Short Polar Codes

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Outline

1. Motivation
2. Improve the Distance Property
3. Simulation Results
4. sum-min Approx.
5. Rate Matching/IR-HARQ
6. Conclusions

Polar codes

- Provable Capacity-Achieving[1]
- Encoding:
  Precoding ($k \rightarrow n$)
  Polar transform ($n \rightarrow n$)
  $\frac{1}{2} n \log n$ "$\oplus$" (parallelizable)
- Successive Cancellation (SC) decoding (sequential):
  $\frac{1}{2} n \log n$ "$+$"
  $\frac{1}{2} n \log n$ "$\boxplus$"

$$x \boxplus y\colon\ 2 \tanh^{-1}[\tanh \frac{x}{2} \tanh \frac{y}{2}] \approx \operatorname{sign}(x) \operatorname{sign}(y) \min(|x|, |y|)$$

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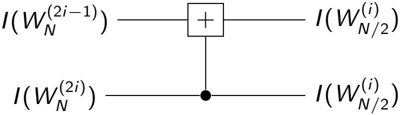
[1] E. Arıkan. "Channel polarization: A method for constructing capacity-achieving codes for symmetric binary-input memoryless channels." *IEEE Trans. on Information Theory*, 2009

Construction of Polar codes (Gaussian Approx.[2] with J-function[3])



$I(W_N^{(2i-1)})$ $\oplus$ $I(W_{N/2}^{(i)})$

$I(W_N^{(2i)})$ $I(W_{N/2}^{(i)})$

- $I(W_1^{(1)}) = J(2/\sigma_n)$
- $I(W_N^{(2i-1)}) = 1 - J\left(\sqrt{2\left[J^{-1}\left(1 - I(W_{N/2}^{(i)})\right)\right]^2}\right)$
- $I(W_N^{(2i)}) = J\left(\sqrt{2\left[J^{-1}\left(I(W_{N/2}^{(i)})\right)\right]^2}\right)$

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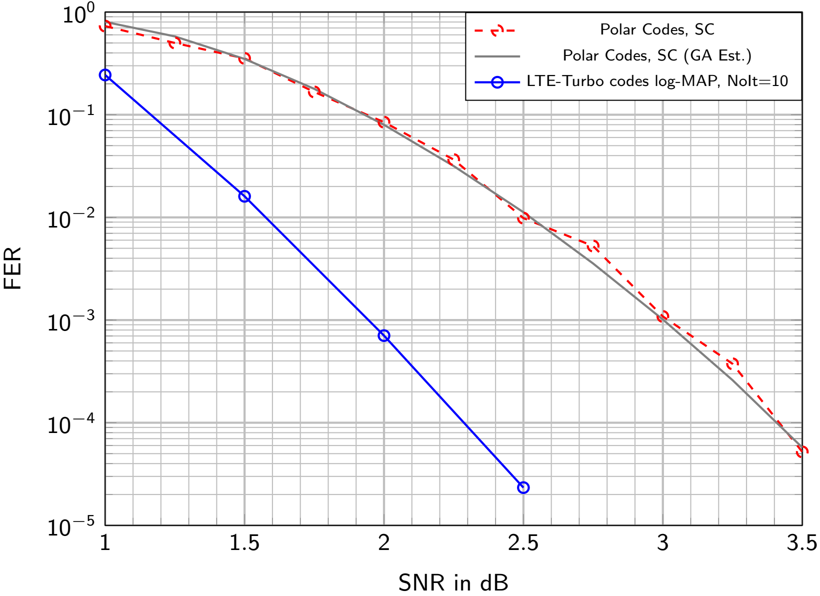
[2] S. ten Brink *et al.* "Design of low-density parity-check codes for modulation and detection." *IEEE Trans. on Communications*, 2004

[3] F. Brännström. "Convergence analysis and design of multiple concatenated codes." Ph.D. dissertation, Chalmers Univ. Technol., Göteborg, Sweden, Mar. 2004.

