Polar Codes for Probabilistic Amplitude Shaping

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Outline

1. Motivation
2. Polar Codes
   - Polarization
   - Polar Codes for BPSK
   - Polar Codes for Higher-Order Modulation
3. Probabilistic Amplitude Shaping (PAS)
4. Numerical Results
5. Conclusions

Polar Codes

- Polar Codes achieve the symmetric capacity of binary-input memoryless channels.
- low-complexity encoding algorithms ($\mathcal{O}(N \log N)$)
- low-complexity decoding algorithms ($\mathcal{O}(N \log N)$)
- very good performance for finite lengths with some modifications (outer CRC-code)

Probabilistic Shaping

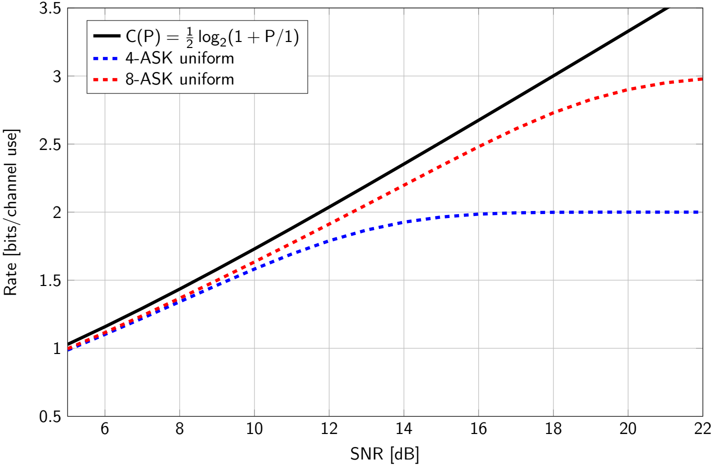
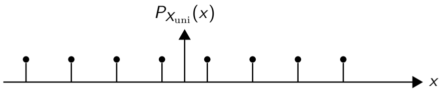


Figure 1 : Rates for uniform and shaped ASK Constellations.

Probabilistic Shaping



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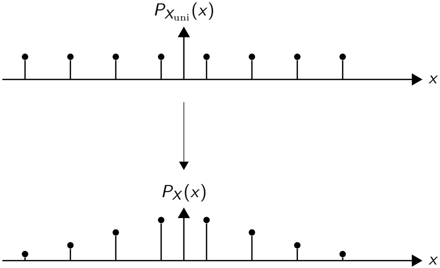

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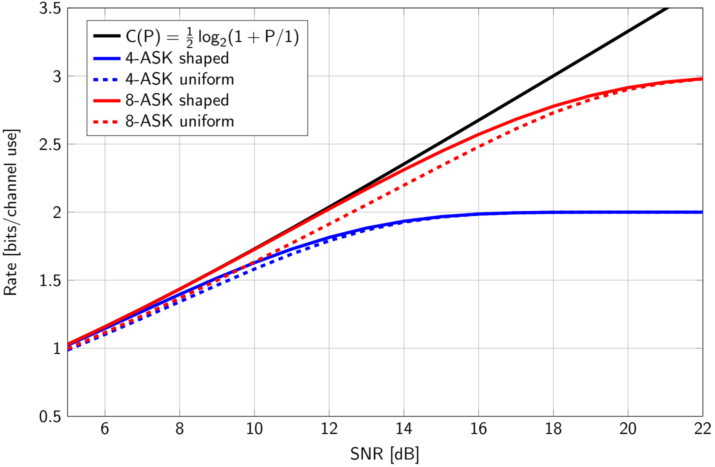

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Polarization[1]

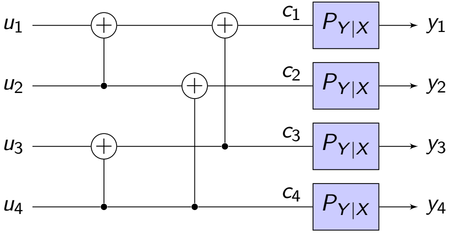


Figure 2 : Polar Transformation: $N = 4$.

