Probabilistic Shaping for Trellis-Coded Modulation

Richard Wesel

Linfang Wang, Dan Song, Felipe Areces, Thomas Wiegart

Munich Workshop on Shannon Coding Techniques

April 4, 2024

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Outline

1. History of Shaping
2. A Trellis Code with a Distribution Matcher (and a CRC)
3. Labeling for both Set Partitioning and Distribution Mapping
4. CRC and sign bits have a uniform distribution
5. Analytical Union Bound for Shaped Trellis Codes
6. Selecting a Distribution Mapper
7. An Automorphism-Enabled Decoder
8. Example Designs, Simulation Results
9. Conclusions

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Maxwell-Boltzmann distribution has better mutual information than uniform signaling.

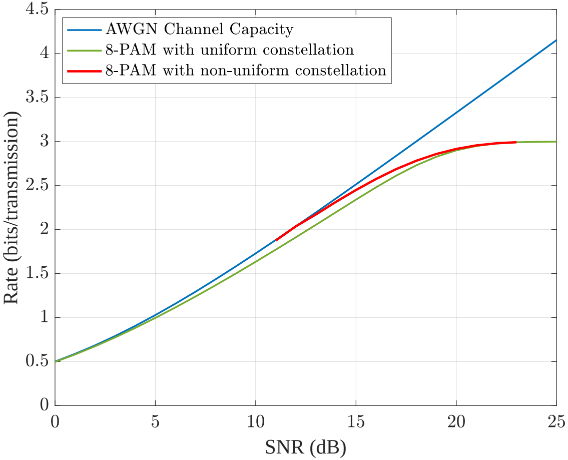
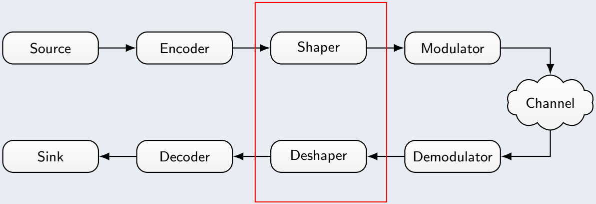


Figure 1: Shaping improves $I(X;Y)$.

Shaping after Coding

Typical encoders produce equally-likely symbols, so classical techniques tried to put the shaper after the encoder.



Examples of Shaping after Coding

Gallager et al. [1968]: Many-To-One Mapping applied to equally-likely encoder outputs.

Forney et al [1984]: try a prefix-free code to map encoder output to non-equally-likely symbols.

Forney [1992] and Laroia, Farvardin, and Tretter [1994]: Trellis Shaping

Forney's Trellis Shaping

TRANSACTIONS ON INFORMATION THEORY, VOL. 38, NO. 2, MARCH 1992

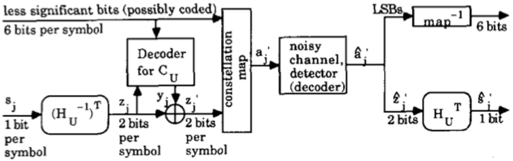


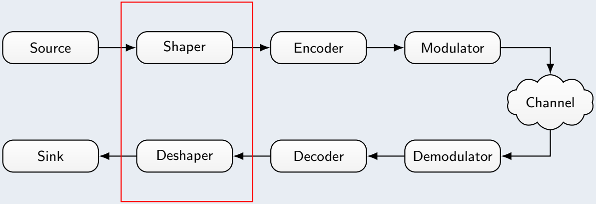
Fig. 3. Sign bit shaping system supporting $R = 7$ bits per symbol, using the 16×16 constellation of Fig. 1 and the rate-1/2 convolutional code C_U.

Nowadays we put the shaping before the coding.

The shape of things to come.

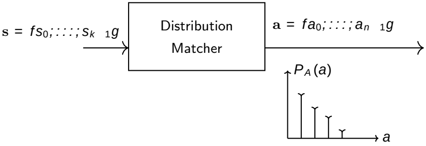
Shaping before Coding

The innovation of Steiner, Bocherer, and Schulte was to place the shaper before the encoder.



Böcherer et al. proposed the Probabilistic-Amplitude Shaping (PAS) framework that improves the spectral efficiency, i.e., transmission rate.

A PAS framework that uses 8-PAM symbols as channel inputs:



G. Böcherer *et al.*, "Bandwidth efficient and rate-matched low-density parity-check coded modulation," *IEEE Transaction on Communication, vol. 63, no. 12, pp. 4651-4665, 2015*

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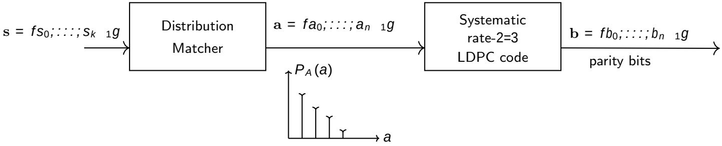

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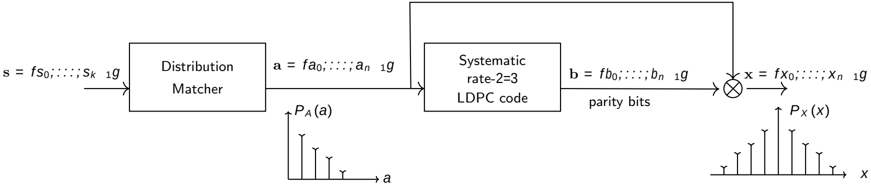