Polar codes vs. LTE-Turbo codes $(1024, 512)$



SC List Decoding[4]

- Time Complexity:
  $\mathcal{O}(Ln \log n)$
- Space Complexity:
  $L(2n-1)$ `float`
  $2L(2n-1)$ `boolean`
- ML-achieving decoding ($L \to 2^k$)

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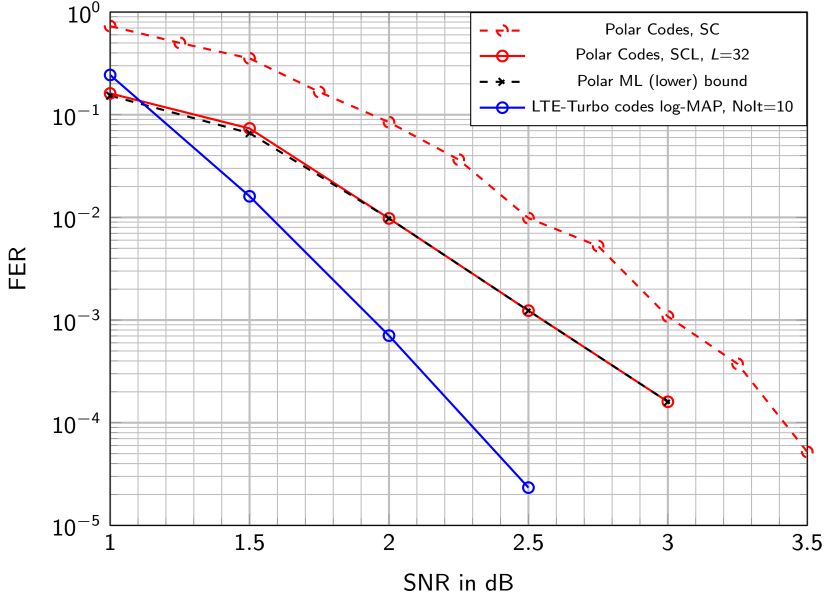
[4] I. Tal and A. Vardy. "List decoding of polar codes." *IEEE Trans. on Information Theory*, 2015

Polar codes vs. LTE-Turbo codes $(1024, 512)$



ML (lower) bound[5]

Algorithm 1 Estimate ML bound

```
$\hat{c} = \mathrm{decode}(y)$
if $\hat{c} \neq c$ then
    error = error + 1;
    if $\hat{c}y^{\mathsf{T}} > cy^{\mathsf{T}}$ then
        error_ml = error_ml + 1;
    end if
end if
```

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[5] I. Tal and A. Vardy. "List decoding of polar codes." *IEEE Trans. on Information Theory*, 2015, Sec. 5

Distance Property of Polar codes

$$F_n = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}^{\otimes m}$$

$$F_{n \times n} \xrightarrow{n-k \text{ row deletions}} G_{k \times n}$$

Minimum Hamming distance of $(1024, 512)$ Polar codes:

- 16 (Design SNR $<$ 5 dB)
- 32 (Design SNR $\geq$ 5 dB)

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Outer codes + Polar codes

CRC-aided (CA)-Polar codes[6]

- Flexible
- HARQ/Adaptive-decoding
- Minimum Distance?

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[6] K. Niu and K. Chen. "CRC-aided decoding of polar codes." *IEEE Communications Letters*, 2012

RM-Polar codes[7]

RM codes and Polar codes are obtrained from same polarization matrix $F_2^{\otimes m}$.