Polar Transformation:

- $\mathbf{c} = \mathbf{u}\mathbf{G}$
- $\mathbf{G} = \mathbf{F}^{\otimes m}$
- $m = \log_2 N$
- $\mathbf{F} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$

Observation (Polarization)

Most of the bit-channels $U_i \rightarrow (U_1^{i-1}, Y_1^N)$ are either "good" channels $I(U_i; Y_1^N | U_1^{i-1}) \approx 1$ or "bad" channels $I(U_i; Y_1^N | U_1^{i-1}) \approx 0$.

[1] E. Arıkan, "Channel polarization: A method for constructing capacity-achieving codes," *IEEE Trans. Inf. Theory*, 2009

Polar Codes

Definition ((N,k) - Polar Code)

An $(N = 2^m, k)$ Polar Code is defined by the Polarization $\mathbf{c} = \mathbf{uG}$, where

$$\mathbf{G} = \mathbf{F}^{\otimes m}, \quad m = \log_2 N, \quad \mathbf{F} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

and a set of $N - k$ indices in $\mathbf{u}$ which correspond to frozen bits. The frozen bits are usually set to zero. The unfrozen bits of $\mathbf{u}$ contain the data to be transmitted.

Performance of Polar Codes

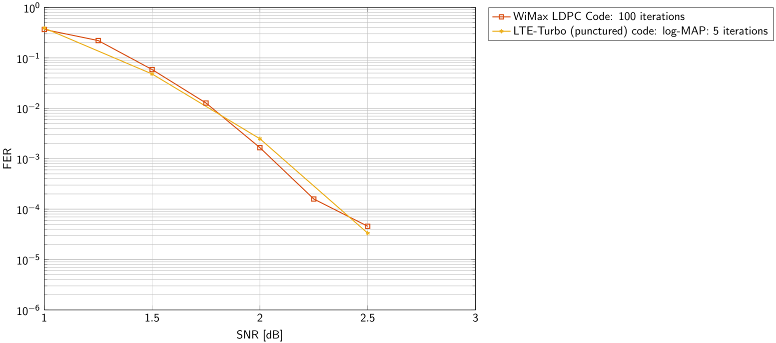
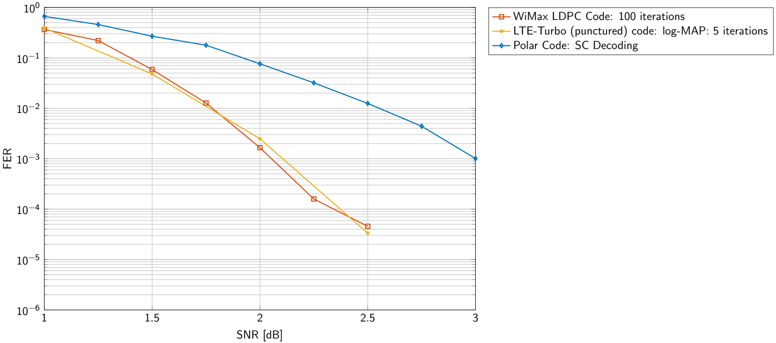


Figure 3 : Performance of Polar Codes of length $N = 1024$ with overall rate $1/2$.

Performance of Polar Codes



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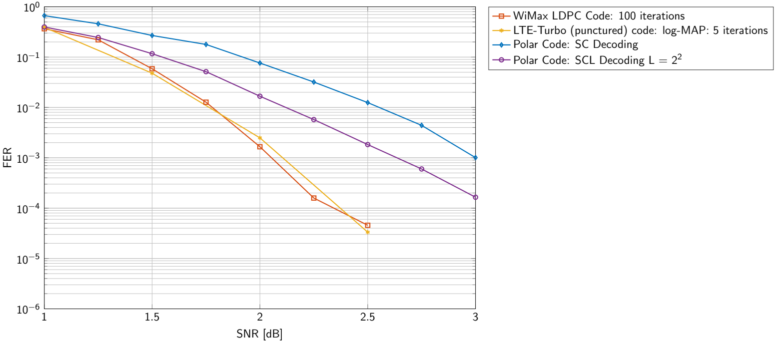

