An Introduction to the **ID**entification

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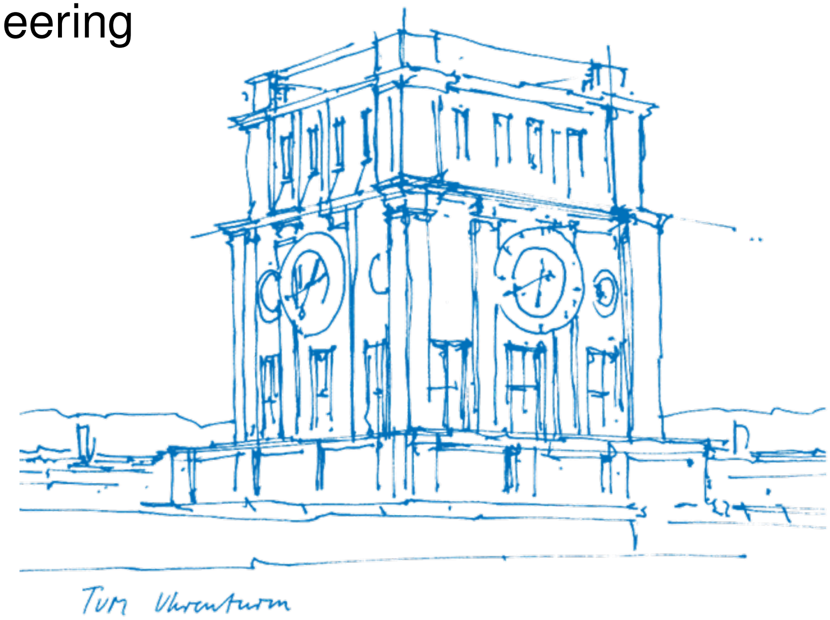
4th Feb 2020



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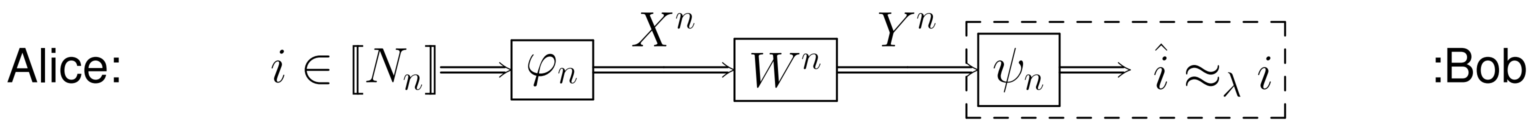
Outline

- Transmission
- **ID**entification
- ID Codes
- Remarks

Transmission

(Shannon 1948):



Transmission (n, N_n, λ) code for W is a system $\{(\boldsymbol{u_i}, \mathcal{D}_i)\}_{i \in [\![N_n]\!]}$:

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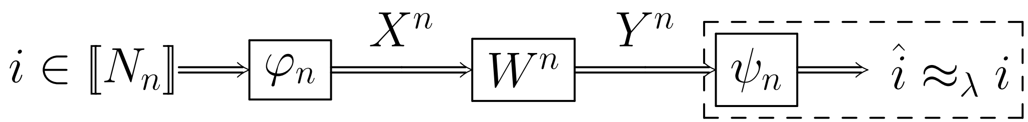

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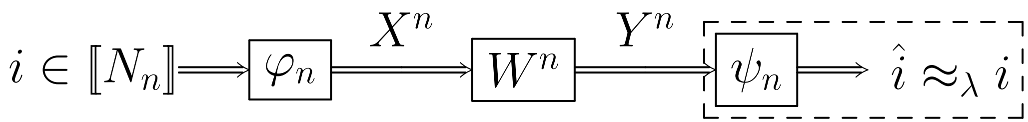

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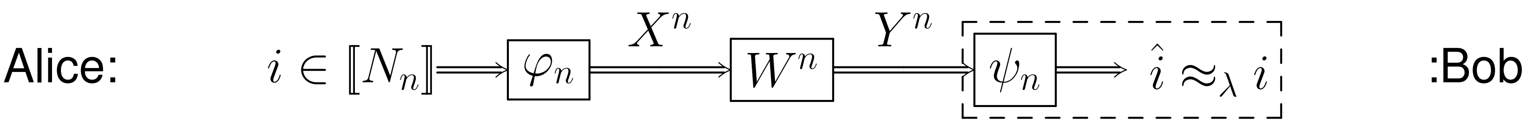

