Probabilistic Constellation Shaping Algorithms: Performance vs. Complexity Trade-offs

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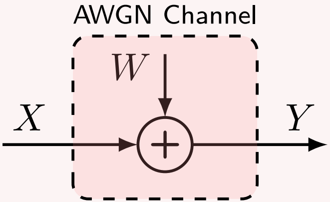
@Technische Universität München

What is This Presentation About?

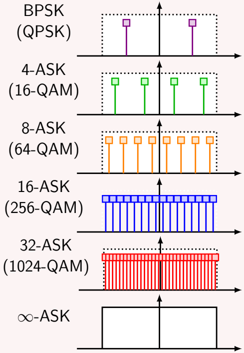
- Shaping
  - Probabilistic shaping
    - Sphere shaping
      - Enumerative sphere shaping (ESS)
      - Applied to the AWGN and NL fiber-optic channels
      - With complexity concerns
      - With an attempt at FPGA implementation

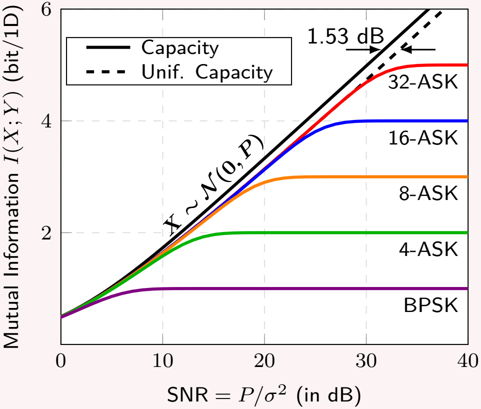
Motivation: The Shaping Gap

Achieving the Capacity of the AWGN Channel



- $E[X^2] = P$
- $W \sim \mathcal{N}(0, \sigma^2)$
- $\text{SNR} = P/\sigma^2$
- $C = \frac{1}{2}\log_2(1 + \text{SNR})$

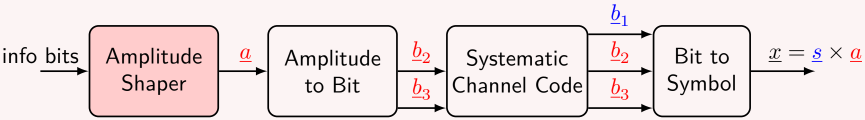


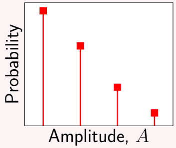


The gap can be closed by transmitting non-uniform (i.e., shaped) X.

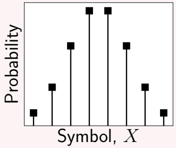
Probabilistic Amplitude Shaping (PAS)

[1] PAS with 8-ASK (64-QAM)





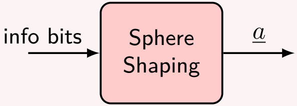
A	7	5	3	1	1	3	5	7
S	-1	-1	-1	-1	1	1	1	1
X	-7	-5	-3	-1	1	3	5	7
B_1	0	0	0	0	1	1	1	1
B_2	0	0	1	1	1	1	0	0
B_3	0	1	1	0	0	1	1	0



[1] Böcherer et al., "Bandwidth Efficient and Rate-Matched LDPC Coded Modulation," IEEE Trans. Commun., 2015.

Sphere shaping (1/3)

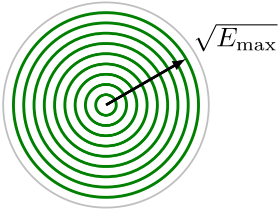
Sphere Shaping

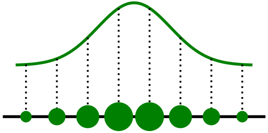


- $a_i \in \mathcal{A}_{M\text{-ASK}}$

$$\left\{ \underline{a} = (a_1, a_2, \ldots, a_N) \middle| \sum_{n=1}^{N} a_n^2 \leqslant E_{\max} \right\}$$

- As $N \to \infty$, $P(a) \to$ Sampled Gaussian (MB)
- As also $M \to \infty$, $P(a) \to$ Gaussian





