Random access on graphs: Capture-or tree evaluation

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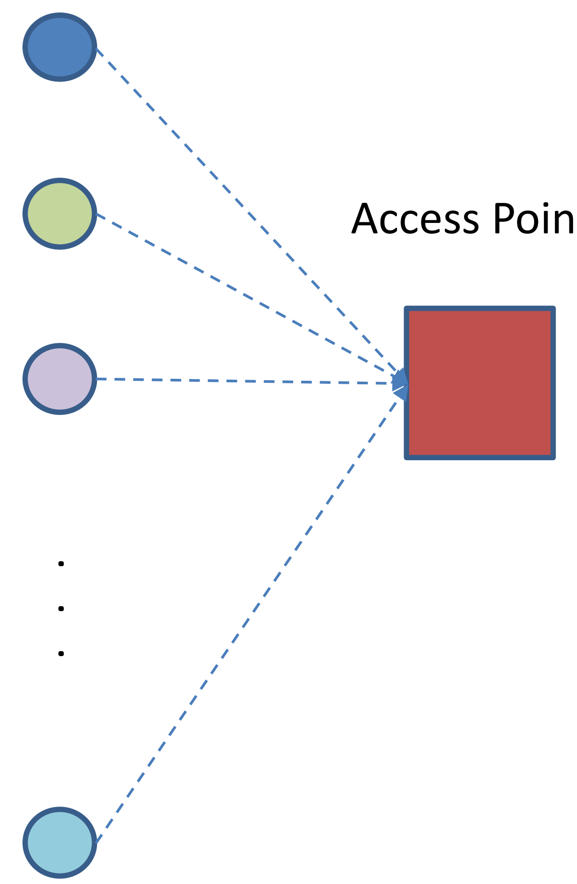
Preliminaries

- N users
  - Each user wants to send a packet over shared medium
  - Equal length packets
  - Users are synchronized
- Random access
  - Distributed, decentralized
  - Users behave in the same way

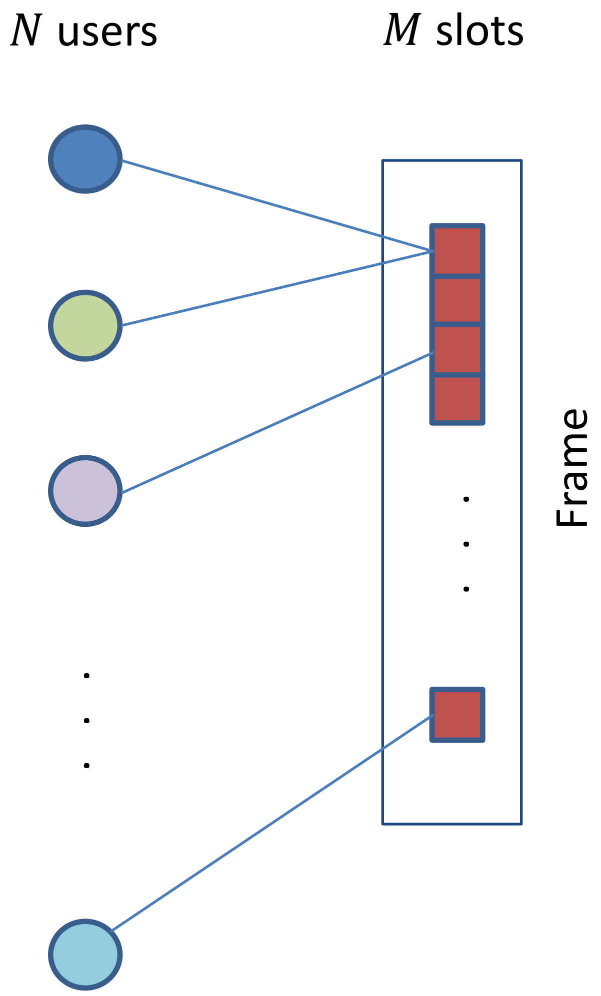


Framed slotted ALOHA

H. Okada, Y. Igarashi, Y. Nakanishi, ”Analysis and application of framed ALOHA channel in satellite packet switching networks”, Electronics and Communications, 1977

- Slots are organized in frames
- Each users transmit (just once) in a randomly selected slot of the frame
- Slots can be idle, singleton, or collision slots
- Collision channel model:
  - singleton slots are perfectly decoded,
  - collision slots can not be decoded at all
- Throughput:
  - No. resolved users vs no. slots:

