An Introduction to IDentification

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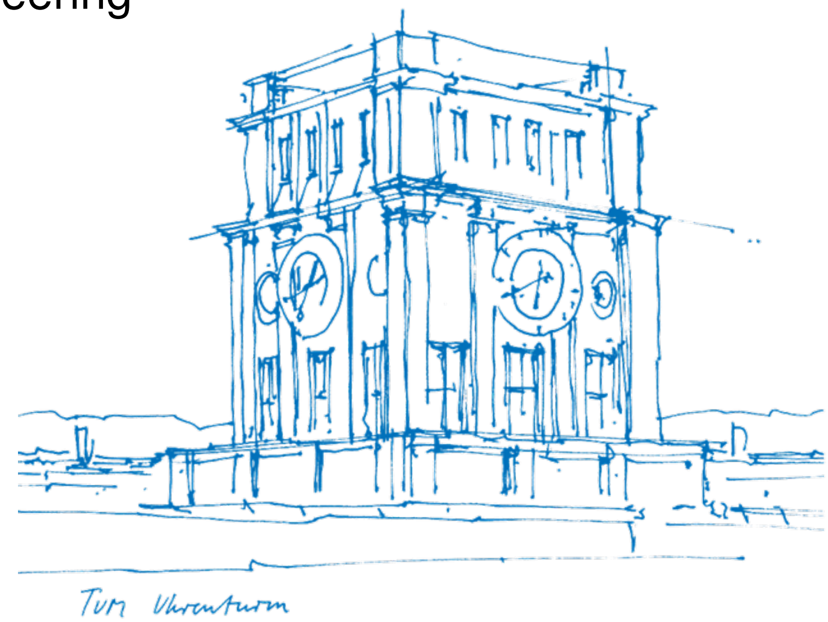

Outline

- Transmission (Shannon)
- IDentification (Ahlswede & Dueck)
- Construction of ID code
- Summary

Transmission

Transmission (Shannon) over W^n:

Alice: $i \in \{1, ..., N_n\} \Longrightarrow \boxed{\text{Enc}} \xRightarrow{X^n} \boxed{W^n} \xRightarrow{Y^n} \boxed{\text{Dec}} \Longrightarrow \hat{i} \approx_\lambda i$:Bob

Classical (n, N_n, λ) code for W is a system $\{(u_i, D_i)\}_{i\in[N_n]}$ such that:

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$$W^n(D_i|u_i) \geq 1 - \lambda$$

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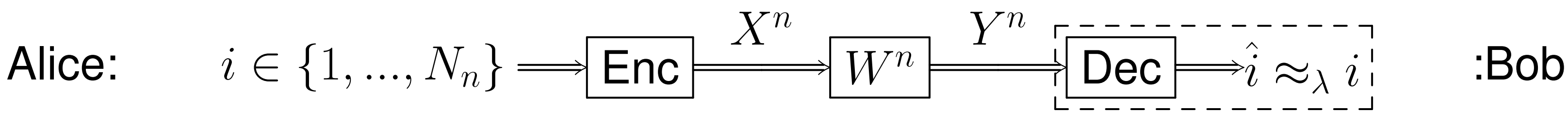

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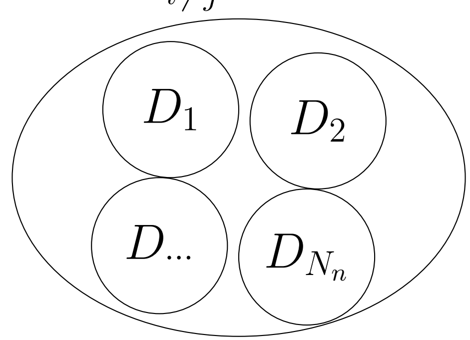

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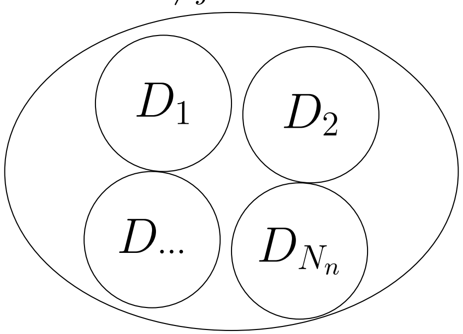

Transmission capacity (Shannon)

$$\lim_{n \to \infty} \frac{1}{n} \log N_{max}(n, \lambda) = C_T \qquad \forall \lambda \in (0, 1)$$

