Gaussian Primitive Diamond Channel: Correlated Noise at Relays and Relevant Applications

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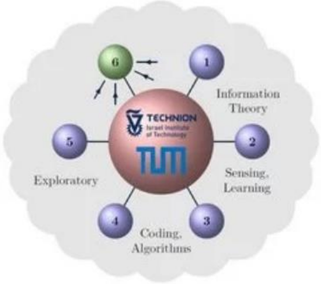

Outline

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    - Information bottleneck problem
    - Relaying Techniques
- Primitive diamond relay channel with correlated noise
- Dirty-paper coding relaying
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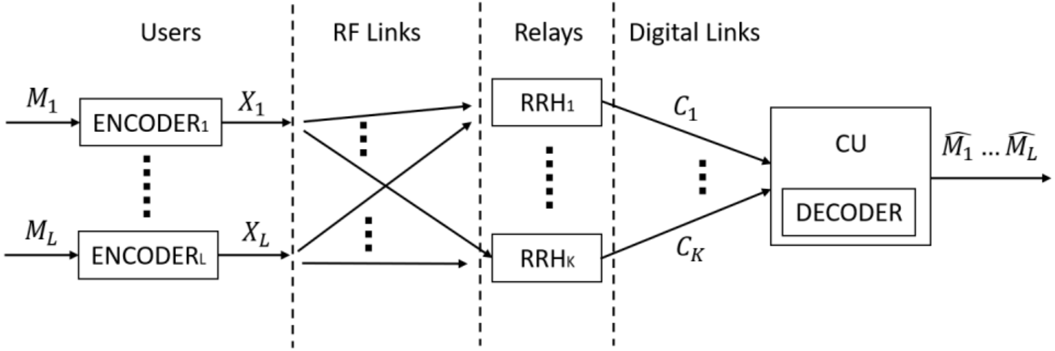
Introduction

Modern communication systems require:

- Higher communication capacity
- Higher spectral efficiency
- Lower power consumption
- Resource pooling
- Scalability



Modern and future communication system infrastructure:

- **Central unit** (CU) - performing signal processing and control
- **Remote radio heads** (RRH) - performing the radio frequency (RF) functions of amplification, reception and basic signal processing (compression), denoted in our work as “Relays”.

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Introduction (cont.)

Examples for systems that use relaying:

- Future wireless networks [Amjad, Musavian, Rehmani, 2019]
- Cloud radio access network (CRAN)
  [Park, Simeone, Sahin, Shamai, 2014], [Quek, Peng, Simeone, Yu, 2017] , [Aguerri, Zaidi, Caire, Shamai, 2019].

Our study is directly associated with these models, with the focus on point-to-point communications over the primitive diamond channel.

- The scalar Gaussian transmission over the oblivious symmetric diamond relay Gaussian channel is shown in [Sanderovich et al., 2008].
- The capacity of oblivious relays with L users and K relays using JDD is provided in [Aguerri et al., 2019].

Introduction (cont.)

Uplink Channel:

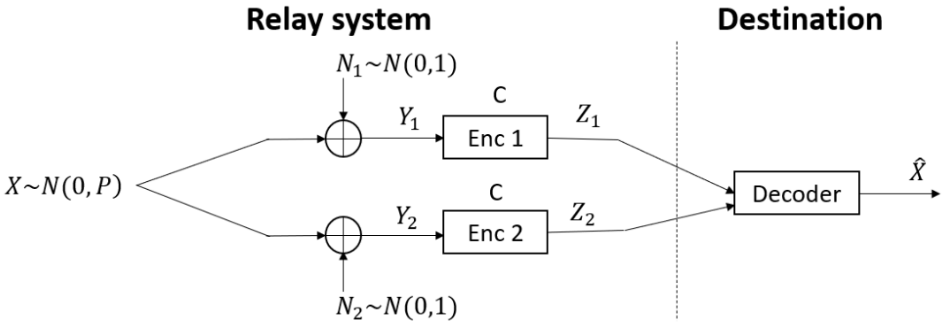
- In [Sanderovich et al., 2008] upper and lower bounds for the single user uplink channel are presented. Both joint decompression and decoding (JDD) and successive decompression and decoding (SDD) are presented. Time-sharing of decode and forward (DF) and compress and forward (CF) is considered for discrete-time model.
  The capacity of the scalar Gaussian case is derived.
- Compute and forward (CoF) relaying is proposed in [Nazer et al., 2011].
- In [Park et al., 2013] it is presented that JDD outperform SDD. A comparison of SDD with CoF is shown in [Park et al., 2014]. It is suggested that CoF outperforms in high SNR.
- In [Aguerri et al., 2019] it is shown that JDD does not provide much gain over SDD, both have the same maximum sum-rate. It is also shown that CoF does not always outperforms in high SNR.
  Also, the capacity of oblivious system with L users and K relays using JDD is provided.
- Tighter upper bound than the cut-set bound is shown in [Wu et al., 2019].
- Uplink-downlink CRAN duality aspects [Liu, Liu, Patil, Yu, 2021].