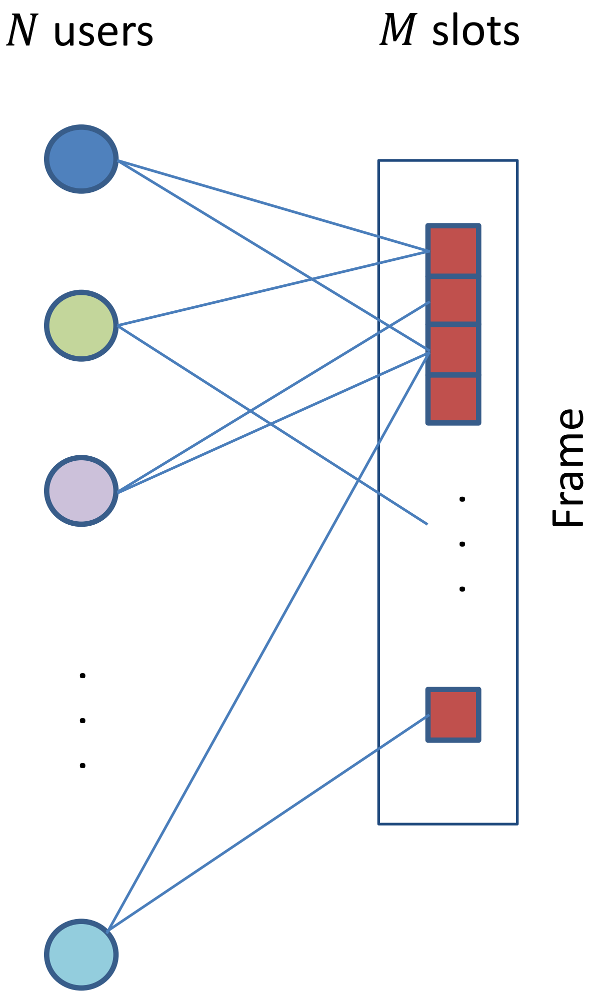
$$T_{max} = \frac{1}{e} \approx 0.37 \text{ (when } N = M\text{)}$$



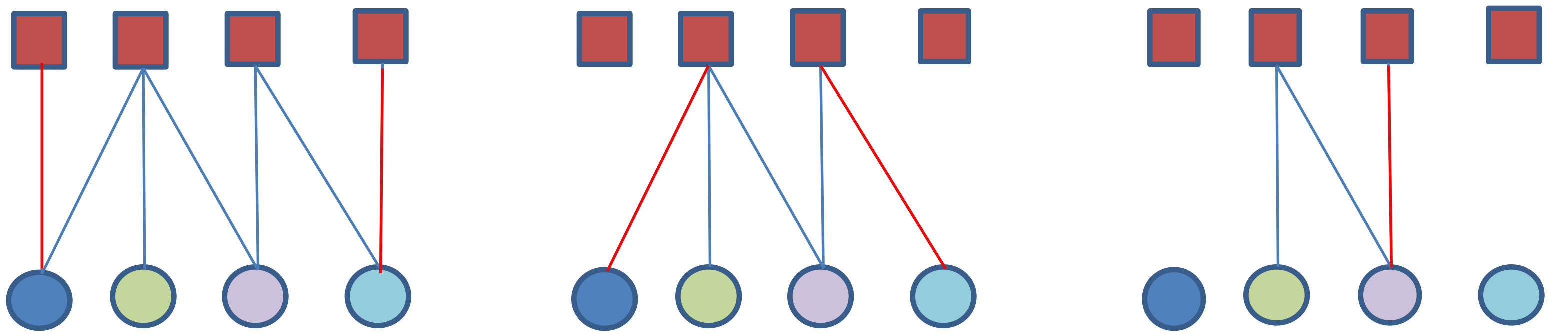
Content resolution diversity slotted ALOHA

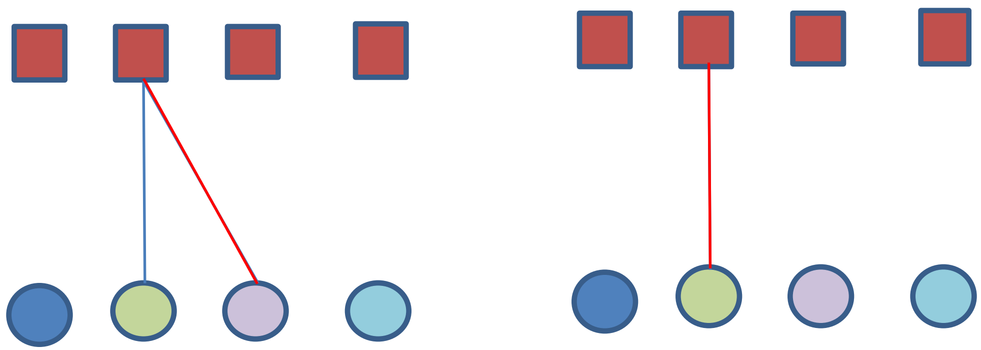
E. Casini, R. De Gaudenzi, O. del Rio Herrero, “Contention Resolution Diversity Slotted ALOHA (CRDSA): An Enhanced Random Access Scheme for Satellite Access Packet Networks”, IEEE Transactions on Wireless Communications, April 2007

- Users repeat their transmission in several randomly chosen slots of the frame
  - Same number of packet replicas per user
- Collisions can be exploited!
  - Successive interference cancellation (assumed to be perfect)
  - Improves throughput
  - $T \approx 0.55$ for CRDSA with two repetitions per user