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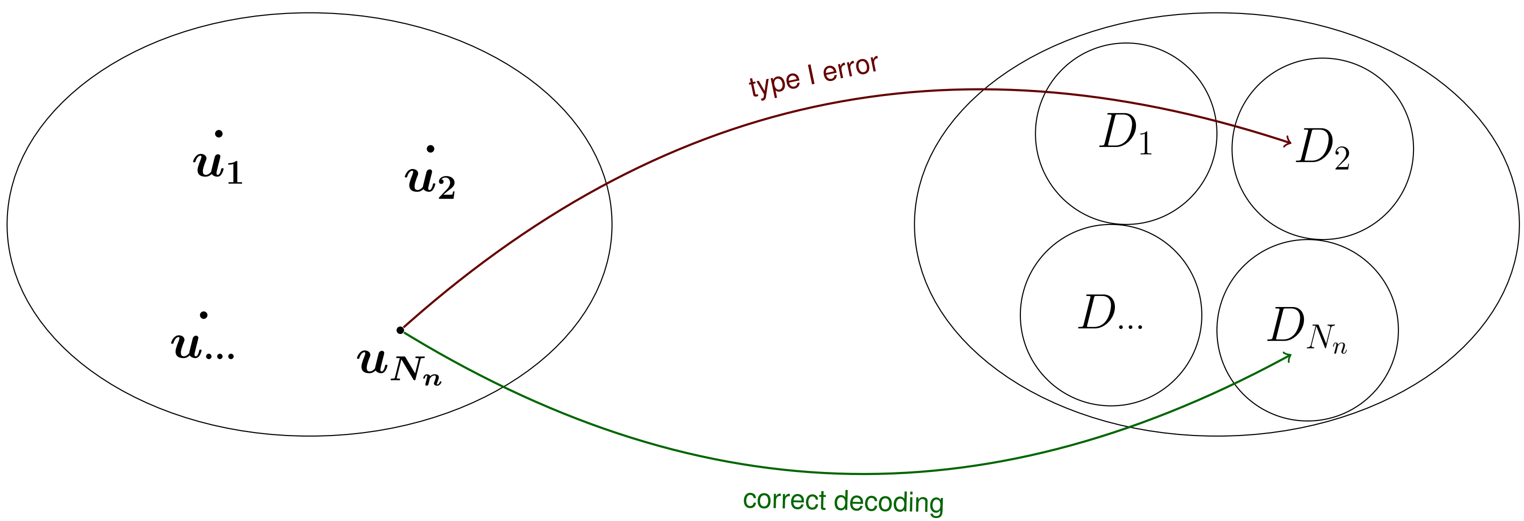
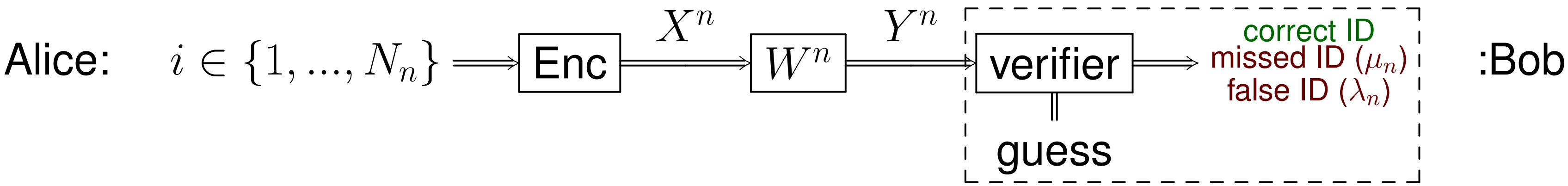


Figure: Geometric depiction of Transmission code

IDentification

IDentification (Ahlswede-Dueck) over W^n:



Randomized $(n, N_n, \mu_n, \lambda_n)$ identification code for W is a system $\{(Q_i, D_i)\}_{i\in[N_n]}$ such that:

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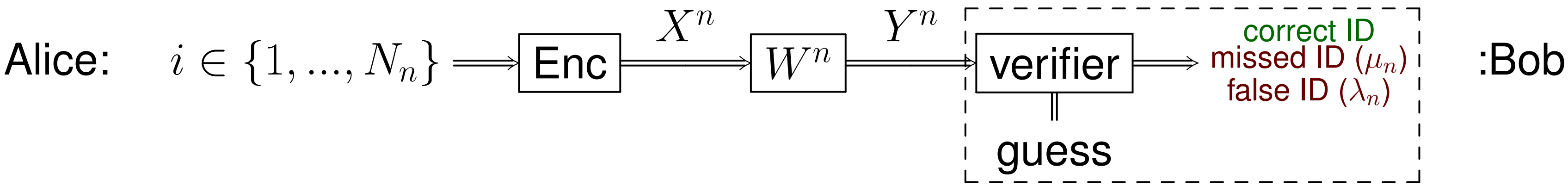