Introduction (cont.)

- In [Katz-Peleg-Shamai, 2019] we examine the single user oblivious uplink primitive diamond channel, extending the single relay results of [Homri-Peleg-Shamai, 2018].
- We extended the system model in [Katz-Peleg-Shamai, 2021] with optimal time-sharing between oblivious and non-oblivious relaying and presented the cost of obliviousness.
- In [Katz-Peleg-Shamai, 2022] we compared the time-sharing with a superposition coding scheme and proved that time-sharing is advantageous.
- The effect of interference on the system rate is of interest. In [Peleg-Shamai, 2017] the single relay channel with interference that is known at the encoder was investigated, using the known dirty paper coding scheme from [Costa, 1983].
- In this work we extend the primitive diamond system model with correlated channel noise, derive a closed form formula and study the effect of positive noise correlation.

System Model – Gaussian primitive diamond relay channel

- We investigate the **uplink** using the case of **Gaussian channel** with correlated noises and limited rate relay to destination ideal fronthaul link.
- For the **oblivious** case the compress and forward (CF) method is used.
  In this there is no a priori knowledge of the modulation or the coding at the relay, thus the relaying system is universal and can serve many diverse users and operators.
- For **non-oblivious** relay we use the decode and forward (DF) method.
- The **combination** of CF and DF relaying is also analyzed.



$$E(N_1 \cdot N_2) = \rho$$

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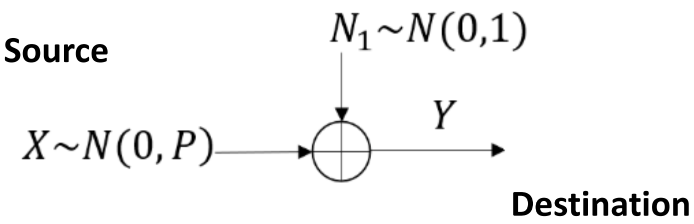
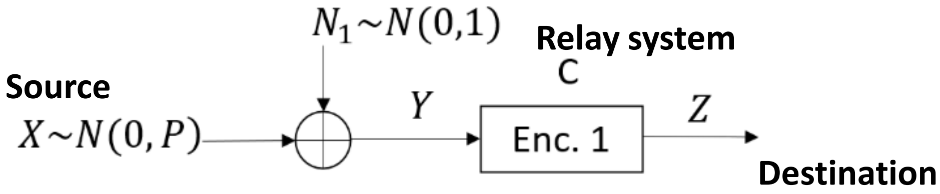
The information bottleneck problem

- The information bottleneck [Tishby et al., 1999], [Chechik et al., 2005] adds a rate constraint to the known AWGN channel model. This is a more precise model which is necessary in order to evaluate CRAN performance.





$$R_{\text{AWGN}} = \frac{1}{2}\log_2\left(1+P\right)$$

$$\max_{P(z|y)} I(X;Z) \quad s.t\ I(Y;Z) \le C \quad \longrightarrow \quad R_{\text{Single Relay}} = \frac{1}{2}\log_2\left(\frac{1+P}{1+P\cdot 2^{-2C}}\right)$$

[Winkelbauer-Matz, 2014]

Distributed information bottleneck

- Distributed IB is an extension of the IB problem for the case of more than one relay [Zaidi et al., 2020]. This extension can be seen as the Chief Executive Officer (CEO) remote source coding problem with logarithmic loss distortion [Ugur et al., 2020], [Aguerri et al., 2018].
- The system rate of [Sanderovich et al., 2008]:

$$\max_{r_1, r_2 \geq 0} \min_{S \in 1,2} \frac{1}{2} \log_2 \left(1 + P \sum_{t \in S^C} (1 - 2^{-2r_t})\right) + \sum_{t \in S} (C - r_t)$$

