The Arbitrarily Varying Multiple-Access Channel with Conferencing Encoders

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***Abstract*—We characterize the capacity region of the arbitrarily varying multiple-access channel with conferencing encoders. This channel exhibits a dichotomy: either it is useless or its capacity region equals the region achievable with random coding. We determine exactly when either case holds. This model can be used to analyze downlink networks with cooperating base stations suffering from exterior interference.**

I. Introduction

Wireless Local Area Network (WLAN) hot spots are seen as an attractive means of making internet access possible everywhere for everybody. As these hot spots become more, though, some problems arise. The WLAN frequency band is not regulated, and different operators can establish their own competing WLAN networks sharing the same spectrum arbitrarily close to each other. This causes inter-network interference. As interference carries information, it cannot be treated as noise in the analysis, so it cannot be included into the channel description yielding a new virtual single-state channel. A more robust channel model is necessary. This is only one example of networks disturbed by exterior interference.

This paper is an information-theoretic study of the problem of exterior interference under the assumption that the base stations of the disturbed network can cooperate at a finite rate, e.g. using a Coordinated Multi-Point (CoMP) scheme [6]. Instead of considering additional interior interference, only single-receiver downlink networks are considered. This is due to the fact that the capacity of interference channels has not yet been characterized completely. We model the problem by the Arbitrarily Varying Multiple-Access Channel (AV-MAC) with conferencing encoders. It goes without saying that due to its generality, this describes considerably more situations than described at the beginning. The capacity region of the AV-MAC provides insights into the interdependence of the channel and interference structure and the amount of information the base stations can exchange.

Discrete memoryless Multiple-Access Channels (MACs) with conferencing encoders were introduced by Willems in [11]. "Conferencing" is an iterative protocol of exchanging information between the two encoders about the messages to be sent through the backbone link. This protocol was further used in [3], [7], [8], among others. It was generalized in [9], where the capacity of the compound MAC with conferencing encoders was characterized. In that paper, conferencing was also used to exchange channel state information (CSI).

The exterior interference requires a model using AV-MACs with conferencing encoders. At each time instant, an AV-MAC is in a certain state which may change arbitrarily from channel use to channel use. This behavior reflects the influence of the exterior interference. As this interference is not known at the encoders except for the range of channel states it may cause, coding must be done in such a way that communication within the network is reliable no matter what the sequence of states is during the transmission of the codewords. Thus we do not assume that the state sequences are generated probabilistically! This gives a very robust channel model.

The paper is organized as follows: Section II introduces the channel model and the main coding theorem. Section III shows the direct part of the coding theorem, Section IV shows the converse. A discussion in Section V concludes the paper.

Notation: For any positive integer m, we write $[1,m]$ for the set $\{1,\dots,m\}$. For a set $A\subset\mathscr{X}$, we denote its complement by $A^c:=\mathscr{X}\setminus A$. For real numbers x and y, we set $x\wedge y:=\min(x,y)$. $\mathscr{P}(\mathscr{X})$ denotes the set of probability measures on the discrete set $\mathscr{X}$.

II. Channel Model and Main Results

The input alphabets of the transmitters are the finite sets $\mathscr{X}$ and $\mathscr{Y}$, respectively. The channel outputs are from the finite alphabet $\mathscr{Z}$. That means that we assume a fixed sampling/quantization scheme. As the interference is supposed to be digital as well, we may assume that the channel under consideration only assumes finitely many states collected in the set $\mathscr{S}$. In state $s\in\mathscr{S}$, the channel is given by a stochastic matrix

$$W(z|x,y|s):\quad (x,y,z)\in\mathscr{X}\times\mathscr{Y}\times\mathscr{Z}.$$

We write $\mathscr{W}:=\{W(\cdot|\cdot,\cdot|s):s\in\mathscr{S}\}$. As the channel states are determined by unknown exterior interference, they may change arbitrarily during the transmission of words of length greater than one. Assuming that words $\mathbf{x}\in\mathscr{X}^n$ and $\mathbf{y}\in\mathscr{Y}^n$ are sent and that the sequence of channel states attained during their transmission is $\mathbf{s}\in\mathscr{S}^n$, the probability that the word $\mathbf{z}\in\mathscr{Z}^n$ is received equals

$$W^n(\mathbf{z}|\mathbf{x},\mathbf{y}|\mathbf{s}):=\prod_{m=1}^n W(z_m|x_m,y_m|s_m).$$

This behavior defines an Arbitrarily Varying Multiple-Access Channel (AV-MAC).

