On Analog Computation of Vector-Valued Functions in Clustered Wireless Sensor Networks

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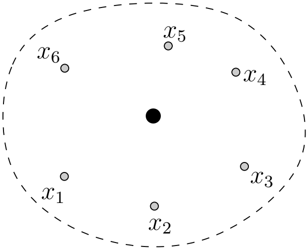
Motivation

- **Key problem:** Efficient computation of **functions** of the measurements
- To combat interference, traditional schemes like TDMA were used.
- The fusion center reconstructs each sensor signal **separately** and **subsequently** computes f.
- Separating **communication** and **computation** can be highly inefficient [Nazer,Gastpar 07].
- To combine the processes, interpret the wireless multiple-access channel (MAC) as a **computer**.
- Every multivariate function is analog-computable via the wireless MAC [Goldenbaum,Boche,Stańczak 11].



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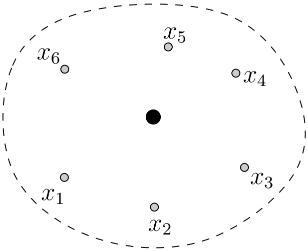

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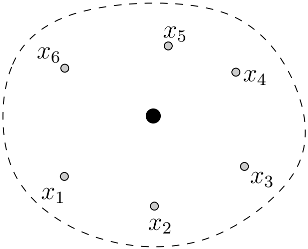

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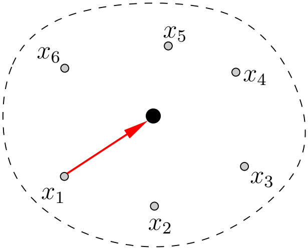

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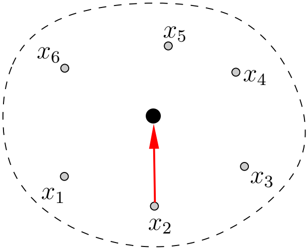

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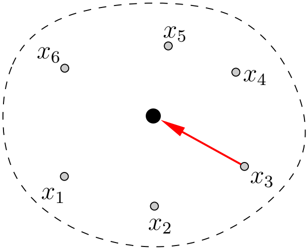

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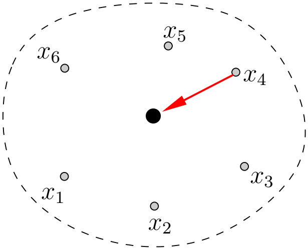

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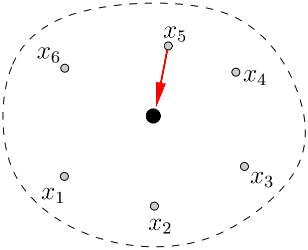

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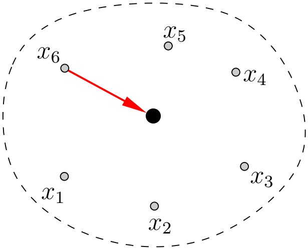

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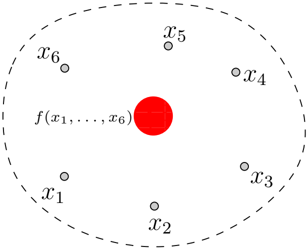

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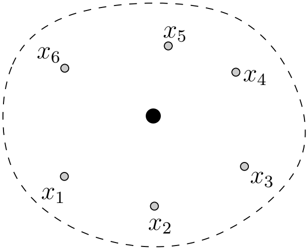

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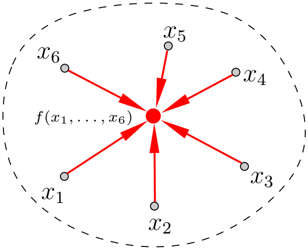

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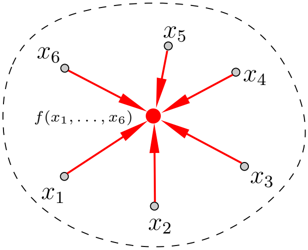

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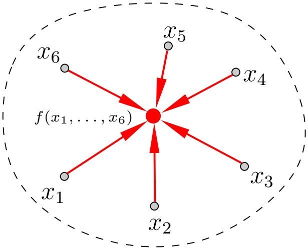


Question:

- What happens in more general networks?