Successive interference cancellation



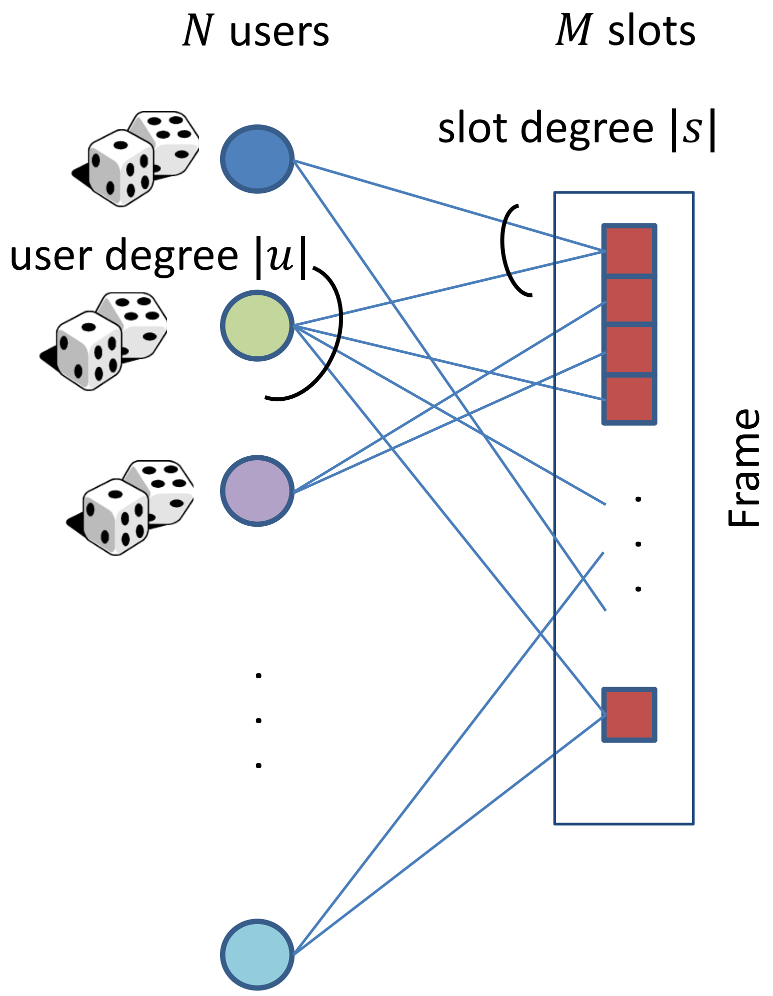


Analogous to iterative belief-propagation decoding of erasure correcting codes!

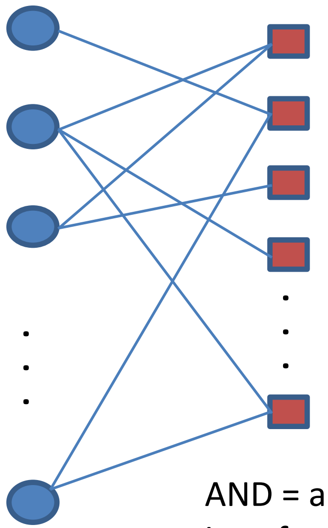
Irregular repetition slotted ALOHA

G. Liva, "Graph-Based Analysis and Optimization of Contention Resolution Diversity Slotted ALOHA," IEEE Trans. Commun., Feb. 2011.

- Generalization of CRDSA
  - No. of replicas varies across users
  - Every user selects its no. of replicas according to a predefined distribution
- Only the distribution of the user degrees can be controlled (designed):
  - $\Lambda_k = P[|u| = k]$
  - $\Lambda(x) = \sum_k \Lambda_k x^k$
- Optimal user degree distributions are designed according to the distribution design principles used for left-irregular LDPC codes
  - Achieve asymptotic throughput close to 1

