Deterministic Identification for Molecular Communications over the Poisson Channel

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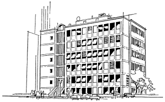

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Outline

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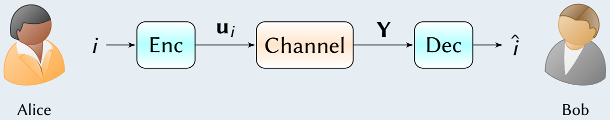
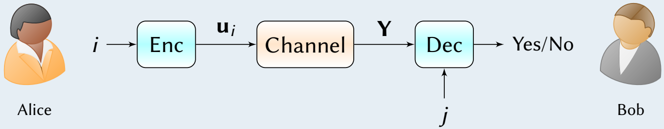
Transmission vs. Identification

- Shannon's setting: Bob recover the message.



- Identification setting: Bob asks if a message was sent or not?



- Event-triggered scenarios
- Molecular communication

Transmission (TR) [1]

- Originally introduced by Shannon (1948)
- Capacity was established without randomness at encoder
- Codebook size $\sim 2^{nR}$

[1] Shannon, "A Mathematical Theory of Communication", Bell Sys. Tech. J., 1948

Transmission (TR) [1]

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Theorem (Shannon, 1948)

Transmission capacity of a DMC $\mathcal{W}$ with exponential codebook size, i.e., $L(n, R) = 2^{nR}$ is given by

$$\mathbb{C}_T(\mathcal{W}, L) = \max_{p_X} I(X; Y)$$

[1] Shannon, "A Mathematical Theory of Communication", Bell Sys. Tech. J., 1948

Randomized Identification (RI) [2]

- Originally introduced by Ahlswede and Dueck (1989)
- Capacity was established with randomness at encoder
- Encoder employs distribution to select codewords

Remarkable Property

- Reliable identification is possible with codebook size $\sim 2^{2^{nR}}$

Theorem (Ahlswede and Dueck, 1989)

RI capacity of a DMC $\mathcal{W}$ in double exponential codebook size, i.e., $L(n,R) = 2^{2^{nR}}$ is given by

$$\mathbb{C}_{RI}(\mathcal{W}, L) = \max_{p_X} I(X;Y)$$

[2] Ahlswede and Dueck, "Identification via Channels", T-IT, 1989

Deterministic Identification (DI) [3] [4]

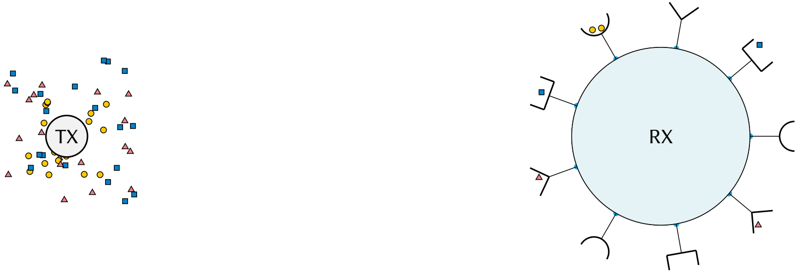
- Encoder uses deterministic mapping for coding
- Simpler implementation (random resource not required)
- Achievable rates **higher** than transmission [a]
- Suitable for molecular communication
  - Olfactory-inspired MC system
  - Cancer treatment and target drug delivery

[a] Salarisedigh *et al.*, "Deterministic Identification Over Channel With Power Constraints", T-IT, 2022

[3] Ahlswede and Cai, "Identification Without Randomization", T-IT, 1999

[4] Salariseddigh *et al.*, "Deterministic Identification Over Channels With Power Constraints," Proc. ICC, 2021

Olfactory-inspired MC System



- A codeword is molecular mixture
- Dedicated type receptors ensure spacial orthogonality

Outline

Main Contributions

- We develop lower and upper bounds on the DI capacity for the memoryless discrete time Poisson channels (DTPC) subject to both average and peak power constraints

- We use the bounds to determine the **correct size** of codebook

- We show that the optimal codebook size scales as $\sim 2^{(n \log n)R}$

Outline

DI Codes

Definition

An $(L(n,R), n, \lambda_1, \lambda_2)$-DI code for DTPC $\mathcal{W}$ is a system $\{(\boldsymbol{u_i}, \mathcal{D}_i)\}_{i \in [1:L(n,R)]}$ subject to