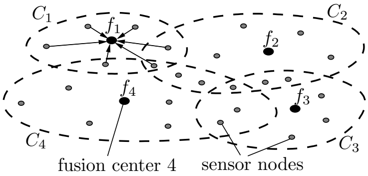
Outline

1. System Model & Problem Statement
2. Preliminaries
3. Analog Computation of Vector-Valued Functions
4. Performance & Properties
5. Summary

Outline

System Model & Problem Statement

- N sensor nodes organized into K clusters C_k ($C_k \cap C_\ell \neq \emptyset$)
- Sensor readings $x_n \in [0, 1]$, $n = 1, \ldots, N$



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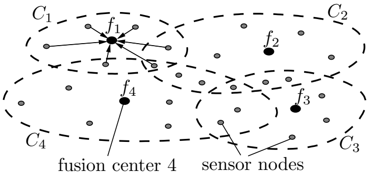

Intra-cluster Communication

$$y_k = \sum_{n \in C_k} h_{kn} x_n + v_k \quad , \quad k = 1, \dots, K$$

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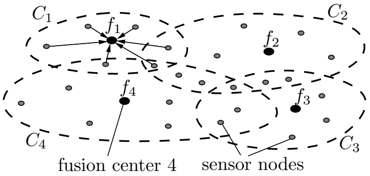

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- **Problem:** Efficiently computing $\boldsymbol{f} : [0,1]^N \to \mathbb{R}^K$

$$\boldsymbol{f}(x_1, \ldots, x_N) = \begin{pmatrix} f_1\left(x_{1_1}, \ldots, x_{1_{|C_1|}}\right) \\ \vdots \\ f_K\left(x_{K_1}, \ldots, x_{K_{|C_K|}}\right) \end{pmatrix}$$

Outline

Preliminaries (single cluster case) [Goldenbaum,Boche,Stańczak 11]

Receive Signal

$$y = \sum_{i=1}^{N} h_i x_i + v \qquad (*)$$

- $\Rightarrow$ The ideal MAC can be used to compute every function $f : [0,1]^N \to \mathbb{R}$ that has a representation $(*)$.
- Examples:
  - Arithmetic Mean: $f(x_1, \dots, x_N) = \frac{1}{N} \sum_i x_i$, $\varphi_i(x) = x$, $\psi(y) = \frac{1}{N} y$
  - Geometric Mean: $f(x_1, \dots, x_N) = \left(\prod_i x_i\right)^{1/N}$, $\varphi_i(x) = \log(x)$, $\psi(y) = \exp(y/N)$
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Observation

The space of functions $(*)$ is exactly the space of nomographic functions.

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- **Question:** Is the theorem valid if pre- and post-processing functions are required to be continuous?

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Theorem [Arnol'd 57], [Buck 82]

The space of nomographic functions with continuous pre- and post-processing functions is nowhere dense in the Banach-space of continuous functions.

Preliminaries (single cluster case) [Goldenbaum,Boche,Stańczak 11]

- Let $\ell \in \mathbb{N}$ and $\{\varphi_{ij}\}_{1 \leq i \leq N, 1 \leq j \leq \ell}$ be a collection of ℓN pre-processing functions and consider the sequence of receive signals

$$\left\{ y_1 = \sum_{i=1}^{N} \varphi_{i1}(x_i), \ldots, y_\ell = \sum_{i=1}^{N} \varphi_{i\ell}(x_i) \right\}$$

- **Question:** Which functions are computable with such sequence of length ℓ?
- The question is closely related to the 13^{th} Hilbert problem formulated in 1900.
- **Hilbert's conjecture:** The computation of continuous multivariate functions is in general **not possible** with **finite** ℓ.
- The conjecture was disproven by Kolmogorov in 1957. He has shown that every continuous $f : [0,1]^N \to \mathbb{R}$ has a representation
- $2N + 1$ can not be reduced [Sternfeld 85].
- $\Rightarrow$ Unfortunately, continuity requires more wireless resources.

