Asymptotic Analysis and Spatial Coupling of Counter Braids

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Joint work with
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Counter braids
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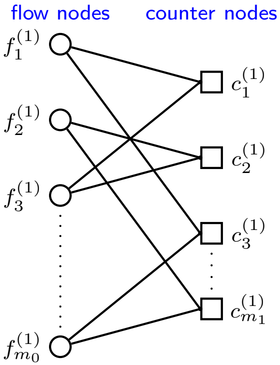
- Cheap high-speed memory-efficient approximate counting.
- Asymptotically optimal, i.e., the average number of bits needed to store the size of a flow tends to the information-theoretic limit (under ML decoding).

Counter braids

- Can be described by a graph.

Counter braids



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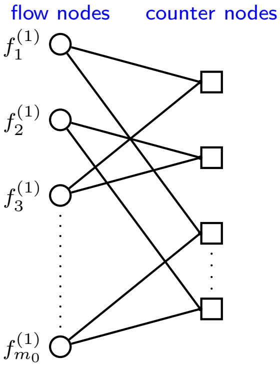

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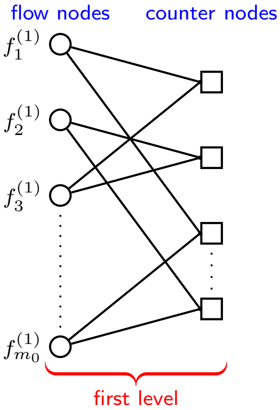

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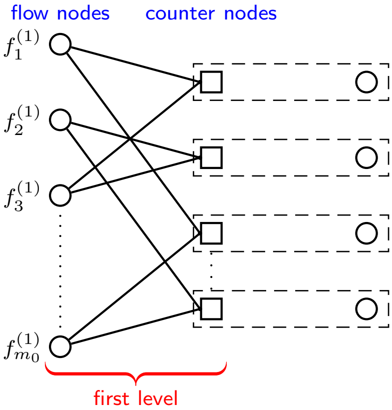

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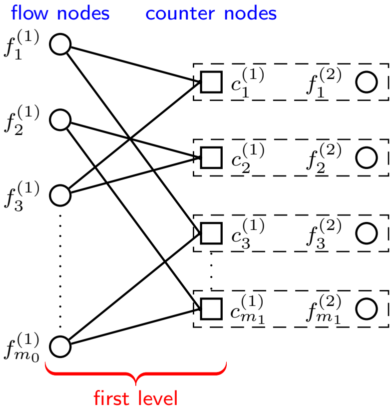

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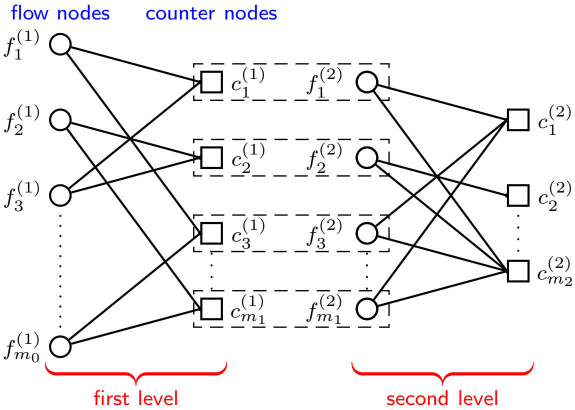

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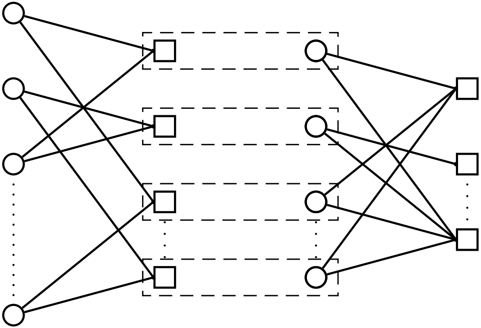

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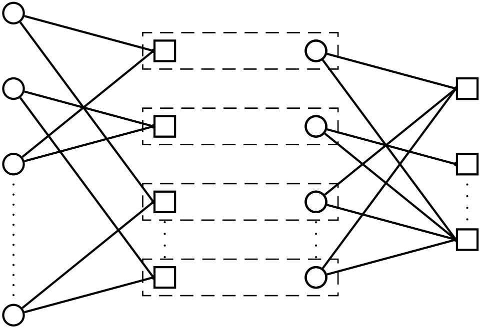