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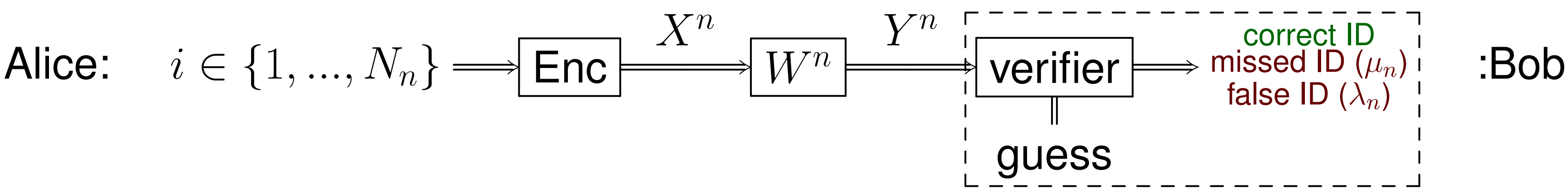

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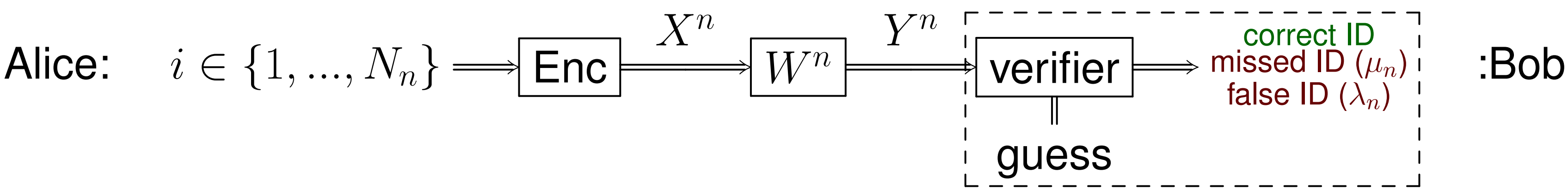

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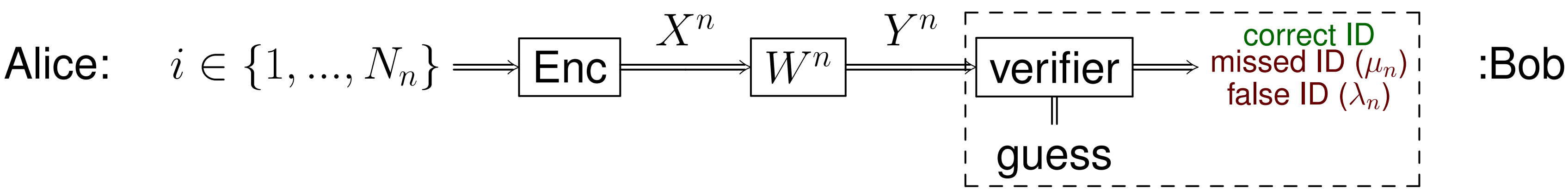

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$$\mu_n^{(i)} = Q_i W^n(D_i^c) = Pr\{Y^n(i) \in \mathcal{Y}^n \setminus D_i\} \xrightarrow{\text{type I error}} \qquad \mu_n = \max_{1 \leq i \leq N_n} \mu_n^{(i)}$$

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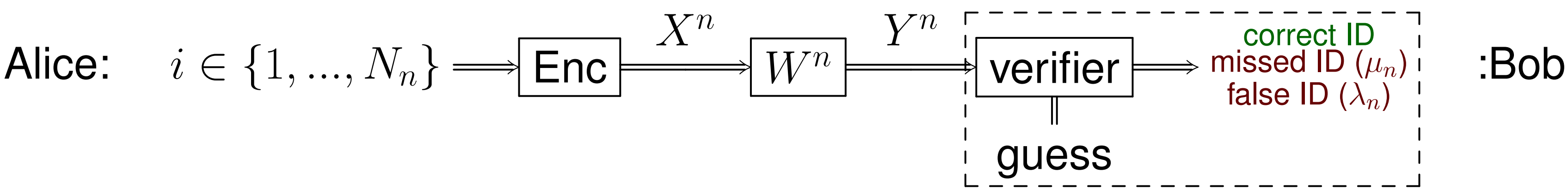