- The same rate can be derived from [Aguerri et al., 2019]:

$$R_X \leq \min_{S \subseteq \{1,2\}} \sum_{k \in S} \left[C_k - I(Y_k; U_k | X)\right] + I(X; U_{S^C})$$

Relaying Techniques - Compress and forward

- Each relay compress the received signal into digital representation, which is then transmitted to the destination.
- The compression method is distributed Wyner-Ziv compression.
- Higher fronthaul rate leads to lower quantization noise.
- At the destination, the compression indices and the user messages are jointly decoded based on the data received from the relays.
- Decoding at the destination is done by JDD.

Relaying Techniques - Compress and forward

- For the **discrete-time real gaussian signal** over oblivious diamond relay Gaussian channel, the optimal solution was shown in [Sanderovich, Shamai, Steinberg and Kramer, 2008].
- $SNR_1 = SNR_2 = P_{CF}$
- Fronthaul rate = C_{CF} [bits/channel use]
- The system rate in [bits/channel use] is

$$R_{\mathrm{CF}} = \frac{1}{2}\log_2\left[1 + 2P_{\mathrm{CF}} \cdot 2^{-4C_{\mathrm{CF}}} \cdot \left(2^{4C_{\mathrm{CF}}} + P_{\mathrm{CF}} - \sqrt{P_{\mathrm{CF}}^2 + (1 + 2P_{\mathrm{CF}}) \cdot 2^{4C_{\mathrm{CF}}}}\right)\right]$$

Relaying Techniques - Decode and forward

- The relays are non-oblivious.
- Each relay decodes the message and sends half of the bits to the CU.
- The CU combines the bits received from the relays.
- SNR_1=SNR_2 = P_{CF}
- Fronthaul rate = C_{DF} [bits/channel use]
- The system rate in [bits/channel use] is the known broadcast channel capacity

$$R_{\mathrm{DF}} = \frac{1}{2}\log_2(1+P_{\mathrm{DF}})$$

s.t. $C_{\mathrm{DF}} \geq \frac{R_{\mathrm{DF}}}{2} = \frac{1}{4}\log_2(1+P_{\mathrm{DF}})$ [bits/channel use]

Relaying Techniques - Time-sharing

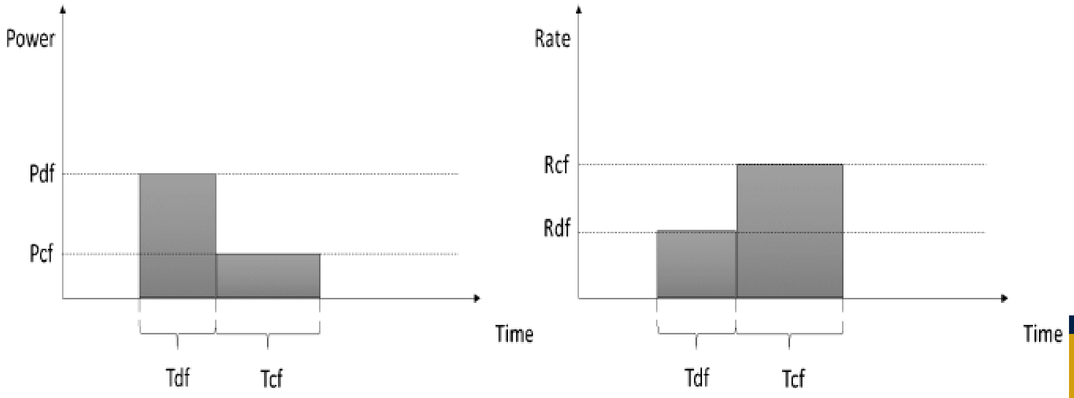
We use time sharing with both CF and DF.

In the first phase, both relays transmit DF over time fraction T_{DF} with power P_{DF} and fronthaul rate C_{DF}.

In the second phase both relays transmit CF over time fraction T_{CF} with power P_{CF} and fronthaul rate C_{CF}.

