# Post Shannon Theory: Deterministic Identification With and Without Feedback

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Session on **Post-Shannon Communications**

Friday 21 May

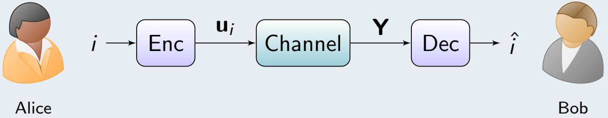
# Outline

1. Motivation
2. Deterministic Identification without Feedback (DI)
3. Deterministic Identification with Feedback (DIF)
4. Conclusions

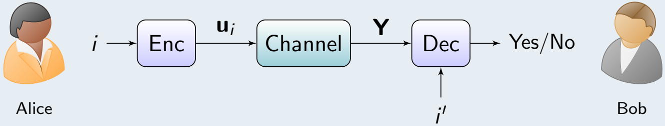
# Outline

# Transmission vs. Identification

- Shannon's setting: Bob recover the message.




- Identification setting: Bob asks if a message was sent or not?




- V2X and P2MP communications
- Molecular communication and Health care
- Any event-triggered scenario

# Randomized Identification (RI) [1]

- Originally introduced by Ahlswede and Dueck (1989)
- Capacity was established with randomness at encoder
- Encoder employs distribution to select codewords

## Remarkable Property

- Reliable identification is possible with code size growth $\sim 2^{2^{nR}}$
- Sharp difference to transmission with code size growth $\sim 2^{nR}$

[1] R. Ahlswede, and G. Dueck, "Identification via channels", 1989

# Deterministic Identification (DI) [2]

- Encoder uses deterministic mapping for coding
- Code size $\sim 2^{nR}$ for DMC as in transmission paradigm
- Achievable rates **higher** than transmission

**Why deterministic?**

- Simpler implementation (random resource not required)
- Suitable for Jamming scenarios
- Suitable for molecular communication

[2] R. Ahlswede and N. Cai, "Identification without randomization", 1999

# Outline

# Main Contributions

- We established the DI capacity for DMC with power constraint
- We show that the optimal code size scales as $\sim 2^{nR}$
- The analysis combines techniques and ideas from works by:
  1. JáJá [3]
  2. Ahlswede [4]
- We develop lower and upper bounds on the DI capacity for Gaussian channels with
  1. Fast fading
  2. Slow fading
- We use the bounds to determine the **correct scale**
- We show that the optimal code size scales as $\sim 2^{n \log(n) R}$

[3] J. J. Ja, "Identification is easier than decoding", 1985

[4] R. Ahlswede, "A method of coding and its application to arbitrarily varying channels", 1980

# Deterministic Identification (DI) over DMCs

## DI codes

A $(L(n,R), n, \lambda_1, \lambda_2)$-DI code for DMC $\mathcal{W}$ is a system $\{(\boldsymbol{u_i}, \mathcal{D}_i)\}_{i\in[1:L(n,R)]}$ subject to

1. Code size: $L(n,R) = 2^{nR}$
2. Code-word: $\boldsymbol{u_i} \in \mathcal{X}^n$, decoding regions: $\mathcal{D}_i \subset \mathcal{Y}^n$
3. Input constraint: $n^{-1}\sum_{t=1}^{n} \phi(u_{i,t}) \leq A$ with $\phi : \mathcal{X} \to [0,\infty)$
4. Error requirement type I: $W^n(\mathcal{D}_i | \boldsymbol{u}_i) \geq 1 - \lambda_1$
5. Error requirement type II: $W^n(\mathcal{D}_i | \boldsymbol{u}_j) \underset{i\neq j}{\leq} \lambda_2$

# DI Codes (Cont.)

## Definition

A $(L(n,R), n, \lambda_1, \lambda_2)$-DI code for DMC $\mathcal{W}$ is a system $\{(\boldsymbol{u_i}, \mathcal{D}_i)\}_{i\in[1:L(n,R)]}$ subject to

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5. Error requirement type II: $W^n(\mathcal{D}_i|\boldsymbol{u}_j) \underset{i\neq j}{<} \lambda_2$

