User Cooperation and Conferencing Encoders for Different Classes of Multiple-Access Channels

Holger Boche
Joint work with Moritz Wiese and Igor Bjelaković

Technische Universität München

WiOpt 2012, Paderborn
May 17, 2012

Outline

Outline

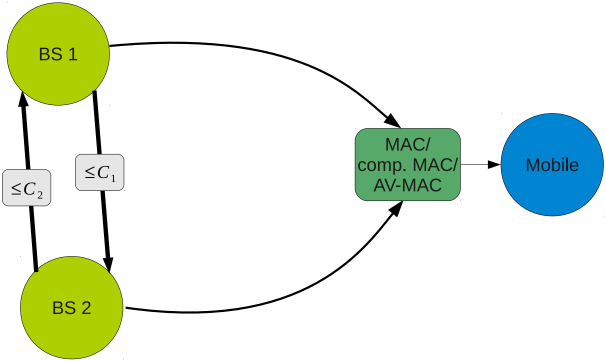
Motivation

- Power reduction and spectrum scarcity cause interference to be the main factor that limits performance of modern wireless systems.
- The potential of pure cellular concepts to deal with interference is close to being exhausted.
- Investigate potential of inter-cell cooperation [Karakayali, Foschini, Valenzuela 2006].
  - To be included in 5G wireless systems like LTE-Advanced.
- *Problem 1:* Full cooperation is too complex and uses too many resources
  - Partial cooperation needs to be investigated.
  - Partial channel state information (CSI) needs to be considered.
- *Problem 2:* How can one cope with interference from co-existing networks run by different providers?
  - Uncoordinated WLAN hot spots
  - Frequency co-sharing in 5G mobile networks

Conferencing Encoders

Willems introduced the concept of conferencing encoders in information theory [Willems 1982].

- models rate-limited cooperation between base stations.



Willems' Conferencing Protocol

- Consists of an interactive exchange of information about the messages m_1, m_2 present at encoder 1 and 2, resp.

An I-iterations conferencing protocol has the form

$$\begin{array}{rr}
\mathcal{M}_1 \xrightarrow{c_{1,1}} \mathcal{V}_{1,1} & \mathcal{M}_2 \xrightarrow{c_{2,1}} \mathcal{V}_{2,1} \\
\mathcal{M}_1 \times \mathcal{V}_{2,1} \xrightarrow{c_{1,2}} \mathcal{V}_{1,2} & \mathcal{M}_2 \times \mathcal{V}_{1,1} \xrightarrow{c_{2,2}} \mathcal{V}_{2,2} \\
\mathcal{M}_1 \times \mathcal{V}_{2,1} \times \mathcal{V}_{2,2} \xrightarrow{c_{1,3}} \mathcal{V}_{1,3} & \mathcal{M}_2 \times \mathcal{V}_{1,1} \times \mathcal{V}_{1,2} \xrightarrow{c_{2,3}} \mathcal{V}_{2,3} \\
\vdots & \vdots \\
\mathcal{M}_1 \times \mathcal{V}_{2,1} \times \ldots \times \mathcal{V}_{2,I-1} \xrightarrow{c_{1,I}} \mathcal{V}_{1,I} & \mathcal{M}_2 \times \mathcal{V}_{1,1} \times \ldots \times \mathcal{V}_{1,I-1} \xrightarrow{c_{2,I}} \mathcal{V}_{2,I}
\end{array}$$

Here

- $\mathcal{M}_\nu$ = message set of sender $\nu \in \{1, 2\}$,
- $\mathcal{V}_{\nu,i}$ finite set with $|\mathcal{V}_{\nu,i}| = V_{\nu,i}, \qquad \nu = 1, 2,\ i = 1, \ldots, I.$

MAC Coding with Rate-Constrained Conferencing

Given a multiple-access channel (MAC) with input alphabets $\mathcal{X}$ and $\mathcal{Y}$.

