Deterministic Identification for Molecular Communications over the Poisson Channel

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Outline

1. Motivation
2. Main Contributions
3. Definitions
4. Main Results
5. Conclusions
Outline

1 Motivation

2 Main Contributions

3 Definitions

4 Main Results

5 Conclusions
Transmission vs. Identification

- **Shannon’s setting**: Bob recover the message.

- **Identification setting**: Bob asks if a message was sent or not?

- **Event-triggered scenarios**

- **Molecular communication**

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Salariseddigh et al. — Deterministic Identification for Molecular Communications over the Poisson Channel
Transmission (TR)\textsuperscript{1}

- Originally introduced by Shannon (1948)
- Capacity was established without randomness at encoder
- Codebook size $\sim 2^{nR}$

\textsuperscript{1}Shannon, ”A Mathematical Theory of Communication”, Bell Sys. Tech. J., 1948
Transmission (TR) \(^1\)

- Originally introduced by Shannon (1948)
- Capacity was established without randomness at encoder
- Codebook size \( \sim 2^{nR} \)

**Theorem (Shannon, 1948)**

Transmission capacity of a DMC \( \mathcal{W} \) with exponential codebook size, i.e., \( L(n, R) = 2^{nR} \) is given by

\[
C_T(\mathcal{W}, L) = \max_{\rho_X} I(X; Y)
\]

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Randomized Identification (RI) \(^2\)

- Originally introduced by Ahlswede and Dueck (1989)
- Capacity was established with randomness at encoder
- Encoder employs distribution to select codewords

**Remarkable Property**

- Reliable identification is possible with codebook size \( \sim 2^{2nR} \)

**Theorem (Ahlswede and Dueck, 1989)**

RI capacity of a DMC \( \mathcal{W} \) in double exponential codebook size, i.e.,

\[ L(n, R) = 2^{2nR} \]

is given by

\[
\mathbb{C}_{RI}(\mathcal{W}, L) = \max_{P_X} I(X; Y)
\]

\(^2\) Ahlswede and Dueck, “Identification via Channels”, T-IT, 1989
Deterministic Identification (DI) \(^3\) \(^4\)

- Encoder uses deterministic mapping for coding
- Simpler implementation (random resource not required)
- Achievable rates \textbf{higher} than transmission \(^a\)
- Suitable for molecular communication
  - Olfactory-inspired MC system
  - Cancer treatment and target drug delivery

\(^a\) Salarisedigh \textit{et al.}, ”Deterministic Identification Over Channel With Power Constraints”, T-IT, 2022

\(^3\) Ahlswede and Cai, ”Identification Without Randomization”, T-IT, 1999
A codeword is molecular mixture

Dedicated type receptors ensure spatial orthogonality
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Main Contributions

- We develop lower and upper bounds on the DI capacity for the memoryless discrete time Poisson channels (DTPC) subject to both average and peak power constraints.

- We use the bounds to determine the correct size of codebook.

- We show that the optimal codebook size scales as $\sim 2^{(n \log n) R}$. 
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DI Codes

Definition

An \((L(n, R), n, \lambda_1, \lambda_2)\)-DI code for DTPC \(\mathcal{W}\) is a system \(\{(u_i, D_i)\}_{i \in [1:L(n,R)]}\) subject to

1. Codebook size: \(L(n, R) = 2^{(n \log n) R}\)
2. Codeword: \(u_i \in \mathcal{X}^n\), decoding sets: \(D_i \subset \mathcal{Y}^n\)
3. Input constraints:
   - \(0 \leq u_{i,t} \leq P_{\text{max}}\)
   - \(n^{-1} \sum_{t=1}^{n} u_{i,t} \leq P_{\text{avg}}\)

- Error requirement type I: \(W_n(D_i|u_i) > 1 - \lambda_1\)
- Error requirement type II: \(W_n(D_i|u_j) < \lambda_2\) for \(i \neq j\)
DI Codes (Cont.)