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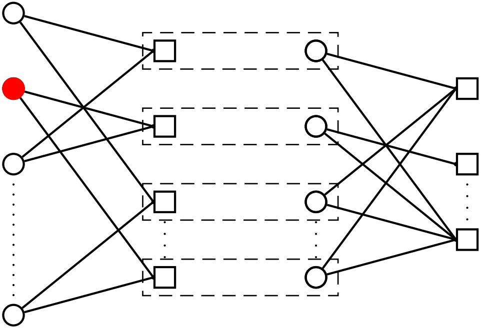

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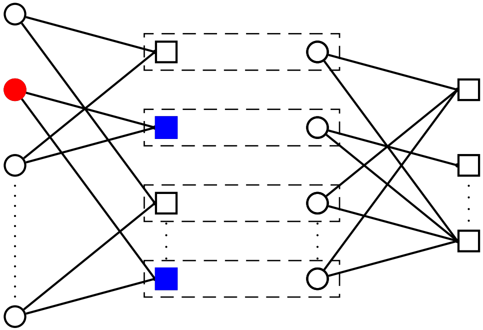

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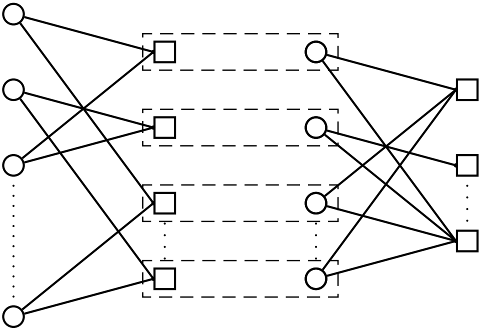

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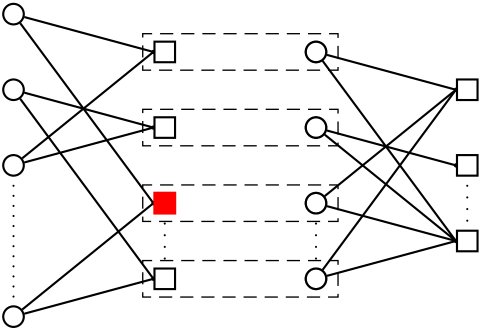

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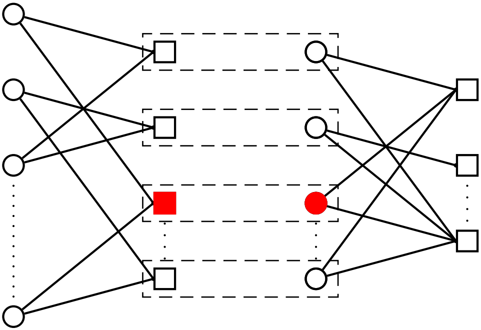

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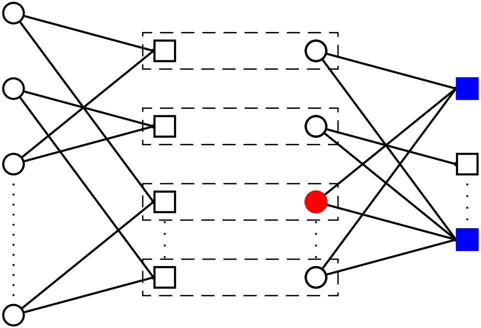

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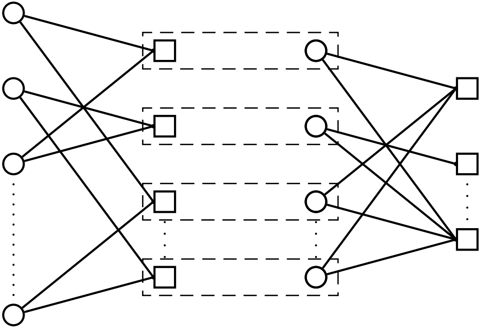

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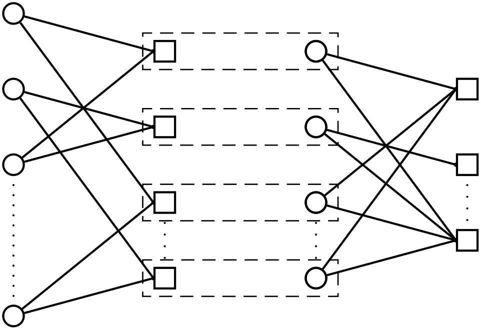