1. Codebook size: $L(n,R) = 2^{(n \log n)R}$
2. Codeword: $\boldsymbol{u_i} \in \mathcal{X}^n$, decoding sets: $\mathcal{D}_i \subset \mathcal{Y}^n$
3. Input constraints:
   - $0 \leq u_{i,t} \leq P_{\max}$
   - $n^{-1} \sum_{t=1}^{n} u_{i,t} \leq P_{\text{avg}}$
4. Error requirement type I: $W^n(\mathcal{D}_i | \boldsymbol{u}_i) > 1 - \lambda_1$
5. Error requirement type II: $W^n(\mathcal{D}_i | \boldsymbol{u}_j) \underset{i \neq j}{<} \lambda_2$

DI Codes (Cont.)

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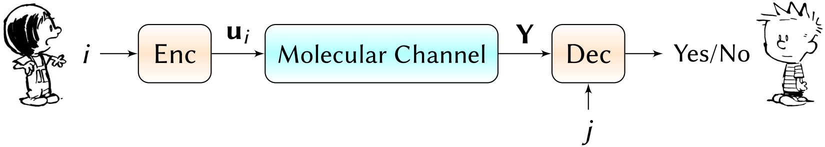
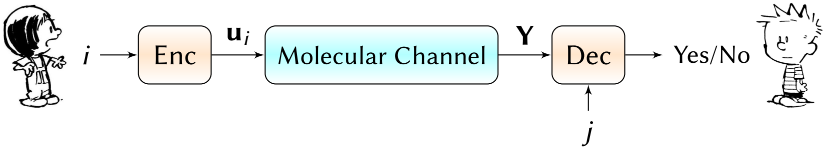
DI for Poisson Channel



DI for Poisson Channel



Definitions

- $Y_t \sim \text{Pois}\left(\lambda + \rho u_{i,t}\right)$
- $\lambda \in \mathbb{R}_{>0} \rightarrow$ Expected # of interfering molecules
- $\rho X \rightarrow$ Expected # observed molecules due to release by TX
- $\mathbf{y} \in \mathbb{N}_0^n \rightarrow$ Output vector
- Power const. $0 \leq u_{i,t} \leq P_{\max}$ and $\frac{1}{n}\sum_{t=1}^{n} u_{i,t} \leq P_{\text{avg}}$
- $W^n(\mathbf{y}|\mathbf{u}_i) = \prod_{t=1}^{n} \frac{e^{-(\lambda+\rho u_{i,t})}(\lambda+\rho u_{i,t})^{y_t}}{y_t!}$

DI for Poisson Channel [5]

Theorem

Let $\mathcal{W}$ be a DTPC with expected interference $\lambda \in \mathbb{R}_{>0}$. Then the DI capacity subject to power constraints $n^{-1}\sum_{t=1}^{n} u_{i,t} \leq P_{avg}$ and $0 \leq u_{i,t} \leq P_{max}$ for $L(n,R) = 2^{(n\log n)R}$ is bounded by

$$\frac{1}{4} \leq \mathbb{C}_{DI}(\mathcal{W}, L) \leq \frac{3}{2}$$

[5] Salariseddigh *et al.*, "Deterministic Identification Over Poisson Channels," Proc. GC, 2021

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- Achiev. proof: sphere pkg. of rad. $\sqrt[4]{n} \Rightarrow 2^{\frac{1}{4}(n \log n)}$ codewords

[5] Salariseddigh *et al.*, "Deterministic Identification Over Poisson Channels," Proc. GC, 2021

Proof Sketch. (Achievability)

- Dense sphere packing arrangement with radius $\sqrt{n\epsilon_n}$
- *Minkowski-Hlawka Theorem* guarantees a density $\Delta_n \geq 2^{-n}$
- $2^{(n \log n)R} \geq \Delta_n \cdot \frac{\text{Vol}(\mathcal{Q}_{\mathbf{0}}[n,A])}{\text{Vol}\left(\mathcal{S}_{\mathbf{u}_1}(n,\sqrt{n\epsilon_n})\right)} \geq 2^{-n} \cdot \frac{A^n}{\text{Vol}\left(\mathcal{S}_{\mathbf{u}_1}(n,\sqrt{n\epsilon_n})\right)}$
- $R \geq \frac{1}{n \log n}\left[\left(\frac{1-b}{4}\right) n \log n + n \log\left(\frac{A}{e\sqrt{a}}\right) + o(n)\right] \xrightarrow[b \to 0]{n \to \infty} \frac{1}{4}$
    1. $A = \min\left(P_{\text{ave}}, P_{\max}\right)$
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Chebyshev's inequality gives following error bounds:

1. $P_{e,1}(i) \leq \frac{c_1}{n\epsilon_n^2} = \mathcal{O}(\frac{1}{n^b})$
2. $P_{e,2}(i,j) \leq \frac{c_2}{n\epsilon_n^2} = \mathcal{O}(\frac{1}{n^b})$

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- Error bounds (1 and 2) gives: $\epsilon_n = a n^{\frac{1}{2}(b-1)}$

Proof Sketch. (Converse)

Lemma

- *For every pair of codewords* $\mathbf{u}_i$ *and* $\mathbf{u}_j$ $1 \leq \exists t \leq n$ *such that*

$$\left| 1 - \frac{\rho u_{i,t} + \lambda}{\rho u_{j,t} + \lambda} \right| > \epsilon'_n$$

then exploiting continuity of Poisson distribution gives:

$$P_{e,1}(i) + P_{e,2}(i,j) \geq 1 - \kappa_n \qquad \lightning$$

- $\epsilon'_n, \kappa_n \xrightarrow{n \to \infty} 0$

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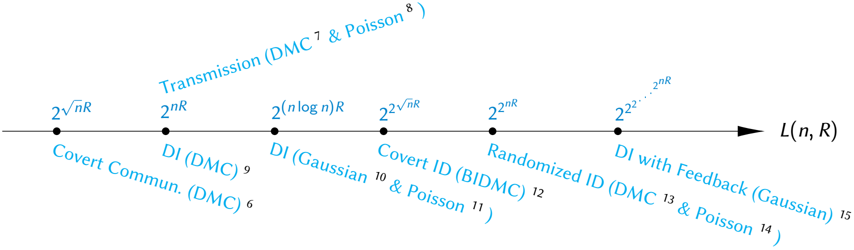
- $\left| 1 - \frac{\rho u_{i,t}+\lambda}{\rho u_{j,t}+\lambda} \right| > \epsilon'_n \Rightarrow \rho \left| u_{i,t} - u_{j,t} \right| > \lambda \epsilon'_n \Rightarrow \left\| \mathbf{u}_i - \mathbf{u}_j \right\| > \frac{\lambda \epsilon'_n}{\rho}$
- $R \leq \frac{1}{n \log n} \left[\left(\frac{1}{2} + (1+b) \right) n \log n + o(n) \right] \xrightarrow[b \to 0]{n \to \infty} \frac{3}{2}$

Codebook Sizes Spectrum



[6] Bloch, "Covert Communication over Noisy Channels: A Resolvability Perspective", T-IT, 2016

[7] Shannon, "A Mathematical Theory of Communication", Bell Sys. Tech. J., 1948

[8] Lapidoth and Moser, "On the Capacity of the Discrete-Time Poisson Channel", T-IT, 2009

[9] Salariseddigh *et al.*, "Deterministic Identification Over Channel With Power Constraints", T-IT, 2021

[10] Salariseddigh *et al.*, "Deterministic Identification Over Fading Channels", Proc. ITW, 2020

[11] Salariseddigh *et al.*, "Deterministic Identification Over Poisson Channels", Proc. GC, 2021

[12] Zhang and Tan, "Covert Identification over Binary-Input Discrete Memoryless Channels", arXiv, 2021

[13] Ahlswede and Dueck, "Identification via Channels", T-IT, 1989

[14] Burnashev, "On Identification Capacity of Infinite Alphabets or Continuous-Time Channels", T-IT, 2000

[15] Labidi *et al.*, "Identification over the Gaussian Channel in the Presence of Feedback", Proc. ISIT, 2021

Outline

Conclusions

- We have determined bounds on the DI capacity for DTPC
- We have found optimal codebook size $\sim 2^{(n \log n)C}$
- We observed that DI codebook size for **DTPC** and **Gaussian** channels is the same

- Future directions:
  1. Extension to multi-user scenarios (e.g., broadcast and multiple access channels) or multiple-input multiple-output channels
  2. Extension to **non-orthogonal** molecule reception / ISI Memory
  3. To obtain better bounds:
     - Enhance sphere packing arrangement
     - Try a different decoder
     - Improve error analysis techniques (sharper conc. inequal.)

Question and Discussion