# DI Capacity of DMC

**Theorem**

[5] *Let $\mathcal{W}$ be a DMC with distinct rows in channel matrix. Then the DI capacity with exponential code size and under input constraint is given by*

$$\mathbb{C}_{DI}(\mathcal{W}) = \max_{p_X \,:\, \mathbb{E}\{\phi(X)\} \leq A} H(X)$$

[5] M. J. Salariseddigh, U. Pereg, H. Boche, and C. Deppe, "Deterministic identification over channels with power constraints," IEEE Int'l Conf. Commun. (ICC), 2021 [arXiv:2010.04239, 2021]

# DI Capacity of DMC

**Theorem (Ahlswede and Dueck, 1989 [6]; Ahlswede and Cai, 1999 [7])**

*For DMC $\mathcal{W}$ let $W : \mathcal{X} \to \mathcal{Y}$ be channel matrix with distinct rows. Then the DI capacity with exponential code size is given by*

$$\mathbb{C}_{DI}(\mathcal{W}) = \log|\mathcal{X}|$$

[6] R. Ahlswede, and G. Dueck, "Identification via channels", 1989

[7] R. Ahlswede and N. Cai, "Identification without randomization", 1999

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- A proof was not provided !
- Consequence of our result with $A = \phi_{max}$ [8]

[6] R. Ahlswede, and G. Dueck, "Identification via channels", 1989

[7] R. Ahlswede and N. Cai, "Identification without randomization", 1999

[8] M. J. Salariseddigh, U. Pereg, H. Boche, and C. Deppe, "Deterministic identification over channels with power constraints," IEEE Int'l Conf. Commun. (ICC), 2021 [arXiv:2010.04239, 2021]

# Proof Sketch (Achievability)

**Lemma**

*Let* $R < H(X)$ *and* $\epsilon > 0$. *Then,* $\exists\, \mathcal{U}^* = \{\boldsymbol{v}_i\}_{i \in \mathcal{M}}$ *such that*

1. $\boldsymbol{v}_i \in \mathcal{T}(p_X) \quad \forall i \in \mathcal{M}$
2. $d_H(\boldsymbol{v}_i, \boldsymbol{v}_j) \geq n\epsilon \quad \forall i \neq j$
3. $|\mathcal{M}| \geq 2^{n(R-\theta)}$

# Proof Sketch (Achievability) Cont.

## Coding Scheme

- **Enc**: given message $i \in \mathcal{M}$ transmit $x^n = \boldsymbol{v}_i$
- **Dec**: $\mathcal{D}_j = \{y^n : (\boldsymbol{v}_j, y^n) \in \mathcal{T}_\delta(p_X W)\}$
- **Error Analysis**
  1. $P_{e,1}(i) \leq 2^{-n\alpha_1(\delta)}$ by standard type class argument
  2. $P_{e,2}(i,j) \leq 2^{-n\alpha_2(\epsilon,\delta)}$ by conditional type intersection lemma
- We extend the **JáJá** approach to a non-binary channel to bound the type II error.

# Proof Sketch (Achievability) Cont.

**Lemma (Ahlswede, 1980)**

[9] *Let* $W : \mathcal{X} \to \mathcal{Y}$ *be a channel matrix of a DMC* $\mathcal{W}$ *with distinct rows. Then, for every* $x^n, x'^n \in \mathcal{T}_\delta(p_X)$ *with* $d(x^n, x'^n) \geq n\epsilon$,

$$\frac{|\mathcal{T}_\delta(p_{Y|X}|x^n) \cap \mathcal{T}_\delta(p_{Y|X}|x'^n)|}{|\mathcal{T}_\delta(p_{Y|X}|x^n)|} \leq e^{-ng(\epsilon)}$$

*with* $p_{Y|X} \equiv W$, *for sufficiently large* $n$ *and some positive function* $g(\epsilon) > 0$ *which is independent of* $n$.

