Nomographic Gossiping for f-Consensus

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Motivation

- Consensus problems have a long history in computer science and form the foundation of the field of distributed computing.
- A consensus protocol specifies the information exchange between nodes in a network and all of its neighbors.

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$\Longrightarrow$ Goal: global consensus based on local data exchange (gossiping) as fast as possible

- Relevance:
  - synchronization issues
  - data fusion in sensor networks
  - distributed coordination of mobile autonomous agents
  - distributed spectrum sensing in cognitive radio systems
  - distributed decision making in control systems
  - etc.
- Most work on consensus algorithms do not take properties of the wireless channel into account.

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Computation and Communication are often viewed as distinct processes.

Outline

1. Network Model & Problem Statement
2. Nomographic Gossiping
   - Deterministic Nomographic Gossiping
   - Randomized Nomographic Gossiping
3. Numerical Examples
4. Summary

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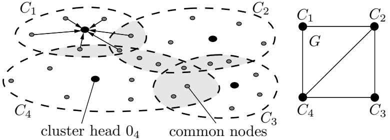
- N wireless nodes organized into K single-hop clusters C_i
- $G = (C, E)$ associated graph with $(i, j) \in E$ iff $C_i \cap C_j \neq \emptyset$
- Connected clustered WN: for any C_i and C_j there is a sequence of connected clusters from C_i to C_j
- Initial states $x_n(0) \in [0, 1]$, $n = 1, \dots, N$



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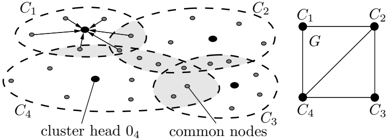

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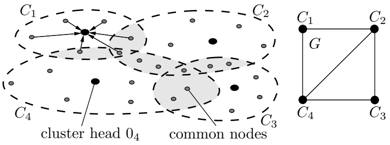

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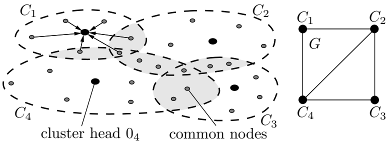

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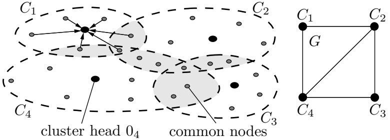

Intra-cluster Communication

$$y_{0_i}(t) = \sum_{n \in C_i \setminus \{0_i\}} h_{in}(t) x_n(t) + v_i(t) \quad , \quad i = 1, \ldots, K$$

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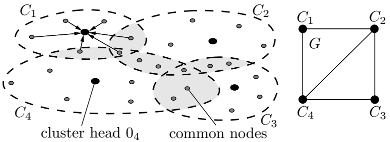

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f-consensus problem

Let $f : [0,1]^N \to \mathbb{R}$ be any desired consensus. Then,

$$\forall n = 1, \dots, N : \lim_{t \to \infty} \left\| x_n(t) - f\big(x_1(0), \dots, x_N(0)\big) \right\| = 0$$

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Preliminaries (single cluster case)

Receive Signal

$$y = \sum_{n=1}^{N} h_n x_n + v \qquad (*)$$

- $\Rightarrow$ The ideal MAC can be used to compute every function $f : [0,1]^N \to \mathbb{R}$ that has a representation $(*)$.
- Examples:
  - Arithmetic Mean: $f(x_1, \ldots, x_N) = \frac{1}{N}\sum_i x_i$, $\varphi_i(x) = x$, $\psi(y) = \frac{1}{N} y$
  - Geometric Mean: $f(x_1, \ldots, x_N) = \left(\prod_i x_i\right)^{1/N}$, $\varphi_i(x) = \log(x)$, $\psi(y) = \exp(y/N)$
  - Euclidean Norm: $f(x_1, \ldots, x_N) = \sqrt{x_1^2 + \cdots + x_N^2}$, $\varphi_i(x) = x^2$, $\psi(y) = \sqrt{y}$

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Observation

The space of functions $(*)$ is exactly the space of nomographic functions.

- The theorem constitutes the basis for Nomographic Gossiping
- Gossiping: nodes have only a local view on network dynamics
- We propose two classes of corresponding algorithms that differ in the way clusters are activated
  - Deterministic: coordinated cluster activation
  - Randomized: clusters randomly wake up

Preliminaries (single cluster case)

Theorem [Goldenbaum,Boche,Stańczak 11]

Every function is universally computable via an ideal MAC, since every $f : [0,1]^N \to \mathbb{R}$ is nomographic.