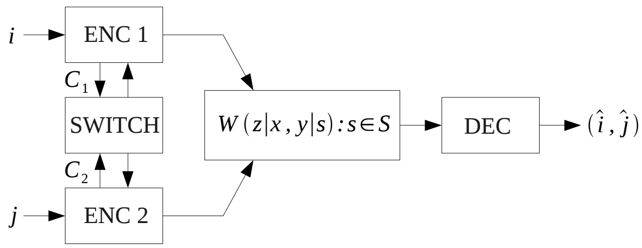

Fig. 1. The AV-MAC $\mathscr{W}$ with conferencing encoders.

Next we introduce conferencing codes$_{\text{CONF}}$, where encoders may have a conference before choosing their codewords. The original conferencing concept was introduced in [11]. We use here the generalized conferencing introduced in [9], which includes Willems' conferencing as a special case. Suppose that the first transmitter has the message set $[1, M_1]$ and that the second transmitter has the message set $[1, M_2]$. A generalized conferencing function is a function c from $[1, M_1] \times [1, M_2]$ to a set $[1, V]$, where V is a positive integer. This function abstractly describes a base station cooperation protocol. The idea is that if the first transmitter would like to transmit message $i \in [1, M_1]$ and the second message $j \in [1, M_2]$, then both encoders have access to the conferencing result $c(i, j)$ and can use this knowledge about the other transmitter's message for encoding, see Fig. 1. We assume that the amount of information that can be exchanged during the conference is rate-constrained. To make this precise, define functions c_j ($j \in [1, M_2]$) and c_i ($i \in [1, M_1]$) by $c_j(i) = c_i(j) = c(i, j)$. We write $\|c_j\|$ and $\|c_i\|$ for the cardinality of the ranges of c_j and c_i, respectively. Then given conferencing capacities $C_1, C_2 \geq 0$ and a coding blocklength n, we require c to satisfy

$$\log\|c_j\| \leq nC_1 \qquad (j \in [1, M_2]), \tag{1}$$
$$\log\|c_i\| \leq nC_2 \qquad (i \in [1, M_1]). \tag{2}$$

For a more detailed discussion of this concept of conferencing, see [9]. The exact definition of encoding (with blocklength n and conferencing capacities C_1, C_2) then is that in addition to a conferencing function c satisfying (1) and (2), each transmitter has an encoding function f_1 and f_2, respectively, where

$$f_1 : [1, M_1] \times [1, V] \to \mathscr{X}^n,$$
$$f_2 : [1, M_2] \times [1, V] \to \mathscr{Y}^n.$$

Given that the pair of messages (i, j) is to be transmitted, the encoders will use the codewords

$$f_1(i, c(i, j)) =: \mathbf{x}_{ij},$$
$$f_2(j, c(i, j)) =: \mathbf{y}_{ij}.$$

Decoding is standard, i.e. the decoder partitions $\mathscr{Z}^n$ into disjoint sets F_{ij} and decides for the message pair (i, j) if the channel output $\mathbf{z} \in F_{ij}$. The pair (M_1, M_2) is called the codelength pair. We say that the average error of such a code$_{\text{CONF}}$ is smaller than $\lambda \in (0, 1)$ if for every $\mathbf{s} \in \mathscr{S}^n$,

$$\frac{1}{M_1 M_2} \sum_{i,j} W^n(F_{ij}^c | \mathbf{x}_{ij}, \mathbf{y}_{ij} | \mathbf{s}) \leq \lambda.$$

This is a very strict criterion: the average error must be small uniformly for all possible state sequences.