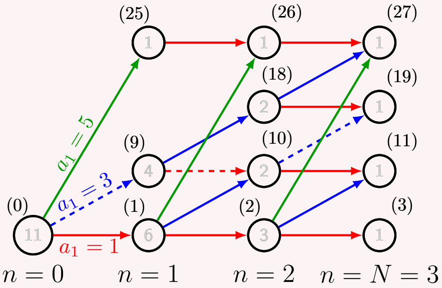
Implementation

- Shell mapping (SM) [2], [3]
- **Enumerative sphere shaping (ESS)** [4], [5]

[2] R. Laroia et al., "On optimal shaping of multidimensional constellations," IEEE Trans. Inf. Theory, 1994.
[3] P. Schulte et al., "Divergence-Optimal Fixed-to-Fixed Length DM with SM," IEEE Wireless Commun. Lett., 2019.
[4] F. Willems et al., "A pragmatic approach to shaped coded modulation," Symp. Commun. Veh. Technol. in the Benelux, 1993.
[5] Y.C. Gültekin et al., "ESS for Wireless Communications with Short Packets," IEEE Trans. Wireless Commun., 2020.

Enumerative Sphere Shaping

$N = 3,\ \mathcal{A} = \{1,3,5\},\ E_{\max} = 27$



	e		e		e
$(1,1,1)$	3	$(1,3,3)$	19	$(3,3,1)$	19
$(1,1,3)$	11	$(1,5,1)$	27	$(3,3,3)$	27
$(1,1,5)$	27	$(3,1,1)$	11	$(5,1,1)$	27
$(1,3,1)$	11	$(3,1,3)$	19		

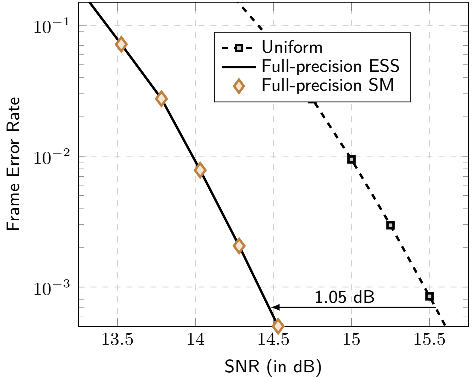
- ESS requires $+, -$, and storage of $\boldsymbol{T}$

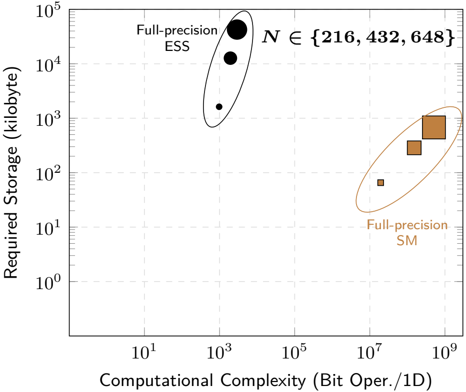
$$\boldsymbol{T} = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 2 & 1 \\ 0 & 4 & 2 & 1 \\ 11 & 6 & 3 & 1 \end{bmatrix} \qquad \boldsymbol{F} = \begin{bmatrix} 0 & 1 & 2 & 4 \\ 0 & 0 & 1 & 3 \\ 0 & 1 & 2 & 3 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

- SM requires $\times, \div$, and $\log N$ columns of $\boldsymbol{F}$

Sphere Shaping (2/3)

- 64-QAM, 4 bit/2D, 648-bit Wi-Fi LDPC codes ($N = 216$), AWGN channel

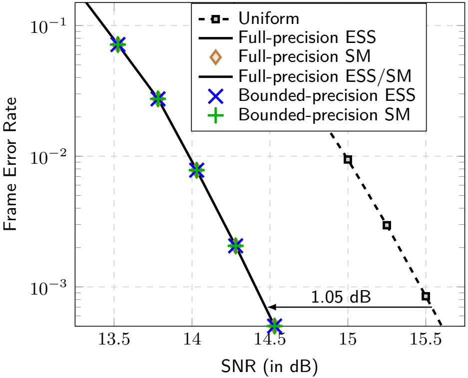


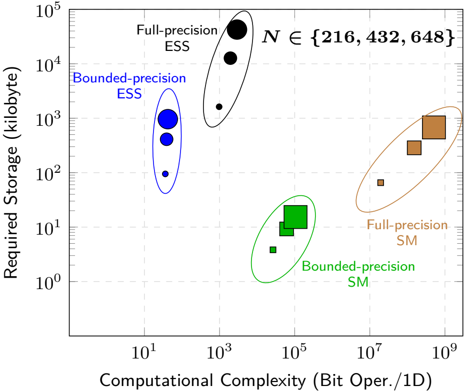


Problem: Too complex!