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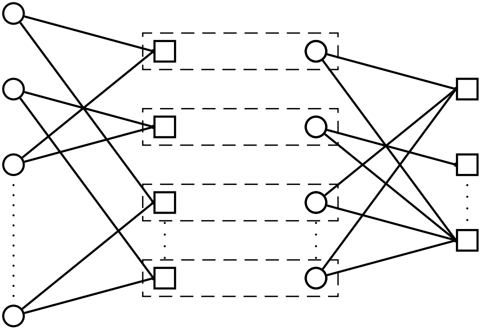

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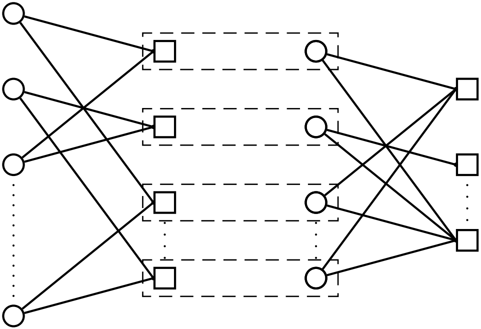

- Counter braids described by a graph → Low-complexity BP decoding! Start with the right-most layer and proceed layer-by-layer until the first layer is decoded.

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where $f_{\min}$ is the minimum flow size (from the flow size distribution) of a flow node.

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- Initialization: $\mu^{(0)}_{f \to c} = f_{\min}$.

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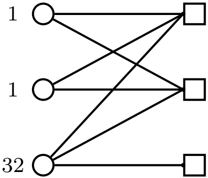

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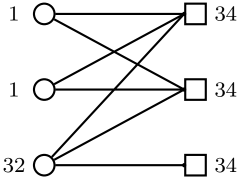

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- In practice...BP decoding entails suboptimality.
- Idea: Use spatial coupling to get to the MAP threshold [Rosnes 2015] $\rightarrow$ Performance improvement, and saturation of the BP threshold to a given value.

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Two relevant questions

1. What is the MAP decoding threshold?
2. Threshold saturation to the MAP threshold?

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We know that...

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- $\beta = m_1 / m_0$ is the number of counters per flow → βd_1 is the average number of bits needed to represent a flow.

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- Alternatively, fix β and find

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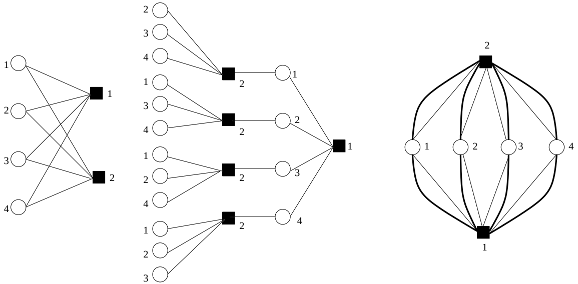
Equivalent graph representation of CBs



(a) (b) (c)

- Same (iteration-by-iteration) finite-length performance and asymptotic behavior.

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- DE on the equivalent bipartite graph:

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- The two DE recursions give the same BP decoding threshold!

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- Extended BP EXIT curve:

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Definition: Area threshold

Let $(\epsilon(x^*), h^{\mathrm{EBP}}(x^*))$ be a point on the EBP EXIT curve h^{EBP} of a single-layer CB such that

$$\int_{x^*}^{1} h^{\mathrm{EBP}}(x)\ \mathrm{d}\epsilon(x) = \int_{0}^{1} h^{\mathrm{EBP}}(x)\ \mathrm{d}\epsilon(x)$$

and there exist no $x' \in (x^*, 1]$ such that $\epsilon(x') = \epsilon(x^*)$. Then, the area threshold is defined as $\bar{\epsilon} = \epsilon(x^*)$.