Given $C_1, C_2 \geq 0$, a rate pair (R_1, R_2) is achievable with conferencing capacities C_1, C_2 if for every $\varepsilon > 0$ and $\lambda \in (0, 1)$, for n sufficiently large, there is a blocklength-n code$_{\text{CONF}}$ with average error smaller than λ, whose conferencing function satisfies (1) and (2) and which has a codelength pair (M_1, M_2) satisfying

$$\frac{1}{n} \log M_\nu \geq R_\nu - \varepsilon \quad (\nu = 1, 2).$$

Note that the conferencing function is part of the code and may be chosen freely as long as (1) and (2) are satisfied.

The main result of this work is the characterization of the capacity region $\mathscr{C}(C_1, C_2)$ achieved with conferencing capacities $C_1, C_2 > 0$. The description of $\mathscr{C}(C_1, C_2)$ requires some additional concepts and notation. First we define a set $\mathscr{C}^*(C_1, C_2)$ as follows. Let $\overline{\mathscr{W}}$ be the convex hull of $\mathscr{W}$. Every $W \in \overline{\mathscr{W}}$ can be assigned (not necessarily uniquely) a "state" $q \in \mathscr{P}(\mathscr{S})$ satisfying

$$W(z|x, y|q) = \sum_s W(z|x, y|s) q(s).$$

Hence we can write $\overline{\mathscr{W}} = \{W(\cdot|\cdot, \cdot|q) : q \in \mathscr{P}(\mathscr{S})\}$. Further, for any finite set $\mathscr{U}$, we consider the probability measures on the set $\mathscr{U} \times \mathscr{X} \times \mathscr{Y}$ which have the form

$$p(u, x, y) = p_0(u) p_1(x|u) p_2(y|u). \tag{3}$$

Every such p and every $q \in \mathscr{P}(\mathscr{S})$ together define a quadruple (U, X, Y, Z_q) of random variables with distribution $p(u, x, y) W(z|x, y|q)$. With these random vectors, we can define the sets $\mathscr{R}(p, q, C_1, C_2)$ consisting of those nonnegative real pairs (R_1, R_2) which satisfy

$$R_1 \leq I(Z_q; X|Y, U) + C_1,$$
$$R_2 \leq I(Z_q; Y|X, U) + C_2,$$
$$R_1 + R_2 \leq (I(Z_q; X, Y|U) + C_1 + C_2) \wedge I(Z_q; X, Y).$$

The set $\mathscr{C}^*(C_1, C_2)$ then is defined to equal

$$\bigcup_p \bigcap_q \mathscr{R}(p, q, C_1, C_2),$$

where the intersection is over all $q \in \mathscr{P}(\mathscr{S})$ and the union is over all finite subsets $\mathscr{U}$ of the integers and probability measures p of the form (3).

We will see that either $\mathscr{C}(C_1, C_2) = \mathscr{C}^*(C_1, C_2)$ or $\mathscr{C}(C_1, C_2) = \{(0, 0)\}$. In order to distinguish these two cases, we need one more type of concepts which is unique to arbitrarily varying channels: symmetrizability. In our case, the most important kind of symmetrizability is $(\mathscr{X}, \mathscr{Y})$-symmetrizability. $\mathscr{W}$ is $(\mathscr{X}, \mathscr{Y})$-symmetrizable if there is a stochastic matrix $\sigma(s|x,y) : (x,y,s) \in \mathscr{X} \times \mathscr{Y} \times \mathscr{S}$ such that for every $x, x' \in \mathscr{X}$, $y, y' \in \mathscr{Y}$, and $z \in \mathscr{Z}$, one has

$$\sum_s W(z|x,y|s)\sigma(s|x',y') = \sum_s W(z|x',y'|s)\sigma(s|x,y).$$

We can now formulate our main result:

Theorem 1. *Let conferencing capacities $C_1, C_2 > 0$ be given. If $\mathscr{W}$ is not $(\mathscr{X},\mathscr{Y})$-symmetrizable, then $\mathscr{C}(C_1,C_2) = \mathscr{C}^*(C_1,C_2)$. Otherwise, $\mathscr{C}(C_1,C_2) = \{(0,0)\}$.*

Remark 1. One can restrict the conferencing functions to have the form

$$c(i,j) = (c^{(1)}(i), c^{(2)}(j)). \tag{4}$$

Thus non-iterative Willems conferencing is optimal. Proceeding like [5], one sees that $\mathscr{C}^*(C_1,C_2)$ is two-dimensional if $\mathscr{W}$ is not $(\mathscr{X},\mathscr{Y})$-symmetrizable, so $(\mathscr{X},\mathscr{Y})$-symmetrizability is the right condition to distinguish useless from useful AV-MACs. A detailed analysis of $\mathscr{C}^*(C_1,C_2)$ can be found in [9]. In particular, if C_1, C_2 exceed a certain finite value, then $\mathscr{C}^*(C_1,C_2)$ equals $\mathscr{C}^*(\infty,\infty)$, the rate region obtained when the base stations can exchange an infinite amount of information in finite time.

III. The direct part of Theorem 1

The proof of the direct part of Theorem 1 builds on two techniques for arbitrarily varying channels developed by Ahlswede: "robustification" [2] and "elimination of correlation" [1]. Robustification uses a coding theorem for the corresponding compound channel (here the compound MAC) in order to find a rate region for the arbitrarily varying channel achievable by random coding. In our case, one can thus show that if random coding is used, whether or not $\mathscr{W}$ is symmetrizable, the complete region $\mathscr{C}^*(C_1,C_2)$ is achievable. This is not yet enough to characterize the deterministic capacity of arbitrarily varying channels, in contrast to random coding for discrete memoryless or compound channels. In fact, if the arbitrarily varying channel fulfills a symmetrizability condition (in our case $(\mathscr{X},\mathscr{Y})$-symmetrizability), then it turns out to be useless. Otherwise, one can use elimination of correlation to show that $\mathscr{C}^*(C_1,C_2)$ is indeed achievable deterministically, i.e. without any randomization at the encoders or the decoder.

A. Robustification

In [9] it was shown that the capacity region of the compound MAC with conferencing capacities $C_1, C_2 > 0$ corresponding to the AV-MAC equals $\mathscr{C}^*(C_1,C_2)$. (The compound MAC corresponding to the AV-MAC does not change its state during the transmission of a codeword. Its possible "states" are all $q \in \mathscr{P}(\mathscr{S})$, not just the elements of $\mathscr{S}$.) It was also shown in [9] that nothing is lost by restricting the conferencing functions to have the form (4). Further, for every achievable rate pair, one finds an approximating sequence of codes$_{\text{CONF}}$ whose average errors tend to zero exponentially in blocklength. We now show that this result implies the achievability of $\mathscr{C}^*(C_1,C_2)$ for the AV-MAC with conferencing encoders and random coding.

Let a code$_{\text{CONF}}$ from such an approximating sequence be given with blocklength n, a codelength pair (M_1, M_2), and using codewords $\mathbf{x}_{ij}, \mathbf{y}_{ij}$ and decoding sets F_{ij}. The result from [9] means that there is a $\zeta > 0$ such that for every $q \in \mathscr{P}(\mathscr{S})$, writing $q^n := (q,\dots,q) \in \mathscr{P}(\mathscr{S})^n$,

$$\frac{1}{M_1 M_2}\sum_{i,j} W^n(F_{ij}^c|\mathbf{x}_{ij},\mathbf{y}_{ij}|q^n) \le 2^{-n\zeta}.$$