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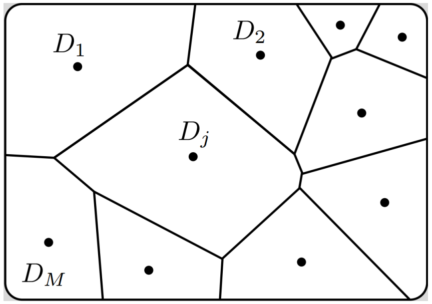

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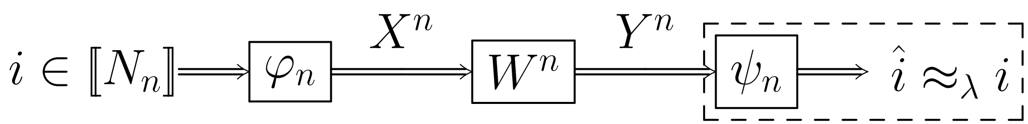

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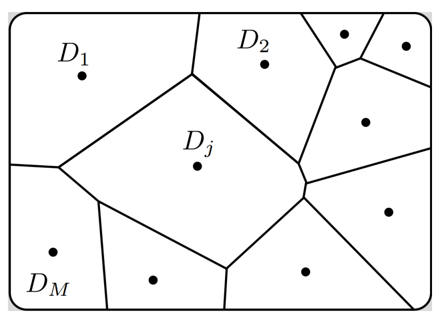

Capacity

$$\lim_{n \to \infty} \frac{1}{n} \log N_{max}(n, \lambda) = C_T \qquad \forall \lambda \in (0, 1)$$

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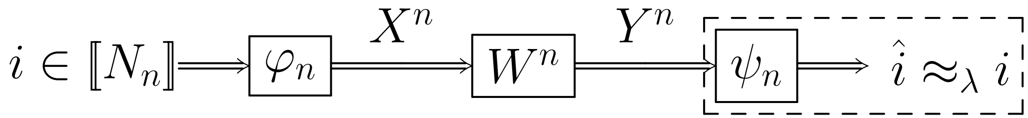
Alice: 

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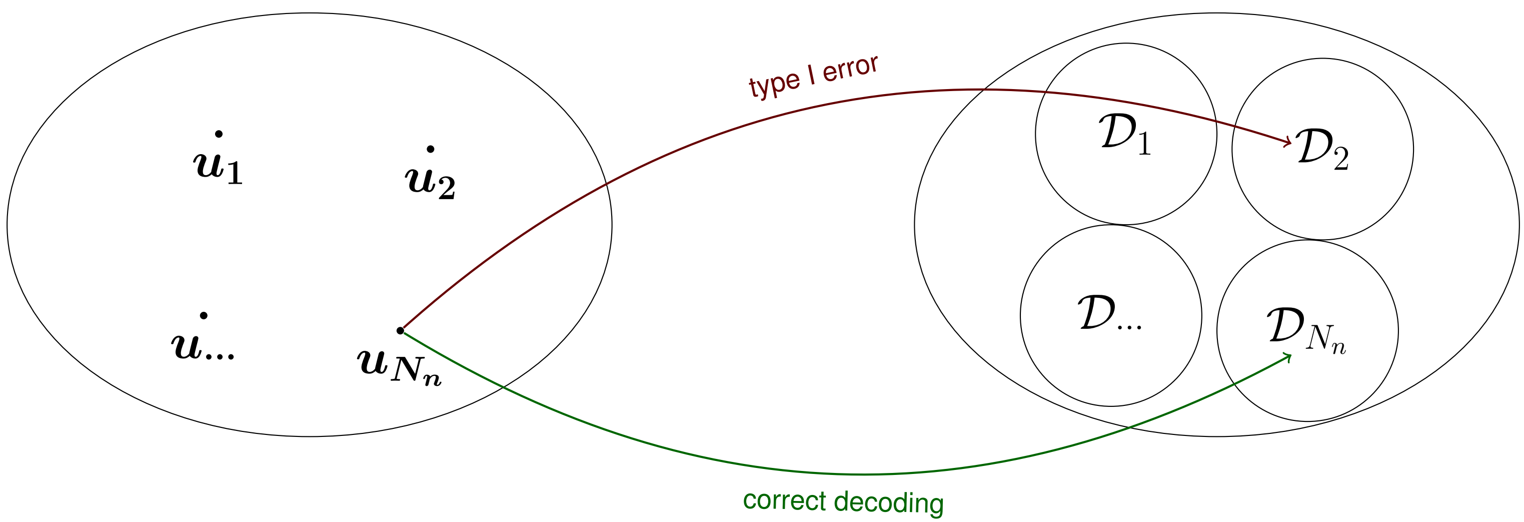