- Conferencing is part of coding.
- With conferencing capacities $C_1, C_2 \geq 0$, for a blocklength-n code, the above sets $\mathcal{V}_{1,1}, \ldots, \mathcal{V}_{2,I}$ need to satisfy

$$\frac{1}{n} \log V_{1,1} \cdots V_{1,I} \leq C_1, \qquad \frac{1}{n} \log V_{2,1} \cdots V_{2,I} \leq C_2.$$

- Describes the rate-constrained iterative exchange of information about the messages present at the encoders.
- The encoding functions have the form

$$f_1 : \mathcal{M}_1 \times \mathcal{V}_{2,1} \times \ldots \times \mathcal{V}_{2,I} \to \mathcal{X}^n,$$
$$f_2 : \mathcal{M}_2 \times \mathcal{V}_{1,1} \times \ldots \times \mathcal{V}_{1,I} \to \mathcal{Y}^n.$$

Channel Models

- The Compound MAC with conferencing encoders models channel state uncertainty in a downlink network with cooperating base stations.
- It is also the key to the solution of the coding theorem for AV-MACs.
- The Arbitrarily Varying MAC (AV-MAC) with conferencing encoders models a downlink network with cooperating base stations suffering from interference from networks operating in the same band.
- New effects occur in AV-MACs – conferencing can change the whole channel structure.

Outline

The Compound MAC

The Compound MAC models channel state uncertainty.

- A Compound MAC with state set $\mathcal{S}$ is a family

$$\mathcal{W} = \{W_s : s \in \mathcal{S}\}$$

  of discrete memoryless MACs $W_s : \mathcal{X} \times \mathcal{Y} \to \mathcal{P}(\mathcal{Z})$.
- The transmission of words $\mathbf{x} \in \mathcal{X}^n$ and $\mathbf{y} \in \mathcal{Y}^n$ is governed by the probabilities

$$W_s^n(\mathbf{z}|\mathbf{x}, \mathbf{y}) = \prod_{i=1}^{n} W_s(z_i|x_1, y_i) \qquad (s \in \mathcal{S}).$$

- The encoders and the decoder only have partial CSI.
- Partial CSI is modeled as partitions of the state set $\mathcal{S}$ for the encoders and the decoder.
- We restrict ourselves here to the case with no CSI at all.
  - arbitrary CSI treated in [W,B,B,Jungnickel 2011, TransIT]

Coding for the Compound MAC

- The conferencing codes are as described before.
- Through conferencing, the codewords of each encoder depend on both messages m_1 and m_2. They are called $\mathbf{x}_{m_1 m_2}$ and $\mathbf{y}_{m_1 m_2}$.
- The decoding sets are called $D_{m_1 m_2}$.
- The average error for the code is given by

$$\max_{s \in \mathcal{S}} \frac{1}{|\mathcal{M}_1||\mathcal{M}_2|} \sum_{m_1, m_2} W_s^n(D_{m_1 m_2}^c | \mathbf{x}_{m_1 m_2}, \mathbf{y}_{m_1 m_2}).$$

- To achieve a rate pair with conferencing capacities $C_1, C_2 \geq 0$, one may use codes whose conferencing protocol satisfies the rate restriction

$$\frac{1}{n} \log V_{1,1} \cdots V_{1,I} \leq C_1, \qquad \frac{1}{n} \log V_{2,1} \cdots V_{2,I} \leq C_2.$$

The Capacity Region

The capacity region of the Compound MAC with conferencing capacities $C_1, C_2 \geq 0$ equals the closure of

$$\bigcup_{U,X,Y} \bigcap_{s \in \mathcal{S}} \Big\{ (R_1, R_2) \in [0,\infty)^2 : \\ R_1 \leq I(Z_s; X|Y,U) + C_1, \\ R_2 \leq I(Z_s; Y|X,U) + C_2, \\ R_1 + R_2 \leq \min\big(I(Z_s; X,Y|U) + C_1 + C_2, I(Z_s; X,Y) \big) \Big\},$$

where $P_{Z_s|U,X,Y} = W_s$ and X, Y are independent given U.

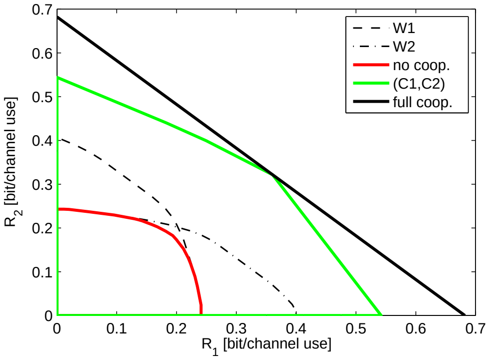
- The region is achieved using non-iterative conferencing protocols, i.e. with $I = 1$.