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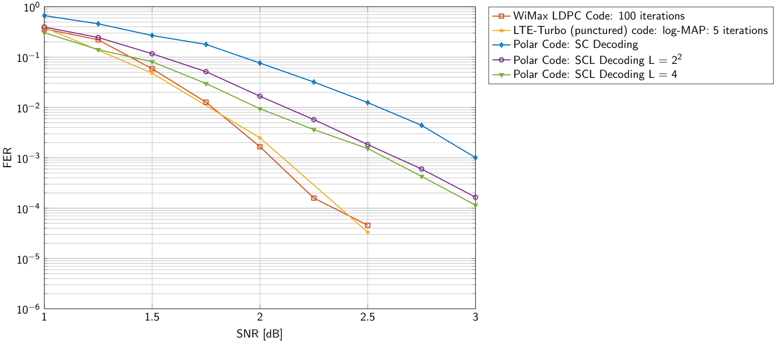

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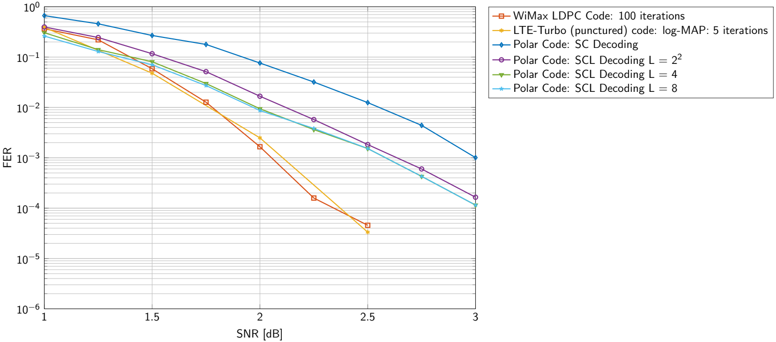

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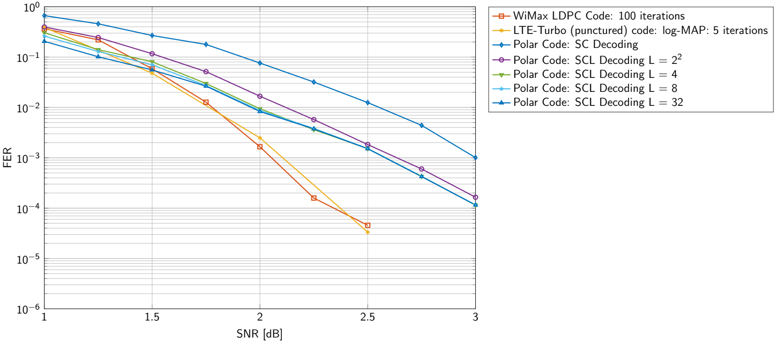

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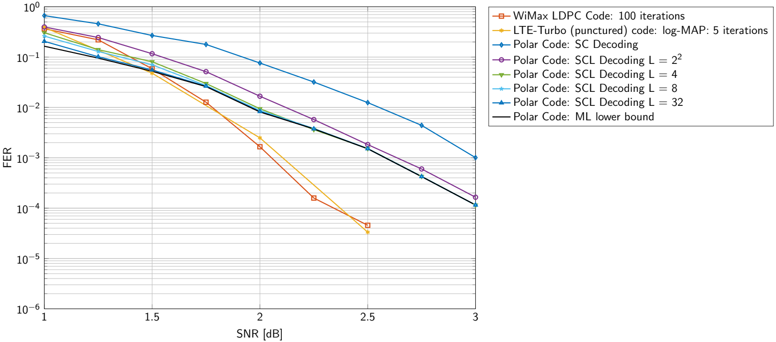

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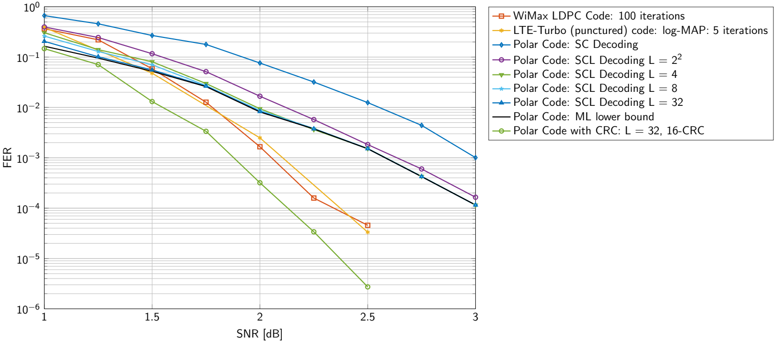