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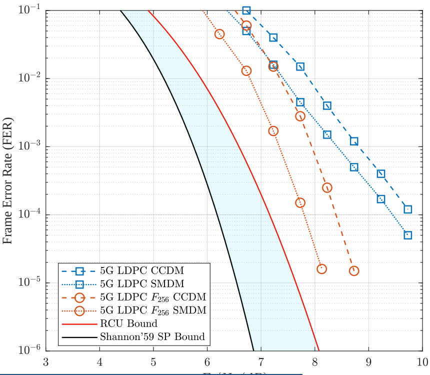
Coskun et al. studied the PAS for short blocklength transmission for LDPC, Polar, and BCH codes:

- PAS system: 96 input bits and 64 8-AM symbols.
- CCDM: Constant Composition distribution matcher;
- SMDM: Shell-mapping distribution matcher;
- Random-Coding Union (RCU) bound: A tight achievable bound for finite length code;
- Shannon's 59 Sphere packing bound: A converse bound for finite length code;



M. C. Coskun *et al.*, "Efficient error-correcting codes in the short blocklength regime," *Physical Communication, vol. 34, pp. 66-79, 2019*

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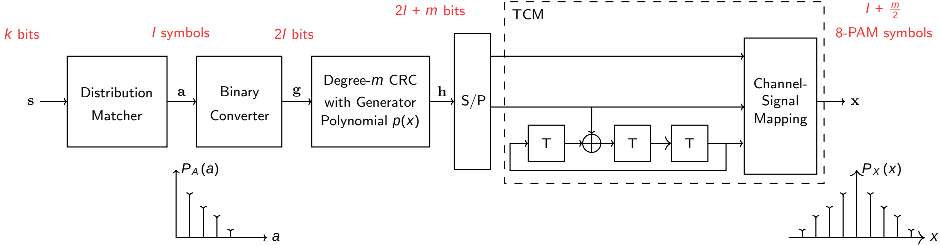
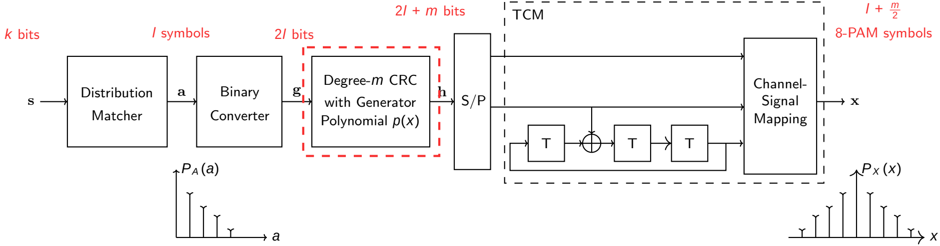


$$\mathbf{s} \in \mathbb{F}_2^k,\ \mathbf{a} \in \mathcal{C}_{\mathrm{DM}} \in \mathcal{A}^l,\ \mathbf{g} \in \mathbb{F}_2^{2l}.$$

CRC-TCM-PAS



$\mathbf{s} \in \mathbb{F}_2^k$, $\mathbf{a} \in \mathcal{C}_{\mathrm{DM}} \in \mathcal{A}^l$, $\mathbf{g} \in \mathbb{F}_2^{2l}$.

m-bits CRC encoding: $\mathbf{h} \in \mathbb{F}_2^{2l+m}$. ($m$ is divisible by 2.)

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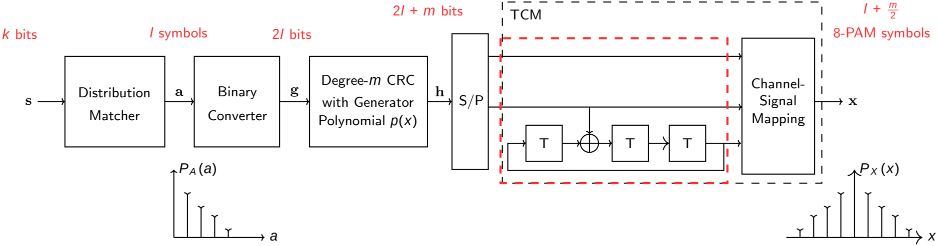

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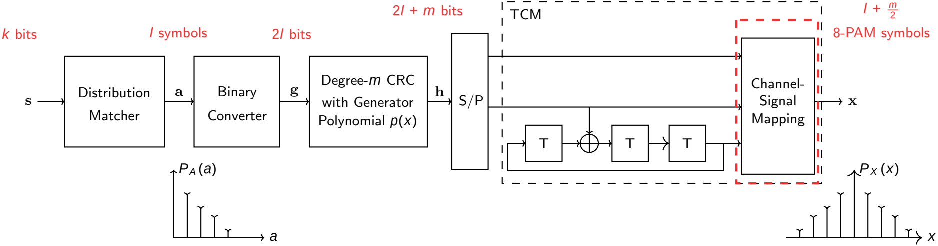
Rate-2/3, systematic, recursive, tail-biting convolutional code (TBCC).