[9] R. Ahlswede. "A method of coding and its application to arbitrarily varying channels". 1980

# Proof Sketch (Converse)

**Lemma**

*Distinct messages have distinct codewords, i.e.,*

$$i_1 \neq i_2 \;\Rightarrow\; \boldsymbol{u}_{i_1} \neq \boldsymbol{u}_{i_2}$$

Proof. If $\boldsymbol{u}_{i_1} = \boldsymbol{u}_{i_2} = x^n$, then

$$P_{e,1}(i_1) + P_{e,2}(i_2, i_1) = W^n(\mathcal{D}_{i_1}^c | x^n) + W^n(\mathcal{D}_{i_1} | x^n) = 1$$

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## Further Steps

- $2^{nR} \leq \left| \{x^n : n^{-1} \sum_{t=1}^n \phi(x_t) \leq A\} \right|$
- $\left| \{x^n : n^{-1} \sum_{t=1}^n \phi(x_t) \leq A\} \right| \leq 2^{n(\max_{p_X : \mathbb{E}\{\phi(X)\} \leq A} H(X) + \alpha_n)}$ since input subspace is a union of type classes
- $R \leq \max_{p_X : \mathbb{E}\{\phi(X)\} \leq A} H(X) + \alpha_n$ for $\alpha_n \xrightarrow{n \to \infty} 0$

# Coding Scale: Randomized Identification

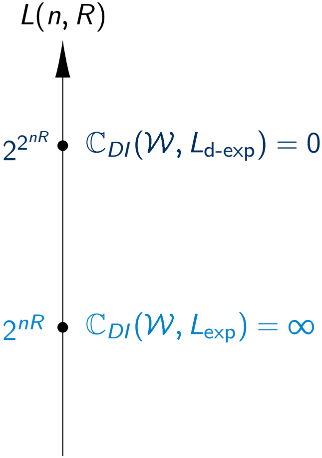
$L(n, R)$

$2^{2^{nR}}$ $\mathbb{C}_{RI}(\mathcal{W}, L_{\text{d-exp}}) = \frac{1}{2}\log\left(1 + \frac{A}{\sigma_Z^2}\right)$

$2^{nR}$ $\mathbb{C}_{RI}(\mathcal{W}, L_{\text{exp}}) = \infty$

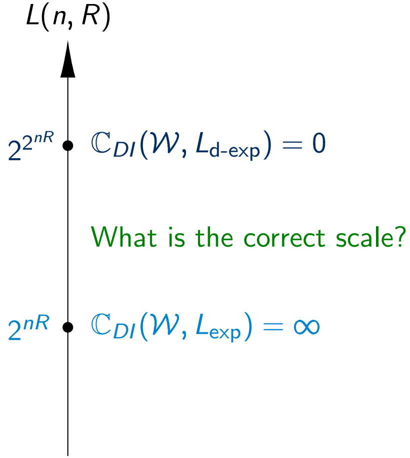
Ahlswede & Dueck, 1989

# Coding Scale: Deterministic Identification



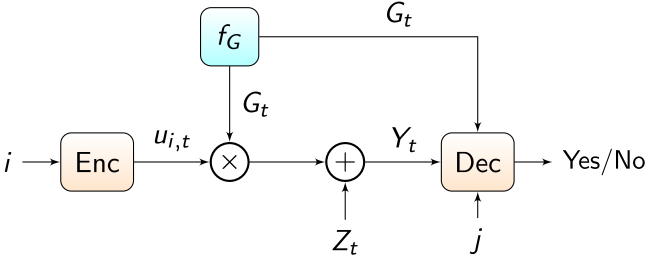

S., Pereg, Boche & Deppe, ICC 2021

# Coding Scale: Deterministic Identification




S., Pereg, Boche & Deppe, ICC 2021

# DI for Fading Channel



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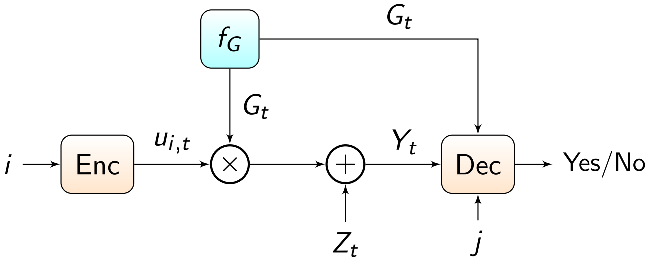