- Polar rule:
  freeze the unreliable bits
- RM rule:
  freeze the bits with low weight of their corresponding rows

RM-Polar codes:
semi-RM semi-Polar rule

- SCL decodable
- Minimum Distance
- not Flexible

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[7] B. Li *et al.* "A RM-polar codes." *arXiv preprint arXiv:1407.5483*, 2014

eBCH-Polar codes[8]

- Dynamic frozen bits
- (k', n, d) eBCH codes $(k' > k)$ with H
- $c = uF_2^{\otimes m}$, $cH^{\mathsf{T}} = 0$
- $uF_2^{\otimes m} H^{\mathsf{T}} = 0$, let $V_{n \times (n-k')} = F_2^{\otimes m} H^{\mathsf{T}}$

$(k, n, \geq d)$ eBCH-Polar codes

- SCL decodable
- Minimum Distance
- Flexible

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[8] P. Trifonov and V. Miloslavskaya. "Polar subcodes." *IEEE Journal on Selected Areas in Communications*, 2016

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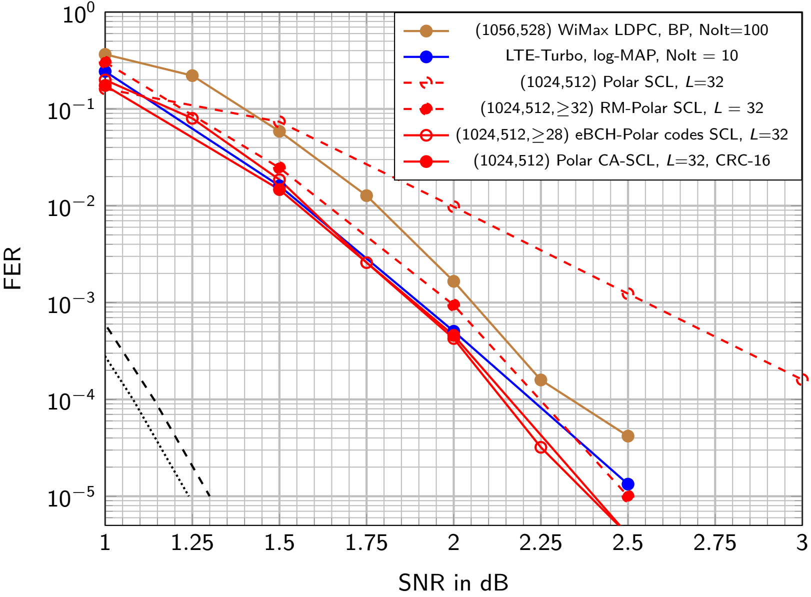
Simulation Results, $R = 1/2, n = 1024$



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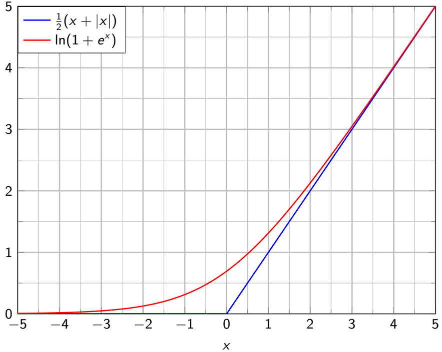
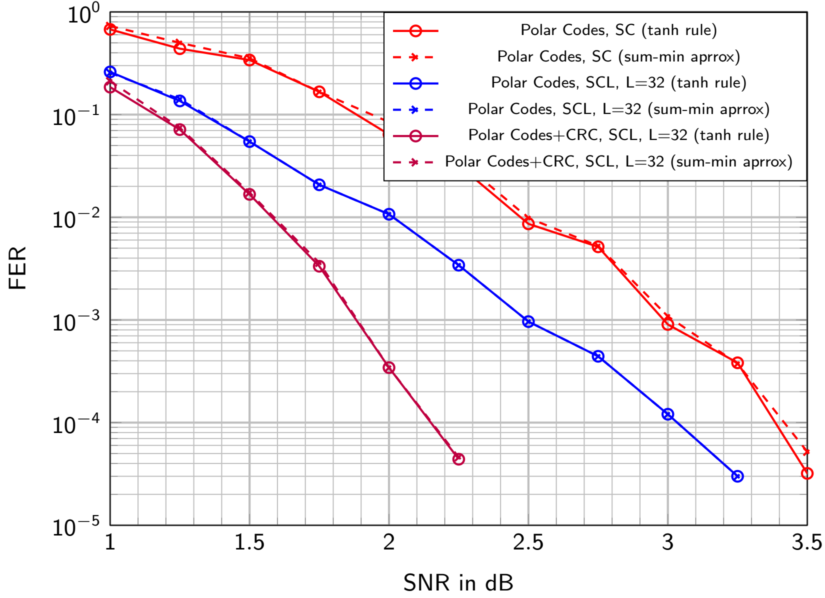
sum-min Approximation Polar codes