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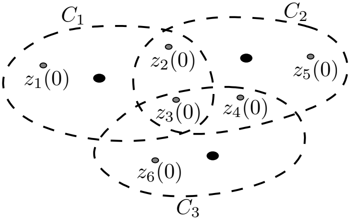

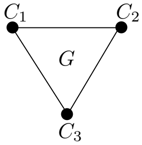

- Let f be the desired consensus and let $\varphi_1, \ldots, \varphi_6, \psi$ be such that

$$f\big(x_1(0), \ldots, x_6(0)\big) = \psi\Big(\sum_{n=1}^{6} \underbrace{\varphi_n(x_n(0))}_{=:z_n(0)}\Big)$$

- Activation sequence: W.l.o.g. $\pi(t) = \{1, 2, 3, 1, 2, 3, 1, \ldots\}$
- Nodes are equipped with a transmission counter and they know their status (standard or common nodes)

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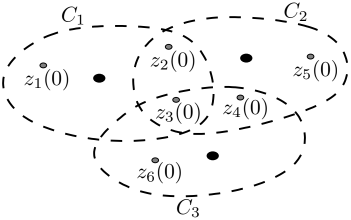

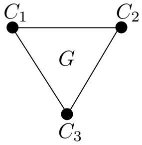

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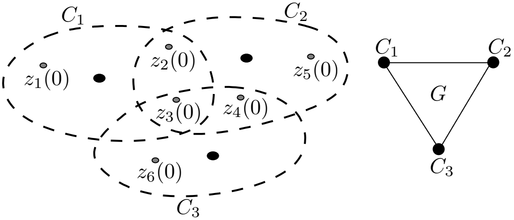

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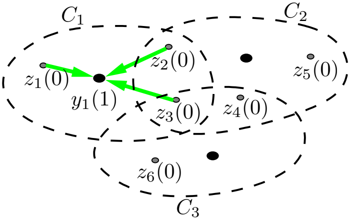

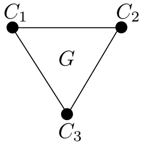

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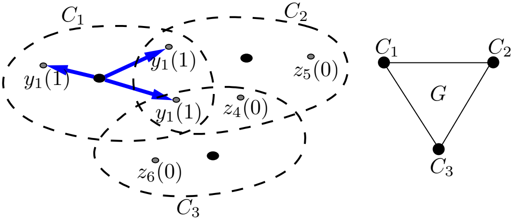

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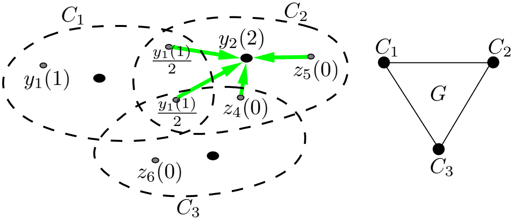

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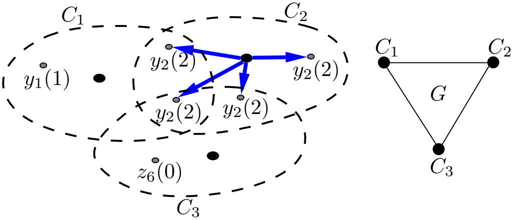

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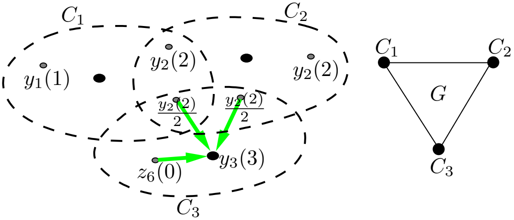
- Activation sequence: W.l.o.g. $\pi(t) = \{1, 2, 3, 1, 2, 3, 1, \dots\}$
- Nodes are equipped with a transmission counter and they know their status (standard or common nodes)

Deterministic Nomographic Gossiping

$t = 3$:

$$y_3(3) = \sum_{n=1}^{6} \varphi_n\big(x_n(0)\big)$$



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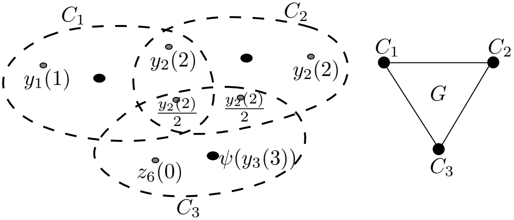