Sphere Shaping (3/3)

- 64-QAM, 4 bit/2D, 648-bit Wi-Fi LDPC codes ($N = 216$), AWGN channel



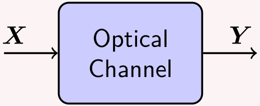


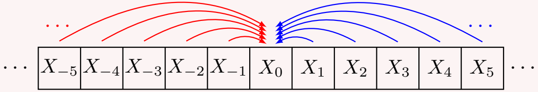
Solution: Bounded-precision implementation to limit the required storage [6].

[6] Y.C. Gültekin et al., “Approximate Enumerative Sphere Shaping,” ISIT, 2018.

What About Nonlinear Fiber-optic Channels?

Optical Channels & Nonlinear Interference





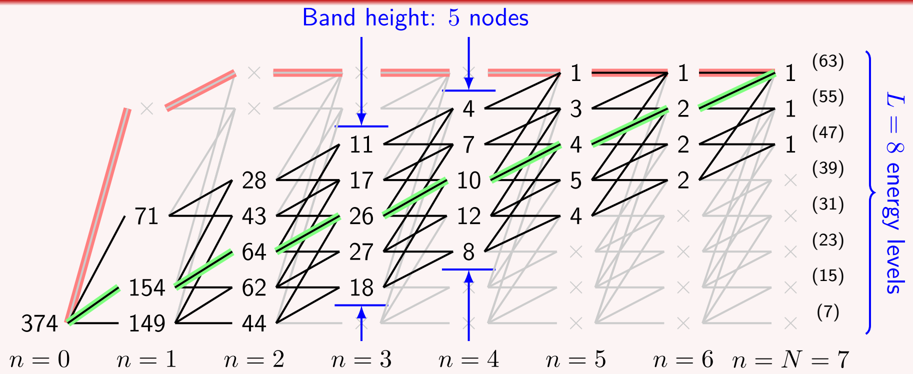
- Can be modeled as a **time-varying ISI channel**:
  - All **past** and **future** symbols create NL interference [7], [8].
  - **Variations in the energy** of the TX signal generate nonlinear distortion [9].

Solution: Modified sphere shaping to decrease energy variations.

[7] M. Secondini et al., "AIR in NL WDM Fiber-Optic Systems w/ Arbitrary Mod and Dispersion Maps," JLT, 2013.
[8] R. Dar et al., "Inter-Channel NLIN in WDM Systems: Modeling and Mitigation," JLT, 2015.
[9] O. Geller et al., "A Shaping Algorithm for Mitigating Inter-Channel NLPN in NL Fiber Systems," JLT, 2016.

Band-trellis ESS (B-ESS) [10] (1/2)

