On Surrogate Channels for Code Design

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July 26, 2016
Second LNT & DLR Summer Workshop on Coding

Outline

- **Popular Surrogate Channels:** BEC and biAWGN
- **Design Challenge:** Polar Codes for 8-ASK
- First Attempt
- Second Attempt
- **Detour:** Mismatched Decoding
- Final Attempt
- Conclusions

BEC and biAWGN

- **Idea:** analyze decoding by tracking mutual information (MI) as it propagates through factor graph.
- Two channels are **"easy"**:
  - Binary Erasure Channel (BEC)
  - biAWGN Channel (with Gaussian Approximation)[1,2]

[1] S. ten Brink, G. Kramer, and A. Ashikhmin, "Design of low-density parity-check codes for modulation and detection," 2004.

[2] F. Brännström, "Convergence analysis and design of multiple concatenated codes," Ph.D. dissertation, 2004.

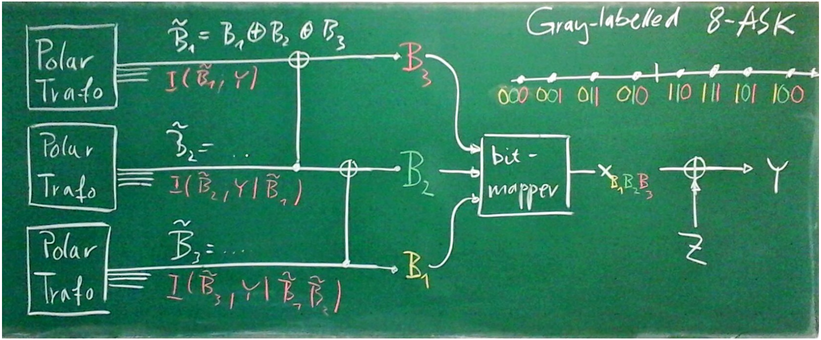
Code Design by Surrogate Channels

- **Observation:** codes often have "universal" properties.[3]
- **Idea:** Design code for surrogate channel, use it for target channel.[4]
- **Higher-order modulation:** use surrogate channel for each bit-channel.[5]

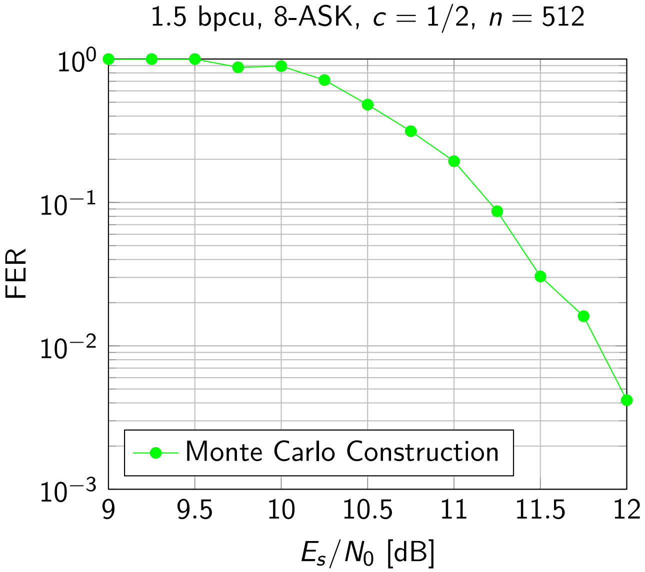
[3] M. Franceschini, G. Ferrari, and R. Raheli, "Does the performance of LDPC codes depend on the channel?" 2006.
[4] F. Peng, W. E. Ryan, and R. D. Wesel, "Surrogate-channel design of universal LDPC codes," 2006.
[5] F. Steiner, G. Böcherer, and G. Liva, "Protograph-based LDPC code design for shaped bit-metric decoding," 2016.

Design Challenge: Polar Code for 8-ASK[6]

For the following modulation scheme, we want to **efficiently** construct Polar codes that perform as good as polar codes constructed by exhaustive search ("Monte Carlo Construction").

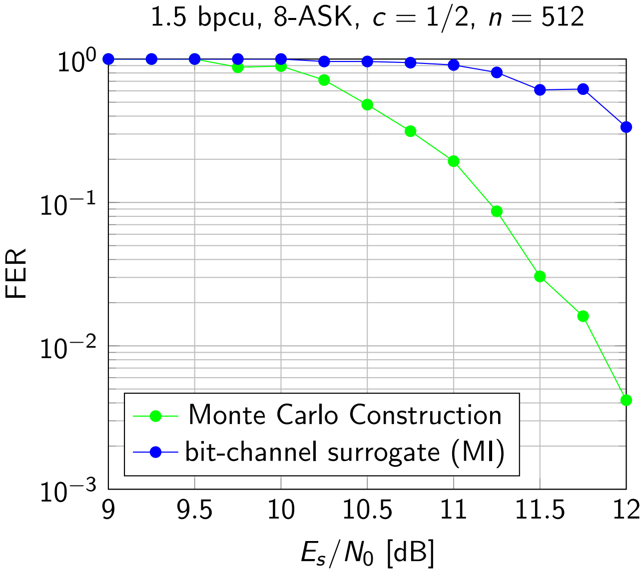


[6] H. Mahdavifar, M. El-Khamy, J. Lee, and I. Kang, "Polar coding for bit-interleaved coded modulation," 2016.