Figure: Geometric depiction of Transmission code

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IDentification

(Ahlswede & Dueck 1989):

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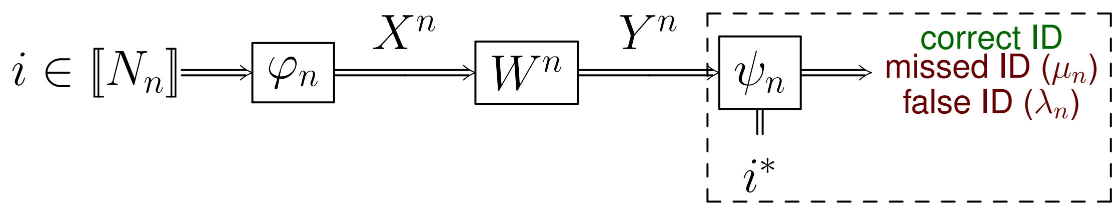

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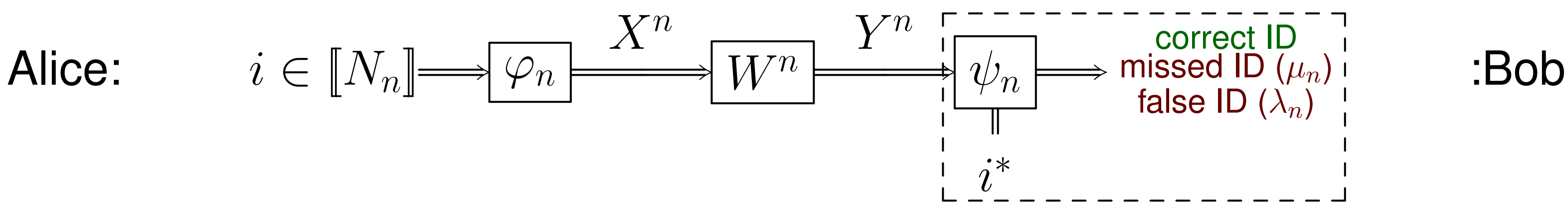

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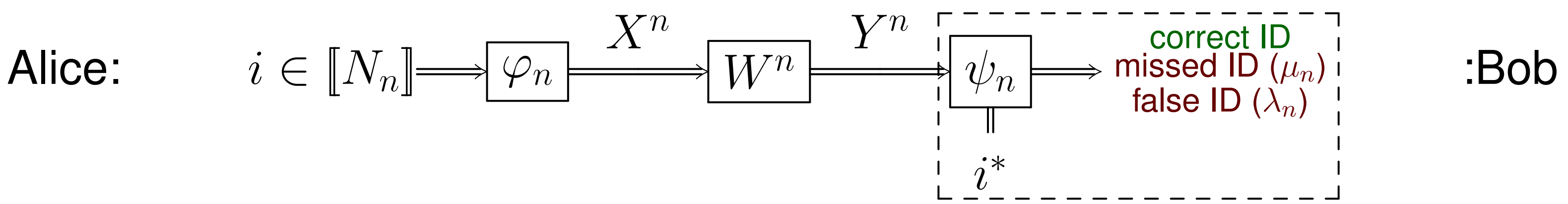

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$$Q_i(x^n) = Pr\{X^n(i) = x^n\},\ x^n \in \mathcal{X}^n,\ \mathcal{D}_i \subset \mathcal{Y}^n$$

$$Q_i W^n \rightarrow Pr\{Y^n(i) = y^n\} \text{ (response of } Q_i)$$

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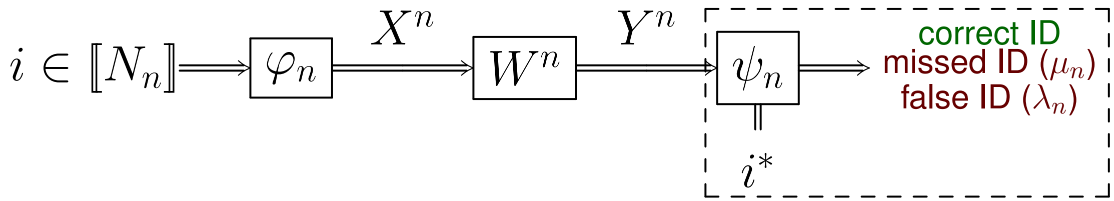