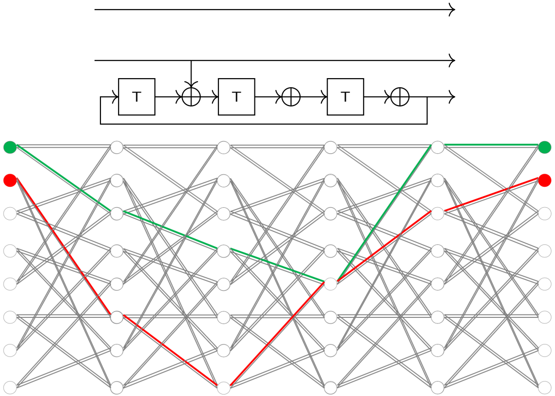


$\mathbf{s}\ 2\ \mathbb{F}_2^k$, $\mathbf{a}\ 2\ C_{DM}\ 2\ A^l$, $\mathbf{g}\ 2\ \mathbb{F}_2^{2l}$.

m-bits CRC encoding: $\mathbf{h}\ 2\ \mathbb{F}_2^{2l+m}$. ($m$ is divisible by 2.)

Rate-2=3, systematic, recursive, tail-biting convolutional code (TBCC).

Channel-signal mapping function: $\mathbf{x}\ 2\ X^{l+\frac{m}{2}}$;. $n = l + m$=2.



The starting state and ending state of a TBCC codeword are the same.

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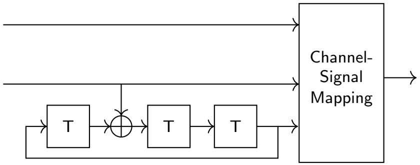
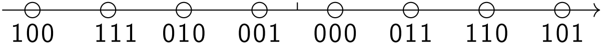
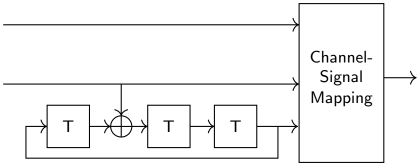


Every three bits are mapped to one of 8 PAM symbols:





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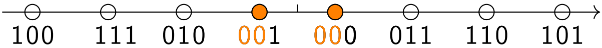

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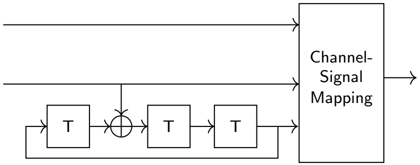

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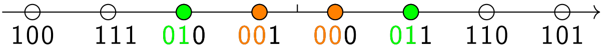

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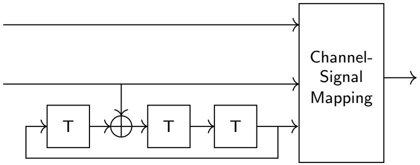

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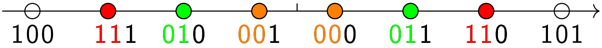

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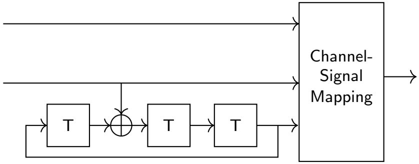

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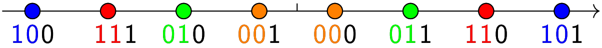

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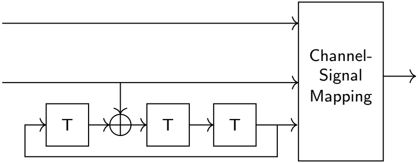

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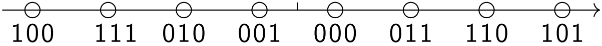

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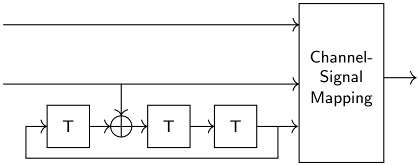

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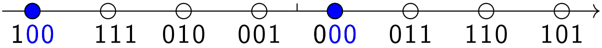

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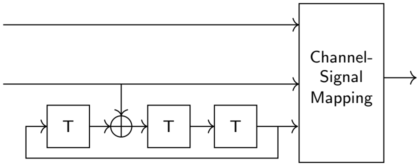

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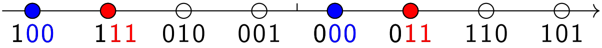

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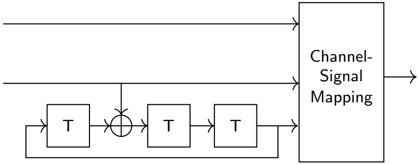

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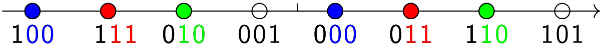

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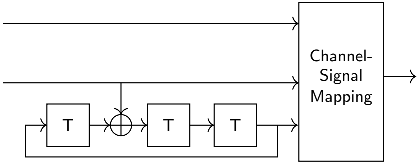

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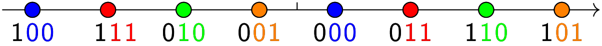

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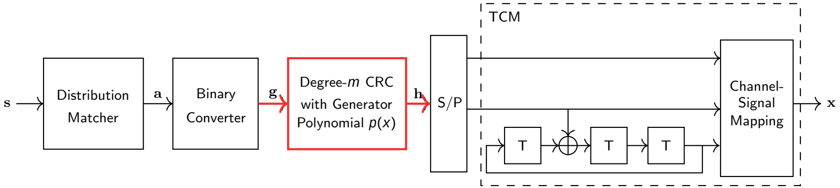


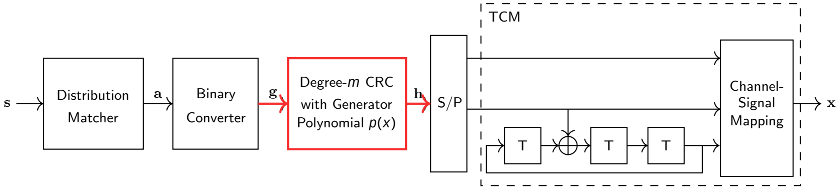
Property 1 (Uniformity of CRC bits)

Even though $\mathbf{g}$ is non-uniform, The m CRC bits, $[h_{2l}, \ldots, h_{2l+m-1}]$, has asymptotic uniform distribution:

$$\lim_{l \to \infty} P_{h_{2l+i}}(0) = 0.5, \quad i = 0, 1, \ldots, m-1.$$

Property 1 can be proved by calculating the distribution of the CRC bits.