Area threshold and Maxwell threshold

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Conjecture

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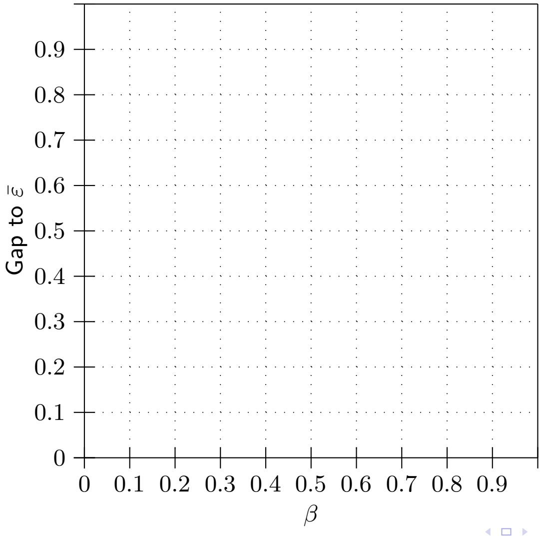

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$k = 6$

$k = 8$

dashed: uncoupled

solid: coupled
($N = 128$, $w = 5$)

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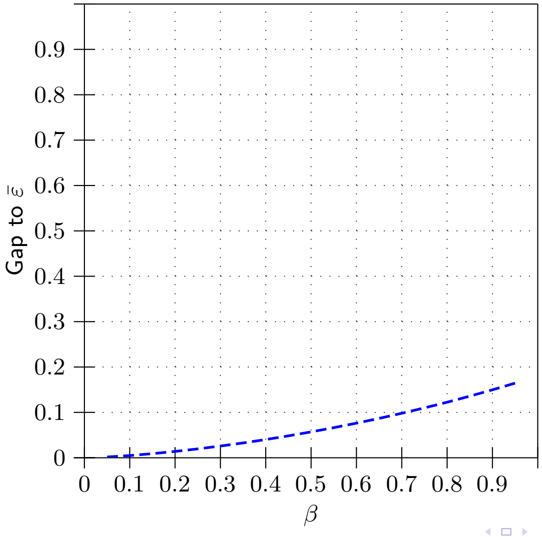

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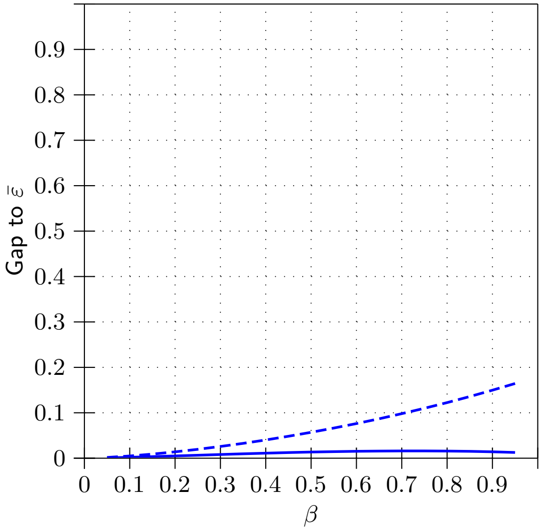

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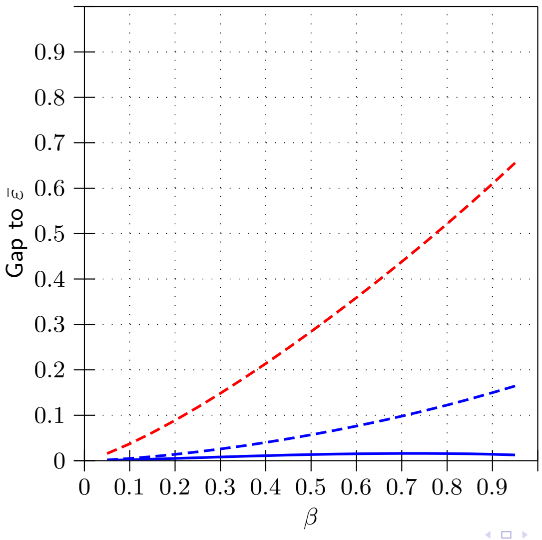