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Compound Polar Codes for Higher-Order Modulation[3]

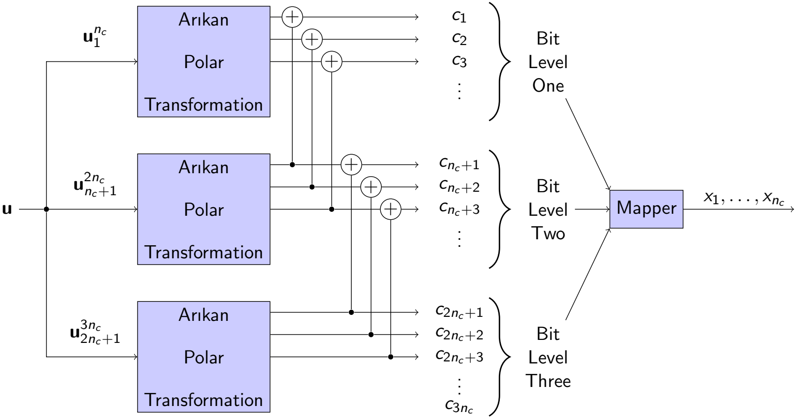
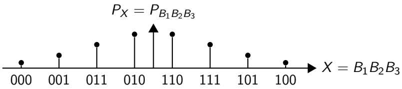


Figure 4 : Compound Polar Code for 8-ASK

[3] H. Mahdavifar *et al.*, "Polar coding for bit-interleaved coded modulation," *IEEE Trans. Veh. Technol.*, 2015

Probabilistic Amplitude Shaping (PAS)[4]

- 2^m-ASK constellation: label X by $B_1 B_2 \dots B_m$ (Gray labeling)



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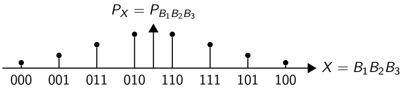

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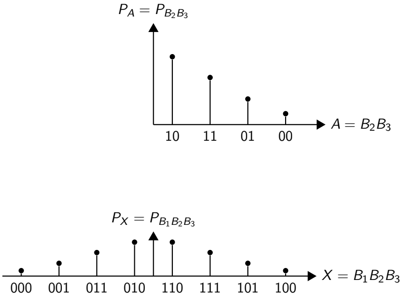
- $P_A(|x|) = 2P_X(|x|)$



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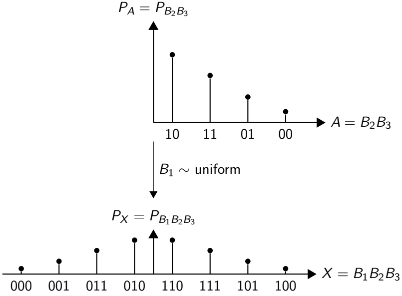
- $P_A(|x|) = 2P_X(|x|)$
- interpret B_1 as uniformly distributed sign



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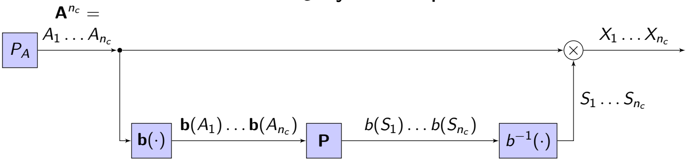
PAS encoding schemes[4]

- block transmission with n_c symbols per block



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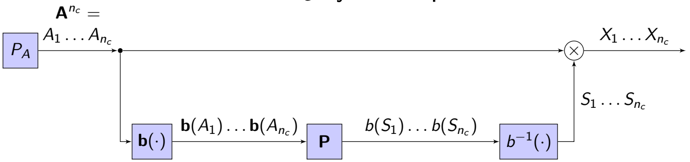