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$$\mu_n^{(i)} = Q_i W^n(\mathcal{D}_i^c) = Pr\{Y^n(i) \in \mathcal{Y}^n \setminus \mathcal{D}_i\} \xrightarrow{\text{type I error}} \quad \mu_n = \max_{1 \le i \le N_n} \mu_n^{(i)}$$

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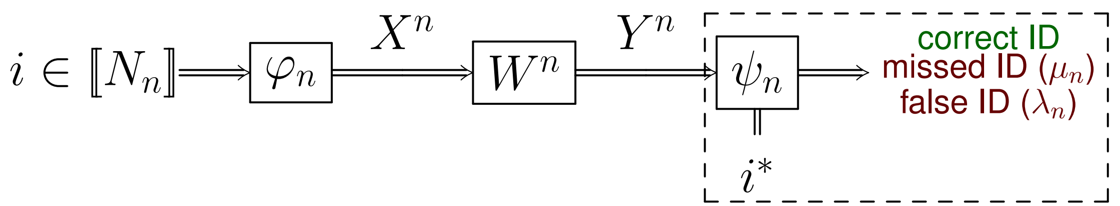

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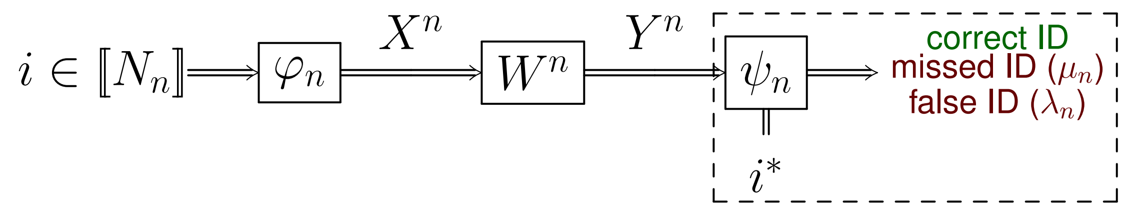

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$$N_n \le 2^{|\mathcal{Y}|^n}$$

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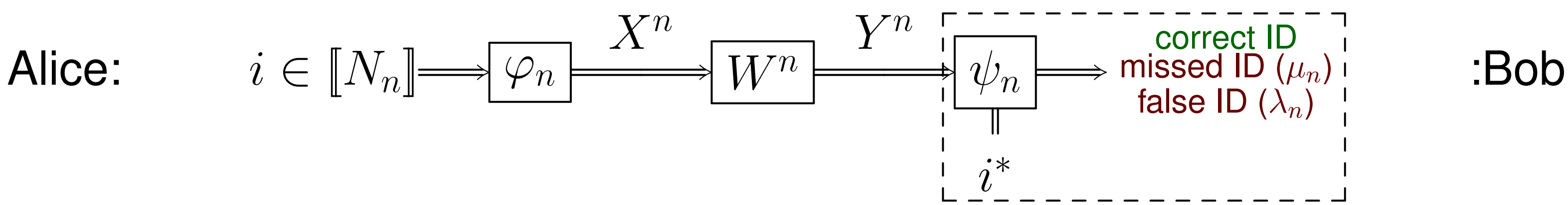

missed ID (due to channel noise)

Alice sent message i, Bob who is interested in test message i^* can decide i^* was not sent

false ID (inherent to the code)

Alice sent message j, Bob who is interested in test message j^* can decide message $i \neq j$ was sent

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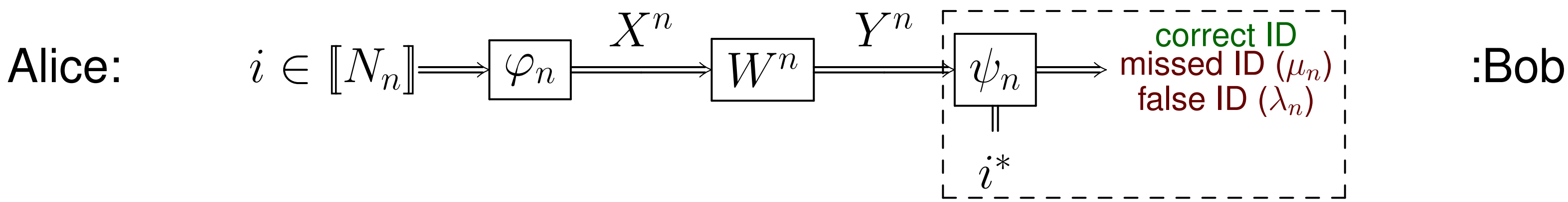