$k = 3$

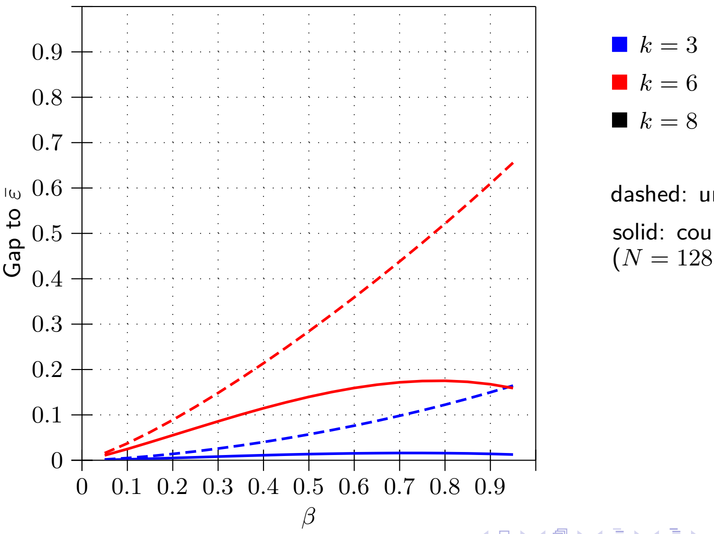
$k = 6$

$k = 8$

dashed: uncoupled

solid: coupled
$(N = 128,\ w = 5)$

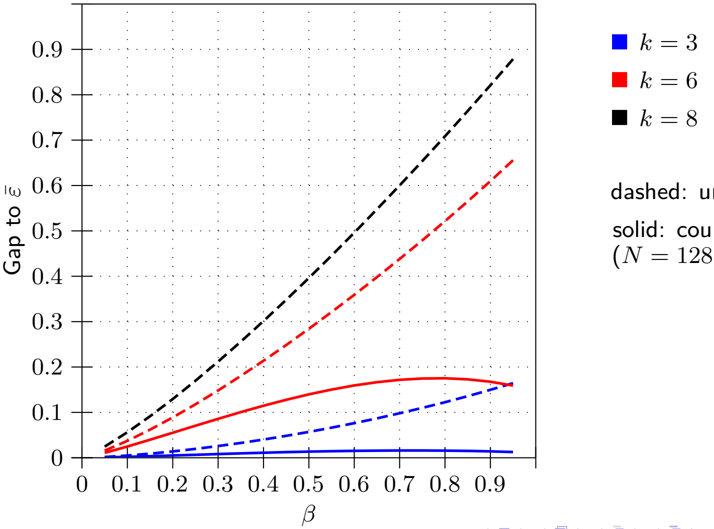
Threshold saturation?



dashed: uncoupled

solid: coupled
($N = 128$, $w = 5$)

Threshold saturation?



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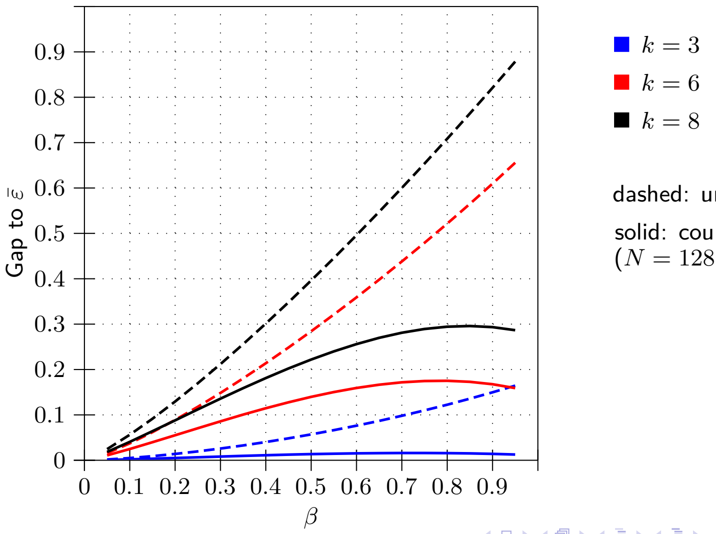
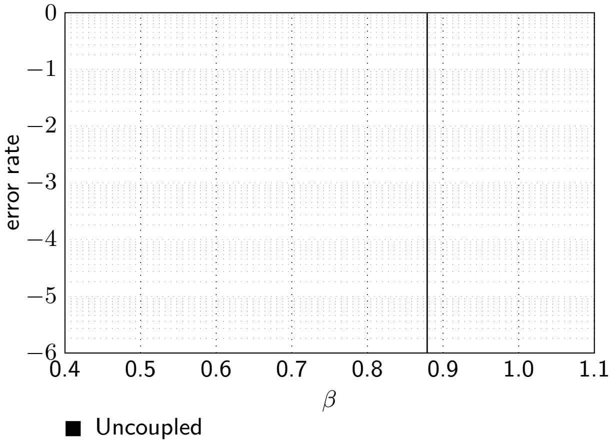