- rate $R = H(A)$
- coderate $c = \frac{m-1}{m}$

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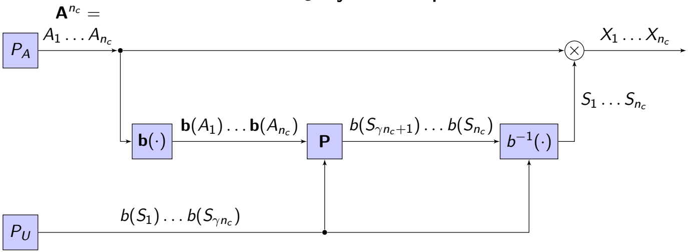

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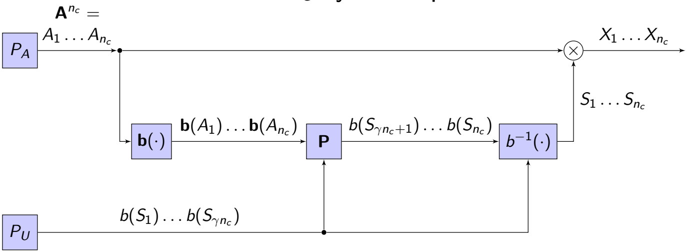

- rate $R = H(A)$
- coderate $c = \frac{m-1}{m}$
- extended PAS scheme: rate $R = H(A) + \gamma$
- coderate $c = \frac{m-1+\gamma}{m}$

[4] G. Böcherer *et al.*, "Bandwidth efficient and rate-matched low-density parity-check coded modulation," *IEEE Trans. Commun.*, 2015

Bit-Metric Decoding

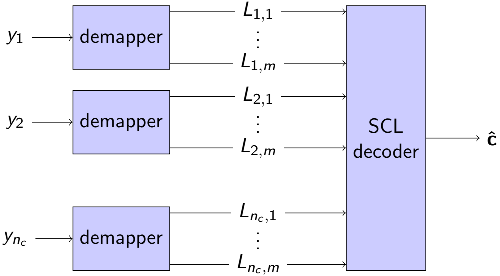

Figure 5 : Bit-Metric Decoding (BMD)

$$L_{i,j} = \underbrace{\log_2 \frac{p_{Y|B_j}(y_i|b_{i,j}=0)}{p_{Y|B_j}(y_i|b_{i,j}=1)}}_{\text{channel likelihood}}$$

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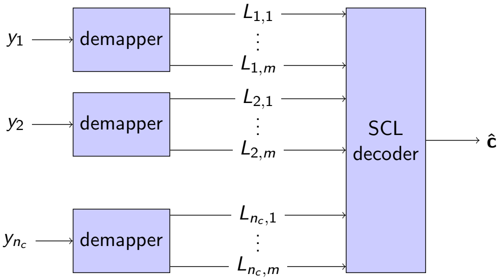

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$$L_{i,j} = \underbrace{\log_2 \frac{p_{Y|B_j(y_i|b_{i,j}=0)}}{p_{Y|B_j(y_i|b_{i,j}=1)}}}_{\text{channel likelihood}} + \underbrace{\log_2 \frac{P_{B_j}(b_{i,j}=0)}{P_{B_j}(b_{i,j}=1)}}_{\text{a priori information}}$$

Results for $n_c = 512$

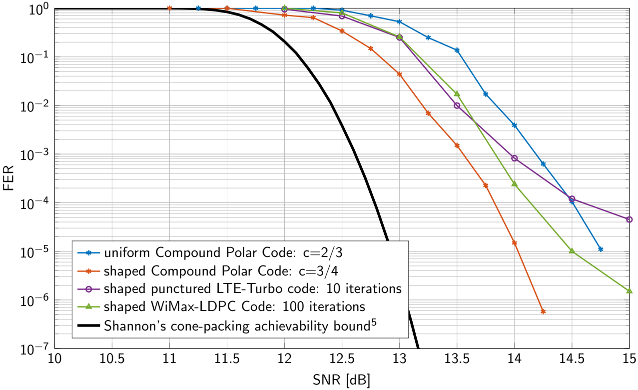

Figure 6 : Frame error rate of a shaped 8-ASK Compound Polar Code with rate $R = 2$ bits per channel use under SCL Decoding with $L = 32$ and 16−CRC.