1.5 bpcu, 8-ASK, $c = 1/2$, $n = 512$
10^0
10^{-1}
10^{-2}
10^{-3}
FER
Monte Carlo Construction
9
9.5
10
10.5
11
11.5
12
E_s/N_0 [dB]

First Attempt

- Three bit-channels $p_{Y|B_i}$, $i = 1, 2, 3$.
- Replace the ith bit-channel by a biAWGN surrogate channel with MI equal to $\mathsf{I}(B_i; Y)$, $i = 1, 2, 3$.



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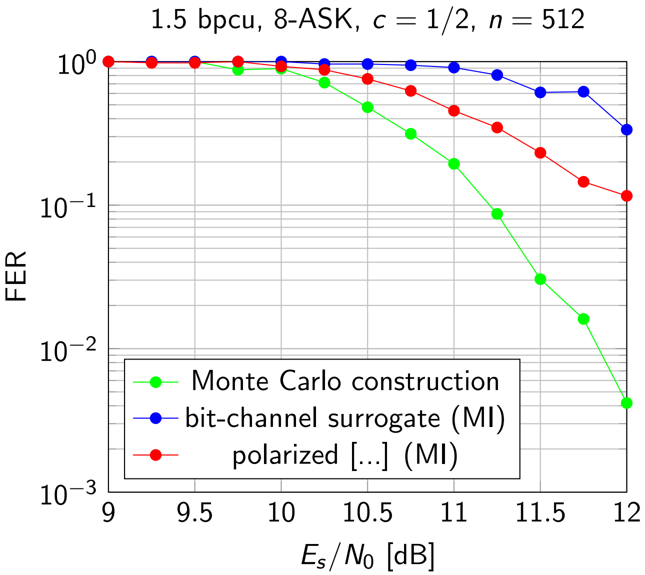
E_s/N_0 [dB]
Monte Carlo Construction
bit-channel surrogate (MI)

Second Attempt

- Replace the polarized bit-channels by biAWGN surrogate channels with MI equal to

$$\mathsf{I}(\tilde{B}_1; Y)$$
$$\mathsf{I}(\tilde{B}_2; Y|\tilde{B}_1)$$
$$\mathsf{I}(\tilde{B}_3; Y|\tilde{B}_1\tilde{B}_2)$$

  respectively.



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polarized [...] (MI)

Detour: Mismatched Decoding

What is mutual information?

- Memoryless channel $p_{Y|X}$.
- Random codebook $\mathcal{C} = \{X^n(1), \ldots, X^n(2^{nR})\}$ with entries iid P_X.
- Message $w \in \{1, 2, \ldots, 2^{nR}\}$
- ML decoder

$$\hat{W} = \operatorname*{argmax}_{w \in \{1,2,\ldots,2^{nR}\}} p_{Y^n|X^n}(Y^n|X^n(w)) = \prod_{i=1}^{n} p_{Y|X}(Y_i|X_i(w))$$

- Error probability $\Pr(W \neq \hat{W}) \overset{n\to\infty}{\to} 0$ if

$$R < \mathrm{I}(X;Y).$$

Detour: Mismatched Decoding[7][8]

- (Random coding as on previous slide).
- Auxiliary channel $q(\cdot|\cdot)$
- Mismatched Decoder

$$\hat{W} = \underset{w \in \{1,2,\ldots,2^{nR}\}}{\operatorname{argmax}} \prod_{i=1}^{n} q_{Y|X}(Y_i|X_i(w))$$

- Achievable rate

$$\mathrm{R}_{\mathrm{LM}} = \max_{s,r} \mathrm{E}\left[\log \frac{q(Y|X)^s r(X)}{q_{s,r}(Y)}\right]$$

  - Auxiliary output distribution $q_{s,r}(\cdot) = E[q(\cdot|X)^s r(X)]$
  - $s \geq 0$, function $r\colon \mathcal{X} \to \mathbf{R}$.

[7] A. Ganti, A. Lapidoth, and E. Telatar, "Mismatched decoding revisited: General alphabets, channels with memory, and the wide-band limit," 2000.

[8] G. Böcherer, "Achievable rates for shaped bit-metric decoding," 2016.