From this code$_{\text{CONF}}$, one constructs for every n-permutation π a new code$_{\text{CONF}}$ with codewords $\pi(\mathbf{x}_{ij}), \pi(\mathbf{y}_{ij})$ and decoding sets $\pi(F_{ij})$, where for $\mathbf{x} = (x_1,\dots,x_n) \in \mathscr{X}^n$, we define $\pi(\mathbf{x}) = (x_{\pi(1)},\dots,x_{\pi(n)})$ and where for a set F, $\pi(F) = \{\pi(\mathbf{z}) : \mathbf{z} \in F\}$. Further, as conferencing only concerns the messages and not the codewords, the conferencing functions of all these codes coincide; they are the same c as that of the code$_{\text{CONF}}$ we started out with.

Then using the Theorem "Robustification Technique" [2], one can show that for every $\mathbf{s} \in \mathscr{S}^n$,

$$\frac{1}{n!}\frac{1}{M_1 M_2}\sum_\pi\sum_{i,j} W^n(\pi(F_{ij})^c|\pi(\mathbf{x}_{ij}),\pi(\mathbf{y}_{ij})|\mathbf{s}) \le (n+1)^{|\mathscr{S}|}2^{-n\zeta}.$$

(The first sum is over all n-permutations.) This expression means that if the transmitters and the receiver together do a random experiment by which they uniformly choose an n-permutation and agree to use the corresponding code$_{\text{CONF}}$, then in the mean, the average error will be exponentially small. This reasoning reveals that $\mathscr{C}^*(C_1,C_2)$ is achievable by the AV-MAC if random coding is allowed.

For later use in the elimination of correlation, we note without proof that instead of using a random code$_{\text{CONF}}$ with $n!$ deterministic component codes, every rate pair in $\mathscr{C}^*(C_1,C_2)$ is achievable using a random code$_{\text{CONF}}$ which only has n^2 components which are chosen uniformly at random. (We must not have more than polynomially many components, but $n!$ grows exponentially.) This fact is proved using a simple Bernstein type inequality.

B. A Positive Rate Pair

Here we show that if $\mathscr{W}$ is not $(\mathscr{X},\mathscr{Y})$-symmetrizable, then the AV-MAC achieves a rate pair (R,R), $R > 0$, using deterministic codes$_{\text{CONF}}$. This will be used in the next subsection. Let $0 < R < C_1 \wedge C_2$. This means that if both transmitters have approximately 2^{nR} messages at blocklength n, then they can use the conference to inform the other transmitter completely about which of their own messages they would like to transmit. In particular, the conferencing function, being the identity on the Cartesian product of the message sets, has form (4).

Thus one can regard the pair of transmitters as a single "super-transmitter" with input alphabet $\mathscr{X} \times \mathscr{Y}$, turning the multiple-access problem into a discrete multiple-input-single-output (MISO) problem. Information-theoretically, $\mathscr{X} \times \mathscr{Y}$ is a perfectly admissible input alphabet for a single transmitter, so if $R < C_1 \wedge C_2$, conferencing transforms the AV-MAC into a single-user arbitrarily varying channel.

For these channels, [5] has shown that they achieve a positive rate in case they are not "symmetrizable". But "symmetrizability" for an arbitrarily varying single-user channel with input alphabet $\mathscr{X} \times \mathscr{Y}$ means nothing but $(\mathscr{X}, \mathscr{Y})$-symmetrizability. Thus we know that the "super-transmitter" can achieve a positive rate through our non-$(\mathscr{X}, \mathscr{Y})$-symmetrizable AV-MAC. As $R < C_1 \wedge C_2$, every single-user code approximating the rate $2R$ can easily be turned into an admissible code$_{\text{CONF}}$ approximating the rate pair (R, R), which has thus been shown to be achievable.