dashed: uncoupled

solid: coupled
$(N = 128,\ w = 5)$

Simulation results

$$k = 6,\ w = 3,\ \varepsilon = 2^{-1.5}$$



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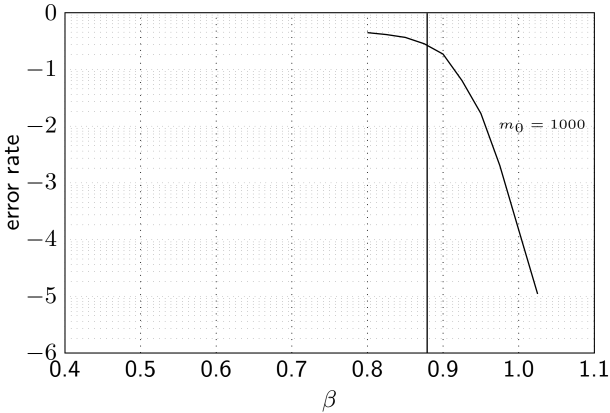
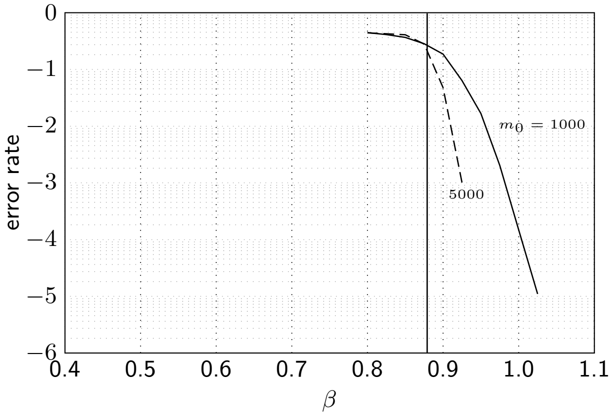
$k = 6,\ w = 3,\ \varepsilon = 2^{-1.5}$



■ Uncoupled

Simulation results

$k = 6,\ w = 3,\ \varepsilon = 2^{-1.5}$



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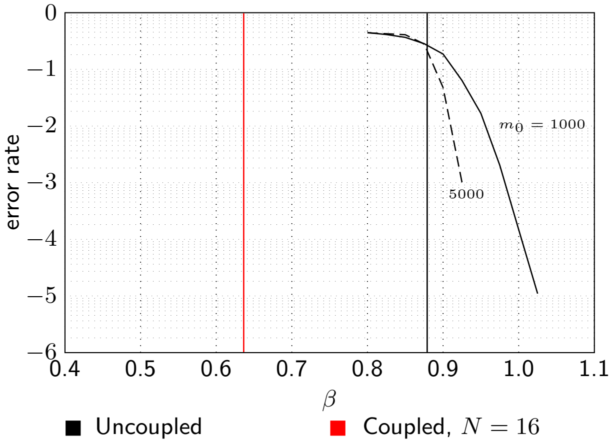

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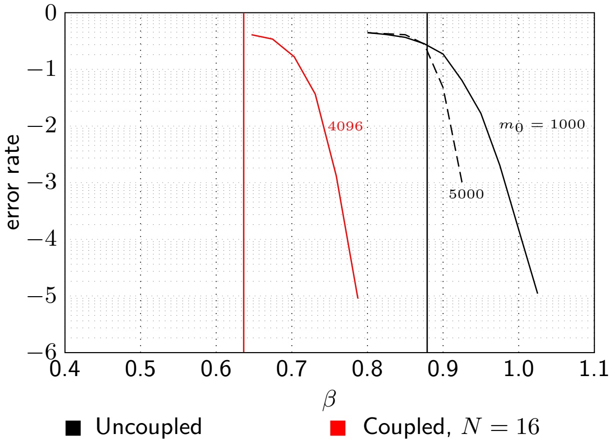

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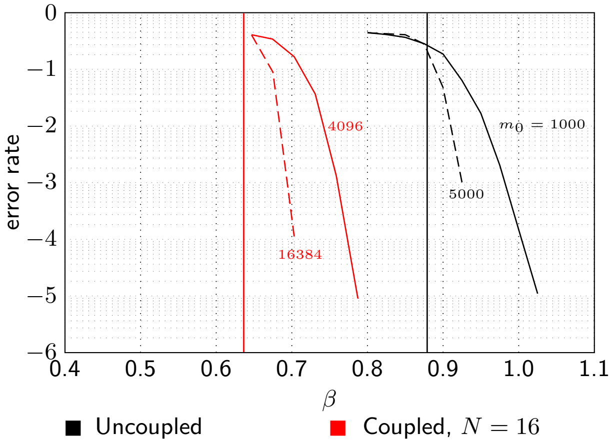