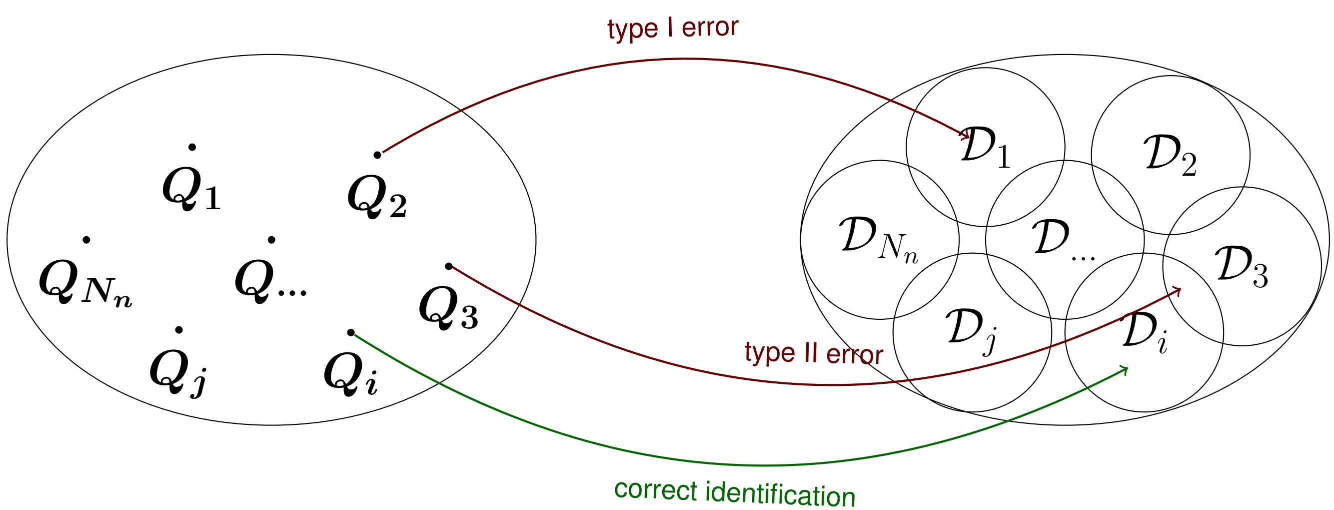

Figure: Geometric depiction of ID code

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IDentification Theorems

Rate

$$r_n \triangleq \frac{1}{n} \log \log N(n, \mu_n, \lambda_n) \tag{1}$$

Capacity

$$C_{ID} = \lim_{n \to \infty} \frac{1}{n} \log \log N_{max}(n, \mu_n, \lambda_n) \tag{2}$$

Direct Part

$$\liminf_{n \to \infty} r_n \geq C_T \quad \forall \mu_n, \lambda_n \in (0, 1] \tag{3}$$

Soft Converse

$$\limsup_{n \to \infty} r_n \leq C_T \quad \mu_n, \lambda_n \leq 2^{-n\varepsilon} \quad \forall \varepsilon > 0 \tag{4}$$

Strong Converse (Han & Verdu 1992)

$$C_{ID} \leq C_T \quad \forall \mu_n, \lambda_n \geq 0 \quad \& \quad \limsup_{n \to \infty} (\mu_n + \lambda_n) < 1 \tag{5}$$

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Transmission vs **ID**entification