C. Elimination of Correlation

We use the results from the previous subsections to show that if $\mathscr{W}$ is not $(\mathscr{X}, \mathscr{Y})$-symmetrizable, then $\mathscr{C}(C_1, C_2) = \mathscr{C}^*(C_1, C_2)$. Let R be as in the previous subsection. Let $\varepsilon > 0$ and let $\lambda \in (0, 1)$. Choose n sufficiently large for there to exist a deterministic code$_{\text{CONF}}$ with conferencing function c^* and codelength pair (n, n) whose average error is at most λ and such that

$$R - \varepsilon \le \frac{1}{m} \log \le R, \tag{5}$$

where m denotes the blocklength of the code. Such an n exists because (R, R) is deterministically achievable. By (5), m is logarithmically small in n. We write the code in self-explanatory notation

$$\{(\mathbf{x}^*_{k\ell}, \mathbf{y}^*_{k\ell}, F^*_{k\ell}) : (k, \ell) \in [1, n]^2\}.$$

Due to the results of Subsection III-A, we may assume that for this n and for some $(R_1, R_2) \in \mathscr{C}^*(C_1, C_2)$, there is a random code$_{\text{CONF}}$ whose n^2 deterministic components, all with blocklength n and codelength pair (M_1, M_2), have the common conferencing function $\tilde{c}$ and the form

$$\{(\tilde{\mathbf{x}}^{(k,\ell)}_{ij}, \tilde{\mathbf{y}}^{(k,\ell)}_{ij}, \tilde{F}^{(k,\ell)}_{ij}) : (i, j) \in [1, M_1] \times [1, M_2]\},$$

where we use $(k, \ell) \in [1, n]^2$ as indices for the n^2 different codes. Further, we may assume that

$$\frac{1}{n} \log M_\nu \ge R_\nu - \varepsilon \qquad (\nu = 1, 2)$$

and that for all $\mathbf{s} \in \mathscr{S}^n$,

$$\frac{1}{n^2} \frac{1}{M_1 M_2} \sum_{k,\ell=1}^{n} \sum_{i,j} W^n((\tilde{F}^{(k,\ell)}_{ij})^c | \mathbf{x}^{(k,\ell)}_{ij}, \mathbf{y}^{(k,\ell)}_{ij} | \mathbf{s}) \le \lambda,$$

and that $\tilde{c}$ has the form (4).

The elimination of correlation technique uses the $*$-code as a prefix code for the $\sim$-code in order to generate a deterministic coding scheme. That means that one defines a code$_{\text{CONF}}$ with blocklength $m + n$ and message sets $[1, n] \times [1, M_\nu]$ $(\nu = 1, 2)$ where, when the transmitters would like to transmit the messages (k, i) and (j, ℓ), respectively, they inform each other via $(c^*(k, \ell), \tilde{c}(i, j))$. If n is sufficiently large, this is an admissible conferencing function satisfying (4). Then they use codewords $(\mathbf{x}^*_{k\ell}, \tilde{\mathbf{x}}^{(k,\ell)}_{ij})$ and $(\mathbf{y}^*_{k\ell}, \tilde{\mathbf{y}}^{(k,\ell)}_{ij})$. The receiver has decoding sets $F^*_{k\ell} \times \tilde{F}^{(k,\ell)}_{ij}$. The average error incurred by this code is bounded by 2λ if n is large using the uniform randomness of the random code: the probability n^{-2} of using a certain deterministic component code of the random code transforms into the probability of transmitting a certain prefix message.

Finally, we also see why the number of component codes$_{\text{CONF}}$ had to be bounded by n^2 in Subsection III-A: the number of messages of transmitter ν equals nM_ν $(\nu = 1, 2)$, and an easy calculation using (5) shows that

$$\frac{1}{m + n} \log(nM_\nu) \ge R_\nu - 2\varepsilon \quad (\nu = 1, 2).$$

Thus the rate pair (R_1, R_2) approximated by the random code$_{\text{CONF}}$ is preserved despite the addition of the prefix code$_{\text{CONF}}$, and we have shown that every rate pair contained in $\mathscr{C}^*(C_1, C_2)$ is deterministically achievable by the AV-MAC if $\mathscr{W}$ is not $(\mathscr{X}, \mathscr{Y})$-symmetrizable.

