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LDPC in Impulsive Noise: Unsupervised LLR Estimation

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Plan

LLR approximation in impulsive noise

Dense and heterogeneous networks

Impulsive interference not encompassed by the Gaussian model [Pinto'10] [Egan'17]

Solution: impulsive noise (SαS, Middleton, ε-contaminated, ...)

Considered setup: Binary transmission in impulsive noise

Log Likelihood Ratios for modern channel coding: $LLR(y_i) = \log \frac{\mathbb{P}(Y_i = y_i | X_i = +1)}{\mathbb{P}(Y_i = y_i | X_i = -1)}$

Challenge: No closed form expression of the LLR in impulsive noise

Objective: Optimizing a LLR approximation in an unsupervised manner in both asymptotic regime and short block length

Impulsive noise: Middleton class A

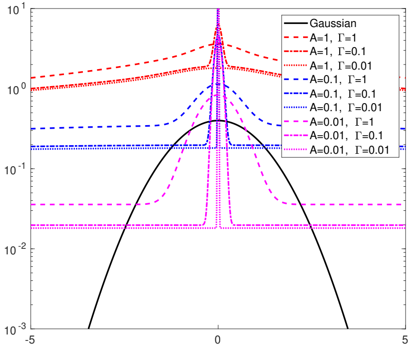
Suited for electromagnetic interference, background noise in

- Power line communications [Andreadou'10]
- Orthogonal Frequency-Division Multiplexing (OFDM) [Ishikawa'07]
- Multiple Input Multiple Output (MIMO) [Chopra'09]

+ Closed-form expression of pdf:
Gaussian mixture with infinite number of components

$$f_{\text{Middleton}}(x) = \sum_{k=0}^{\infty} \frac{A^k e^{-A}}{k!} f_N\left(x; 0; \sigma^2 \left(\frac{k}{A\Gamma} + 1\right)\right)$$

σ^2 Thermal noise variance
Γ Gaussian to impulsive noise power
A Impulsiveness parameter



− Infinite series nature difficult to handle in practice
⇒ **Approximation focusing on the most significant terms**

$f_N(x; \mu; \sigma^2)$: Gaussian pdf with mean μ and variance σ^2

Impulsive noise: Gaussian mixture

Gaussian mixture with finite components

Suited for

- Multi-path in satellite transmission [Nahimana'09]
- Multi-user interference in UWB systems [Erseghe'08]

+ Closed-form expression of pdf

$$f_{\text{GaussianMixt.}}(x) = \sum_{k=0}^{K} \lambda_k f_N(x; \mu_k; \sigma_k^2)$$

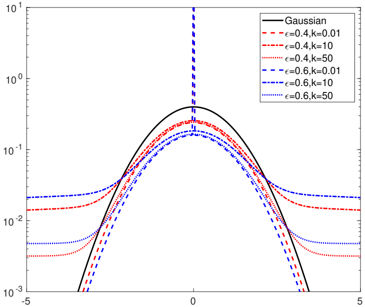
− Require estimation of $\lambda_k, \mu_k, \sigma_k^2$

+ Only 2 or 3 components are enough [Vastola'84]

ε-contaminated Gaussian mixture with 2 components

$$f_{\varepsilon-\text{contaminated}}(x) = (1-\varepsilon) f_N(x; 0; \sigma^2) + \varepsilon f_N(x; 0; k\sigma^2)$$

- ε Probability of impulsive occurrence
- k Impulsive strength



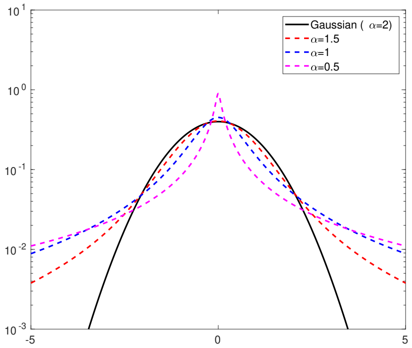
Impulsive noise: SαS model

Suited for interference modeling in wireless communications
[Egan'17], [Pinto'10], [Win'09], [Egan'18]