Removing sequences w/ high energy variations: Band-trellis

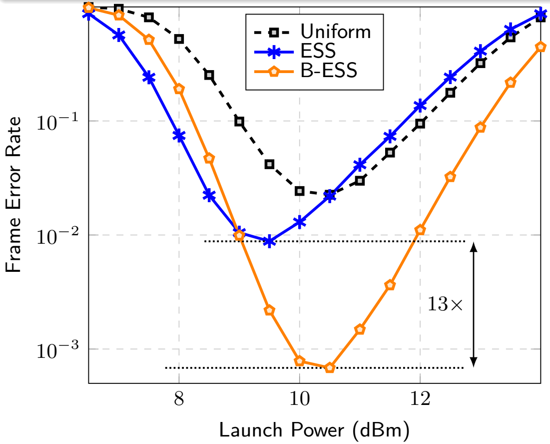


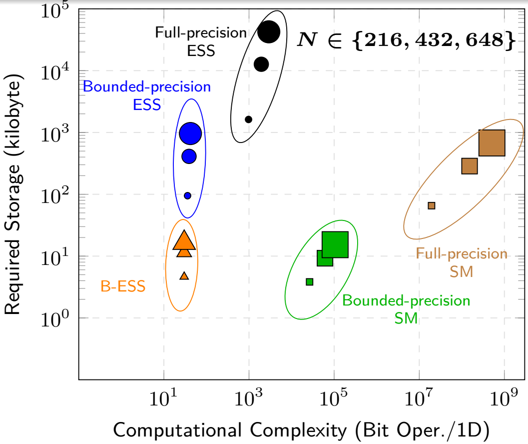
- $\underline{a} = (3, 3, 3, 3, 3, 3, 3) \rightarrow \text{Var}(A^2) = 0$ (in the sphere & in the band)
- $\underline{a} = (7, 3, 1, 1, 1, 1, 1) \rightarrow \text{Var}(A^2) = 274$ (in the sphere but **not** in the band)

[10] Y.C. Gültekin et al., "Mitigating Nonlinear Interference by Limiting Energy Variations in Sphere Shaping," OFC, 2022.

Band-trellis ESS (B-ESS) (2/2)

Narrow-band B-ESS $\rightarrow$ Good for nonlinear fiber-optic channels!

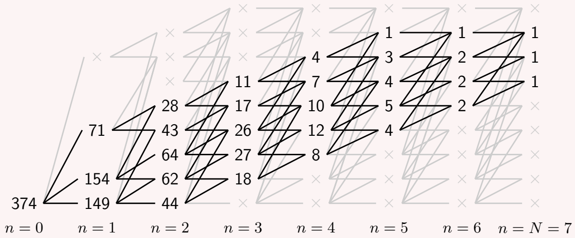




Can we further simplify? $\rightarrow$ **Binary-shift-based implementation**

Shift-based B-ESS [11] (1/2)

$$\boldsymbol{T} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 4 & 3 & 2 & 1 \\ 0 & 0 & 0 & 11 & 7 & 4 & 2 & 1 \\ 0 & 0 & 28 & 17 & 10 & 5 & 2 & 0 \\ 0 & 71 & 43 & 26 & 12 & 4 & 0 & 0 \\ 0 & 0 & 64 & 27 & 8 & 0 & 0 & 0 \\ 0 & 154 & 62 & 18 & 0 & 0 & 0 & 0 \\ 374 & 149 & 44 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$



$$\boldsymbol{t}_n = \boldsymbol{A}\boldsymbol{t}_{n+1} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix} \boldsymbol{t}_{n+1}$$

- $\boldsymbol{t}_n \approx \rho(\boldsymbol{A})\boldsymbol{t}_{n+1}$ (for N large enough)

Shift-based B-ESS:

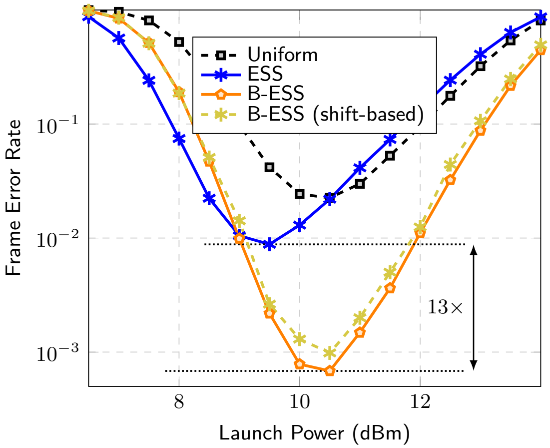
$$\boldsymbol{t}_n \approx \rho^m \boldsymbol{t}_{n+m}$$

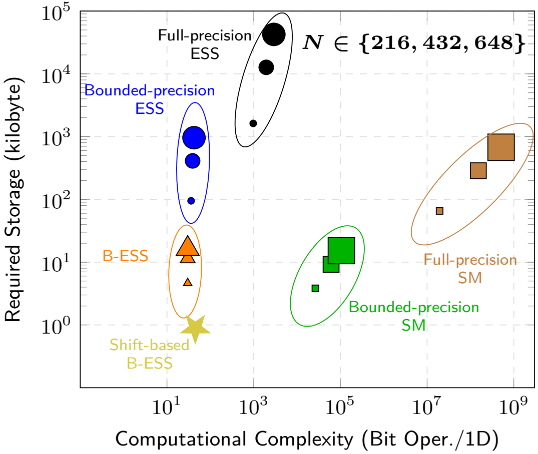
- **Select** m s.t. $\rho^m \approx 2^\alpha$ for integer α
- **Then:** Multiplication $\rightarrow$ α-bit shift

[11] Y.C. Gültekin et al., "Band-ESS: Streaming Enumerative Coding with Applications to Prob. Shaping," GLOBECOM, 2022.