IV. The Converse

Here we show a converse for Theorem 1. We distinguish whether or not $\mathscr{W}$ is $(\mathscr{X}, \mathscr{Y})$-symmetrizable. For the case where $\mathscr{W}$ is $(\mathscr{X}, \mathscr{Y})$-symmetrizable, we use that every code$_{\text{CONF}}$ also is a legitimate code for the "super-transmitter" introduced in Subsection III-B. And as was shown in [5], any code with more than two codewords used by the super-transmitter incurs an error at least 1/4 if $\mathscr{W}$ is $(\mathscr{X}, \mathscr{Y})$-symmetrizable. This gives a "strong converse" if $\mathscr{W}$ is $(\mathscr{X}, \mathscr{Y})$-symmetrizable.

If $\mathscr{W}$ is not $(\mathscr{X}, \mathscr{Y})$-symmetrizable, assume that a blocklength-n code$_{\text{CONF}}$ with codelength pair (M_1, M_2) is given satisfying that $(\frac{1}{n} \log M_1, \frac{1}{n} \log M_2)$ is at least distance $\varepsilon > 0$ from $\mathscr{C}^*(C_1, C_2)$. We now show a "weak converse", i.e. that this code incurs an average error at least $\lambda(\varepsilon) > 0$, where $\lambda(\varepsilon)$ only depends on the AV-MAC and ε, but not on the code.

For any (not necessarily conferencing) code with codewords $\mathbf{x}_{ij}, \mathbf{y}_{ij}$ and decoding sets F_{ij},

$$\sup_{\mathbf{s} \in \mathscr{S}^n} \frac{1}{M_1 M_2} \sum_{i,j} W^n(F^c_{ij} | \mathbf{x}_{ij}, \mathbf{y}_{ij} | \mathbf{s}) \ge \sup_{q \in \mathscr{P}(\mathscr{S})} \frac{1}{M_1 M_2} \sum_{i,j} W^n(F^c_{ij} | \mathbf{x}_{ij}, \mathbf{y}_{ij} | q^n).$$

The proof of this inequality is similar to the proof of [4, Lemma 2.6.3]. Thus when used for the AV-MAC, the code incurs at least the same average error as when used for the corresponding compound MAC. Then one can infer from [9] that the converse for the compound MAC also holds for the AV-MAC, thus showing that $\mathscr{C}^*(C_1, C_2)$ not only is an achievable rate region if $\mathscr{W}$ is not $(\mathscr{X}, \mathscr{Y})$-symmetrizable, but also an outer bound on $\mathscr{C}(C_1, C_2)$. This finishes the proof of Theorem 1.

V. Conclusion

In this paper, we characterized the capacity region of the AV-MAC with conferencing encoders. We showed a dichotomy: either the channel is useless or it achieves the same rate region as the corresponding compound MAC. The difference lies in the fact that when random coding is used, derandomization is possible only if a certain symmetrizability condition is violated. Due to this dichotomy, the possibility of conferencing brings about new effects as compared to single-state or compound conferencing MACs, whose respective capacity regions were determined in [11] and [9]. One especially finds that conferencing may in some cases be a technique to enable reliable communication which is not possible in AV-MACs without encoder cooperation. These new effects are discussed in detail in [10].

The proof of the direct part of the main theorem bases on two techniques developed by Ahlswede. The first technique constructs a random code for the AV-MAC from a deterministic code for the corresponding compound MAC. The second technique is used for derandomization and is much more involved than the derandomization traditionally used for discrete memoryless or compound channels. This is due to the fact that the number of possible channel state sequences of the AV-MAC grows exponentially in blocklength, whereas it remains constant for discrete memoryless or compound channels.

We used the AV-MAC to describe a single-receiver downlink network with base station cooperation, with transmissions from the senders to the receiver disturbed by exterior interference. The analysis shows the dependence of the capacity region on the limited base station cooperation capacity. It turns out that optimal conferencing protocols are simple and do not even need iterative steps.

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