**Definition**

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4. **Error requirement type I**: \(W^n(D_i|u_i) > 1 - \lambda_1\)
5. **Error requirement type II**: \(W^n(D_i|u_j) < \lambda_{2} \quad i \neq j\)
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DI for Poisson Channel

\[ \text{Expected # of interfering molecules} \]

\[ \text{Expected # observed molecules due to release by TX} \]

\[ \text{Output vector} \]

\[ \text{Power const.} = 0 \]

\[ \text{P}_{\text{max}} \text{ and } 1 \]

\[ \text{P}_{\text{avg}} \text{ (y)} = Q_{\text{t}} = 1 \]

\[ e^{(u_i; t) + (u_i; t) y_t} \]
DI for Poisson Channel

**Definitions**

- \( Y_t \sim \text{Pois}(\lambda + \rho u_{i,t}) \)
- \( \lambda \in \mathbb{R}_{>0} \rightarrow \text{Expected} \# \text{ of interfering molecules} \)
- \( \rho \chi \rightarrow \text{Expected} \# \text{ observed molecules due to release by TX} \)
- \( y \in \mathbb{N}_0^n \rightarrow \text{Output vector} \)
- Power const. \( 0 \leq u_{i,t} \leq P_{\text{max}} \) and \( \frac{1}{n} \sum_{t=1}^{n} u_{i,t} \leq P_{\text{avg}} \)
- \( W^n(y|u_i) = \prod_{t=1}^{n} \frac{e^{-(\lambda + \rho u_{i,t})(\lambda + \rho u_{i,t})} y_t}{y_t!} \)
DI for Poisson Channel

Theorem

Let $\mathcal{W}$ be a DTPC with expected interference $\lambda \in \mathbb{R}_{>0}$. Then the DI capacity subject to power constraints $n^{-1}\sum_{t=1}^{n} u_{i,t} \leq P_{\text{avg}}$ and $0 \leq u_{i,t} \leq P_{\text{max}}$ for $L(n, R) = 2^{(n \log n)R}$ is bounded by

$$\frac{1}{4} \leq C_{\text{DI}}(\mathcal{W}, L) \leq \frac{3}{2}$$

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Salariseddigh et al., "Deterministic Identification Over Poisson Channels," Proc. GC, 2021
**Theorem**

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$$
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- Achiev. proof: sphere pkg. of rad. $\sqrt{n} \Rightarrow 2^{\frac{1}{4}(n \log n)}$ codewords

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5 Salariseddigh et al., "Deterministic Identification Over Poisson Channels," Proc. GC, 2021
Proof Sketch. (Achievability)

- Dense sphere packing arrangement with radius $\sqrt{n\epsilon_n}$
- **Minkowski-Hlawka Theorem** guarantees a density $\Delta_n \geq 2^{-n}$
- $2^{(n \log n)R} \geq \Delta_n \cdot \frac{\text{Vol}(Q_0[n,A])}{\text{Vol}(S_{u_1}(n,\sqrt{n\epsilon_n}))} \geq 2^{-n} \cdot \frac{A^n}{\text{Vol}(S_{u_1}(n,\sqrt{n\epsilon_n}))}$
- $R \geq \frac{1}{n \log n} \left[ \left( \frac{1-b}{4} \right) n \log n + n \log \left( \frac{A}{e\sqrt{a}} \right) + o(n) \right] \xrightarrow{n \to \infty} \frac{1}{4}$
  1. $A = \min(P_{\text{ave}}, P_{\text{max}})$
  2. $a = \Omega(A^2)$ is constant
  3. $b$ is arbitrarily small constant
Proof Sketch. (Achievability)

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  1. $A = \min(P_{\text{ave}}, P_{\text{max}})$
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Chebyshev’s inequality gives following error bounds:

1. $P_{e,1}(i) \leq \frac{c_1}{n\epsilon_n^2} = \mathcal{O}\left( \frac{1}{n^b} \right)$
2. $P_{e,2}(i, j) \leq \frac{c_2}{n\epsilon_n^2} = \mathcal{O}\left( \frac{1}{n^b} \right)$
Proof Sketch. (Achievability)