Numerical Example

Let $\mathcal{X} = \mathcal{Y} = \mathcal{Z} = \mathcal{S} = \{0, 1\}$ and the Compound MAC $\{W_1, W_2\}$ with

$$W_1 = \frac{1}{10}\begin{pmatrix} 9 & 1 \\ 4 & 6 \\ 6 & 4 \\ 0 & 10 \end{pmatrix} \quad \text{and} \quad W_2 = \frac{1}{10}\begin{pmatrix} 9 & 1 \\ 6 & 4 \\ 4 & 6 \\ 0 & 10 \end{pmatrix},$$

where the output corresponding to the input combination (a, b) is written in row $2a + b + 1$. With $C_1 = C_2 \approx 0.301$,



Proof Strategy

Achievability:

- Generate a common message by one-shot conferencing, i.e. with $I = 1$.
- Apply the coding result for the Compound MAC with common message.

Converse:

- Show the converse for all possible conferences satisfying the rate constraints, i.e. with I arbitrary.
- Outcome of the conference is treated as a random variable, exact structure is not relevant.

Proof of Achievability

Reduced to the Compound MAC with Common Message. Given a rate pair (R_1, R_2),

- Set

$$\mathcal{M}_1 = \mathcal{M}_{1,p} \times \mathcal{M}_{1,c}, \qquad \mathcal{M}_2 = \mathcal{M}_{2,p} \times \mathcal{M}_{2,c}$$

  with $\frac{1}{n}\log|\mathcal{M}_\nu| = 2^{nR_\nu}$ and $\frac{1}{n}\log|\mathcal{M}_{\nu,c}| = \min(R_\nu, C_\nu)$.
  - Uniform partitions of $\mathcal{M}_1$ and $\mathcal{M}_2$.
- Set $c_\nu(m_\nu) = c_\nu(m_{\nu,p}, m_{\nu,c}) = m_{\nu,c} \quad (\nu = 1, 2)$.
- The joint result of conferencing $(m_{1,c}, m_{2,c})$ is a uniformly distributed common message from $\mathcal{M}_{1,c} \times \mathcal{M}_{2,c}$.
- Use codes for the Compound MAC with Common Message for the message set $(\mathcal{M}_{1,c} \times \mathcal{M}_{2,c}) \times \mathcal{M}_{1,p} \times \mathcal{M}_{2,p}$.

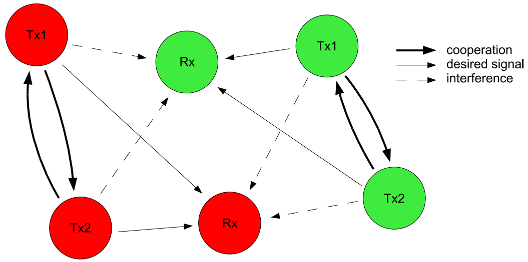
Weak and Strong Converse

- There is a weak converse.
- A strong converse for compound channels can only be shown for the maximal error [Ahlswede 1967].
- For MACs, the maximal error capacity region differs from the average error capacity region in general [Dueck 1978].
- For MACs, the capacity region is only known for the average error.

Outline

The Arbitrarily Varying MAC (1)

The AV-MAC models a MAC under the interference of networks it does not cooperate with, but which operate in the same band.



For example:

- Uncoordinated WLAN hot spots
- Frequency co-sharing in 5G mobile networks

The Arbitrarily Varying MAC (2)

- The AV-MAC with state set $\mathcal{S}$ is also given by a family

$$\mathcal{W} = \{W_s : s \in \mathcal{S}\}.$$

- However, the states may change at every time instant, so the set of n-block transition probabilities consists of

$$W^n(\mathbf{z}|\mathbf{x},\mathbf{y}|\mathbf{s}) = \prod_{i=1}^{n} W_{s_i}(z_i|x_i,y_i)$$

for $\mathbf{s} \in \mathcal{S}^n$.