[5] C. Shannon, "Probability of error for optimal codes in a Gaussian channel," *Bell Systems Tech. Journal*, 1959

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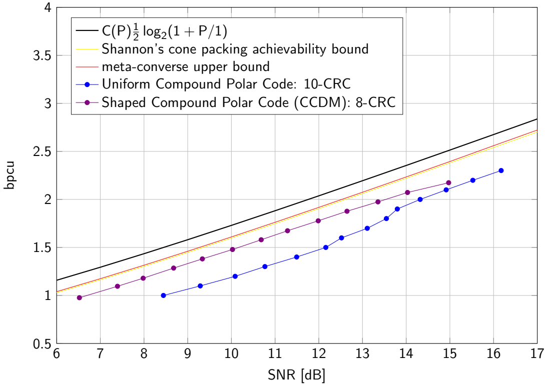

Figure 7 : Rate Curves for 8-ASK uniform and shaped Compound Polar Codes of blocklength $n_c = 512$ with coderate $c = 0.75$ under SCL Decoding with $L = 32$.

[6] Y. Polyanskiy *et al.*, "Channel coding rate in the finite blocklength regime," *IEEE Trans. Inf. Theory*, 2010

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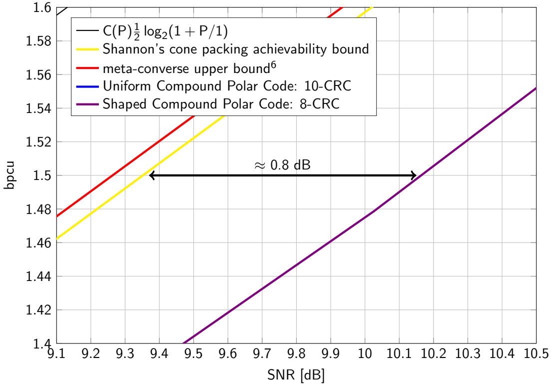

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Results for $n_c = 128$

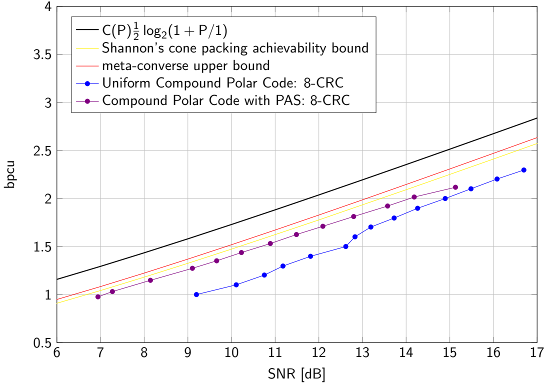

Figure 8 : Rate Curves for 8-ASK uniform and shaped Compound Polar Codes of blocklength $n_c = 128$ with coderate $c = 0.75$ under SCL Decoding.

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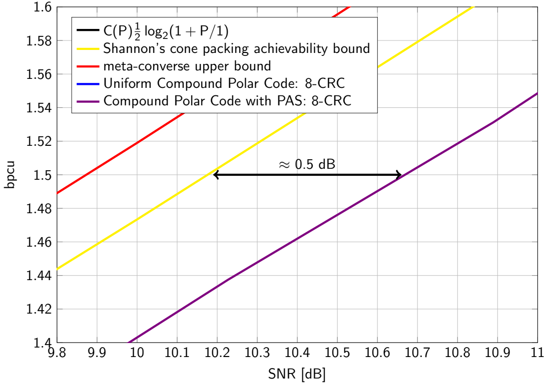

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Conclusions

- Polar Codes for PAS perform very close to the theoretical limits for intermediate lengths.
- Polar Codes for PAS can be constructed very efficiently.

Future work:

- Code length constricted to powers of two $\Rightarrow$ combine Polar Codes for PAS with punctering schemes.
- Consider Polar Codes with multilevel coding and multistage decoding for PAS.

Thank you! Questions?