Detour: Mismatched Decoding

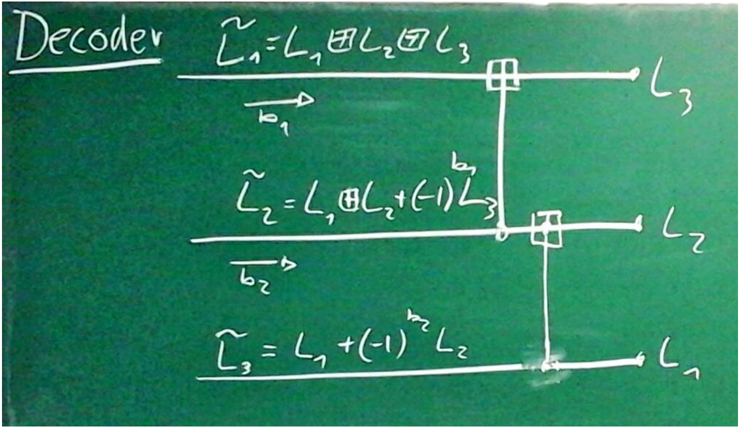
- Let's account for what the decoder is actually doing!
- L-value defines auxiliary channel via

$$L = \log \frac{q_{B|Y}(0|y)}{q_{B|Y}(1|y)}$$

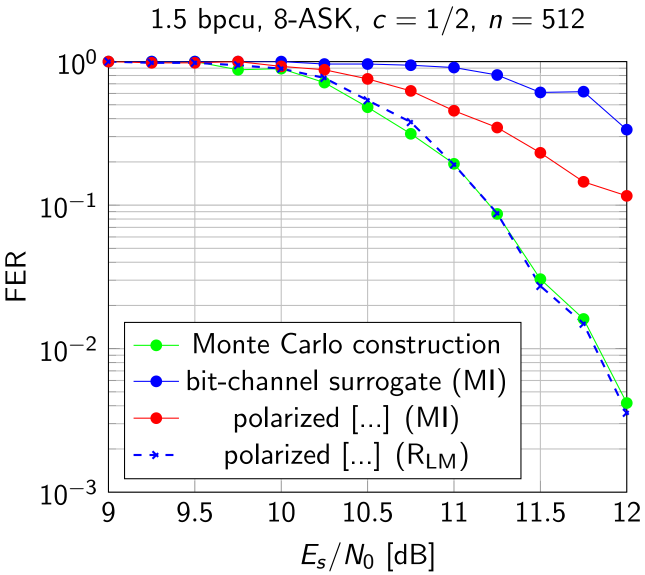
$$q_{B|Y}(b|y) = \frac{e^{-L\cdot b}}{1+e^{-L}}$$

⇒ Estimate R_{LM} for L-value as decoding metric.

Final Attempt



- Replace each polarized bit-channel by a biAWGN surrogate channel with MI matched to R_{LM} estimated from $\tilde{L}_i, i = 1, 2, 3$.



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Monte Carlo construction
bit-channel surrogate (MI)
polarized [...] (MI)
polarized [...] (R_{LM})

Conclusions

- Monte Carlo construction: half an hour (implementation in C).
- Surrogate channel design: some milliseconds.
- Construction with appropriate surrogate channel yields performance as good as Monte Carlo construction.
- Tool: information theory for mismatched decoding.

Literature

[1] S. ten Brink, G. Kramer, and A. Ashikhmin, "Design of low-density parity-check codes for modulation and detection," *IEEE Trans. Commun.*, vol. 52, no. 4, pp. 670–678, 2004.

[2] F. Brännström, "Convergence analysis and design of multiple concatenated codes," Ph.D. dissertation, Chalmers University of Technology, 2004.

[3] M. Franceschini, G. Ferrari, and R. Raheli, "Does the performance of LDPC codes depend on the channel?" *IEEE Trans. Commun.*, vol. 54, no. 12, pp. 2129–2132, Dec. 2006.

[4] F. Peng, W. E. Ryan, and R. D. Wesel, "Surrogate-channel design of universal LDPC codes," *IEEE Commun. Lett.*, vol. 10, no. 6, pp. 480–482, Jun. 2006.

[5] F. Steiner, G. Böcherer, and G. Liva, "Protograph-based LDPC code design for shaped bit-metric decoding," *IEEE J. Sel. Areas Commun.*, vol. 34, no. 2, pp. 397–407, Feb. 2016.

[6] H. Mahdavifar, M. El-Khamy, J. Lee, and I. Kang, "Polar coding for bit-interleaved coded modulation," *IEEE Trans. Veh. Technol.*, vol. 65, no. 5, pp. 3115–3127, May 2016.

[7] A. Ganti, A. Lapidoth, and E. Telatar, "Mismatched decoding revisited: General alphabets, channels with memory, and the wide-band limit," *IEEE Trans. Inf. Theory*, vol. 46, no. 7, pp. 2315–2328, Nov. 2000.

[8] G. Böcherer, "Achievable rates for shaped bit-metric decoding," *arXiv preprint*, 2016. [Online]. Available: http://arxiv.org/abs/1410.8075