- $2\tanh^{-1}[\tanh\frac{x}{2}\tanh\frac{y}{2}] =$
  $\text{sign}(x)\,\text{sign}(y)\min(|x|,|y|) + \ln(1+e^{-|x+y|}) - \ln(1+e^{-|x-y|})$

sum-min Approximation for Polar codes



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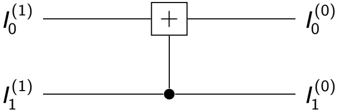
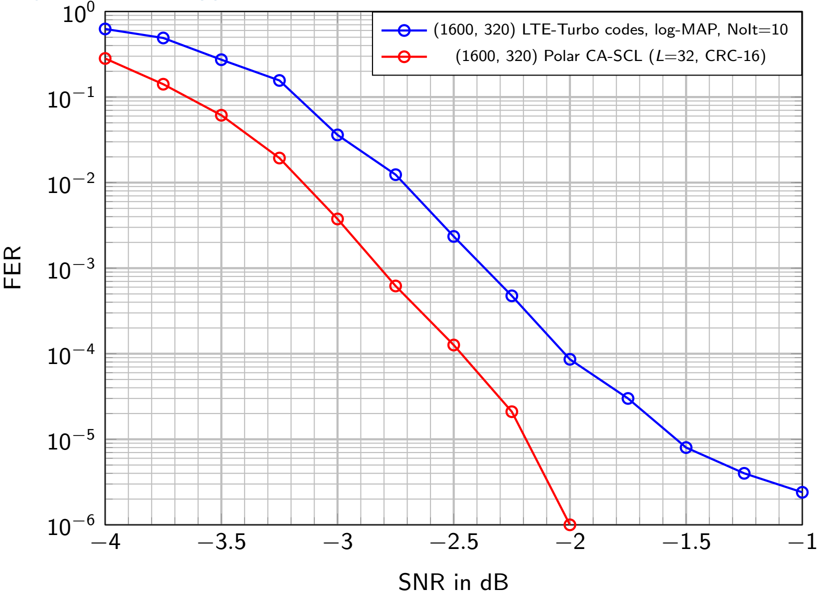
Rate Matching/IR-HARQ

- k is controlled via bit-freezing
- n is controlled via puncturing
  'mother' code length $N = 2^{\lceil \log_2 n \rceil}$,
  the first $N - n$ bits will be punctured
- IR: equivalent puncture pattern



- $I_0^{(1)} = 1 - J\left(\sqrt{\left[J^{-1}\left(1 - I_0^{(0)}\right)\right]^2 + \left[J^{-1}\left(1 - I_1^{(0)}\right)\right]^2}\right)$
- $I_1^{(1)} = J\left(\sqrt{\left[J^{-1}\left(I_0^{(0)}\right)\right]^2 + \left[J^{-1}\left(I_1^{(0)}\right)\right]^2}\right)$

Rate 0.2 Turbo & Polar Codes



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Conclusions

- Pros:
    - Good Performance
    - Efficient Design
    - Low Complexity Encoding/Decoding
- Cons:
    - no High-Throughput VLSI Architecture
    - no Adaptive Decoding