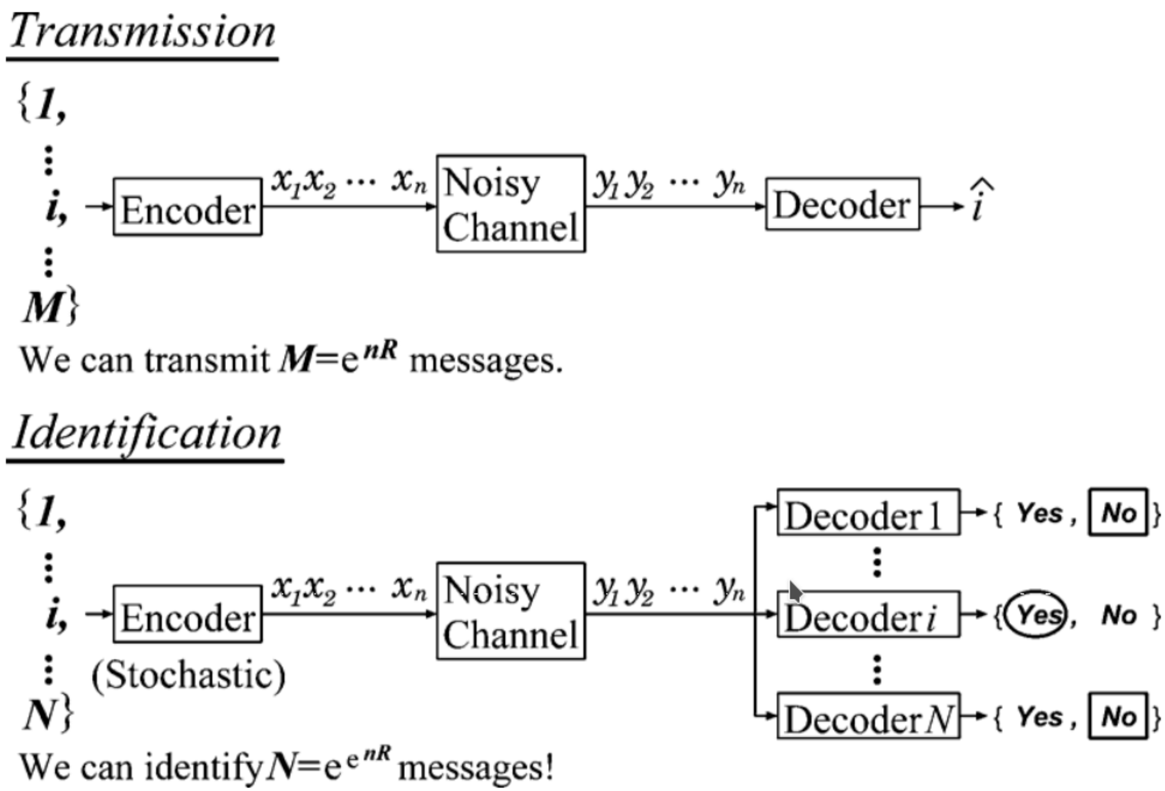

Figure: difference between Transmission and **ID**entification [1]

[1] Y. Oohama, "Converse coding theorems for identification via channels," IEEE Trans. Inform. Theory, vol. 59, pp. 744-759, Feb. 2013.

Application

- Scenario → Radio Networks, LAN, and Downlink Satellite Communications
- Goal → Delivery a sequences of messages, each intended for one receiver

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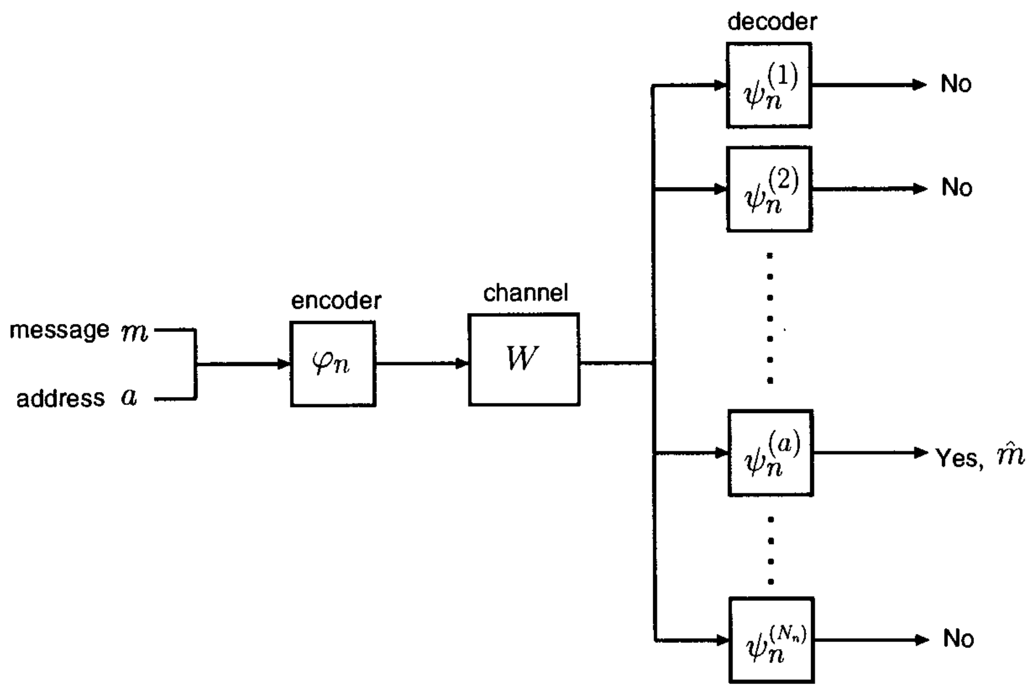

Figure: realization of identification-transmission communication [2]

[2] adopted from "Information-Spectrum Methods in Information Theory", T. S. Han, Tokyo, 2003, p. 436.