The time allocation is done according to the total average power and rate constraints.

$$R_{TS} = T_{DF} \cdot R_{DF}(P_{DF}) + T_{CF} \cdot R_{CF}(P_{CF}, C_{CF})$$



$$0 \leq T_{\mathrm{DF}} \leq 1$$

$$0 \leq T_{\mathrm{CF}} \leq 1$$

$$0 \leq T_{\mathrm{DF}} + T_{\mathrm{CF}} \leq 1$$

$$0 \leq T_{\mathrm{DF}} \cdot P_{\mathrm{DF}} + T_{\mathrm{CF}} \cdot P_{\mathrm{CF}} \leq P$$

$$0 \leq T_{\mathrm{DF}} \cdot C_{\mathrm{DF}} + T_{\mathrm{CF}} \cdot C_{\mathrm{CF}} \leq C$$

Diamond relay channel with correlated noise

$$Y_1 = X_{\text{CF}} + N_1$$
$$Y_2 = X_{\text{CF}} + N_2$$

- A correlated noise can be caused, for example, by a jammer that interfere both relays, a scheme that was previously investigated for infinite fronthaul rate in [Sanderovich et al., 2011].
- For DF operation, correlation between N_1 and N_2 does not affect the system rate because the message is decoded separately at each relay so there is no information about the other relay signal that could be used during decoding.
- For CF method, correlation between N_1 and N_2 does affect the system performance. We next calculate the CF system rate for correlated noise

$$R_{\text{CF,corr}}(P_{\text{CF}}, C_{\text{CF}}, \rho)$$

Compress and forward with correlated noise (cont.)

- We first express the CF system rate for non correlated noise as an equivalent AWGN:

$$I(X;\hat{X}) = R_{CF} = \frac{1}{2}log_2\left[1 + \underbrace{2P_{CF}\cdot 2^{-4C_{CF}}\cdot\left(2^{4C_{CF}} + P_{CF} - \sqrt{P_{CF}^2 + (1+2P_{CF})\cdot 2^{4C_{CF}}}\right)}_{SNR_{eq}}\right]$$

$$\hat{X} = \sqrt{SNR_{eq}}\cdot\frac{X}{\sqrt{P_{CF}}} + N_3 \qquad N_3 \sim N(0,1)$$

- For positive correlation ρ, we can express the noises as

$$N_1 = \sqrt{\rho}\cdot L + M_1 \qquad L \sim N(0,1) \qquad E(M_1\cdot M_2) = 0$$
$$N_2 = \sqrt{\rho}\cdot L + M_2 \qquad M_1, M_2 \sim N(0, 1-\rho) \qquad E(L\cdot M_i) = 0$$

- Then the equivalent channel is

$$Y_1 = X + \sqrt{\rho}\cdot L + M_1$$
$$Y_2 = X + \sqrt{\rho}\cdot L + M_2$$

Compress and forward with correlated noise (cont.)

- We now combine X and L and treat them as a single CF user W, so we can use the CF system rate for non correlated noise.

$$Y_1 = X + \sqrt{\rho} \cdot L + M_1$$
$$Y_2 = \underbrace{X + \sqrt{\rho} \cdot L}_{\text{CF user}} + M_2$$

- We then normalize the independent noise power in Y_i, so the equivalent channel is

$$Y_{1\rho} = \sqrt{SNR_\rho} \cdot W + \nu_1 \qquad \nu_1, \nu_2 \sim N(0,1)$$
$$Y_{2\rho} = \sqrt{SNR_\rho} \cdot W + \nu_2 \qquad W \sim N(0,1)$$

$$W = \frac{X + \sqrt{\rho} \cdot L}{\sqrt{P_{CF} + \rho}} \qquad SNR_\rho = \frac{P_{CF} + \rho}{1 - \rho}$$

Compress and forward with correlated noise (cont.)

