



Soft-aided Decoding of Product Codes for Optical Communications

Laurent Schmalen, Sisi Miao, Lukas Rapp



Overview



Motivation

Iterative Bounded Distance Decoding of PCs

Decoder with Dynamic Reliability Scores

List-Based Decoder with Dynamic Reliability Scores



Motivation



Product codes (PCs) [Eli55] are powerful code construction for high speed optical communication systems
 Conventional decoding of PCs

Soft-decision decoding (SDD)

e.g.: turbo product decoding (TPD) [Pyn98]

- ✓ large decoding gain
- X high complexity
- X high internal decoder data flow

Hard-decision decoding (HDD)

e.g.: iterative bounded distance decoding (iBDD)

- X small decoding gain
- ✓ low complexity
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- [Eli55] P. Elias, "Coding for noisy channels," in IRE Convention Record, Part IV, Mar. 1955.
- [Pyn98] R. M. Pyndiah, "Near-optimum decoding of product codes: Block turbo codes, "IEEE Trans. Commun., vol. 46, no. 8, 1998.
- [LSA19] G. Liga, A. Sheikh, and A. Alvarado, "A novel soft-aided bit-marking decoder for product codes," in *Proc. ECOC.* 2019.
- [SAA21] A. Sheikh, A. Graell i Amat, and A. Alvarado, "Novel High-Throughput Decoding Algorithms for Product and Staircase Codes Based on Error-and-Erasure Decoding," *J. Lightw. Technol.*, vol. 39, no. 15, 2021.
- [MRS22] S. Miao, L. Rapp, and L. S., "Improved soft-aided decoding of product codes with dynamic reliability scores," J. Lightw. Technol., vol. 40, no. 22, pp. 7279-7288, 2022.



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This work: **soft-aided HDD** with

Hard-decision decoding (HDD)

e.g.: iterative bounded distance decoding (iBDD)

- X small decoding gain
- ✓ low complexity
- ✓ low internal decoder data flow
- ✓ low complexity and reduced internal decoder data flow with hard message passing
- ✓ provide largest decoding gain among existing soft-aided algorithms [LSA19],[SAA21],[MRS22]

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PC of rate $r = k^2/n^2$ with (n, k, t) component codes, typically being BCH/RS codes





































PC of rate r = k²/n² with (n, k, t) component codes, typically being BCH/RS codes
 Rows and columns of a PC block are decoded alternately with a component code decoder D_C



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Problem: What degrades the performance of iBDD?





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- High miscorrection rate and error propagation
 - A miscorrection of $m{y}$ happens when $\mathsf{D}_\mathsf{C}(m{y}) = m{c} \in \mathcal{C}$ but $m{c}
 eq m{x}$, the transmitted codeword
 - PCs for optical communication consist of high rate BCH/RS codes which entails high miscorrection rate [MS86][Jus11]

- [MS86] R. McEliece and L. Swanson, "On the decoder error probability for Reed Solomon codes (Corresp.)," *IEEE Trans. Inf. Theory*, vol. 32, no. 5, 1986.
- [Jus11] J. Justesen, Performance of Product Codes and Related Structures with Iterated Decoding," IEEE Trans. Commun., vol. 59, no. 2, 2011.





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Problem: What degrades the performance of iBDD?

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- Component code decoder D_C limited to bounded distance, small number of correctable errors

Our solution:

- Introduce dynamic reliability score (DRS) to ensure near miscorrection free decoding
- Enhance D_C to have large decoding radius as BDD is not enough
- Find trade-off between decoding radius and miscorrection rate





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- Such bits can be identified with
 - Output channel reliability higher than threshold T_a, as in soft-aided bit marking (SABM) [LSA19], [LCL⁺19]



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[LCL⁺19] Y. Lei, B. Chen, G. Liga, X. Deng, Z. Cao, J. Li, K. Xu, and A. Alvarado, "Improved decoding of staircase codes: The soft-aided bit-marking (SABM) algorithm", *IEEE Trans. Commun.*, 2019.

[HP18] C. Häger and H. D. Pfister, "Approaching miscorrection-free performance of product codes with anchor decoding", *IEEE Trans. Commun.*, 2018.

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- Output channel reliability higher than threshold T_a, as in soft-aided bit marking (SABM) [LSA19], [LCL⁺19]
- Successfully decoded bits that rarely conflict with later component code decoding decisions, as in anchor decoding (AD) [HP18]



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- Successfully decoded bits that rarely conflict with later component code decoding decisions, as in anchor decoding (AD) [HP18]
- We introduce a dynamic reliability score (DRS) which reflect the overall reliability of a bit from both aspects



- [LSA19] G. Liga, A. Sheikh, and A. Alvarado, "A novel soft-aided bit-marking decoder for product codes", in *Proc. ECOC* (2019).
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Dynamic Reliability Score (DRS) [MRS22]



Channel model:

- BPSK signal $\tilde{x}_i \in \{0, 1\}$ transmitted over binary input AWGN channel.
- Channel output $\tilde{y}_i = (-1)^{\tilde{x}_i} + n_i$



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Channel model:

- BPSK signal $\tilde{x}_i \in \{0, 1\}$ transmitted over binary input AWGN channel.
- Channel output $\tilde{y}_i = (-1)^{\tilde{x}_i} + n_i$
- DRSs Reflect the reliability of the bits in a PC codeword
- Represented with 5 bit integers, i.e., range [0, 31]
- Initialized with soft channel information
 - Sort and divide channel outputs into even groups
 - Assign high DRS to the bits with high channel outputs



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Updating DRS



Updated with hard messages

- DRSs are increased (+1) for bits that stay constant during the decoding
- DRSs are decreased (-1) for bits where row and column decoder disagrees
- Clip to [0, 31]



Miscorrection Detection with DRSs





- The decision should not conflict with bits with high DRS (similar idea used in [HP18][LSA19])
- The sum of the DRSs of the bits that are flipped should be small



- [HP18] C. Häger and H. D. Pfister, "Approaching miscorrection-free performance of product codes with anchor decoding", IEEE Trans. Commun., vol. 66, no. 7, 2018.
- [LSA19] G. Liga, A. Sheikh, and A. Alvarado, "A novel soft-aided bit-marking decoder for product codes," in Proc. ECOC. 2019.



Enhanced Component Decoder D_C



Step 1: Introduce erasures (Ternary decision):

$$y_i = \begin{cases} ? \text{ (erasure)} & \text{if } |\tilde{y}_i| < T \\ \text{sign}(\tilde{y}_i) & \text{if } |\tilde{y}_i| \geq T \end{cases}$$





Error-and-Erasure Decoding



- Classical bounded distance error-and-erasure decoding of BCH codes by extending Gorenstein-Zierler decoding [For65]
- Can decode if D errors and E erasures if $2D + E < d_{\min}$
- **Example:** Decoding success rate of code with $d_{\min} = 6$ (BCH even-weight subcode)



[For65] G. Forney, "On decoding BCH codes," IEEE Trans. Inf. Theory, vol. 11, no. 4, pp. 549–557, 1965.



















Step 1: Introduce erasures

 $y_i = \begin{cases} ? \text{ (erasure)} & \text{if } |\tilde{y}_i| < T \\ \text{sign}(\tilde{y}_i) & \text{if } |\tilde{y}_i| \geq T \end{cases}$

Step 2: Replace the erasures with complementary binary values to generate test patterns $(p^{(1)}, p^{(2)})$

Typically
$$p^{(1)} = (0 \cdots 0),$$

 $p^{(2)} = (1 \cdots 1)$

Step 3:

Decode the test patterns with BDD
 Select the decoding results





- Textbook-style list decoding [Moo05]
- Can decode some patterns with D errors and E erasures when $2D + E \ge d_{\min}$
- **Example:** Decoding success rate of code with $d_{\min} = 6$ (BCH even-weight subcode)







Step 1: Introduce erasures

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- Modified Step 2: Replace the erasures with complementary binary random values to generate test patterns $(p_1^{(1)}, p_1^{(2)})$
 - Each decoding iteration, use different random patterns $(p_1^{(1)}, p_1^{(2)})$

Step 3:

Decode the test patterns with BDD
 Select the decoding results



[Moo05] T. K. Moon, Error Correction Coding - Mathematical Methods and Algorithms. John Wiley & Sons, Inc., 2005.



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- Textbook-style list decoding [Moo05]
- Can decode some patterns with D errors and E erasures when $2D + E \ge d_{\min}$
- Example: Decoding success rate of code with $d_{\min} = 6$ (BCH even-weight subcode) after $\ell = 5$ decoding trials















 $y_i = \begin{cases} ? \text{ (erasure)} & \text{if } |\tilde{y}_i| < T \\ \text{sign}(\tilde{y}_i) & \text{if } |\tilde{y}_i| \geq T \end{cases}$

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 Miscorrection (MC) detection







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Flow Chart of the Proposed Algorithm







Parameter Analysis – Simulation Results



Optimized erasure threshold T_{opt} for different PCs with BCH component codes (different t and even-weight subcodes)





Parameter Analysis – Simulation Results



- Optimized erasure threshold T_{opt} for different PCs with BCH component codes (different t and even-weight subcodes)
- Threshold gains compared to iBDD with 20 iterations







PC of rate 0.87, component code [255, 238, 2] even weight subcode of BCH code















- PC of rate 0.87, component code [255, 238, 2] even weight subcode of BCH code
- Percentage of marked anchor bits, percentage of wrongly marked anchor bits, and average number of miscorrections in every iteration













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- Step 2: Replace the erasures with \mathcal{J} pairs of complementary binary random values to generate test patterns $(\boldsymbol{p}_i^{(1)}, \boldsymbol{p}_i^{(2)})$
 - J is a configurable parameter for complexity-performance trade-off

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Conclusions & Outlook



- Proposed a novel soft-aided HDD algorithm with increased number of correctable errors with additional miscorrection control
- Provides comparable decoding performance with soft decision decoding
- Low complexity and internal decoder data flow
- Analysis and extension to generalized PCs and zipper codes [MMR⁺24]

[MMR ⁺ 24]	S. Miao, J. Mandelbaum, L. Rapp, H. Jäkel, L. S., "Performance analysis of generalized product codes with irregular degree distribution,"
	submitted to ISIT, https://arxiv.org/abs/2401.16977
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