## Definitions

- Fast fading $\rightarrow \mathbf{Y} = \mathbf{G} \circ \mathbf{x} + \mathbf{Z}$ where $\mathbf{G} = (G_t)_{t=1}^{\infty} \overset{iid}{\sim} f_G$
- Slow fading $\rightarrow Y_t = Gx_t + Z_t$ where $G \sim f_G$
- Power const. $\rightarrow \|\mathbf{x}\| \leq \sqrt{nA}$ , Noise $\rightarrow \mathbf{Z} \overset{iid}{\sim} \mathcal{N}(0, \sigma_Z^2)$

# DI for Fast Fading Channel

**Theorem**

[10] *Let $\mathscr{G}_{fast}$ be fast fading channel with positive fading coefficients. Then the DI capacity for $L(n, R) = 2^{n \log(n) R}$ is bounded by*

$$\frac{1}{4} \leq \mathbb{C}_{DI}(\mathscr{G}_{fast}, L) \leq 1$$

[10] M. J. Salariseddigh, U. Pereg, H. Boche, and C. Deppe, "Deterministic identification over fading channels," IEEE Inf. Theory Workshop (ITW), 2020 [arXiv:2010.10010, 2021]

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## Corollary (Traditional Scales)

[10] *DI capacity in traditional scales is given by*

$$\mathbb{C}_{DI}(\mathscr{G}_{fast}, L) = \begin{cases} \infty & \text{for } L(n,R) = 2^{nR} \\ 0 & \text{for } L(n,R) = 2^{2^{nR}} \end{cases}$$

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- Standard Gaussian channel is a special case
- Achiev. proof: sphere pkg. of rad. $n^{\frac{1}{4}} \Rightarrow 2^{\frac{1}{4} n \log(n)}$ codewords

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# Proof Sketch. (Achievability)

- Dense sphere packing arrangement with radius $\sqrt{n\epsilon_n}$
- *Minkowski-Hlawka Theorem* guarantees a density $\Delta \geq 2^{-n}$
- $2^{n\log(n)R} = \Delta \cdot \frac{\text{Vol}(\mathcal{S}_{\mathbf{0}}(n,\sqrt{A}))}{\text{Vol}(\mathcal{S}_{\mathbf{u}_1}(n,\sqrt{\epsilon_n}))} \geq 2^{-n} \cdot \left(\frac{\sqrt{A}-\sqrt{\epsilon_n}}{\sqrt{\epsilon_n}}\right)^n$
- $R \geq \frac{1}{\log(n)} \log\left(\frac{\sqrt{A}-\sqrt{\epsilon_n}}{\sqrt{\epsilon_n}}\right) - \frac{1}{\log(n)} \xrightarrow{n\to\infty} \frac{1}{4}$

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Chebyshev's inequality leads to the following error bounds:

1. $P_{e,1}(i) \leq \frac{c_1}{n\epsilon_n^2}$
2. $P_{e,2}(i,j) \leq \frac{c_2}{n\epsilon_n^2}$

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2. $P_{e,2}(i,j) \leq \frac{c_2}{n\epsilon_n^2}$

- Cond. 1 & 2 $\rightarrow \epsilon_n = \frac{A}{n^{\frac{1}{2}(1-b)}}$ for $b > 0$ arbitrarily small

# Proof Sketch. (Converse)

- We show that if two code-words satisfy $\|\mathbf{u}_{i_1} - \mathbf{u}_{i_2}\| < \sqrt{n\epsilon_n}$ then using the continuity of the Gaussian PDF, we obtain

$$P_{e,1}(i) + P_{e,2}(i,j) \geq 1 - \kappa_n$$

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- Tight upper-bound requires:
  1. $\epsilon_n$ large as possible
  2. $\kappa_n$ tends to zero
- Cond. 1 & 2 $\rightarrow \epsilon_n = \frac{A}{n^2}$