Deterministic Coding for the AV-MAC

- The deterministic conferencing codes are as described before.
- The average error of a deterministic conferencing code for transmission over the AV-MAC is given by

$$\sup_{\mathbf{s}\in\mathcal{S}^n} \frac{1}{|\mathcal{M}_1||\mathcal{M}_2|} \sum_{m_1,m_2} W^n(D^c_{m_1m_2}|\mathbf{x}_{m_1m_2},\mathbf{y}_{m_1m_2}|\mathbf{s}).$$

- To achieve a rate pair with conferencing capacities $C_1, C_2 \geq 0$, one may use deterministic codes whose conferencing protocol satisfies the rate restriction

$$\frac{1}{n}\log V_{1,1}\cdots V_{1,I} \leq C_1, \qquad \frac{1}{n}\log V_{2,1}\cdots V_{2,I} \leq C_2.$$

Random Coding for the AV-MAC (1)

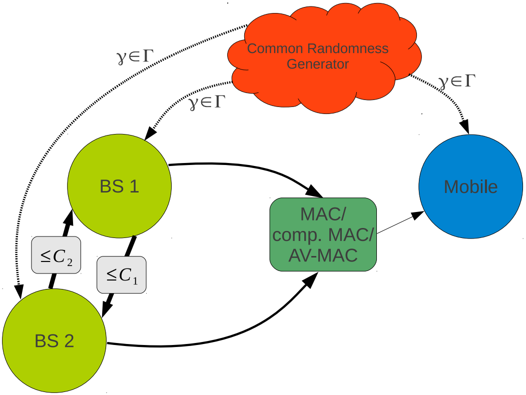
- Let a family $\{C(\gamma) : \gamma \in \Gamma\}$ of deterministic codes $C(\gamma)$ be given.
- Assume that every $C(\gamma)$ has conferencing capacities C_1, C_2.
- Let G be a random variable assuming values in Γ.

This is a random code with conferencing capacities C_1, C_2. Its average error is

$$\sup_{\mathbf{s}\in\mathcal{S}^n} \frac{1}{|\mathcal{M}_1||\mathcal{M}_2|} \sum_{\gamma} \sum_{m_1,m_2} W^n(D_{m_1 m_2}(\gamma)^c|\mathbf{x}_{m_1 m_2}(\gamma), \mathbf{y}_{m_1 m_2}(\gamma)|\mathbf{s}) P_G(\gamma).$$

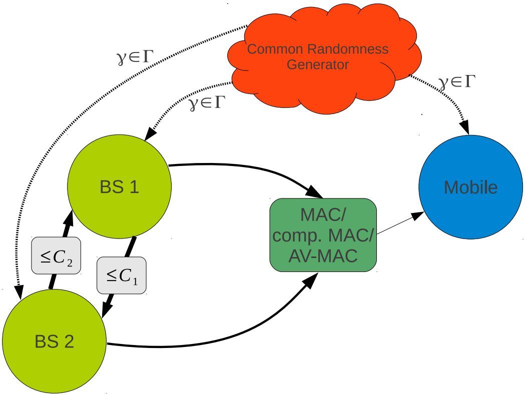
Random Coding for the AV-MAC (2)

Random coding requires common randomness shared by the encoders and the decoder.



Practically, random coding is less relevant. But mathematically it is important!

Random Coding for the AV-MAC (3)



Random coding can be interpreted as a distributed randomized algorithm.

Random Coding Theorem for the AV-MAC

Theorem [W,B 2011]

The capacity region of $\mathcal{W}$ with conferencing capacities $C_1, C_2 \geq 0$ and with random coding equals the capacity region of the compound MAC with conferencing capacities C_1, C_2 given by

$$\overline{\mathcal{W}} := \{W_q = \sum_{s \in \mathcal{S}} W_s q(s) : q \in \mathcal{P}(\mathcal{S})\}.$$

- Optimal conferencing is as simple as for Compound MACs.

New Effects in Uncoordinated Networks

The deterministic capacity region of the AV-MAC exhibits a dichotomy. This is characterized by symmetrizability [Ericson 1985], [Gubner 1990].