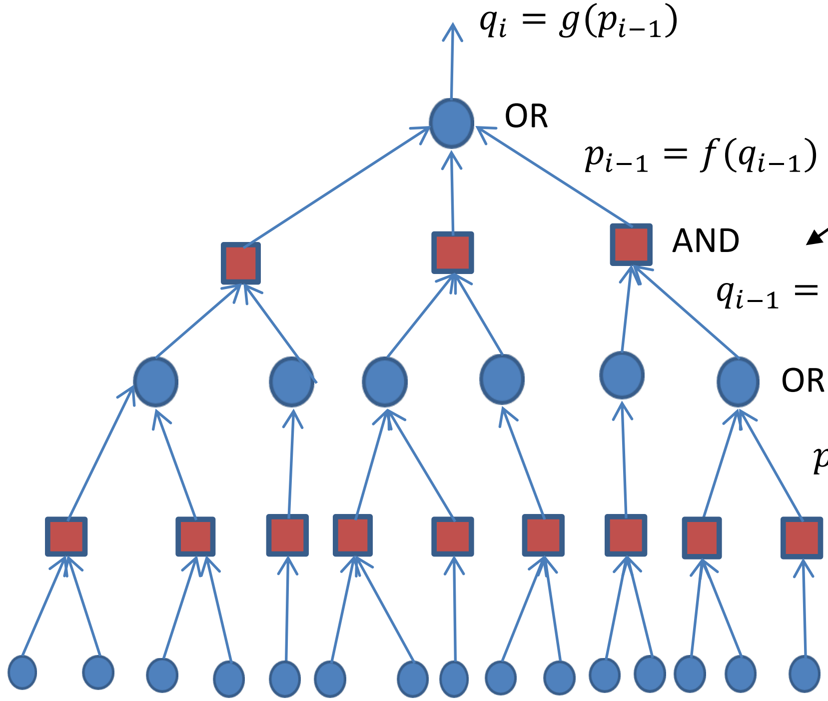


And-or tree evaluation: A tool for the asymptotic analysis



OR = any replica decoded

AND = all interferers decoded and removed by IC



Average probability that any packet replica is not decoded

Average probability that a packet replica is not decoded in a slot

iteration no.

M. G. Luby , M. Mitzenmacher, M. A. Shokrollahi, "Analysis of Random Processes via And-Or Tree Evaluation", in Proc. of 9th ACM-SIAM SODA, 1998

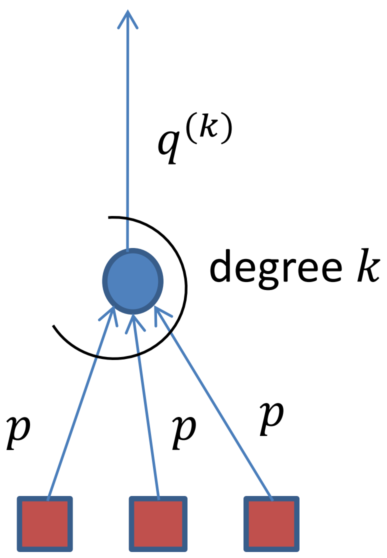
Probability of a packet recovery:

$P_R = 1 - \lim_{i \to \infty} q(i)$, where $q(0) = 1$

Our graphs are not trees! There are loops, i.e., there are interdependencies among messages. The results obtained by the and-or tree evaluation provide upper bound on the performance.

And-or tree evaluation: Message update probabilities

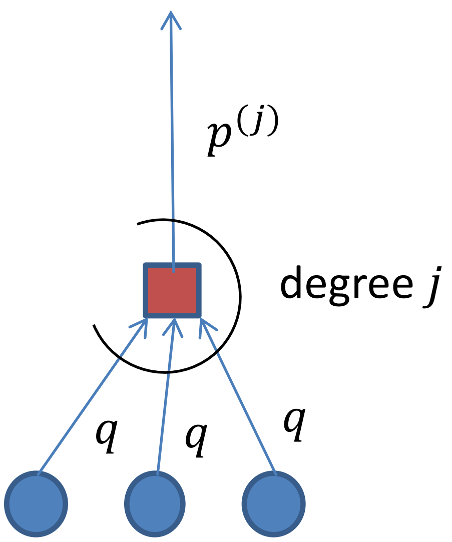
OR nodes



$$q^{(k)} = p^{k-1}$$

$$q = \sum_k \lambda_k p^{k-1} = \lambda(p)$$

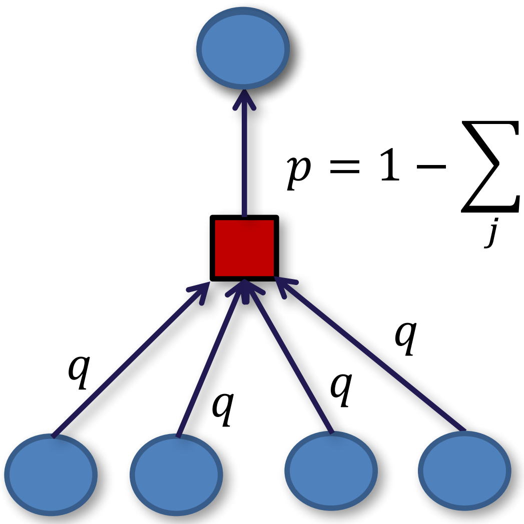
AND nodes



$$p^{(j)} = 1 - (1-q)^{j-1}$$

$$p = 1 - \sum_j \omega_j (1-q)^{j-1}$$

Departing from the collision channel model



$$p = 1 - \sum_j \omega_j \sum_{t=0}^{j-1} \pi_{t,j} \binom{j-1}{t} q^t (1-q)^{j-t-1}$$

- probability that user packet is decoded in a slot of degree j , when t interfering packets remain and $j-t-1$ are cancelled
  - i.e., capture probability