Properties of Channel Input



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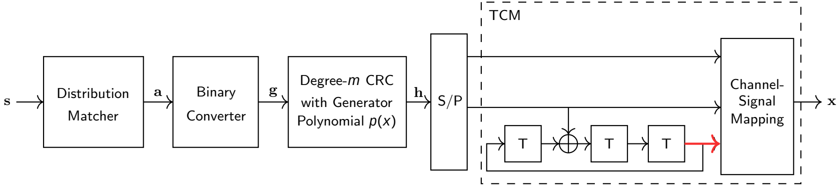
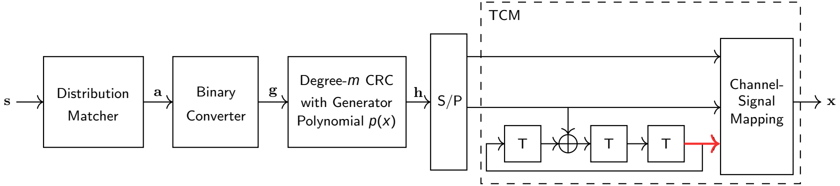
Property 1 can be proved by calculating the distribution of the CRC bits.

- The m CRC bits corresponds to $m/2$ magnitude symbols;
- These $m/2$ magnitude symbols have uniform distribution.



Property 2 (Uniformity of TBCC parity bits)

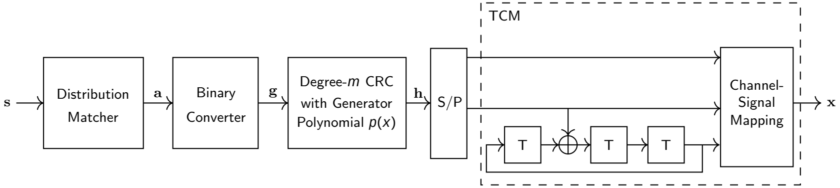
For the proposed CRC-TCM-PAS system, it can be proved that the parity bits have uniform distribution.



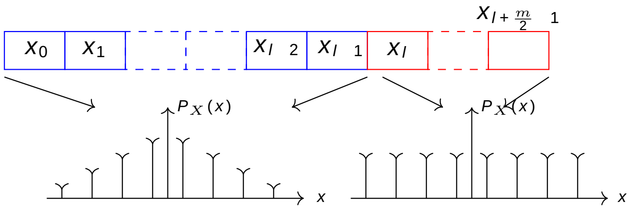
Property 2 (Uniformity of TBCC parity bits)

For the proposed CRC-TCM-PAS system, it can be proved that the parity bits have uniform distribution.

The parity bits indicate the signs of channel inputs. Hence the channel inputs have symmetric distributions.



Channel Input:



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Question

How to design the CRC and TBCC such that the CRC-TCM-PAS system delivers its best performance?

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How to design the CRC and TBCC such that the CRC-TCM-PAS system delivers its best performance?

Answer

Use a generating function union bound on codeword error rate that includes both the CRC and the (ideal distribution of the) distribution matcher.

Given a CRC and TBCC pair, we consider the frame error rate (FER) union bound:

$$P_e \quad \sum_{\mathbf{x}_c \in C_T} P(X^n = \mathbf{x}_c) \sum_{\substack{\mathbf{x}_e \in C_T \\ \mathbf{x}_e \neq \mathbf{x}_c}} P(e_{\mathbf{x}_c;\mathbf{x}_e})$$

$$\sum_{\mathbf{x}_c \in C_T} P(X^n = \mathbf{x}_c) \sum_{\substack{\mathbf{x}_e \in C_T \\ \mathbf{x}_e \neq \mathbf{x}_c}} Q\left(\frac{\sqrt{d^2_{\mathrm{prox}}(\mathbf{x}_c;\mathbf{x}_e)}}{2}\right)$$

$$Q\left(\frac{\sqrt{d^2_{\mathrm{free}}}}{2}\right) \exp\left(\frac{d^2_{\mathrm{free}}}{8\ {}^2}\right) \quad \sum_{\mathbf{x}_c \in C_T} \sum_{\substack{\mathbf{x}_e \in C_T \\ \mathbf{x}_e \neq \mathbf{x}_c}} \prod_{i=1}^{n} \left[\exp\left(\ \frac{d^2_{\mathrm{prox}}(x_{c;i};x_{e;i})}{8\ {}^2}\right) P_{X_i}(x_{c;i})\right] :$$

Assumption:

- the channel inputs are independent;
- First l channel input symbols: symmetric distribution with magnitude distribution defined by DM;
- Last n l channel input symbols: uniform distribution.