The AV-MAC $\mathcal{W}$ is $(\mathcal{X}, \mathcal{Y})$-symmetrizable if there is a stochastic matrix $\sigma : \mathcal{X} \times \mathcal{Y} \to \mathcal{P}(\mathcal{S})$ such that for all $x, x' \in \mathcal{X}$ and $y, y' \in \mathcal{Y}$ and $z \in \mathcal{Z}$

$$\sum_{s \in \mathcal{S}} W_s(z|x, y)\sigma(s|x', y') = \sum_{s \in \mathcal{S}} W_s(z|x', y')\sigma(s|x, y).$$

$\mathcal{W}$ is $\mathcal{X}$-symmetrizable if there is a stochastic matrix $\sigma_1 : \mathcal{X} \to \mathcal{P}(\mathcal{S})$ such that for all $x, x' \in \mathcal{X}$ and $y \in \mathcal{Y}$ and $z \in \mathcal{Z}$

$$\sum_{s \in \mathcal{S}} W_s(z|x, y)\sigma_1(s|x') = \sum_{s \in \mathcal{S}} W_s(z|x', y)\sigma_1(s|x).$$

$\mathcal{Y}$-symmetrizability is defined analogously.

Deterministic Capacity Region of the AV-MAC

The mathematical importance of random coding becomes clear in the following theorem.

Theorem [W,B 2011]

The deterministic capacity region of $\mathcal{W}$ with conferencing capacities $C_1, C_2 > 0$ equals $\{(0,0)\}$ if and only if it is $(\mathcal{X}, \mathcal{Y})$-symmetrizable. Otherwise it equals the random coding capacity region.

- Optimal conferencing remains as simple as for Compound MACs.

Proof of the Random Coding Theorem

- Turn good codes for the compound MAC with cooperating encoders into good random codes for the AV-MAC.

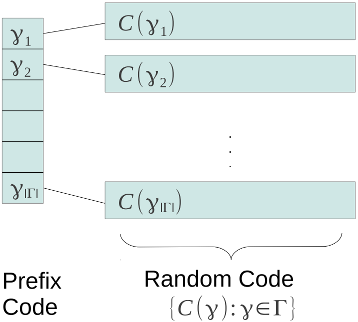
[Ahlswede 1980]: Good deterministic codes for the compound MAC $\overline{\mathcal{W}}$ can be turned into good random codes for the AV-MAC $\mathcal{W}$ by randomizing over permutations of the codewords and the decoding sets.

This is Ahlswede's Robustification Technique.

Proof of the Deterministic Coding Theorem (1)

- Turn a good random code into a good deterministic code.

Idea: Elimination of Correlation. Use a random code with the desired rates. Prefix a deterministic code to it which specifies which random code will be used. [Ahlswede 1978]



This coding strategy shows the practical relevance of random coding.

Proof of the Deterministic Coding Theorem (2)

How small can $|\Gamma|$ be chosen, i.e. how much common randomness is needed in the Random Coding Theorem?

- $|\Gamma|$ can be chosen to grow polynomially in blocklength.
- prefix code has blocklength $\log n$ and – asymptotically – arbitrarily small rate.

Thus if any positive rate is achievable deterministically, then this derandomization method can be used.

Proof of the Deterministic Coding Theorem (3)

- A small positive rate pair is achievable if $\mathcal{W}$ is not $(\mathcal{X}, \mathcal{Y})$-symmetrizable.

$0 < \tilde{R} < 2\min\{C_1, C_2\}$ is deterministically achievable by the single-user AVC $\mathcal{W}$ with input alphabet $\mathcal{X} \times \mathcal{Y}$ if $\mathcal{W}$ is not $(\mathcal{X}, \mathcal{Y})$-symmetrizable [Csiszár, Narayan 1988, capacity of single-user AVCs].

$|\mathcal{M}_1| = |\mathcal{M}_2| = 2^{n\tilde{R}/2} =: 2^{nR}$

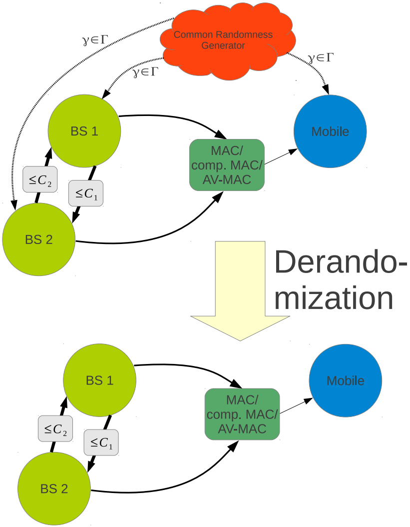
- every encoder informs the other encoder completely about the message it would like to send
- The encoders use the codeword corresponding to the message pair from the single-user code achieving $\tilde{R}$.