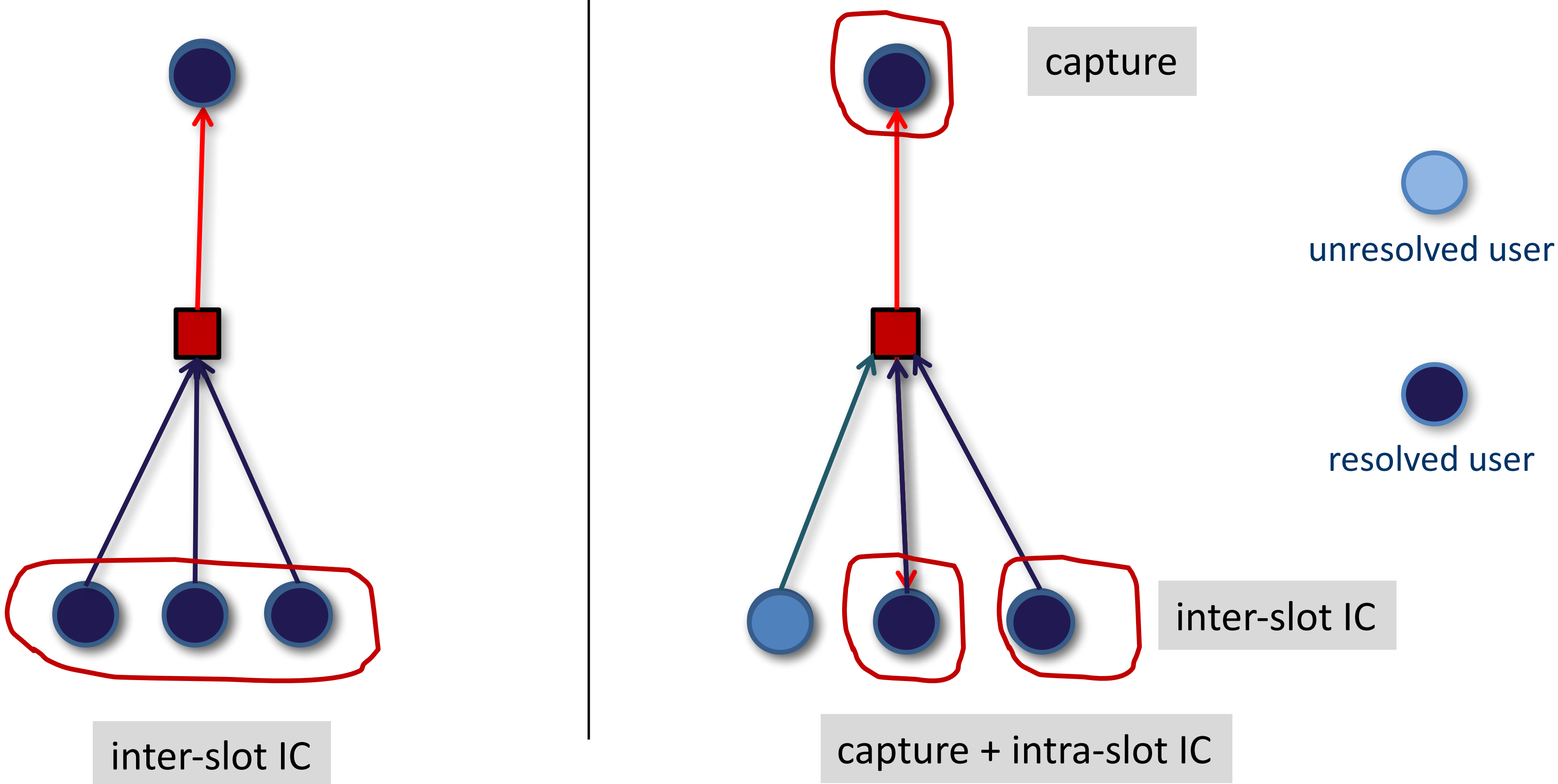
G. Liva, "Graph-Based Analysis and Optimization of Contention Resolution Diversity Slotted ALOHA," *IEEE Trans. Commun.*, Feb. 2011.

$$p = 1 - \sum_j \omega_j (1-q)^{j-1}$$

Departing from the collision channel model

No capture effect

With capture effect



Departing from the collision channel model: Threshold-based capture effect

- Non-equal channel gains (due to fading)
- Noise is not neglected
- SIC is not perfect
- Threshold-based model of the capture-effect:
  - Packet of user u_i is captured in slot if the following condition is satisfied:

$$\frac{P_i}{P_n + \sum_j P_j + \sum_k Q_k} \geq b$$

Received power of u_i → P_i

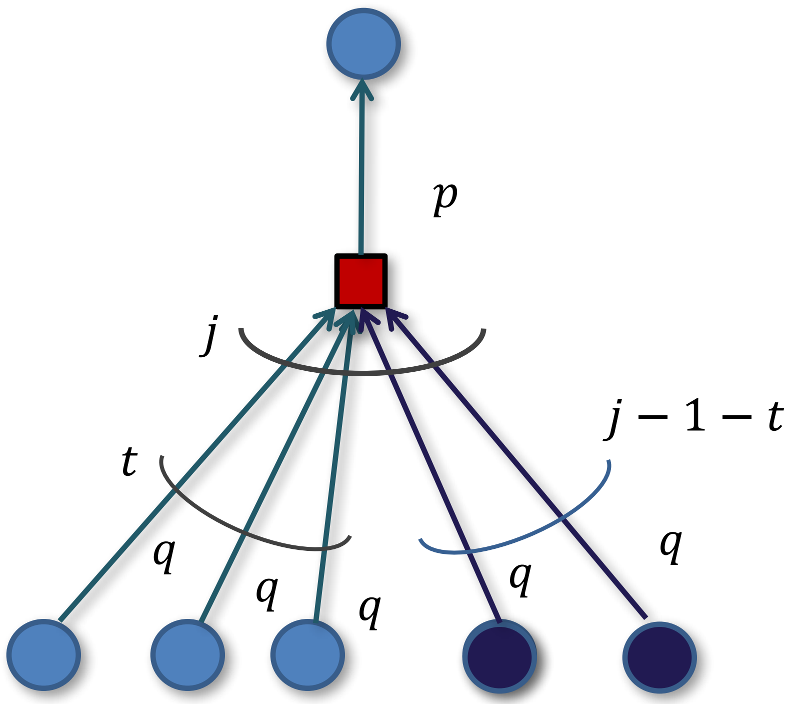
Capture threshold → b

Noise power → P_n

Received power of interfering users → $\sum_j P_j$

Residual power of cancelled interfering users → $\sum_k Q_k$

Capture-Or Tree Evaluation



AND operation becomes CAPTURE operation:

$$p = 1 - \sum_{j} \omega_j \sum_{t=0}^{j-1} \pi_{t,j} \binom{j-1}{t} q^t (1-q)^{j-1-t}$$

- There is a (fairly involved) method to derive capture probabilities in a case of:
  - Equal expected powers at the point of reception for all users,
  - Proportional residual interference power
  - Threshold-based model of the capture effect
- The method is based on the approach presented in:

  A. Zanella and M. Zorzi, "Theoretical Analysis of the Capture Probability in Wireless Systems with Multiple Packet Reception Capabilities," IEEE Trans. Commun. Apr. 2012.
- For some special cases, it can be done in a simpler way

Capture-Or Tree Evaluation

- C. Stefanovic, M. Momoda, and P. Popovski, "Exploiting Capture Effect in Frameless ALOHA for Massive Wireless Random Access," in Proc. of IEEE WCNC 2014, Istanbul, Turkey, May 2014:
  - Capture threshold $b \geq 1$ (narrowband single-antenna system)
  - Rayleigh fading scenario:
    - pdf of SNR user u_i at the reception point:

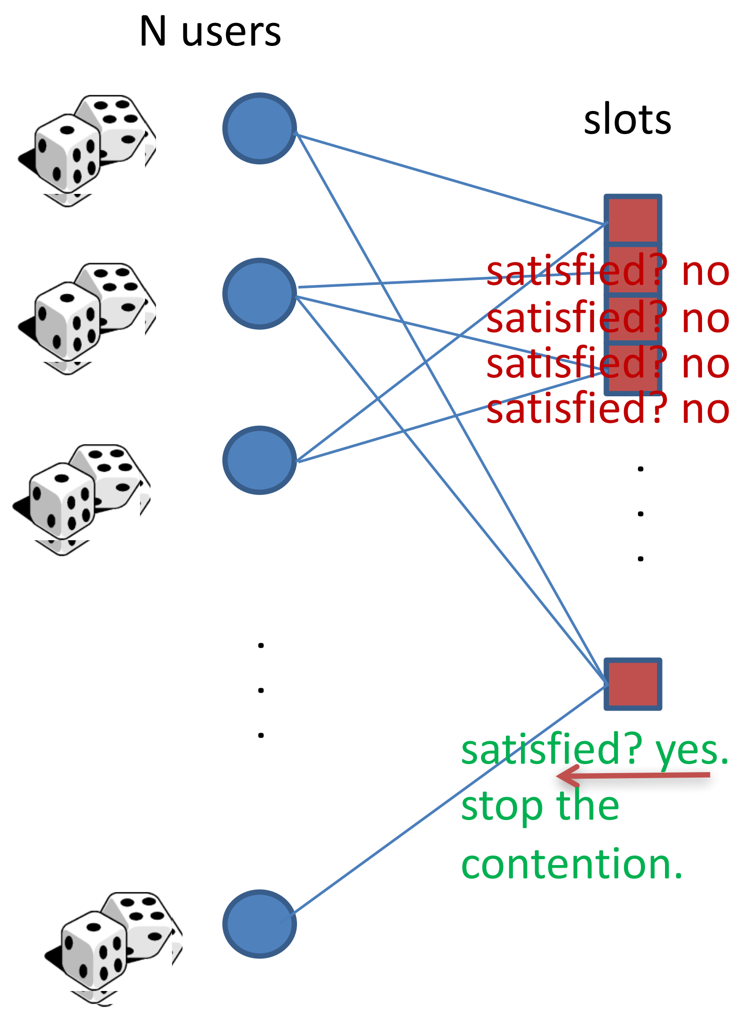
$$p_{X_i}(x) = \frac{1}{\gamma} e^{-\frac{x}{\gamma}}, x \geq 0$$