- Now as before, we can treat this CF system as an equivalent AWGN

$$\hat{X}_\rho = \sqrt{SNR_{eq,\rho}} \cdot W + \nu_3$$

$$SNR_{eq,\rho} = 2 \cdot SNR_\rho \cdot 2^{-4C_{CF}} \cdot \left(2^{4C_{CF}} + SNR_\rho - \sqrt{SNR_\rho^2 + (1 + 2 \cdot SNR_\rho) \cdot 2^{4C_{CF}}}\right)$$

- The destination then extracts X from W, treating L as noise.

$$\hat{X}_\rho = \sqrt{SNR_{eq,\rho}} \cdot W + \nu_3 = \frac{\sqrt{SNR_{eq,\rho}}}{\sqrt{P_{CF} + \rho}} \cdot X + \frac{\sqrt{SNR_{eq,\rho}}}{\sqrt{P_{CF} + \rho}} \cdot \sqrt{\rho} \cdot L + \nu_3$$

- Therefore, the final SNR is

$$SNR_F = \frac{SNR_{eq,\rho} \cdot P_{CF}}{(P_{CF} + \rho) \cdot \left(\frac{SNR_{eq,\rho} \cdot \rho}{P_{CF} + \rho} + 1\right)} = \frac{SNR_{eq,\rho} \cdot P_{CF}}{SNR_{eq,\rho} \cdot \rho + P_{CF} + \rho}$$

- And the CF system rate is then

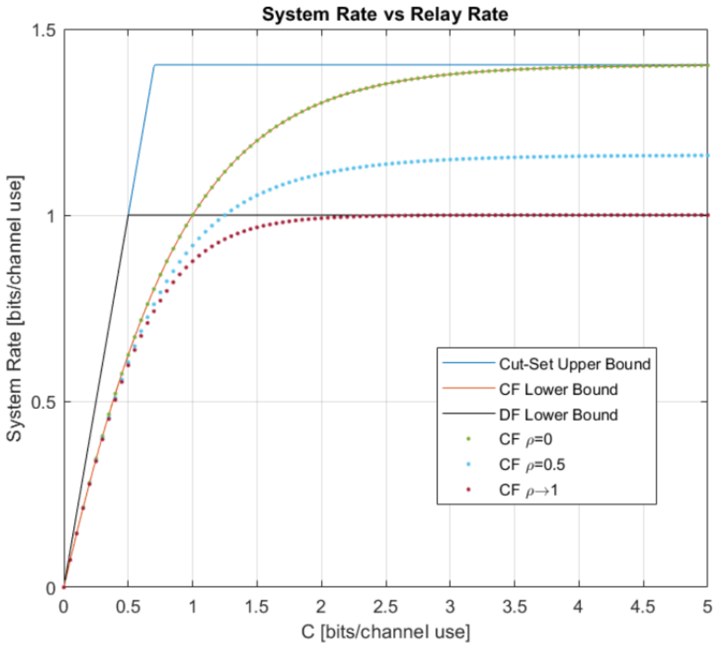
$$R_{CF,\rho} = I(X; \hat{X}_\rho) = \frac{1}{2} \log_2(1 + SNR_F)$$

Compress and forward with correlated noise (cont.)

- For $\rho = 0$ we get the expected CF rate for non correlated noise.
- For $\rho = 1$ we get the classical standard bottleneck result with fronthaul capacity $2C$

$$R_{CF,\rho=1} = \frac{1}{2}\log_2(1+SNR) - \frac{1}{2}\log_2(1+SNR\cdot 2^{-4C})$$

- The noise correlation reduces the CF system rate, with higher affect in high fronthaul rates.