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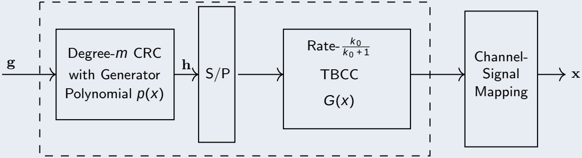
$$\sum_{\mathbf{x}_c \in C_{\mathrm{T}}} \sum_{\substack{\mathbf{x}_e \in C_{\mathrm{T}} \\ \mathbf{x}_e \neq \mathbf{x}_c}} \prod_{i=1}^{n} \left[\exp\left(-\frac{d^2_{\mathrm{prox}}(x_{c;i}, x_{e;i})}{8\sigma^2} \right) P_{X_i}(x_{c;i}) \right]$$

Remark

The concatenation of a CRC with generator polynomial $p(x)$ and a rate-$\frac{k_0}{k_0+1}$ convolutional code with generator matrix $\mathbf{G}(x)$ is equivalent to a rate-$\frac{k_0}{k_0+1}$ convolutional code with generator matrix $\mathbf{G}_{\mathrm{eq}}(x)$.



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FER Upper Bound for CRC-TCM-PAS System

$$\sum_{\mathbf{x}_c \in C_T} \sum_{\substack{\mathbf{x}_e \in C_T \\ \mathbf{x}_e \neq \mathbf{x}_c}} \prod_{i=1}^{n} \left[\exp\left(-\frac{d^2_{\text{prox}}(x_{c,i}, x_{e,i})}{8\sigma^2} \right) P_{X_i}(x_{c,i}) \right]$$

The above term can be calculated using the generating function of the trellis code defined by $G_{eq}(x)$:

$T_{\text{TBCC}}(W)$

$$T_{\text{TBCC}}(W) = 1 + \sum_{i=0}^{S_e} \mathbf{e}_i \mathbf{G}_A^{l}(W) \mathbf{G}_{\text{uni}}^{n-l}(W) \mathbf{e}_i^{\text{T}},$$

Where $\mathbf{G}_A(W)$ and $\mathbf{G}_{\text{uni}}(W)$ are square matrices of size the number of states of $G_{eq}(x)$:

$$\mathbf{G}_A(W)_{s_e, s_e^0} = \sum_{e_o} \sum_{q} P_A(q) W^{d^2_{\text{prox}}(q, e_o)},$$

$$\mathbf{G}_{\text{uni}}(W)_{s_e, s_e^0} = \sum_{e_o} \sum_{q} \frac{1}{|A|} W^{d^2_{\text{prox}}(q, e_o)}.$$

$\mathbf{G}_A(W)$ and $\mathbf{G}_{\text{uni}}(W)$ enumerates all transitions of the error-state diagram.

$$P_e \leq Q\left(\frac{\sqrt{d_{\text{free}}^2}}{2\sigma}\right) \exp\left(\frac{d_{\text{free}}^2}{8\sigma^2}\right) \sum_{\mathbf{x}_c \in \mathcal{C}_T} \sum_{\substack{\mathbf{x}_e \in \mathcal{C}_T \\ \mathbf{x}_e \neq \mathbf{x}_c}} \prod_{i=1}^{n} \left[\exp\left(-\frac{d_{\text{prox}}^2(x_{c,i}, x_{e,i})}{8\sigma^2}\right) P_{X_i}(x_{c,i})\right]$$

$$= Q\left(\frac{\sqrt{d_{\text{free}}^2}}{2\sigma}\right) \exp\left(\frac{d_{\text{free}}^2}{8\sigma^2}\right) T_{\text{TBCC}}\left(W = e^{-\frac{1}{8\sigma^2}}\right).$$

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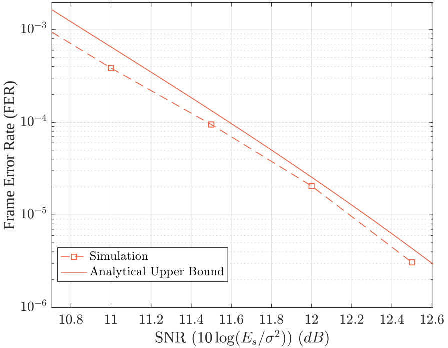
Channel Input: 64 8-AM symbols.

Magnitude (0:45; 1:35; 2:25; 3:15) with P.M.F. (0:58; 0:31; 0:01; 0:09).

2-bit CRC $p(x) = x^2 + 1$.

Rate-2/3 TBCC:

$$G(D) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & D^2 = D^3 + D + 1 \end{bmatrix} :$$



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Rules of the game:

- We only consider fixed-to-fixed distribution mappers.
- We follow Schulte and Böcherer and seek to minimize the normalized KL divergence between our DM and the desired Maxwell-Boltzman distribution $P(\hat{A})$.

P. Schulte and G. Böcherer, "Constant composition distribution matching," IEEE Trans. on Info. Theory, vol. 62, no. 1, pp. 430–434, 2015.

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DMs are defined by the Types that comprise them

The type (or empirical distribution) $P_{\mathbf{a}}$ of a sequence $\mathbf{a} = [a_0, a_1, \ldots, a_{l-1}]$ is the relative proportion of occurrence of each symbol in A, i.e., $P_{\mathbf{a}}(i) = \frac{\sum_{j=0}^{l-1} \mathbb{1}(a_j = i)}{l}$, $i \in A$. Define the set of sequences of length l and type P as set class of P, denoted by T_P^l:

$$T_P^l = \{\mathbf{a} \in A^l : P_{\mathbf{a}} = P\}. \quad (1)$$

Constant Composition Distribution Matcher (CCDM)

Choose the type that is closest in KL divergence to $P(\hat{A})$ among those that have sufficient cardinality, $|T_{P}^{l}| \geq 2^k$.