    - γ – the expected SNR
  - Perfect IC

Case study: Frameless ALOHA

C. Stefanovic, P. Popovski, D. Vukobratovic, "Frameless ALOHA Protocol for Wireless Networks", IEEE Communication Letters, Dec. 2012

- Idea: Apply paradigm of rateless codes to slotted ALOHA:
  - No predefined frame length
  - Slots are successively added until a criterion related to performance parameters of the scheme is satisfied
  - Optimization of the slot-access probability and termination criterion



Frameless ALOHA: Optimization of the slot access probability

- The simplest case:
  - All users use the same slot access probability p_a for all the slots
  - $p_a = \frac{\beta}{N}$
  - β is the average slot degree

- Goal: Maximize throughput T

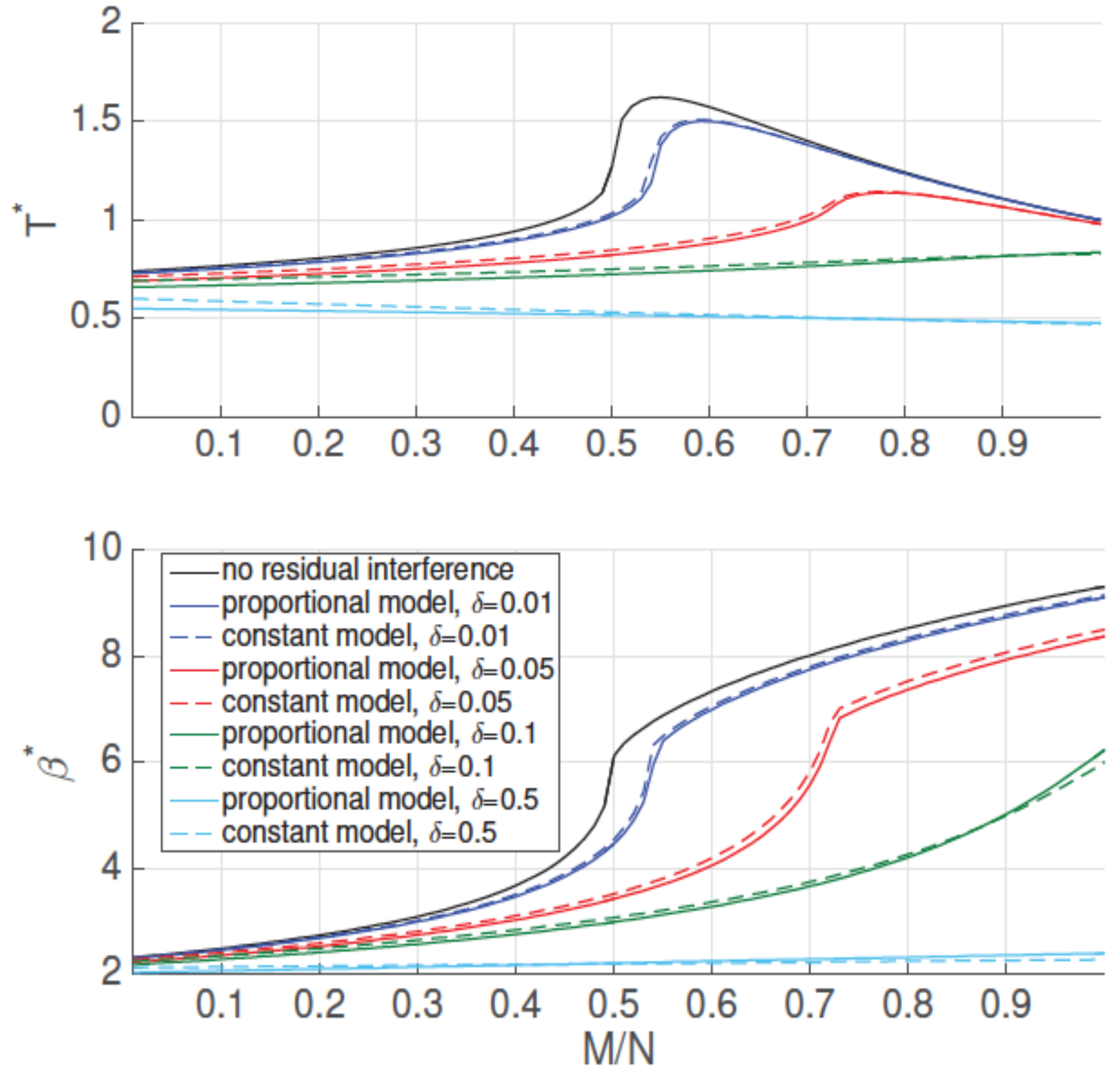
$$T = \frac{N_R}{M} = \frac{P_R\, N}{M}$$