Construction 1 (Random Coding Argument)

(Ahlswede & Dueck):

- Shannon's coding theorem gaurantees existence of two transmission codes:
  - $\mathcal{L}' = \{(\boldsymbol{u}'_j, \mathcal{D}'_j) | j \in [\![M']\!]\}$ / $(n, \lceil 2^{n(C-\varepsilon)} \rceil, 2^{-n\delta})$
  - $\mathcal{L}'' = \{(\boldsymbol{u}''_k, \mathcal{D}''_k) | k \in [\![M'']\!]\}$ / $(\lceil \sqrt{n} \rceil, \lceil 2^{\varepsilon\sqrt{n}} \rceil, 2^{-\sqrt{n}\delta})$

- Let $\mathcal{T}$ be a family of maps $\mathcal{T} = \{T_i | i \in [\![N]\!]\}$ where $T_i : [\![M']\!] \to [\![M'']\!]$
- Let $\mathcal{U}_i := \{\boldsymbol{u}'_j.\boldsymbol{u}''_{T_i(j)} | j \in [\![M']\!]\}$ and $\mathcal{D}_i = \bigcup_{j=1}^{M'} \mathcal{D}'_j \times \mathcal{D}''_{T_i(j)}$
- Let $Q(i)$ be uniform distribution on set of codewords $\mathcal{U}_i$

- Obviously $\mathcal{ID}_{\mathcal{L}',\mathcal{L}''} = \{(Q_i, \mathcal{D}_i)\}_{i \in [\![N]\!]}$ is an $(n + \sqrt{n}, N, \lambda_1^{\mathbb{L}',\mathbb{L}''}, \lambda_2^{\mathbb{L}',\mathbb{L}''})$ ID code.

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Construction 1 (Random Coding Argument)

(Ahlswede & Dueck):

- Let $\forall i \in [\![N]\!]$ and $\forall j \in [\![M']\!]$, U_{ij} be independent RVs s.t. $\Pr\{U_{ij} = \boldsymbol{u}'_j.\boldsymbol{u}''_k\} = \frac{1}{M''}$, $\quad \exists T^* \in$ **T** s.t. $T^*_i(j) = k \in [\![M'']\!]$
- Let random set $\overline{\mathcal{U}}_i = \{U_{i1}, \cdots, U_{iM'}\}$ be a vector of concatenated codeword
- Let random decoding set $\mathcal{D}(\overline{\mathcal{U}}_i) = \bigcup_{j=1}^{M'} \mathcal{D}(U_{ij})$ where $\mathcal{D}(U_{ij}) = \mathcal{D}'_j \times \mathcal{D}''_k$
- System $\{(\overline{Q}(i),\mathcal{D}(\overline{\mathcal{U}}_i))|i \in [N]\}$ is $(M'(n + \lceil\sqrt{n}\rceil), N, \lambda_1, \lambda_2)$ ID code and achieves acceptible maximal error probabilities

Construction 2 (Concatenation)

(Verdu & Wei):

- Sequence of binary constant-weight code $\{C_i\} = (S_i, N_i, M_i, \mu_i M_i)$ with weight factor β_i, second order rate ρ_i and pairwise overlap fraction μ_i is optimal for identification if:
  - $\beta_i \to 1$, $\rho_i \to 1$, $\mu_i \to 0$

- 3 layer concatenated code $C_1 \circ C_2 \circ C_3$ denoted by $[q, k, t]$ with:
  - $C_1 = [q]$ PPM (all binary q-vectors of unit weight)
  - $C_2 = [q, k]$ RS Code
  - $C_3 = [q^k, q^t]$ RS Code
  - $t \leq k \leq q =$ prime

  is a $(q^{k+2}, (q^k)^{q^t}, q^{k+1}, kq^k + q^{1+t})$ binary constant-weight code

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Construction 2 (Concatenation)

(Verdu & Wei):