Shift-based B-ESS (2/2)

Narrow-band B-ESS → Good for nonlinear fiber-optic channels!

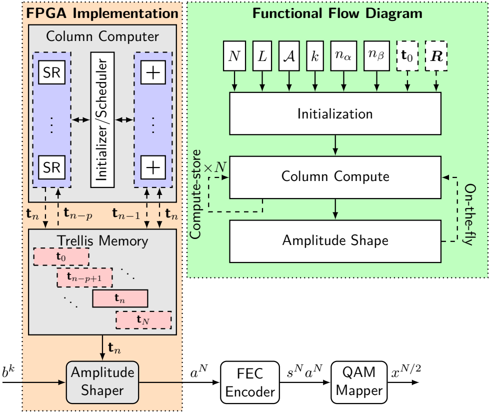




FPGA Implementation

	Technique	FPGA (Xilinx)	f_{CLK} (MHz)	Thr. (Mbps)	Thr./Area (Kbps/LUT)	LUT #	Reg. #
Literature	HiDM [12]	XCVU9P	240	2260	1.91	1182k	2364k
	PCDM [13]	XCKU040	146	1680	124	13.6k	3.5k
	HiDM [14]	XCVU125	100	3200	586	5.5k	4.9k
	ESS [15]	XCVU125	100	1600	18.9	95k	140k
	BP BS-B-ESS [16]	XCZU5CG	770	1540	50.2	4.4k	1.5k

†Some values are deduced from limited information provided in the referenced papers.



[12] T. Yoshida et al., "Compressed shaping: concept and FPGA demonstration," JLT, 2021.
[13] Q. Yu et al., "FPGA implementation of rate-adaptable prefix-free code DM for PCS," JLT, 2021.
[14] L. Zhang et al., "Real-time FPGA investigation of interplay between PS and FEC," JLT, 2022.
[15] L. Zhang et al., "FPGA investigation of ESS for PS 64QAM," CLEO, 2022.
[16] Y.C. Gültekin et al., "Band-ESS: Streaming enumerative coding with applications to PS," GLOBECOM, 2022.

Discussion

Take-home Messages

1. **Probabilistic shaping** increases achievable rates for both the AWGN channel and nonlinear fiber-optic channels.
2. Improved performance can be obtained via different techniques, creating a trade-off between **performance**, **storage complexity**, and **computational complexity**.
3. Recently introduced **binary-shift-based B-ESS** provides improved performance with very small storage and computational complexity.

Advertisement Time

2024 European School of Information Theory (ESIT)

July 1-5, 2024, Eindhoven, The Netherlands

Confirmed tutorial speakers (dates tentative):

- **Tutorial 1:** Prof. Prakash Narayan, University of Maryland, USA
- **Tutorial 2:** Prof. Osvaldo Simeone, King's College, UK
- **Tutorial 3:** Prof. Muriel Medard, Massachusetts Institute of Technology, USA, and Prof. Ken Duffy,
- **Tutorial 4:** Prof. Albert Guillén i Fàbregas, University of Cambridge, UK

Confirmed seminar speakers (dates tentative):

- **Seminar 1:** Prof. Marco Secondini, Sant'Anna School of Advanced Studies, Italy
- **Seminar 2:** Prof. Mario Berta, RWTH Aachen University, Germany
- **Seminar 3:** Prof. Laurent Schmalen, Karlsruhe Institute of Technology, Germany
- **Seminar 4:** Prof. Si-Hyeon Lee, Korea Advanced Institute of Science & Technology, South Korea

Confirmed high-tech industry speakers:

- Dr. Roel Maes, TU/e and Intrinsic ID, the Netherlands
- Prof. Andreas Burg, École Polytechnique Fédérale de Lausanne, Switzerland
- Dr. Gianluigi Liva, Deutsches Zentrum für Luft- und Raumfahrt, Germany*