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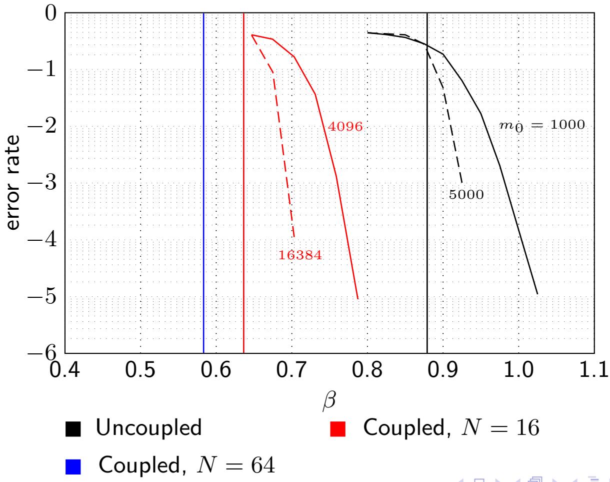

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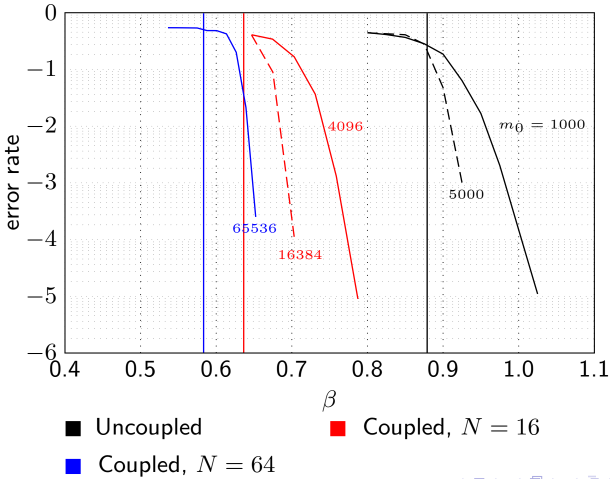

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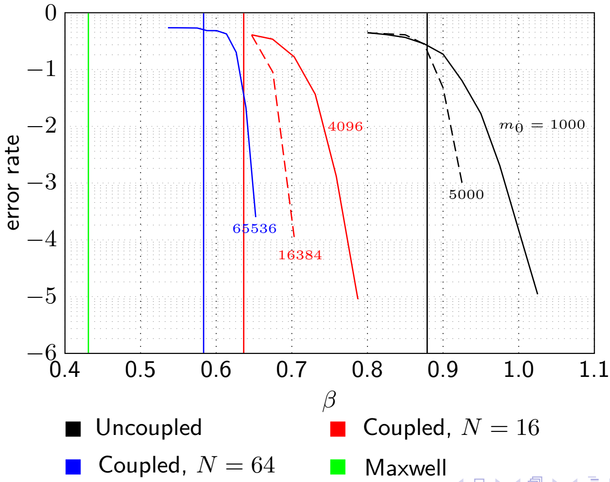

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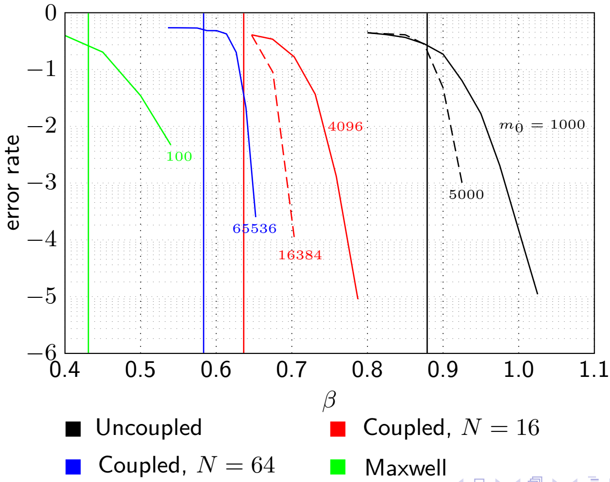

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Take-home messages

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E. Rosnes and A. Graell i Amat, "Asymptotic analysis and spatial coupling of counter braids," submitted to *IEEE Trans. Inf. Theory*. Available on arXiv.