- Let $\{C_i\} = [q_i, k_i, t_i]$ be sequence of 3 layer concatenated codes, then $\{C_i\}$ is optimal for identification if:
  - $t_i \to \infty$
  - $\frac{t_i}{k_i} \to 1$
  - $\frac{k_i}{q_i} \to 0$
  - $q_i^{t_i - k_i} \to 0$
- Coupling 3 layer concatenated code with a transmission code (n, e^{nR}, λ) gives an IT code which subsequently ID code can be extracted from !
- Error exponents of resulting ID code $\to$ $(\frac{1}{n}\log\frac{1}{\lambda}, \frac{1}{n}\log\frac{1}{\lambda+\mu})$

Construction 3 (1 Layer RS Code)

(Moulin & Koetter):

- Let $C = (n, |C|, d)_q$ be an EC code
- For word $c_i = (c_i^1, \cdots, c_i^n)$ let enc/dec set $A_i = D_i$ is $\{(u, c_i^u) | u \in [n]\}$
- $|A_i| = n, \quad |A_i \cap A_j| \leq n - d, \quad \forall i, j \in [|C|] \quad \rightarrow \mu_n = 1 - \frac{d}{n}$
- Let RS code $(n \leq q - 1, k)$ over $\mathbb{F}_q$ map $(x_0, \cdots, x_{k-1}) \in \mathbb{F}_q^k$ to $(y_1, \cdots, y_n) \in \mathbb{F}_q^n$ where $y_i = \sum_{j=0}^{k-1} x_j \alpha_i^j$ where $\alpha_i \in F = \{\alpha_1, \cdots, \alpha_n\} \subset \mathbb{F}_q$
- $(x_0, \cdots, x_{k-1}) \sim P = \sum_{j=0}^{k-1} x_j X^j \in \mathbb{F}_q[X]$
- Set $A_P = \{(j, P(\alpha_j))\} | j \in [n]\} \quad$ for $P \in \mathbb{F}_q[X]$
- Now M-K-RS ID code is defined by $\{(A_p, A_p) | P \in \mathbb{F}_q[X], deg(P) < k\}$
- Corresponding ID code is $(\log_2 n + \log_2 q, q^k, 0, \frac{k-1}{n})$
- Application in ContactLess Device (RFID tags) identification [**Private Interrogation**]

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Construction 3 (1 Layer RS Code)

(Moulin & Koetter):

- Let message set $\mathbb{M}$ have cardinality 2^{rK}, where $r = \varepsilon n$, for some $\varepsilon \in (0,1)$
- Partition message m into K submessages $m_1, \cdots, m_K$
- Binary representation of m as $r \times K$ matrix where m_u sits in the u-th coloumn.

$$\mathbf{m} \sim \left.\underbrace{\begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ \vdots & & & \ddots & \\ 0 & 0 & 0 & \cdots & 0 \end{pmatrix}}_{K}\right\} r$$

- Encoding:
  - Generate RV u uniformly distributed over $\{1, \cdots, K\}$
  - Transmit ID word (u, m_u)
- Parameters:
  - # of bits to represent **a** $= (u, m_u) = \log K + r$ where $\log K = \frac{1-\varepsilon}{\varepsilon} r$
  - $\frac{1}{n} \log\log \mathbb{M} = 1 - \varepsilon + \frac{\log(\varepsilon n)}{n}$

Construction 3 (1 Layer RS Code)

(Moulin & Koetter):

- Decoding:
  - RX observes output of noiseless channel **b** = **a** $= (u, m_u)$
  - To test for the presence of message m^*, decoder compares if $m_u = m_u^*$
- Performance:
  - $\mu_n = 0$ ☺
  - $\lambda_n = 1 - \frac{1}{K}$ ☹
- The need for redundancy $\rightarrow$ representation of message s.t. increase distance between different messages (measured via distinct coloumns)
- Simple idea $\rightarrow$ apply (L, K) RS code with alphabet size $q = 2^r$ to message m
- Performance:
  - $\mu_n = 0$ ☺
  - $\lambda_n = \frac{K}{L}$ ☺

Remarks

- ID codes outperform one exponential order more than transmission codes by gaining reliable transmission of double exponential messages in bloklength
- Double Exponent Coding Theorem have been developed
- ID performance is measured by two errors namely type I and type II
- ID application $\rightarrow$ P2MP, remote alarm service,
- For infinite alphabet channel (white Gaussian with bandwith constraint) or DMC ID and Shannon capacity coincide
- ID code for noisy channel $\rightarrow$ concatenation of standard transmission code and an ID code for noiseless channel
- Need for explicit construction of ID codes and Practical algorithms for implementation continues!

Questions

Thanks For Attendance