_{c1}, J_1, \ldots, m_n, m_n l_{cn}, J_n\right] \quad 10n \text{ parameters!}$$

however, in general not all parameters can be independently identified:

- if the column of Y corresponding to one parameter is zero, then the parameter can
- be removed.
- if two columns in Y are linearly dependent, then only a linear combination of the two parameters can be identified.

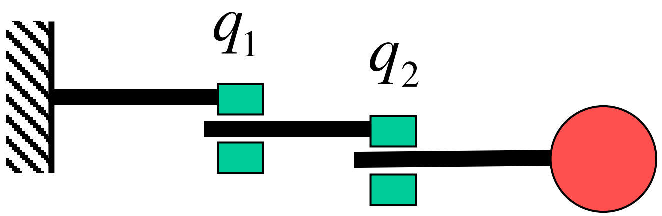
$$\tau = \left[Y_1, \alpha Y_1, \cdots Y_k\right] \begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_k \end{bmatrix} \equiv \quad \tau = \left[Y_1, \cdots Y_k\right] \begin{bmatrix} p_1 + \alpha\, p_2 \\ \vdots \\ p_k \end{bmatrix}$$

By eliminating all linear dependencies, on can reach a minimal parameter set, for which Y can have full rank (if the measurements are independent). This minimal parameter set can be completely identified.

In **[Khalil02]** one can find detailed rules for the formulation of the minimal parameter set.

Example

(the world‘s simplest redundant manipulator)

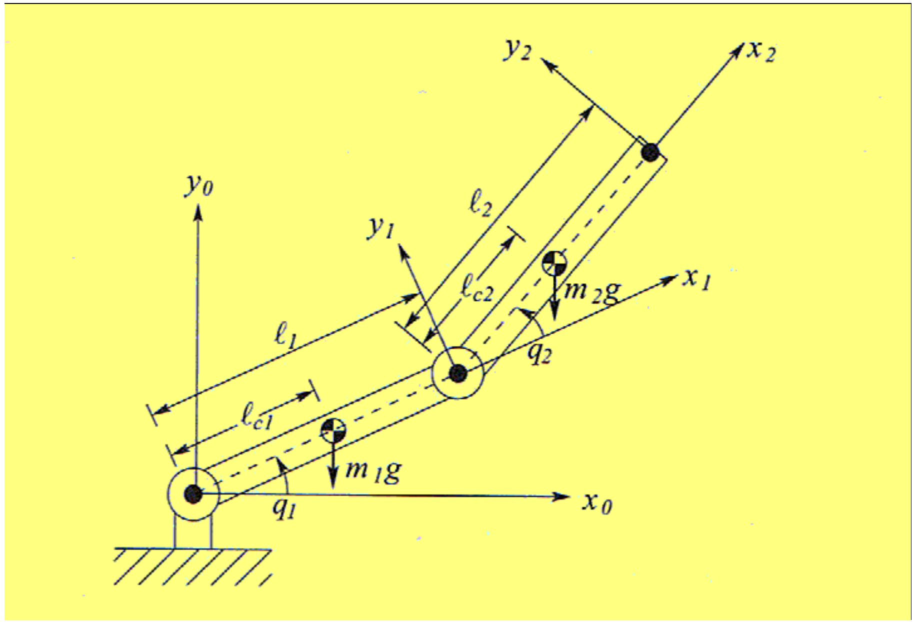


$$\begin{cases} \tau_1 = (m_1 + m_2)\ddot{q}_1 + m_2\ddot{q}_2 \\ \tau_2 = m_2(\ddot{q}_1 + \ddot{q}_2) \end{cases}$$

a) Write the regressor for the identification of the dynamics parameters for this system.
b) Which conditions must be fulfilled in this case in order to identify all parameters? What is the minimal number of robot configurations in which measurements have to be performed for the identification? Why should one choose in practice substantially more points?

Additional Slide: 2DoF Manipulator

find the minimal parameter set and the regressor for the following system:



$$\begin{cases} \tau_1 = M_{11}\ddot{q}_1 + M_{12}\ddot{q}_2 + C_{11}\dot{q}_1 + C_{12}\dot{q}_2 + g_1 \\ \tau_2 = M_{21}\ddot{q}_1 + M_{22}\ddot{q}_2 + C_{21}\dot{q}_1 + C_{22}\dot{q}_2 + g_2 \end{cases}$$

$$M_{11} = m_2(l_1^2 + 2l_1 l_{c2}\cos(q_2)) + I_1 + I_2$$

$$M_{12} = M_{21} = 2m_2 l_{c2} l_1 \cos(q_2) + I_2$$

$$M_{22} = I_2$$

$$h = -m_2 l_1 l_{c2}\sin(q_2)$$

$$C_{11} = h\dot{q}_2 \qquad C_{12} = h(\dot{q}_1 + \dot{q}_2)$$

$$C_{21} = -h\dot{q}_1 \qquad C_{22} = 0$$

$$g_1 = (m_1 l_{c1} + m_2 l_1) g\cos(q_1) + m_2 l_{c2} g\cos(q_1 + q_2)$$

$$g_2 = m_2 l_{c2} g\cos(q_1 + q_2)$$

Hint: The columns of the regressor will be linearly dependent – reduce the dimension of the parameter vector

Practical Aspects

- In practice, one will choose $l >> k$.
- The identification can be also formulated as a minimization problem:

$$\min_{p}\{\tau - Yp\}$$

If Y has full rank, then the pseudoinverse provides the solution of the optimization problem.

- It is very important to choose measurement points (trajectories), which excite all parameters well - each parameter should have a strong influence on the measurements. A measure therefore is the conditioning index of *Y*

$$cond(Y) = \frac{\lambda_{\max}}{\lambda_{\min}} \geq 1$$

λ-singular values

which should be as close as possible to 1.

- It is useful to group the parameters in small sets (wrist joints/ shoulder joints, static parameters / constant velocities, …) and to design special purpose trajectories for excitation of each set separately.
- signals must mostly be filtered for good results.