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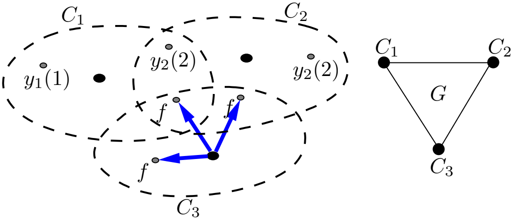

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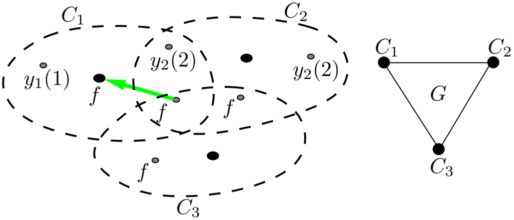

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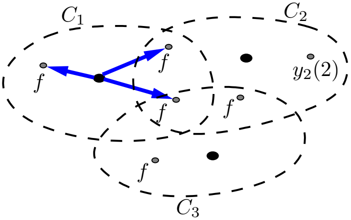

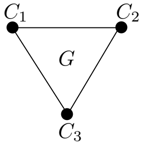

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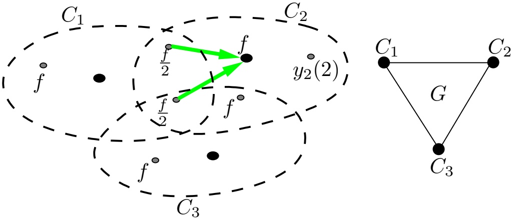

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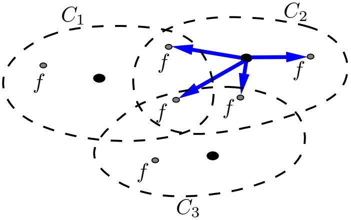

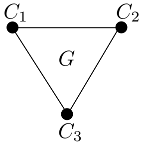

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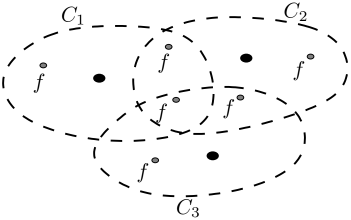

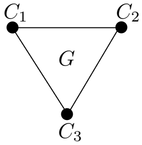

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Deterministic Nomographic Gossiping (Convergence)

- The fast convergence to the exact desired consensus comes at the cost of some coordination
- Due to the universality property, the pre-processing at nodes do not have to be updated if the desired consensus changes

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Theorem

Let f be any desired consensus. Deterministic nomographic gossiping always converges to f in a finite number of steps. If the associated graph is hamiltonian, then convergence can be achieved in at most $2K-1$ steps.

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2. Nomographic Gossiping
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3. Numerical Examples
4. Summary

Randomized Nomographic Gossiping

- Each cluster head has a clock that ticks independently at a rate $\mu_i \in \mathbb{R}_+$ Poisson process, $i = 1, \dots, K$
- The μ_i are chosen such that with high probability two cluster heads do not wake up simultaneously
- Local averaging in each cluster: $z_{0_i}(t) = \frac{1}{N_i - 1} \sum_{n \in C_i} z_n(t-1)$, $t \in \mathbb{Z}_+$

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Theorem

Let f be any desired consensus with continuous pre- and post-processing functions. Then, randomized nomographic gossiping converges to f almost surely.

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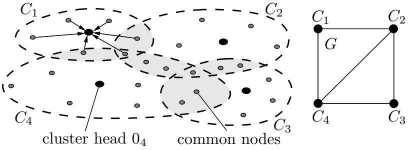
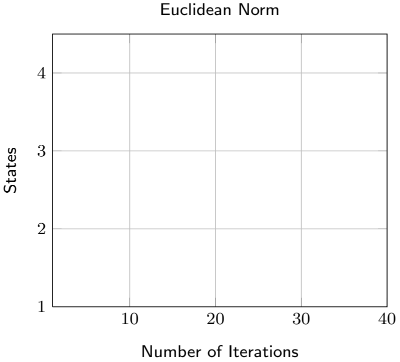


- $N = 25$ nodes
- $K = 4$ clusters
- $x_n(0)$ uniformly drawn from $[0, 1]$



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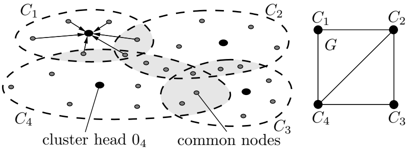