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- Cond. 1 & 2 $\rightarrow \epsilon_n = \frac{A}{n^2}$

$$\text{rate} \uparrow \iff \epsilon_n \downarrow$$

# DI for Slow Fading Channel

## Theorem

[11] *Let $\mathscr{G}_{slow}$ be slow fading channel with positive fading coefficients. Then DI capacity for $L(n, R) = 2^{n \log(n) R}$ is bounded by*

$$\frac{1}{4} \leq \mathbb{C}_{DI}(\mathscr{G}_{slow}, L) \leq 1$$

## Corollary (Traditional Scales)

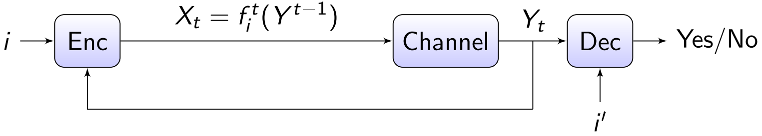
[11] *DI capacity in traditional scales is given by*

$$\mathbb{C}_{DI}(\mathscr{G}_{slow}, L) = \begin{cases} \infty & \text{for } L(n, R) = 2^{nR} \\ 0 & \text{for } L(n, R) = 2^{2^{nR}} \end{cases}$$

[11] M. J. Salariseddigh, U. Pereg, H. Boche, and C. Deppe, "Deterministic identification over fading channels," IEEE Inf. Theory Workshop (ITW), 2020 [arXiv:2010.10010, 2021]

# Outline

# DI with Noiseless Feedback




- Previous work [12] focused on channels with discrete alphabets
- We extend the results to the Gaussian channel [13]

[12] R. Ahlswede and G. Dueck, "Identification in the presence of feedback-a discovery of new capacity formulas," IEEE Trans. Inf. Theory, 1989

[13] W. Labidi, H. Boche, C. Deppe and M. Wiese, "Identification over the Gaussian Channel in the Presence of Feedback," IEEE Int'l Symp. Inf. Theory (ISIT), 2021 [arXiv:2102.01198, 2021]

# DIF Capacity of a DMC

## Theorem

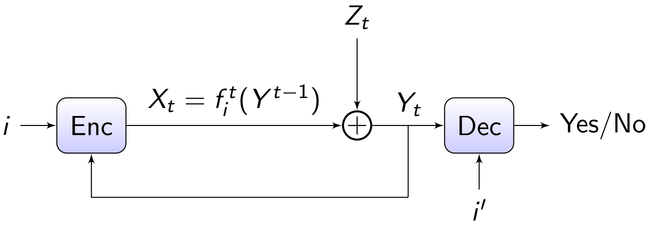
[14] *Let $\mathbb{C}_{DIF}(\mathcal{W})$ and $\mathbb{C}(\mathcal{W})$ be the DIF capacity and the Shannon capacity of the DMC $\mathcal{W}$, respectively. Then the deterministic identification capacity with feedback is given by*

$$\mathbb{C}_{DIF}(\mathcal{W}) = \begin{cases} \max_{x\in\mathcal{X}} H\left(W(\cdot|x)\right) & \textit{if } \mathbb{C}(\mathcal{W}) > 0 \\ 0 & \textit{iff } \mathcal{W} \textit{ is noiseless or } \mathbb{C}(\mathcal{W}) = 0 \end{cases}$$

- Feedback allows a double exponential growth of the identities
- Noise can increase the identification feedback capacity

[14] R. Ahlswede and G. Dueck, "Identification in the presence of feedback-a discovery of new capacity formulas," IEEE Trans. Inf. Theory, 1989

# DIF Over Gaussian Channels: System Model




- $Z_t, \ t = 1, \ldots, n \overset{iid}{\sim} \mathcal{N}(0, \sigma^2)$
- The channel is denoted by $W_{\sigma^2}$

## DIF code for Gaussian channels under average power constraint

A $(L(n,R), n, \lambda_1, \lambda_2)$-DIF code for $W_{\sigma^2}$ with $\lambda_1 + \lambda_2 < 1$ is a system $\{(\boldsymbol{f}_i, \mathcal{D}_i)\}_{i \in [1:L(n,R)]}$ subject to