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$$\lambda_n^{(i,j)} = Q_j W^n(D_i) = Pr\{Y^n(j) \in D_i\} (j \neq i) \xrightarrow{\text{type II error}} \qquad \lambda_n = \max_{1 \leq j, i \leq N_n, j \neq i} \lambda_n^{(j,i)}$$

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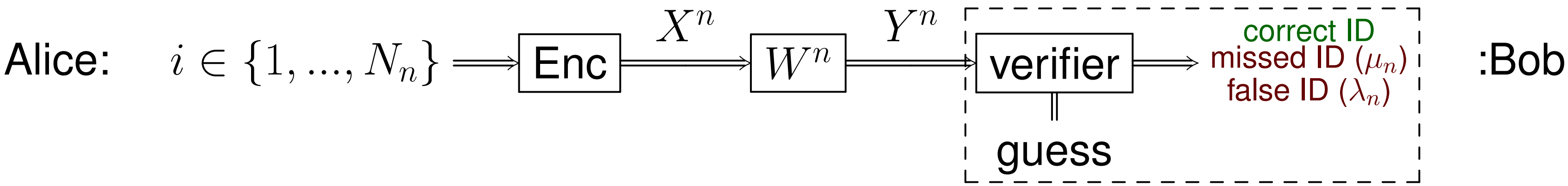

missed identification

Alice sent message i, Bob who is interested in message i (guess i) can decide his message was not sent / caused by transmission errors

false identification

Alice sent message i, Bob who is interested in message $j \neq i$ (guess j) can decide message j was sent / inherent to the code

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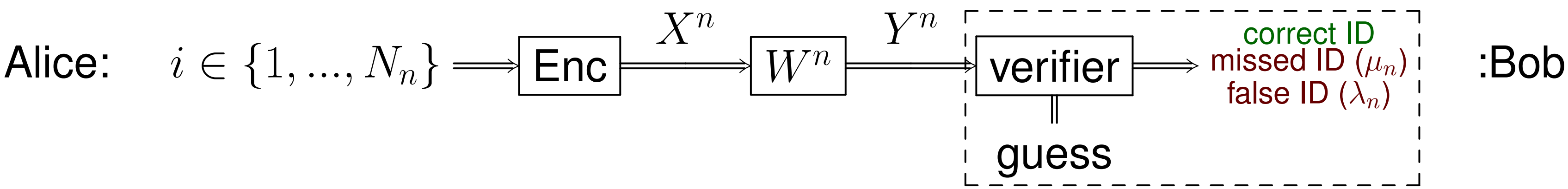