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Choose the type that is closest in KL divergence to $P(\hat{A})$ among those that have sufficient cardinality, $|T_P^l| > 2^k$.

MCDM that selects the typical sets, referred to as "MCDM with $\mathcal{C}_{TS}$"

This MCDM chooses the typical sets as its component constant composition sub-codes. i.e.

$$P(A_i) = \underset{P(A)\in\mathcal{P}\setminus\{P(A_1),\ldots,P(A_{i-1})\}}{\arg\min} \mathrm{D}_{KL}(P(A)||P(\hat{A})). \qquad (2)$$

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MCDM that selects the highest probability sets, referred to as "MCDM with C_{HP}"

This MCDM chooses the highest probability types as its component constant composition sub-codes. i.e.

$$P(A_i) = \underset{P(A) \in \mathcal{P} \setminus \{P(A_1),\ldots,P(A_{i-1})\}}{\arg\max} \prod_{a=1}^{|A|} (P_{\hat{A}}(a))^{\ell P_A(a)} \tag{3}$$

The Enumerative Sphere Shaper (ESS)

Given a symbol sequence $\mathbf{a} = [a_1 \dots a_l]$, the energy of $\mathbf{a}$ is defined as $\sum_{i=1}^{l} a_i^2$. ESS considers the sequences whose energies are less than or equal to a threshold $E_{\max}$ as codeword candidates of the distribution matcher. Given an $E_{\max}$, ESS indexes the qualified sequences lexicographically, and an energy-bounded trellis is built to index the sequences.

A. Amari, S. Goossens, Y. C. Gültekin, O. Vassilieva, I. Kim, T. Ikeuchi, C. M. Okonkwo, F. M. Willems, and A. Alvarado, "Introducing enumerative sphere shaping for optical communication systems with short blocklengths," Journal of Lightwave Technology, vol. 37, no. 23, pp. 5926–5936, 2019.

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KL Divergence Comparison

The table below gives the normalized KL divergence of CCDM, MCDM, and ESS. CCDM delivers the largest normalized KL divergence, while ESS delivers the smallest normalized KL divergence. The MCDM with C_{HP} delivers a comparable normalized KL divergence with ESS, and the MCDM with C_{TS} is slightly larger than that of MCDM with C_{HP}

	ESS	MCDM with C_{HP}	MCDM with C_{TS}	CCDM
normalized KL divergence	0.074	0.077	0.096	0.213
required storage (bits)	3.6e5	3e5	3e4	24

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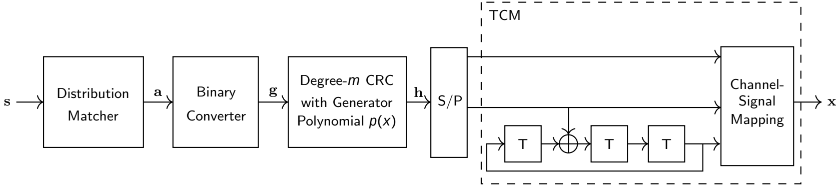
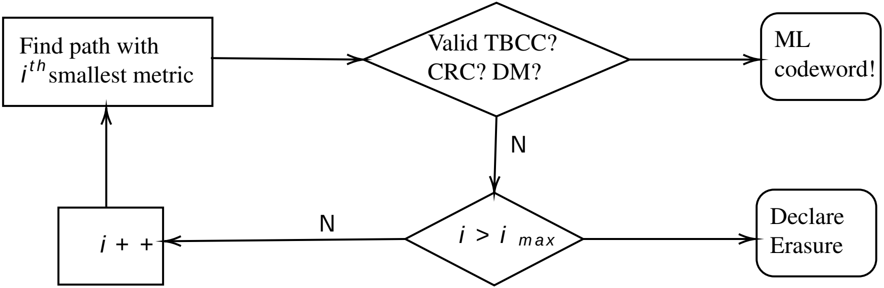


$\mathbf{x}$ 2 X^n is a *CRC-TCM-PAS codeword* if it satisfies all of the following conditions:

1. $\mathbf{x}$ is a (tail-biting) codeword of TCM.
2. $\mathbf{h}$, the dataword of TCM that generates $\mathbf{x}$, passes the CRC check.
3. The information bits $\mathbf{g}$ of the CRC codeword $\mathbf{h}$, are the binary representation of a valid distribution-matcher output, i.e. codeword in C_{DM}.

Denote the codebook of CRC-TCM-PAS system by C_{CTP}.

Serial List Viterbi Decoding (S-LVD):



The Serial List Viterbi Decoder (SLVD) with sufficiently large list size is an ML decoder.

For the TBCC, the start/end state of a codeword could be any one of all 2 states;

The list decoder needs to search the codewords that start/end at all 2 states;

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Decoding (-states Decoder)

For the TBCC, the start/end state of a codeword could be any one of all 2   states;

The list decoder needs to search the codewords that start/end at all 2   states;

One sub-optimal decoder with lower complexity is to consider a subset $\mathcal{V}$ that only contains   states.

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- -state decoder:

$$\hat{\mathbf{x}} = \arg\min_{\substack{\mathbf{x} \in C_{CTP} \\ v(\mathbf{x}) \in \mathcal{V}}} ||\mathbf{x} \quad \mathbf{y}||_2^2.$$

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-state decoder:

$$\hat{\mathbf{x}} = \arg\min_{\substack{\mathbf{x} \in C_{CTP} \\ v(\mathbf{x}) \in \mathcal{V}}} ||\mathbf{x} - \mathbf{y}||_2^2.$$

The states are found using the wrap-around Viterbi algorithm (WAVA).