1. Code size: $L(n,R)$
2. Feedback strategy: $\boldsymbol{f}_i = [f_i^1, f_i^2 \ldots, f_i^n] \in \mathcal{F}_n$,
   decoding region: $\mathcal{D}_i \subset \mathcal{Y}^n$
3. $\sum_{t=1}^n (f_i^t)^2 \leq n \cdot P_{tot}, \quad \forall i \in \{1, \ldots, N\}$
4. Error requirement type I: $W^n(\mathcal{D}_i | \boldsymbol{u}_i) > 1 - \lambda_1$
5. Error requirement type II: $W^n(\mathcal{D}_i | \boldsymbol{u}_j) \underset{i \neq j}{<} \lambda_2$

- $\mathcal{F}_n$ is set of all encoding functions $f_i$, where $f_i^1 \in \mathcal{X}$ and $f_i^t : \mathcal{Y}^{t-1} \rightarrow$ for $t > 1$

# DIF Capacity of Gaussian Channel

**Theorem**

[15] Let $\lambda \in (0,1)$, $\sigma^2 \geq 0$ and $P_{tot} > 0$. Then for all $R > 0$, there exists a blocklength $n_0$ such that for every $n \geq n_0$ there exists a deterministic identification feedback code $(L(n,R), n, \lambda_1, \lambda_2)$ for $W_{\sigma^2}$ of blocklength $n$ with $L(n,R) = 2^{2^{nR}}$ identities and with $\lambda_1, \lambda_2 \leq \lambda$, i.e.,

$$\mathbb{C}_{DIF}(\sigma^2, P_{tot}) = +\infty$$

- **Change the scaling? Choose higher scaling?**
- Without feedback [16], code size growth $\sim 2^{(n \log n)R}$

[15] W. Labidi, H. Boche, C. Deppe and M. Wiese, "Identification over the Gaussian Channel in the Presence of Feedback," IEEE Int'l Symp. Inf. Theory (ISIT), 2021 [arXiv:2102.01198, 2021]

[16] M. J. Salariseddigh, U. Pereg, H. Boche, and C. Deppe, "Deterministic identification over channels with power constraints," IEEE Int'l Conf. Commun. (ICC), 2021 [arXiv:2010.04239, 2021]

# Infinite DIF Capacity regardless of the Scaling

## Theorem

[17] *Let* $\lambda \in (0,1)$, $\sigma^2 \geq 0$ *and* $P_{tot} > 0$. *Then there exists a blocklength* $n_s$ *such that for every positive integer* $L(n,R)$ *and every* $n \geq n_s$ *there exists a deterministic identification feedback code* $(L(n,R), n, \lambda_1, \lambda_2)$ *for* $W_{\sigma^2}$ *of blocklength* $n$ *with* $L(n,R)$ *identities and with* $\lambda_1, \lambda_2 \leq \lambda$

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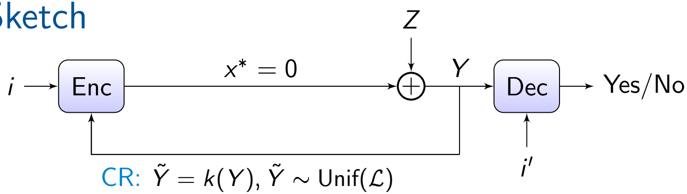
# Proof Sketch ($\sigma^2 > 0$)

1. To send a message $i$, we prepare a set of coloring functions $\{F_i, \quad i = 1, \ldots, L(n, R)\}$ known by the sender and the receiver

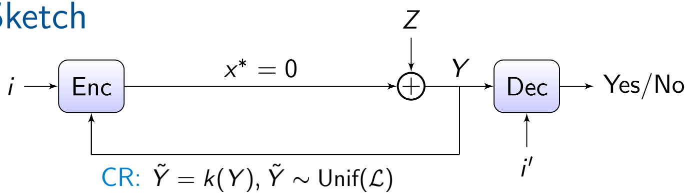
$$F_i : \{1, \ldots, |\mathcal{L}|\} \longrightarrow \{1, \ldots, M\}$$
$$: \underbrace{l}_{\text{coloring}} \mapsto \underbrace{F_i(l)}_{\text{color}}$$