Converse

The converse follows from the converse for the compound channel together with the result of [Csiszár, Narayan 1988].

- If $\mathcal{W}$ is $(\mathcal{X}, \mathcal{Y})$-symmetrizable, then by [Csiszár, Narayan 1988], every code incurs at least an error of $1/4$.
  - "Almost" a strong converse.
- If $\mathcal{W}$ is not $(\mathcal{X}, \mathcal{Y})$-symmetrizable, then at least the corresponding compound region is achievable. Clearly, this cannot be exceeded by the AV-MAC.
  - Weak converse.

Derandomization



Derandomization either

- preserves the random coding region, or
- nothing at all can be transmitted.

This is a dichotomy, an "all-or-nothing-law".

Open problems for the AV-MAC

- Is the direct approach to the AVC capacity [Csiszár, Narayan 1988] feasible for the AV-MAC?
  - Conjecture: Impossible for the Quantum AVC
    $\rightsquigarrow$ Ahlswede's approach must be taken [Ahlswede,B,B,Nötzel 2010].
- The maximal error for arbitrarily varying channels leads to the unsolved problem of zero-error capacity.

For average errors –

- can we obtain a strong converse for non-zero capacity regions?
- still no complete description of the non-conferencing capacity region.

Outline

Gains for Compound MACs

Comparison of Compound MAC without encoder cooperation with Compound MAC with $C_1, C_2 > 0$.

- Gains, if existent, are continuous in C_1, C_2 including $C_1 = C_2 = 0$.
- If single-user (sum) capacity of Compound MAC $\mathcal{W} : \mathcal{X} \times \mathcal{Y} \to \mathcal{P}(\mathcal{Z})$ equals zero
  $\rightsquigarrow$ nothing is gained by conferencing.
- If single-user capacity greater then zero
  $\rightsquigarrow$ capacity region grows from non-cooperative to full-cooperation region
  $\rightsquigarrow$ linear in C_1, C_2 until cutoff.

Gains for AV-MACs: Background

Theorem [Ahlswede, Cai 1999]

The capacity region of $\mathcal{W}$ without encoder cooperation contains a pair (R, R) with $R > 0$ if $\mathcal{W}$ is neither $\mathcal{X}$- nor $\mathcal{Y}$- nor $(\mathcal{X}, \mathcal{Y})$-symmetrizable.

In that case

Theorem [Jahn 1981]

If the capacity region of $\mathcal{W}$ without encoder cooperation contains a pair (R, R) with $R > 0$, then it equals the capacity region of the compound channel $\overline{\mathcal{W}}$ without conferencing.

- If an AV-MAC is $(\mathcal{X}, \mathcal{Y})$-symmetrizable or both $\mathcal{X}$- and $\mathcal{Y}$-symmetrizable, then its capacity region equals $\{(0, 0)\}$.
- If it is either $\mathcal{X}$- or $\mathcal{Y}$-symmetrizable, then its capacity region is at most one-dimensional.

Gains for AV-MACs

Comparison of AV-MAC without encoder cooperation with Compound MAC with $C_1, C_2 > 0$.

- If $\mathcal{W}$ is $(\mathcal{X}, \mathcal{Y})$-symmetrizable, then conferencing does not help.
- If $\mathcal{W}$ is
  - not $(\mathcal{X}, \mathcal{Y})$-symmetrizable,
  - but $\mathcal{X}$- and $\mathcal{Y}$-symmetrizable,

  then
  - its capacity region without conferencing equals zero,
  - its capacity region with $C_1, C_2 > 0$ equals the capacity region of the compound MAC $\overline{\mathcal{W}}$ with C_1, C_2.

  $\rightsquigarrow$ discontinuous gains possible when enabling conferencing.
- Example: The AV-MAC with $\mathcal{X} = \mathcal{Y} = \mathcal{S} = \{0, 1\}$, $\mathcal{Z} = \{0, \ldots, 3\}$,

$$z = x + y + s,$$

  is not $(\mathcal{X}, \mathcal{Y})$-symmetrizable but both $\mathcal{X}$- and $\mathcal{Y}$-symmetrizable.