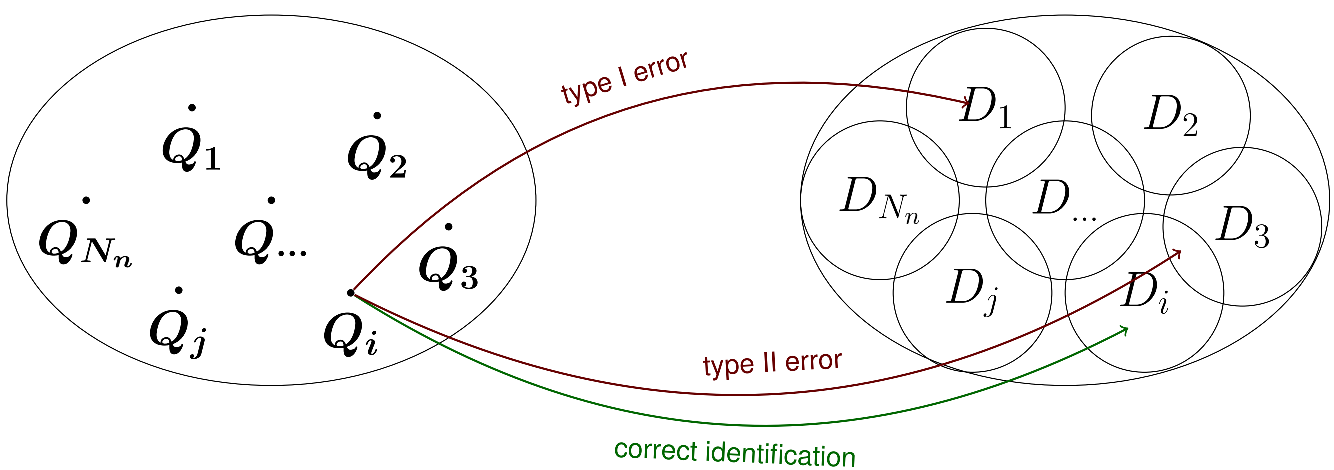

Figure: Geometric depiction of IDentification code

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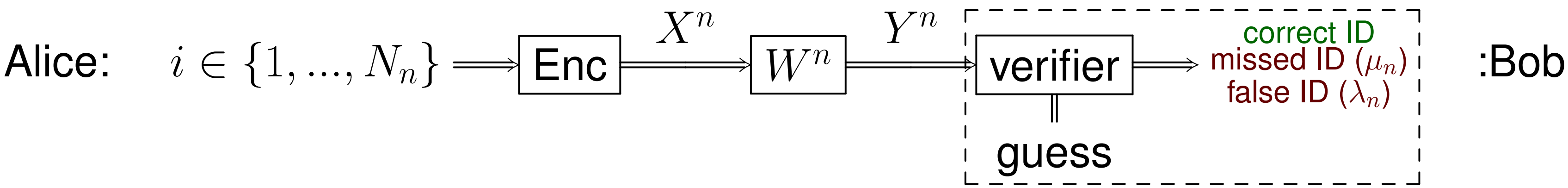

Rate

$$r_n = \frac{1}{n} \log \log N_n \tag{1}$$

Capacity

$$\lim_{n\to\infty} \frac{1}{n} \log \log N_{max}(n, \mu_n, \lambda_n) = C_{ID} \tag{2}$$

ID Coding Theorem

$$C_{ID} = C_T \tag{3}$$

Theorem

ID capacity of any channel is greater than or equal its Shannon capacity

Technical Application of IDentification

- Scenario $\rightarrow$ local-area/radio networks and downlink satellite communications
- Goal $\rightarrow$ centeral station wants to deliver sequences of messages, each intended for one of the receivers

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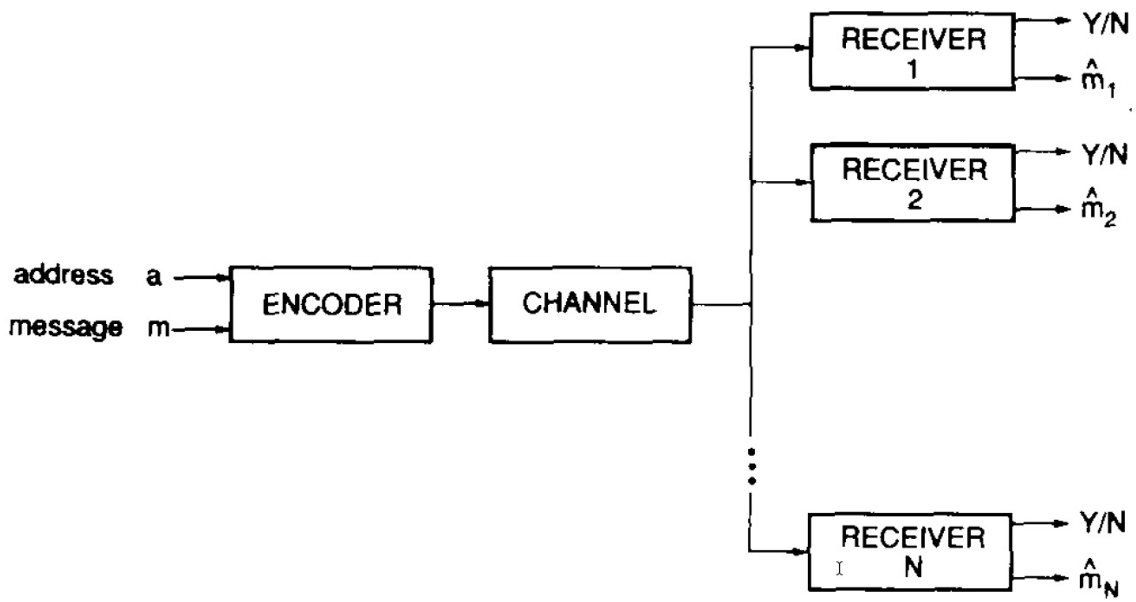

Figure: Identification plus transmission through a noisy channel [2]

[2] T. S. Han and S. Verdb, "New results in the theory and applications of identification via channels," IEEE Trans. Inform. Theory, vol. 38, pp. 14-25, Jan. 1992.