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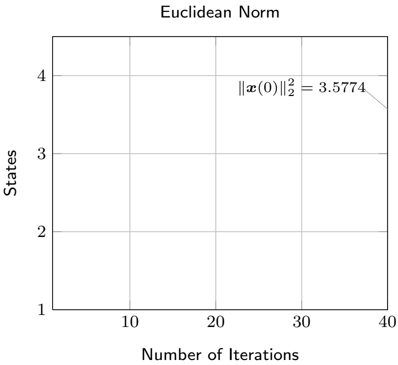

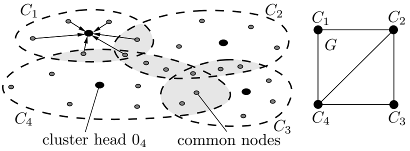

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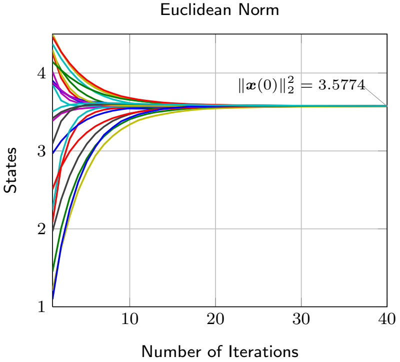

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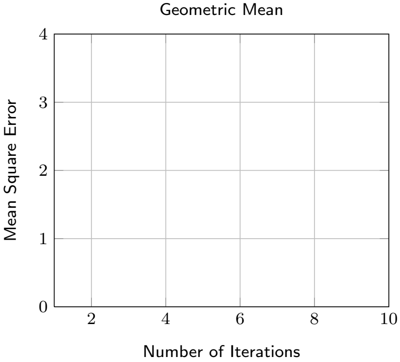

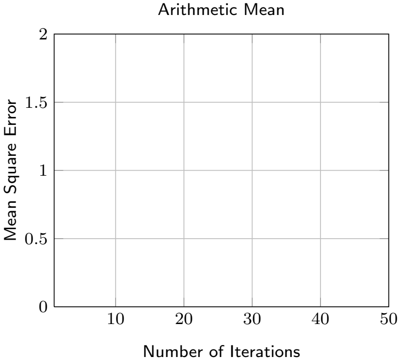

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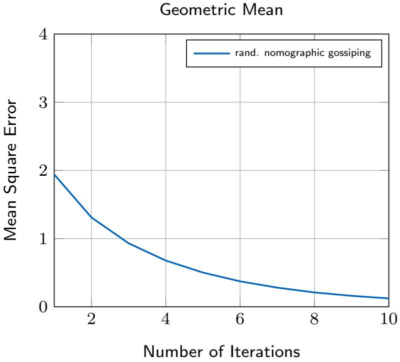

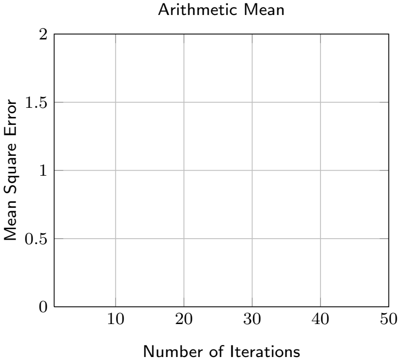

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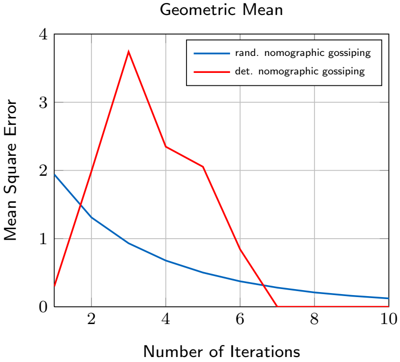

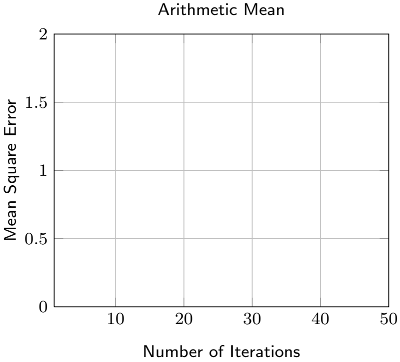

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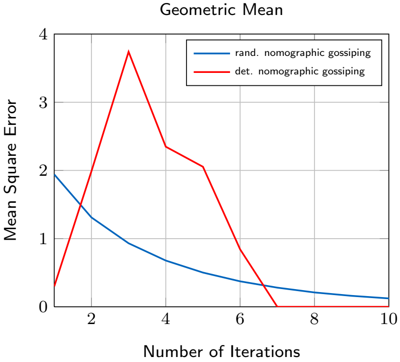