R. Y. Shao *et al.*, "Two decoding algorithms for tail-biting codes," *TCOM, vol. 51, no. 10, pp. 1658-1665, 2003*

Any cyclic shift of a TBCC codeword, say by $_{i}$ symbols, is still a TBCC codeword.

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Automorphism Ensemble Decoding (AED): Employ M parallel independent and identical sub-optimal decoders, each decoding the received word cyclically shifted by a different $_i$, and proposing its best result using states.

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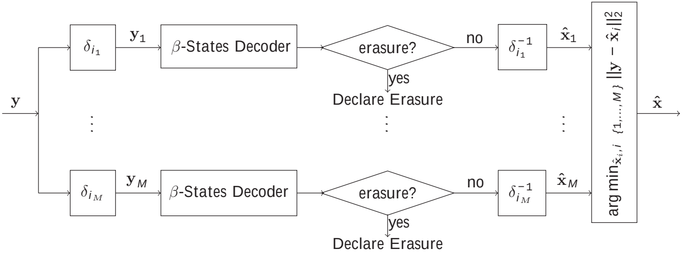

Figure 2: The diagram of an AE decoder with M parallel -States decoders, i.e., AED(M,).

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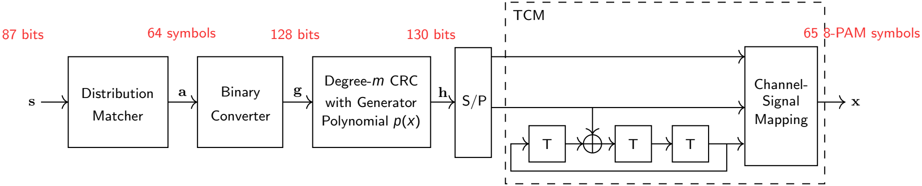
Input bits k = 87 bits, output n = 65 8-PAM symbols;

Degree-2 CRC;

Rate-2/3 TBCC;

Distribution Matcher: Multi-Composition Distribution Matcher (MCDM)*.



* L. Wang *et al.*, "Probabilistic Shaping for Trellis-Coded Modulation With CRC-Aided List Decoding," *IEEE Transactions on Communications, vol. 71, no. 3, pp. 1271-1283, March 2023*

Brute force all possible CC and CRC pairs, and calculate the FER bound at 11 dB.

Table 1: Optimized Convolutional Code and CRC Pairs. All the parameters are optimized while SNR equals 11 dB.

		$H^0(D)$	$H^1(D)$	$H^2(D)$	$p(x)$	FER bound
= 3	Ungerboeck	13	04	00	7	6.65e-4
	Optimized	13	06	00	5	5.80e-4
= 5	Ungerboeck	45	10	00	5	8.20e-5
	Optimized	43	26	00	5	6.58e-5
= 7	Ungerboeck	235	126	000	5	1.15e-5
	Optimized	211	142	000	5	8.96e-6

* G. Ungerboeck *et al.*, "Channel coding with multilevel/phase signals ," *IEEE Transactions on Information Theory*

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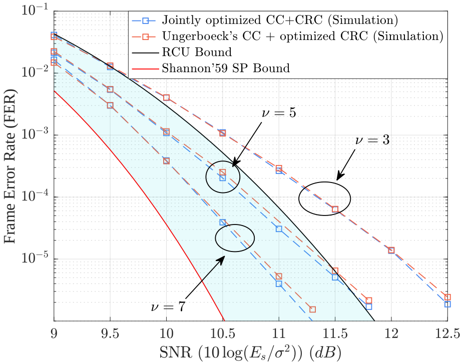


Decoding Algorithm: ML Decoding

Code -2

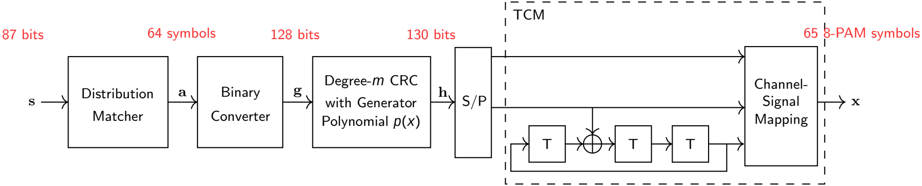
Input bits k = 96 bits, output n = 64 8-PAM symbols;

Degree-2 CRC;

Rate-2/3 TBCC;

Distribution Matcher: We will consider all competitors.

CRC-TCM-PAS Decoder: We will compare ML decoding and AED(5,2) with max list size 100 for each 2-states decoder.



FER performance for ML and AED(5,2) decoding of each DM.

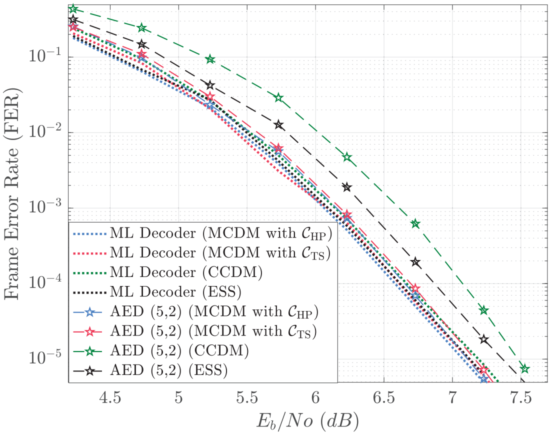


Figure 3: ML decoding makes all DMs look good. AED decoding favors the MCDMs.