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- (Decoupled Coding) $\rightarrow$
  juxtaposing ID code and transmission code to send address and message respectively
  - (address identification rate) $\frac{1}{n}\log\log N_n \rightarrow C_T$ is achievable if $\frac{1}{n}\log M \rightarrow 0$
  - (message transmission rate) $\frac{1}{n}\log M \rightarrow C_T$ is achievable if $\frac{1}{n}\log\log N_n \rightarrow 0$
- (Coupled Coding) $\rightarrow$
  IDentification plus Transmission problem code (IT) can achieve both rates simultaneously

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Construction of ID Code (random selection)

1. Let $\mathscr{L}'$ and $\mathscr{L}''$ be two transmission codes, $\xrightarrow[\text{gaurantees existence of}]{\text{Shannon's coding theorem}}$
   - $(n, M', 2^{-n\delta})$ code $\mathscr{L}' = \{(u_j^{'}, D_j^{'})|j \in [M']\}$ with $M' = \lceil 2^{n(C-\varepsilon)} \rceil$
   - $(\lceil \sqrt{n} \rceil, M'', 2^{-\sqrt{n}\delta})$ code $\mathscr{L}'' = \{(u_k^{''}, D_k^{''})|k \in [M'']\}$ with $M'' = \lceil 2^{\varepsilon\sqrt{n}} \rceil$

2. Let T be a family of maps $T = \{T_i|i \in [N]\}$ where $T_i : [M'] \to [M'']$
3. Let $\mathscr{U}_i := \{u_j^{'}.u_{T_i(j)}^{''}|j \in [M']\}$ and $D_i = \bigcup_{j=1}^{M'} D_j^{'} \times D_{T_i(j)}^{''}$
4. System $\{(Q(.|i), D_i)|i \in [N]\}$ is an ID code constructed from $\mathscr{L}'$ and $\mathscr{L}''$
5. Let U_{ij} be independent RV s.t. $Pr\{U_{ij} = u_j^{'}.u_k^{''}\} = \frac{1}{M''}$
   $i \in [N], j \in [M'], k \in [M'']$
6. Let $\overline{\mathscr{U}}_i = \{U_{i1}, \cdots, U_{iM'}\} \qquad \forall i \in [N]$
7. Let $D(\overline{\mathscr{U}}_i) = \bigcup_{j=1}^{M'} D(U_{ij})$ where $D(U_{ij}) = D_j^{'} \times D_k^{''}$
8. System $\{(\overline{Q}(.|i), D(\overline{\mathscr{U}}_i))|i \in [N]\}$ achieves small maximal error probabilities

Construction of ID Code (concatenation)

- Sequence of binary constant-weight code $\{C_i\} = (S_i, N_i, M_i, \mu_i M_i)$ with weight factor β_i, second order rate ρ_i and pairwise overlap fraction μ_i is optimal for identification if:
- $\beta_i \to 1$, $\rho_i \to 1$, $\mu_i \to 0$
- Three-layer concatenated code $C_1 \circ C_2 \circ C_3$ denoted by $[q, k, t]$ with:
  - $C_1 = [q]PPM$
  - $C_2 = [q, k]$ Reed-Solomon
  - $C_3 = [q^k, q^t]$ Reed-Solomon
  - $t \leq k \leq q =$ prime

  is a $(q^{k+2}, q^{kq^t}, q^{k+1}, kq^k + q^{1+t})$ binary constant-weight code
- Let $\{C_i\} = [q_i, k_i, t_i]$ be sequence of three-layer concatenated codes, then $\{C_i\}$ is optimal for identification if:
  - $t_i \to \infty$, $\frac{t_i}{k_i} \to 1$, $\frac{k_i}{q_i} \to 0$, $q_i^{t_i - k_i} \to 0$
- Coupling three-layer concatenated code with a transmission code (n, e^{nR}, λ) gives an IT code which subsequently ID code can be constructed from

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Concluding Remarks

1. In contrast to single exponential behavior of conventional transmission codes, in IDentification for any channel (not necessarily discrete or memoryless) ID codes can asymptotically gaurantee transmission of $e^{e^{nC}}$ messages with n uses of a noisy channel while keeping error probabilities arbitrarily zero
2. Transmission capacity and IDentification capacity coincides (DMC)
3. Despite of Shannons's formulation, ID decoder selects a list of messages
4. List size grows doubly exponential in block length
5. Decoding reliability is expressed in terms of type I/II errors probabilities
6. IDentification can be used where the recipient is only interested in verify if a certain message is the transmitted message or not (P2MP, remote alarm service)

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Questions

Thanks For Attendance