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- **Minkowski-Hlawka Theorem** guarantees a density $\Delta_n \geq 2^{-n}$
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  1. $A = \min(P_{\text{ave}}, P_{\text{max}})$
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Error bounds (1 and 2) gives: $\epsilon_n = an^{\frac{1}{2}}(b-1)$
Proof Sketch. (Converse)

Lemma

For every pair of codewords $u_i$ and $u_j$ $1 \leq \exists t \leq n$ such that

$$1 - \frac{\rho u_{i,t} + \lambda}{\rho u_{j,t} + \lambda} > \epsilon'_n$$

then exploiting continuity of Poisson distribution gives:

$$P_{e,1}(i) + P_{e,2}(i, j) \geq 1 - \kappa_n$$

$$\epsilon'_n, \kappa_n \xrightarrow{n \to \infty} 0$$
Proof Sketch. (Converse)

Lemma

- For every pair of codewords $u_i$ and $u_j$, $1 \leq \exists t \leq n$ such that
  \[
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  \]

  then exploiting continuity of Poisson distribution gives:
  \[
  P_{e,1}(i) + P_{e,2}(i, j) \geq 1 - \kappa_n
  \]

- $\epsilon'_n, \kappa_n \xrightarrow{n \to \infty} 0$

- $\left| 1 - \frac{\rho u_i,t + \lambda}{\rho u_j,t + \lambda} \right| > \epsilon'_n \Rightarrow \rho \left| u_i,t - u_j,t \right| > \lambda \epsilon'_n \Rightarrow \| u_i - u_j \| > \frac{\lambda \epsilon'_n}{\rho}$

- $R \leq \frac{1}{n \log n} \left[ \left( \frac{1}{2} + (1 + b) \right) n \log n + o(n) \right] \xrightarrow{n \to \infty} \frac{3}{2}$
Codebook Sizes Spectrum

\[ L(n, R) \]

- 2\(\sqrt{nR}\)
- 2\(nR\)
- \(2(n \log n)R\)
- \(2^{2\sqrt{nR}}\)
- \(2^nR\)
- \(2^{2^nR}\)

**Covert Commun. (DMC)**

- Salarisedigh et al., "Deterministic Identification Over Channel With Power Constraints", T-IT, 2021
- Salarisedigh et al., "Deterministic Identification Over Fading Channels", Proc. ITW, 2020
- Salarisedigh et al., "Deterministic Identification Over Poisson Channels", Proc. GC, 2021

**DI (DMC)**

- Zhang and Tan, "Covert Identification over Binary-Input Discrete Memoryless Channels", arXiv, 2021

**DI (Gaussian)**

- Ahlswede and Dueck, "Identification via Channels", T-IT, 1989

**DI (Poisson)**

- Burnashev, "On Identification Capacity of Infinite Alphabets or Continuous-Time Channels", T-IT, 2000

**DI with Feedback (Gaussian)**

- Labidi et al., "Identification over the Gaussian Channel in the Presence of Feedback", Proc. ISIT, 2021

**Transmission (DMC)**


**Covert ID (BIDMC)**

- Bloch, "Covert Communication over Noisy Channels: A Resolvability Perspective", T-IT, 2016
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Conclusions

- We have determined bounds on the DI capacity for DTPC
- We have found optimal codebook size $\sim 2^{(n \log n)C}$
- We observed that DI codebook size for DTPC and Gaussian channels is the same

Future directions:

1. Extension to multi-user scenarios (e.g., broadcast and multiple access channels) or multiple-input multiple-output channels
2. Extension to **non-orthogonal** molecule reception / ISI Memory
3. To obtain better bounds:
   - Enhance sphere packing arrangement
   - Try a different decoder
   - Improve error analysis techniques (sharper conc. inequal.)
Question and Discussion