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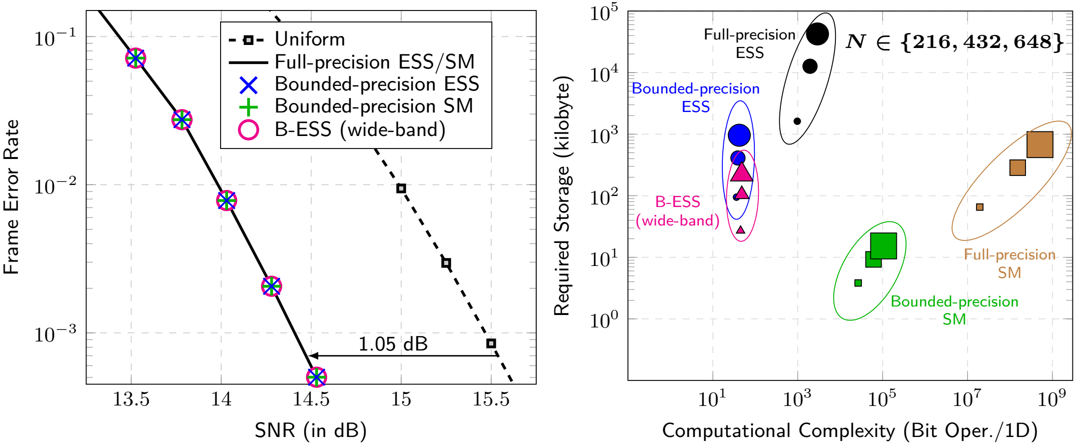
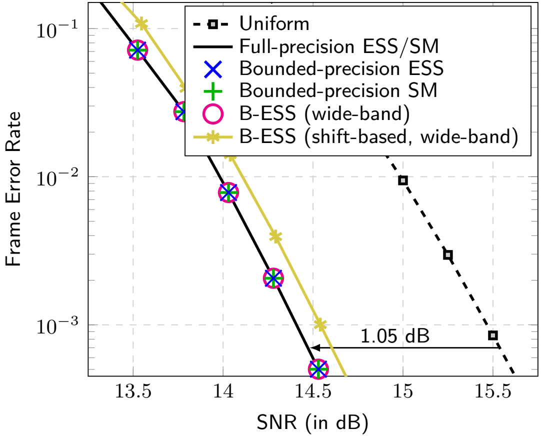
Band-trellis ESS (B-ESS) (AWGN) (1/2)

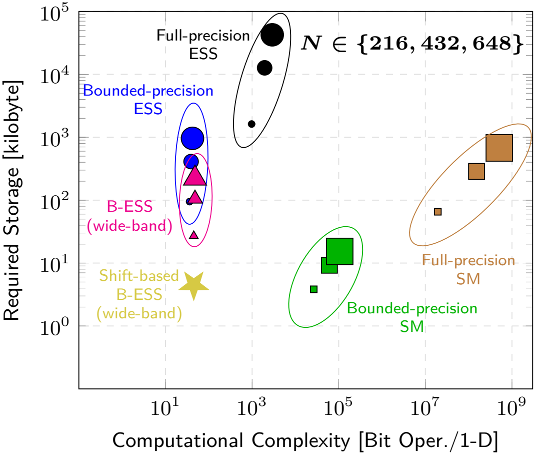
Wide-band B-ESS $\approx$ **ESS** $\rightarrow$ Good for the AWGN channel!



Band-trellis ESS (B-ESS) (AWGN) (2/2) (Shift-based)

Wide-band B-ESS $\approx$ **ESS** $\rightarrow$ Good for the AWGN channel!





SSMF via SSFM: Setup

Parameter	Value
# Channels	1
Attenuation	0.2 dB/km
Dispersion	17 ps/nm/km
Distance	1-span, 205 km
NL	1.3 1/W/km
EDFA NF	5 dB
Polarization	Dual
Symbol rate	50 GBd
RRC roll-off	0.1
Constellation	64-QAM
Raw data rate	600 Gbps
TX rate	4 bit/2D
Code rate	2/3 (U), 5/6 (PS)
PS rate	1.5 bit/amp.
PS N	108
Net data rate	400 Gbps
FEC	648-bit 802.11 LDPC