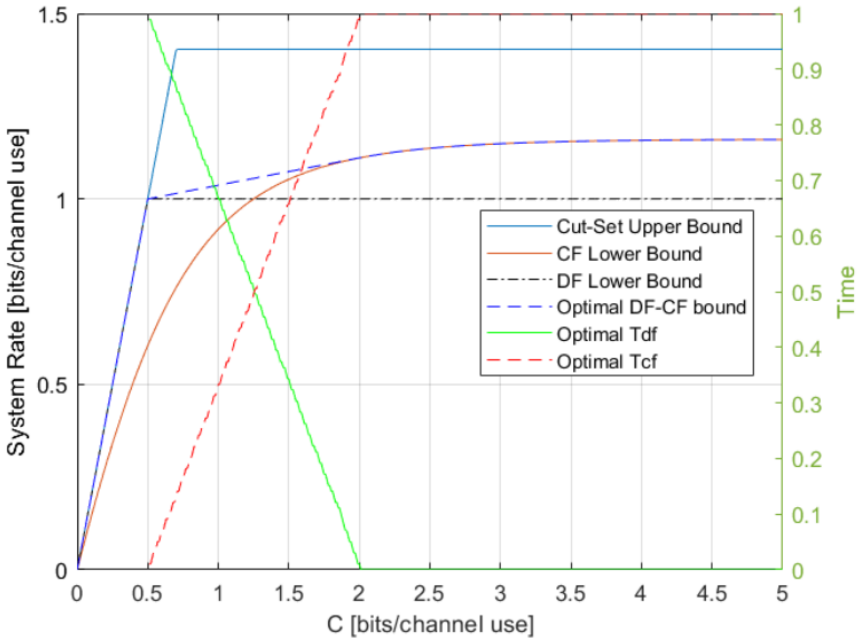


$P = 3$

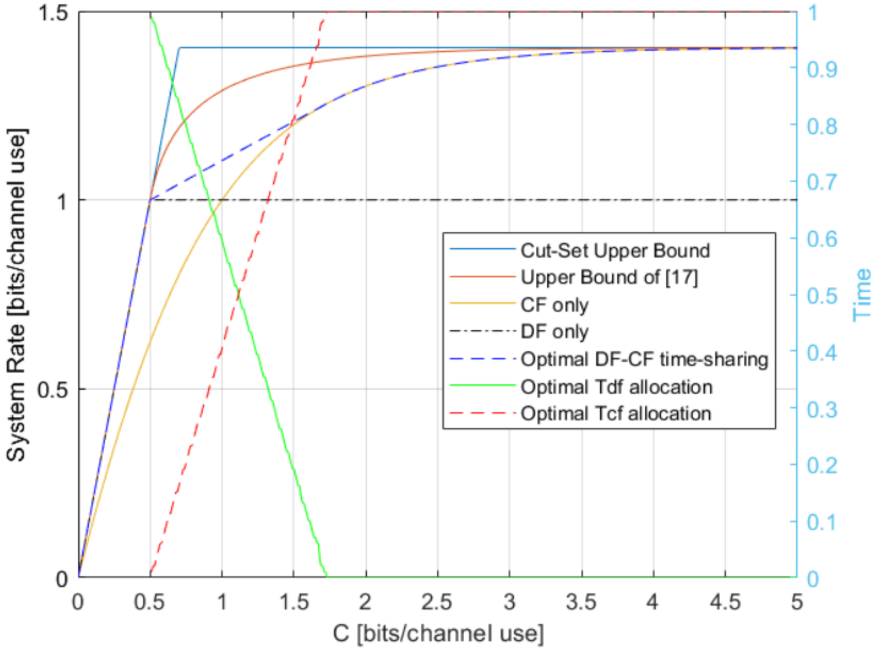
Diamond relay channel with correlated noise – Time-sharing solution for positive correlation

As the correlation increases DF is being preferred over CF.

The system rate is lower than the non correlated case.



P=3 , $\rho = 0.5$



P=3 , $\rho = 0$

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Dirty paper coding relaying

- This scheme, proposed in [Katz-Peleg-Shamai, 2023], is based on the writing on dirty paper scheme [Costa, 1983].

$$X \sim N(0,P) \qquad Y = X + S + N \qquad \alpha^* = \frac{P}{P+N}$$
$$S \sim N(0,Q) \qquad U = X + \alpha S$$
$$N \sim N(0,N)$$

- Transmit DF letter along with CF letter:

$$Y_1 = X_{\mathrm{DF,DPC}} + X_{\mathrm{CF}} + N_1$$
$$Y_2 = X_{\mathrm{DF,DPC}} + X_{\mathrm{CF}} + N_2$$

- The DF letter is encoded such that the CF letter is treated as interference, known noncausally at the transmitter.

- Decoding the DF message at the relays achieves the DF performance, as there is no CF letter (interference).

$$R_{\mathrm{DF}}\left(P_{\mathrm{DF}}\right) = I(U;Y) - I(U;S) = \frac{1}{2}\log_2\left(1 + P_{\mathrm{DF}}\right)$$

Dirty paper coding relaying (cont.)

- The DF message is decoded at the relays and sent to the destination, but the DF signal cannot be fully revealed (for the classical dirty-paper coding $I(Y,U;S) = I(Y;S)$), so it acts as correlated noise for the CF operation.

$$Y_1 = X_{\mathrm{CF}} + X_{\mathrm{DF,DPC}} + N_1$$

$$Y_2 = X_{\mathrm{CF}} + \underbrace{X_{\mathrm{DF,DPC}} + N_2}_{\text{Noise for CF at the relay}}$$

- The CF operation is performed, considering correlated noise. The CF message is then decoded from the information received from both relays.
- The CF system rate for correlated noise (correlation index ρ) is derived in this work.

$$R_{\mathrm{CF,corr}}(P_{\mathrm{CF}}, C_{\mathrm{CF}}, \rho)$$

Dirty paper coding relaying (cont.)