# Proof Sketch



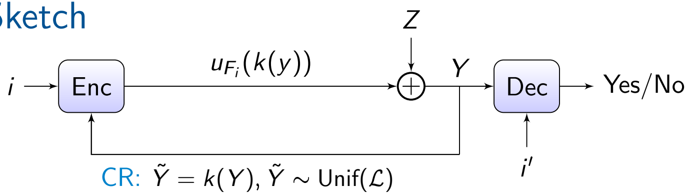

2. We send **one symbol** $x^* = 0$ over the forward channel

# Proof Sketch



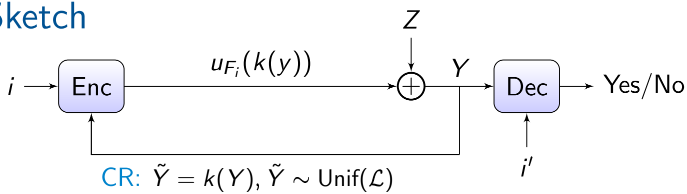

3. We generate the RV $\tilde{Y} = k(Y) \sim Unif(\mathcal{L})$, $|\mathcal{L}|$ determines the growth of $L(n, R)$

# Proof Sketch




4. $\mathcal{C} = \{(u_j, \mathcal{D}_j),\ j = 1, \ldots, M\}$ is an $(m, M, 2^{-m\delta})$ transmission code, we send $u_{F_i(k(y))}$, $k(y) \in \mathcal{L} \implies (n, L(n,R), \lambda_1, \lambda_2)$ DIF code with $n = 1 + m$

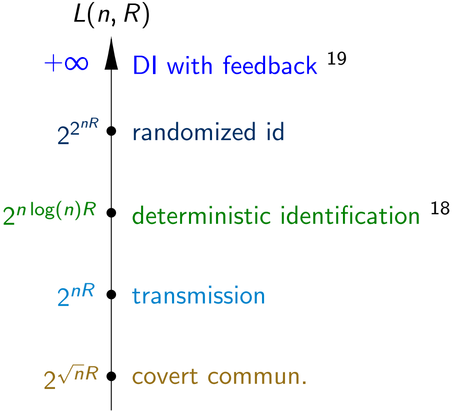
# Proof Sketch




5. If $F_i(k(y)) = F_{i'}(k(y))$, then $i = i'$

# Outline

# Conclusions: Coding Scale




[18] M. J. Salariseddigh, U. Pereg, H. Boche, and C. Deppe, "Deterministic identification over channels with power constraints," IEEE Int'l Conf. Commun. (ICC), 2021 [arXiv:2010.04239, 2021]

[19] W. Labidi, H. Boche, C. Deppe and M. Wiese, "Identification over the Gaussian Channel in the Presence of Feedback," IEEE Int'l Symp. Inf. Theory (ISIT), 2021 [arXiv:2102.01198, 2021]

# Conclusions Cont.

- We have determined DI capacity for
  - Fading $\rightarrow 2^{n\log(n)C} = n^{nC}$ behavior

  As opposed to $2^{2^{nR}}$ for randomized identification
- We have provide a coding scheme that generates infinite common randomness between TX-RX and showed that the DIF capacity is infinte regardless of the scaling
- Future directions
  - Molecular communication channel
  - Study scenario with noisy feedback over continuous channels
- Applications: 6G [20]

[20] Juan Cabrera, Holger Boche, Christian Deppe, Rafael F. Schaefer, Christian Scheunert, Frank H. P. Fitzek, "6G and the Post-Shannon-Theory,", In: Emmanuel Bertin, Noel Crespi, Thomas Magedanz (Hrsg.): "Shaping Future 6G Networks: Needs, Impacts and Technologies. Wiley-Blackwel", 2021

# Acknowledgements

## Thank You !


- We thank the German Research Foundation (DFG) within the Gottfried Wilhelm Leibniz Prize under grant BO 1734/20-1.
- Thanks also goes to the German Federal Ministry of Education and Research (BMBF) within the national initiative for Post Shannon Communication (New-Com) with the project Basics, Simulation and Demonstration for New Communication Models under grant 16KIS1003K and with the project Coding Theory and Coding Methods for New Communication Models under grant 16KIS1005.
- Additionally, we thank the German Federal Ministry of Education and Research (BMBF) within the national initiative for Molecular Communications (MAMOKO) under grant 16KIS0914.