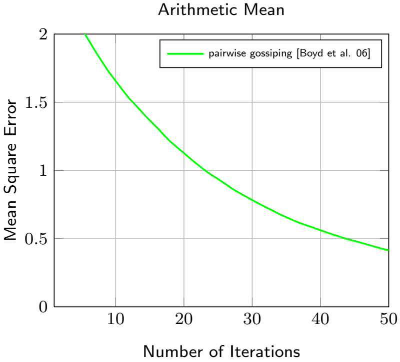

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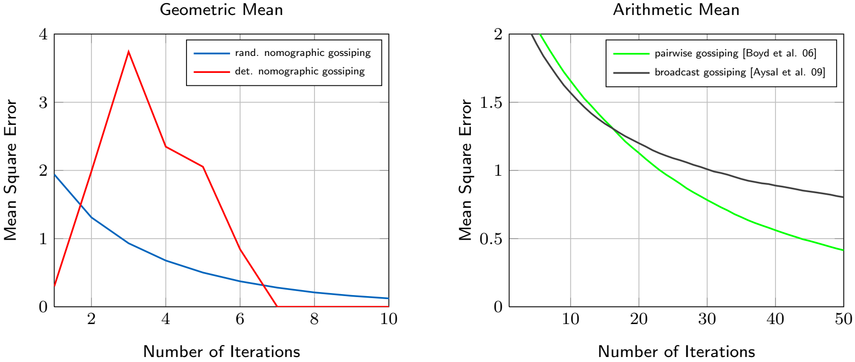

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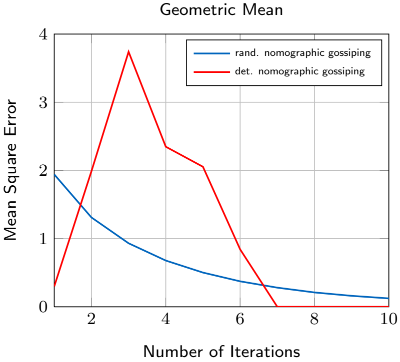

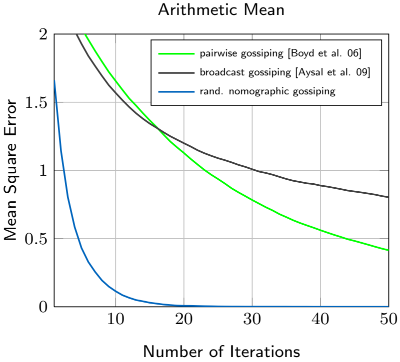

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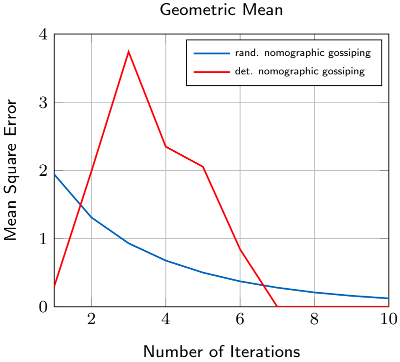

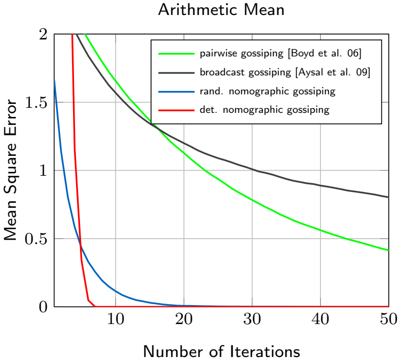

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Summary

- Nomographic gossip algorithms partly allow to achieve a consensus with respect to an arbitrary function of the initial states
- The algorithms rely on the representation of multivariate functions as post-processed superpositions of pre-processed initial states
- Superpositions can be efficiently achieved via the channel by letting nodes transmit simultaneously
- The class of nomographic gossip algorithms consists of deterministic and randomized approaches
- Deterministic nomographic gossiping always converges in a finite number of iterations
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