Expected list size for ML and AED(5,2) decoding of each DM.

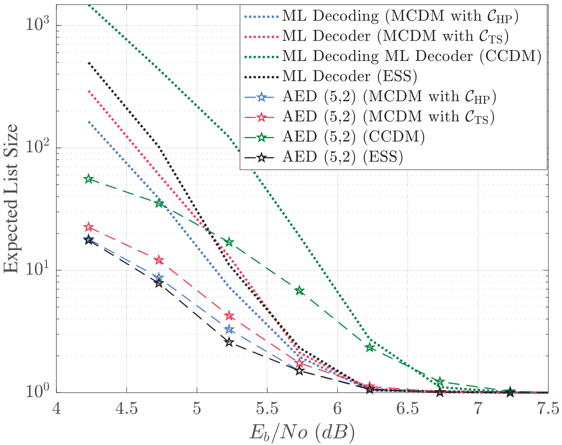


Figure 4: AED improves complexity. CCDM has the worst expected list size. MCDM is looking good for both FER and complexity.

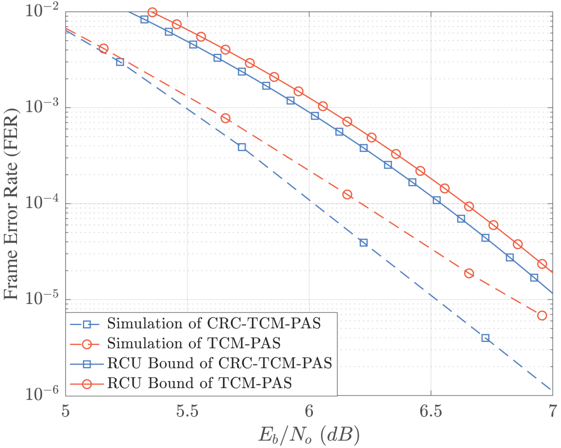


Figure 5: Simulated FER and RCU bounds with and without the 2-bit CRC reveals that it improves performance.

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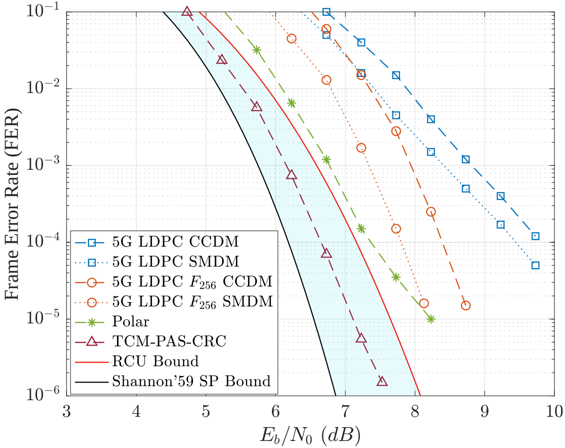
code -2



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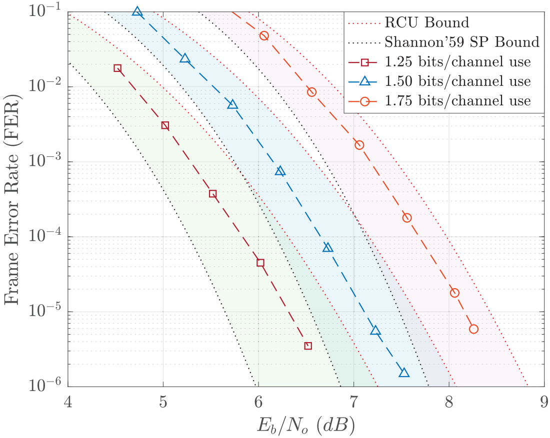
The CRC-TCM-PAS systems generate 64 8-AM symbols, with transmission rates of 1.25, 1.5, and 1.75 bit/real channel use, respectively.



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This talk presents CRC-TCM-PAS: A probabilistic amplitude shaping system that uses TCM with CRC-Aided list decoding.

An analytical upper bound on the FER of the CRC-TCM-PAS system allows selection of the very best convolutional code and CRC.

However, Ungerboeck's original designs still work well if the lableing is selected to support both set partitioning and shaping.

The best CRC is only two bits, but the CRC did help. The DM seems to expurgate in a way that avoids the need for a longer CRC.

CRC-TCM-PAS admits Automorphism-enabled (AE) decoding through the cyclic nature of TBCC codes.

We explored CCDM, MCDM and ESS approaches to distribution matching. MCDM give the best trade-off of performance and complexity in the context of AE decoding.

Simulations show that the CRC-TCM-PAS system has rate flexibility and achieves frame error rates well below the random coding union bound in AWGN channel.

Related Publications

L. Wang, D. Song, F. Areces, and R. D. Wesel. "Achieving Short-Blocklength RCU bound via CRC List Decoding of TCM with Probabilistic Shaping." IEEE ICC, Seoul, South Korea, May 16–20, 2022.

D. Song, F. Areces, L. Wang, and R. D. Wesel, "Shaped TCM with List Decoding that Exceeds the RCU Bound by Optimizing a Union Bound on FER", IEEE GLOBECOM, Rio de Janeiro, Brazil, Dec. 04-08, 2022.

L. Wang, D. Song, F. Areces, T. Wiegart, R. D. Wesel, "Probabilistic Shaping for Trellis-Coded Modulation with CRC-aided List Decoding", IEEE Transactions on Communications, January 2023.

Thank you!