  - N_{R} is the number of resolved users (transmissions)
  - P_R is the probability of user resolution

- Select β such that throughput is maximized

Frameless ALOHA:
Optimization of the slot access probability

- $b = 1$
- $\gamma = 5dB$

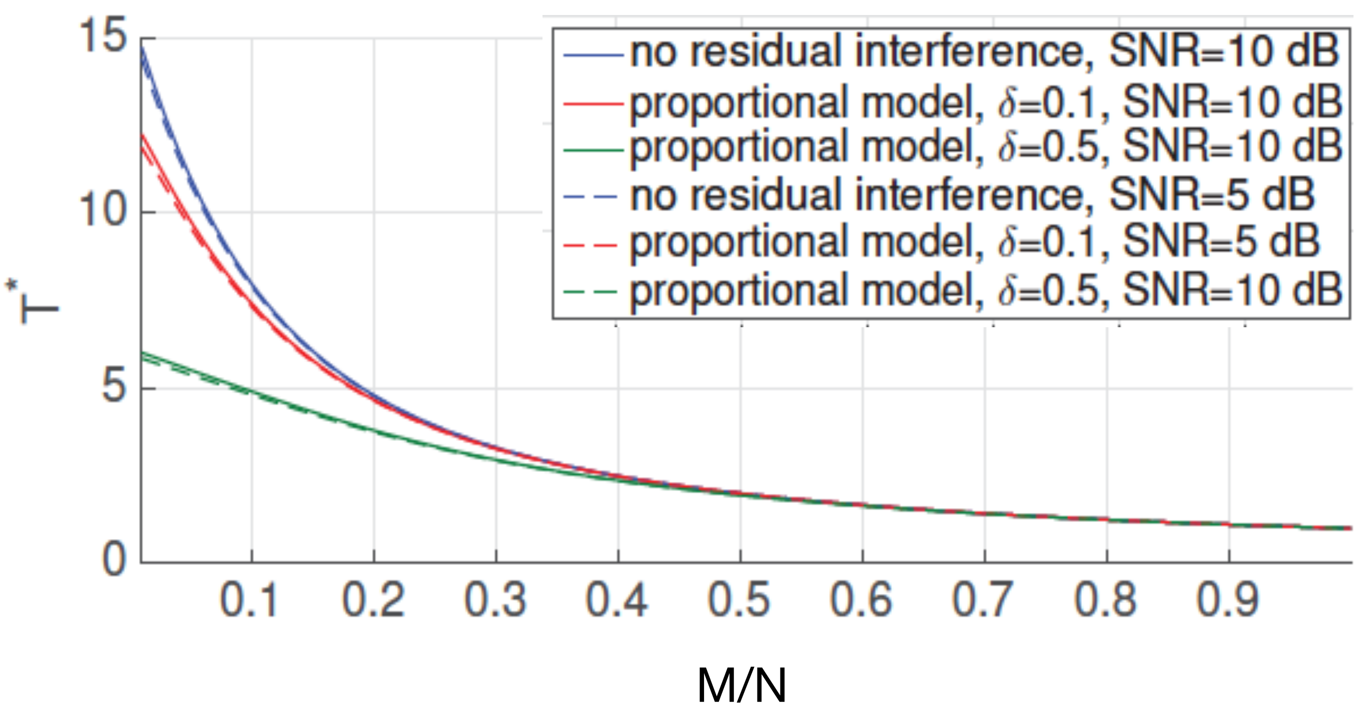


Frameless ALOHA: Optimization of the slot access probability

	b	1					
	SNR [dB]	10			5		
	δ	0	0.1	0.5	0	0.1	0.5
Frameless ALOHA	T_F	2.37	1.08	0.72	1.62	0.83	0.55
	β_F	7.21	5.37	2.05	6.88	6.51	2.02
Slotted ALOHA with intra-slot SIC only	T_S	1.13	0.93	0.72	0.74	0.66	0.55
	β_S	2.5	2.25	2.05	2.29	2.16	2.02
Slotted ALOHA “with” capture effect	T_C	0.67			0.54		
	β_C	2			2		

Frameless ALOHA: Optimization of the slot access probability

- $b = 0.1$
  - Spread spectrum system



Conclusions

- There is a way to analytically assess the asymptotic performance of SIC-enabled slotted ALOHA schemes beyond the collision channel model
- Results show that, in the cases with $b \geq 1$ and low residual interference power, the scheme favors collisions
- For $b \ll 1$, there seems to be no gain to use protocol designed to exploit inter-slot IC (intra-slot IC is enough)
- Finite-length performance?

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