Outline

Conclusion

- Base station cooperation is used in 5G networks to mitigate interference.
- Conferencing is an information-theoretic model of rate-limited base station cooperation.
- The Compound MAC with conferencing encoders models partial CSI and is the key to the AV-MAC with conferencing encoders.
- The capacity region of the Compound MAC with conferencing encoders can is completely characterized.
- The AV-MAC models interference from the same band as occurring in frequency co-sharing.
- The capacity region of the AV-MAC with conferencing encoders is also characterized completely for both random and deterministic coding.

Conclusion

- We can exactly say when derandomization is possible.
- In every case, the optimal conferencing protocol is a simple non-iterative protocol.
- The amount of cooperation necessary to achieve the full cooperation sum capacity or the full cooperation capacity region can be characterized completely.
- For AV-MACs, conferencing may enable discontinuous transmission gains.
- Still open problems regarding maximal error criterion and strong converses.

References

- R. Ahlswede, “Elimination of Correlation in Random Codes for Arbitrarily Varying Channels,”, *Z. Wahrscheinlichkeitstheorie verw. Gebiete*, no. 44, pp. 159–175, 1978.
- R. Ahlswede, “Coloring Hypergraphs: A New Approach to Multi-user Source Coding—II,” *J. Comb. Inform. Syst. Sci.*, vol. 5, no. 3, pp. 220–268, 1980.
- R. Ahlswede, “Certain Results in coding theory for compound channels I,” *Proc. Colloquium Inf. Th. Debrecen (Hungary)*, pp. 35–60, 1967.
- R. Ahlswede, I. Bjelaković, H. Boche, J. Nötzel, “Quantum capacity under adversarial quantum noise: arbitrarily varying quantum channels”, submitted to *Commun. Math. Phys.*, 2010.
- R. Ahlswede and N. Cai, “Arbitrarily Varying Multiple-Access Channels Part I—Ericson's Symmetrizability Is Adequate, Gubner's Conjecture Is True,”, *IEEE Trans. Inf. Theory*, vol. 45, no. 2, pp. 742–749, 1999.
- I. Csiszár and P. Narayan, “The Capacity of the Arbitrarily Varying Channel Revisited: Positivity, Constraints”, *IEEE Trans. Inf. Theory*, vol. 34, no. 2, pp. 181–193, 1988.
- G. Dueck, “Maximal Error Capacity Regions are Smaller Than Average Error Capacity Regions for Multi-User Channels,” *Probl. Contr. Inform. Theory*, vol. 7, no. 1, pp. 11–19, 1978.

References

- E T. Ericson, “Exponential Error Bounds for Random Codes in the Arbitrarily Varying Channel,” *IEEE Trans. Inf. Theory*, vol. IT-31, pp. 42–48, 1985.
- G J.A. Gubner, “On the Deterministic-Code Capacity of the Multiple-Access Arbitrarily Varying Channel,” *IEEE Trans. Inf. Theory*, vol. 36, no. 2, pp. 262–275, 1990.
- J.-H. Jahn, “Coding of Arbitrarily Varying Multiuser Channels,” *IEEE Trans. Inf. Theory*, vol. IT-27, no. 2, pp. 212–226, 1981.
- M. Wiese, H. Boche, I. Bjelaković, and V. Jungnickel, “The Compound Multiple Access Channel with Partially Cooperating Encoders”, *IEEE Trans. Inf. Theory*, vol. 57, no. 5, pp. 3045–3066, 2011.
- M. Wiese, H. Boche, “The Arbitrarily Varying Multiple-Access Channel with Conferencing Encoders”, submitted to *IEEE Trans. Inf. Theory*, available at http://arxiv.org/abs/1105.0319, 2011.
- F.M.J. Willems, “Informationtheoretical Results for the Discrete Memoryless Multiple Access Channel,” Ph.D. dissertation, Katholieke Universiteit Leuven, Belgium, 1982.
- F.M.J. Willems, “The Discrete Memoryless Multiple Access Channel with Partially Cooperating Encodrs,” *IEEE Trans. Inf. Theory*, vol. 29, no. 3, pp. 441–445, 1983.