- Transmit DF letter along with CF letter:

$$Y_1 = X_{\text{DF,DPC}} + X_{\text{CF}} + N_1$$
$$Y_2 = X_{\text{DF,DPC}} + X_{\text{CF}} + N_2$$

- The total system performance is

$$R_{\text{DPC}} = R_{\text{DF}}\left(P_{\text{DF}}\right) + R_{\text{CF,corr}}(P_{\text{CF}}, C_{\text{CF}}, \rho)$$

Dirty paper coding relaying (cont.)

- Three modifications for the proposed dirty paper coding are considered in [Katz-Peleg-Shamai, 2023]:

    1. Optimize the user transmission by choosing α that maximizes the total system rate (and not just the DF rate). For $\alpha = \alpha^*$ we get the classical DPC approach.
    2. Modify the relay input after decoding the DF message, using the reconstructed signal $\mathrm{U} = X_{DF} + \alpha X_{CF}$. One example is subtracting this signal from the relay input before the CF operation, which results in eliminating the DF signal, but also affecting the CF signal and reducing its SNR.
       For $\alpha = 0$ we get the SPC scheme as a special case.
    3. Optimize the CF operation by using the DPC signal $X_{DF} + \alpha X_{CF}$ as a user that can be decoded so we can exploit more information at the destination, this is instead of using the DF signal as a virtual user.

We evaluated the performance of those modified schemes and compared to the suggested DPC scheme. **No improvement was seemed in the system performance**.

Conclusions and outlook

- In this work we extended the diamond relay channel model to the case of correlated noises.
- We derived a closed form formula of the oblivious CF system rate for the positive correlated noise case.
- We examined the effect of the noise correlation on the CF system rate, showing that the CF system rate is reduced with higher positive correlation.
- An optimal time-sharing solution for the discrete time case is presented, showing that DF is being more preferred because CF rate is reduced.

Conclusions and outlook (cont.)

- The effect of negative correlation is presented in [Katz-Peleg-Shamai, 2023], where it is proved that time-sharing is advantageous over SPC also in the correlated noise case.
- Using the results of CF rate for correlated noise, a DPC scheme that is based on [Costa, 1983] is proposed in [Katz-Peleg-Shamai, 2023], extending the single relay DPC scheme of [Peleg-Shamai, 2017].
  It is proved in [Katz-Peleg-Shamai, 2023] that time-sharing is advantageous over the DPC scheme.
- The effect of the noise correlation on the optimal frequency allocation is also presented in [Katz-Peleg-Shamai, 2023]. For positive correlation the optimal frequency allocation changes so that DF is being more preferred, but the allocated bandwidth remains the same.

Conclusions and outlook (cont.)

- For future work we suggest to consider the correlated noise case for the asymmetric diamond channel.
- It be interesting to consider in future work a non-Gaussian input that might have better performance than Gaussian, as demonstrated in [Sanderovich et al., 2008].
- An interesting challenge is to improve the cut-set upper bound and compare the achievable rates to a tighter upper bound. An example of an improved upper bound for the non correlated noise case is presented in [Wu et al., 2019]

Thank you!

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"Gaussian Primitive Diamond Channel: Correlated Noise at Relays and Relevant Applications,"

Abstract—We investigate the special case of a symmetric Gaussian primitive diamond relay channel comprising a correlated Gaussian noise channel, and fronthaul links with limited rate from the relays to the destination. We use a combination of oblivious compress and forward (CF) with distributed Wyner-Ziv compression, and decode and forward (DF) techniques, where each relay decodes the whole message and sends half of its bits to the destination. We derive the CF achievable rate and investigate the effect of positive noise correlation on the system performance. It is shown that CF-DF time-sharing scheme is advantageous over a CF-DF superposition coding (SPC) approach, as well as on a combination of CF and DF, based on dirty paper coding (DPC). The optimal time-sharing proportion between CF and DF is calculated for positive noise correlation.

Joint work with: A. Katz, M. Peleg, H.V. Poor.

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