Message Transmission over Classical Quantum Channels with a Jammer with Side Information: Message Transmission Capacity and Resources

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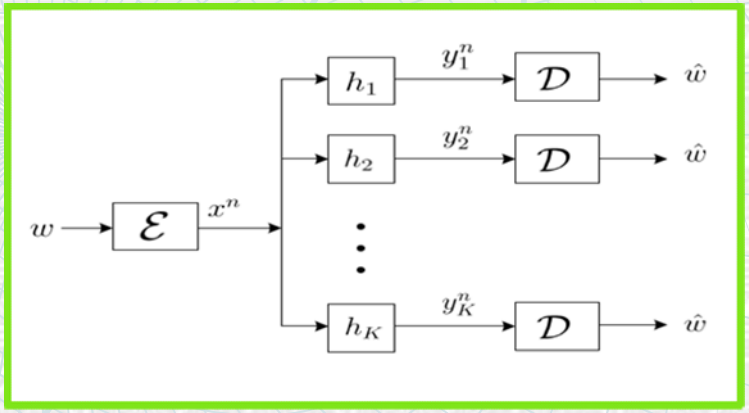
Quantum Arbitrarily Varying Channel When the Jammer Knows Input Codeword

Robustness

Let S := {1;…;T} be a finite set.
For every s ∈ S let $\rho(\,.\,,s)$ be a quantum channel X → S(H).
The set of $\{\rho(x,s): x \in X, s \in S\}$ is an **arbitrarily varying classical-quantum channel** (AVCQC) when s varies in an arbitrary manner.



Quantum Arbitrarily Varying Channel When the Jammer Knows Input Codeword

Robustness

The jammer may know

- coding scheme
- input codeword
- message

Jammer knows only coding scheme:

- R. Ahlswede and V. Blinovsky, 2007
- R. Ahlswede, I. Bjelaković , H. Boche, and J. Nötzel, 2013

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Communication Scenarios

Definition:

- A code $\gamma := (\mathcal{U}, \{\mathcal{D}(i), i \in \mathcal{I}\})$ of length n consists of its code book $\mathcal{U} := \{\mathbf{u}(i), i \in \mathcal{I}\} \subset \mathcal{X}^n$ and the decoding measurement $\{\mathcal{D}(i), i \in \mathcal{I}\} \subset \mathcal{S}(\mathcal{H}^{\otimes n})$: $\mathcal{D}(i) \geq 0$ for all i and $\sum_{i \in \mathcal{I}}, \mathcal{D}(i) = id.$

- A random correlated code Γ is a uniformly distributed random variable taking values in a set of codes $\{(\mathcal{U}(k), \{\mathcal{D}(j,k), j \in \mathcal{J}\}), k \in \mathcal{K}\}$ with a common message set $\mathcal{J}$, where $|\mathcal{K}|$ is called the key size.

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Random Code

Deterministic capacity:

- Jammer does not know codeword ⇨ Ahlswede dichotomy
- Jammer knows codeword ⇨ no dichotomy (example in extended version)

Deterministic code

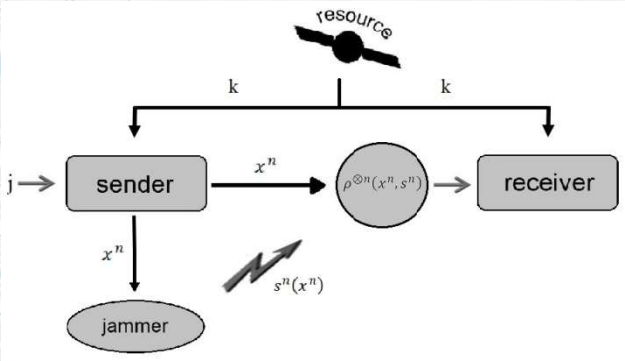
⇨ Knowing the message = knowing the input of the channel

Random code

⇨ Knowing the message ≠ knowing the input of the channel.

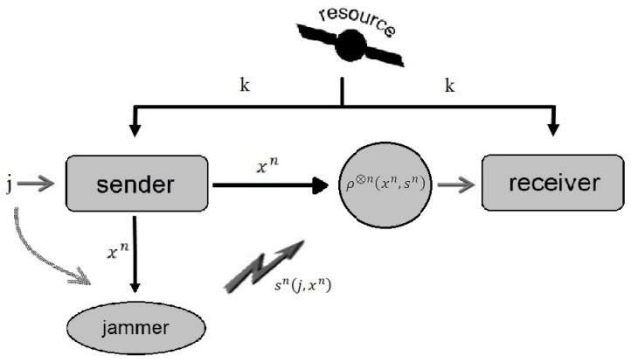
Quantum Arbitrarily Varying Channel When the Jammer Knows Input Codeword Communication Scenarios

Scenario 1:



In this scenario jammer knows input codeword

Scenario 2:



Now the jammer knows both input codeword and message

Quantum Arbitrarily Varying Channel When the Jammer Knows Input Codeword Communication Scenarios

Definition:

- We define the average probability of error in scenario 1 by

$$p_a(\Gamma) = \max_s Etr[\rho^{\otimes n}(\mathbf{u}(J,K), s(\mathbf{u}(J,K)))(I_{\mathcal{H}} - \mathcal{D}(J,K))]$$

- The maximum probability of error is defined as

$$p_m(\Gamma) = \max_{j \in \mathcal{J}} \max_s Etr[\rho^{\otimes n}(\mathbf{u}(j,K), s(\mathbf{u}(j,K)))(I_{\mathcal{H}} - \mathcal{D}(j,K))].$$

Quantum Arbitrarily Varying Channel When the Jammer Knows Input Codeword

Communication Scenarios

Definition:

We define the average probability of error in scenario 2 by

$$p_a(\Gamma) = \max_s Etr[\rho^{\otimes n}(\mathbf{u}(J,K), s(\mathbf{u}(J,K),J))(I_{\mathcal{H}} - \mathcal{D}(J,K))]$$

The maximum probability of error is defined as

$$p_m(\Gamma) = \max_{j \in \mathcal{J}} \max_s Etr[\rho^{\otimes n}(\mathbf{u}(j,K), s(\mathbf{u}(j,K),j))(I_{\mathcal{H}} - \mathcal{D}(j,K))].$$

the jammer knows not only the coding scheme but also the message

Quantum Arbitrarily Varying Channel When the Jammer Knows Input Codeword

Communication Scenarios

2 jamming scenarios + 2 error criteria

⇨ 4 combinations:

- random correlated capacity under the average error criterion in scenario 1, denoted by $C^*(W)$
- random correlated capacity under the maximal error criterion in scenario 1, denoted by $C_m^*(W)$
- random correlated capacity under the average error criterion in scenario 2, denoted by $C^{**}(W)$
- random correlated capacity under the maximal error criterion in scenario 2, denoted by $C_m^{**}(W)$

It is easy to show that $C^*(W) \geq C_m^*(W) = C^{**}(W) = C_m^{**}(W)$

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Main Results

Theorem:

For an AVCQC W={ρ(x,s): x∈X, s∈S} let

$$\bar{\bar{\mathcal{W}}} := \{\{\bar{\bar{\rho}}_Q(x) := \sum_s Q(s|x)\rho(x,s), x \in \mathcal{X}\} : \text{ for all } Q : \mathcal{X} \to \mathcal{S}\}$$

We have

$$C^*(\mathcal{W}) = C_m^*(\mathcal{W}) = C^{**}(\mathcal{W}) = C_m^{**}(\mathcal{W}) = \max_P \min_{\bar{\bar{\rho}}(\cdot)\in\bar{\bar{\mathcal{W}}}} \chi(P, \bar{\bar{\rho}}(\cdot))$$

Moreover all capacities can be achieved by codes with vanishing key rates.

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Previous Works

Capacity of classical arbitrarily varying channels in this scenario was first considered by Sarwate in 2008

- List decoding
- Vanishing key rate

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Previous Works

How to apply list decoding for quantum channels is still an open problem

We need a different approach

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Outline of Proof

The proof to the converse is simple.
The idea of the proof to the direct part:

- If the jammer knew the random key *k*, The best strategy for the jammer would be to choose the most dangerous state to attack the *k*-th deterministic coding, which we do not want.
- To this end every used codeword must be used by “many” outcomes

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Outline of Proof

vanishing key rate

Instead of generating codebooks from the whole product set of the alphabet or the typical set

⇨ we randomly generate a ground set B with a cardinality $|I_n|$ "slightly" (polynomially) larger that our desired size of codebooks

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Outline of Proof

randomly uniformly generate $|K_n|$ codebooks
U(k) := {u(j,k), j∈ J_n, k∈ K_n} with size $|J_n|$ from this ground set

$|J_n|$ smaller than $2^{-n[\min_{\bar{\bar{\rho}}(\cdot)\in\bar{\bar{\mathcal{W}}}} \chi(P_X,\bar{\bar{\rho}}(\cdot))]}$
$|K_n|$ = Poly(n)

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Outline of Proof

We show that with a high probability
every codeword x(i) appears in “sufficiently many” codebooks
and each state sequence is “bad” only in “very few” of those codebooks.

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Outline of Proof

By some modification we can show:

$C^*(W)=C^{**}(W)$

Further knowing message to be sent, may not help a jammer to reduce the capacity.

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Further perspective

Extending the scenario that the jammer knows input codeword to:

- **AVCQC with quantum jammer**
- **correlation as resource instead of randomness**
- **secrecy capacity of an AVCQC with an eavesdropper**
- **deterministic capacity**


Thank you