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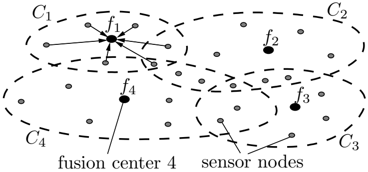
Outline

Analog Computation of Vector-Valued Functions (Geometry)

$$\boldsymbol{f}(x_1, \ldots, x_N) = \begin{pmatrix} f_1(x_{1_1}, \ldots, x_{1_{|C_1|}}) \\ \vdots \\ f_K(x_{K_1}, \ldots, x_{K_{|C_K|}}) \end{pmatrix}$$



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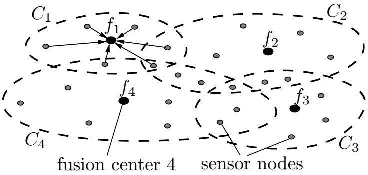

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- $\Rightarrow$ There is a bijection between all continuous $f(x_1,\ldots,x_N)$ on $[0,1]^N$ and all continuous $F(y_0,\ldots,y_{2N})$ on Γ

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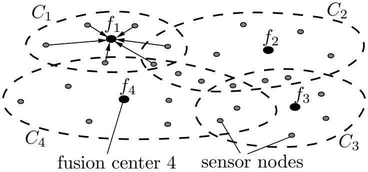

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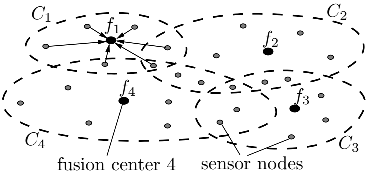

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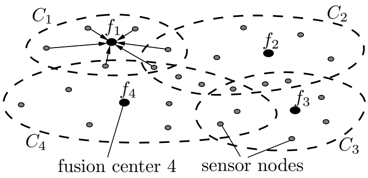

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Geometric interpretation (multiple cluster case):

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Outline

Performance & Properties

Standard TDMA

Analog Computation via Channels

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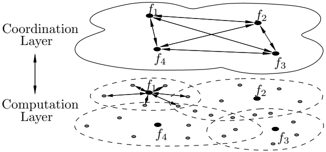

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$$\mathcal{O}\big(KL \geq K \max_k |C_k|\big)$$

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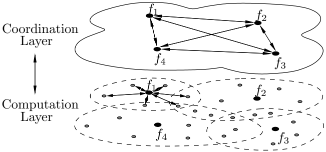

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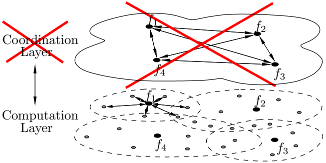

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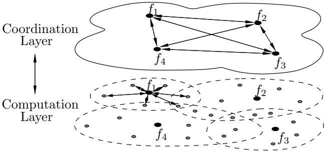

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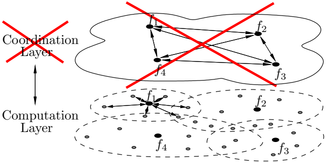

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Due to the universality property

- The hard- and software of sensor nodes is independent of $\boldsymbol{f}$
- Only dumb (cheap) nodes are required
- Feedback about functions is not necessary

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Summary

- By a geometric interpretation of Kolmogorov's theorem, we proposed an efficient analog computation scheme for clustered wireless networks
- Analog systems are gaining more attention for sensor networks (digitial signal processing has some fundamental limits [Boche,Mönich 11])
- The scheme exploits the superposition property of wireless channels
- With and without restrictions on pre- and post-processing functions, huge performance gains are possible compared with orthogonal approaches
- Coordination of nodes and clusters is not necessary
- No information about the functions at nodes is necessary (i.e., no feedback, dumb sensor nodes).

- Realistic MACs: power constraints, fading, noise, synchronization for some Nomographic examples [Goldenbaum,Stańczak 09]
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- Analog systems are gaining more attention for sensor networks (digitial signal processing has some fundamental limits [Boche,Mönich 11])
- The scheme exploits the superposition property of wireless channels
- With and without restrictions on pre- and post-processing functions, huge performance gains are possible compared with orthogonal approaches
- Coordination of nodes and clusters is not necessary
- No information about the functions at nodes is necessary (i.e., no feedback, dumb sensor nodes).

- Realistic MACs: power constraints, fading, noise, synchronization for some Nomographic examples [Goldenbaum,Stańczak 09]
- Node failures/loss or inclusions of new nodes [Goldenbaum,Boche,Stańczak 12]

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Thanks for your attention

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