- + Gaussian distribution as special case
- − No closed form expression of the pdf
- + Closed form expression of the characteristic function

$$\phi(t) = \exp(-\gamma^{\alpha}|t|^{\alpha})$$

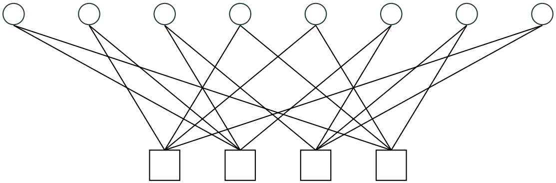
- α Thickness of the distribution tail
- γ Similar role as the variance for Gaussian noise



LDPC codes and iterative decoder

LDPC

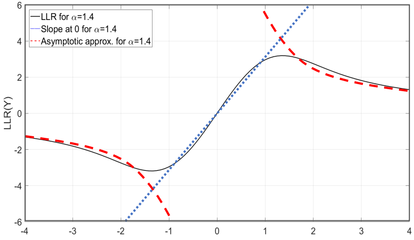
- Proposed by Gallager in the 60s [Gallager'62]
- Rediscovered by MacKay in the 90s [MacKay'97], [MacKay'03]
- Widely used in wireless communication [Chung'01], [Richardson'01], [Fang'21], [Tarver'21]
- Near-capacity performance in Gaussian noise
- Belief propagation decoder based on LLR computations [Gallager'62], [MacKay'97], [Kschischang'01], [Tanner'81]



Received signal: $Y = X + Z$

- Binary channel input $X \in \{0,1\}^n$
- Additive noise Z
- Log Likelihood Ratio $L_i = \log \frac{\mathbb{P}(Y_i=y|X_i=+1)}{\mathbb{P}(Y_i=y|X_i=-1)} = \log \frac{f_Z(y-1)}{f_Z(y+1)}$

LLR approximation in impulsive $S\alpha S$ noise



$S\alpha S$ noise

- No closed-form expression of the LLR
- Non-linear LLR

Approximation of the LLR

- Parametric linear piece-wise approximations: hole puncturer [Ambike'94], clipping [Maad'13]
- Non-parametric non-linear approximation for $S\alpha S$ noise [Dimanche'14]

+ Two terms-based parametric non-linear LLR approximation [Mestrah'20]

$$L_\theta(y) = \mathrm{sign}(y) \min\left(a|y|, \frac{b}{|y|}\right) \text{ with } \theta = (a,b) \in \mathbb{R}_+^2$$

IT-based LLR estimation

LLR estimation:

- Direct estimation of the noise model parameters [Koutrouvelis'81] [Mcculloch'86]
- Noise distribution estimation [Dimanche'16]
- Maximizing the mutual information between the channel input and output [Yazdani'09]

+ IT-based LLR estimation yields the best performance [Dimanche'16]

Lower bound on the capacity under approximated LLR L_θ [Yazdani'09]

$$C_{L_\theta} = 1 - \mathbb{E}_{X,Y}[\log_2(1 + e^{-XL_\theta(Y)})] = 1 - \underbrace{\mathbb{E}_{X,Y}[\log_2(1 + e^{-L_\theta(XY)})]}_{\text{to be minimized}} \leq C_L$$

Considered optimization problem

$$\hat{\theta}^* = \arg\min_\theta \frac{1}{n} \sum_{i=1}^{n} \log_2 \left(1 + e^{-L_\theta(X_i Y_i)}\right)$$

— Not convex

Plan

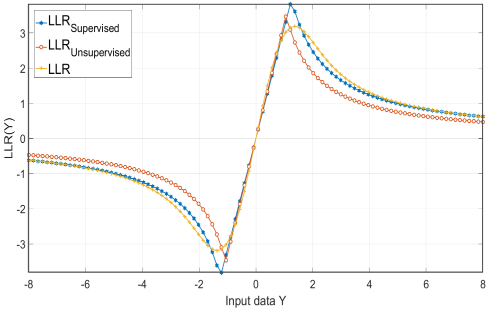
Supervised vs. unsupervised estimation

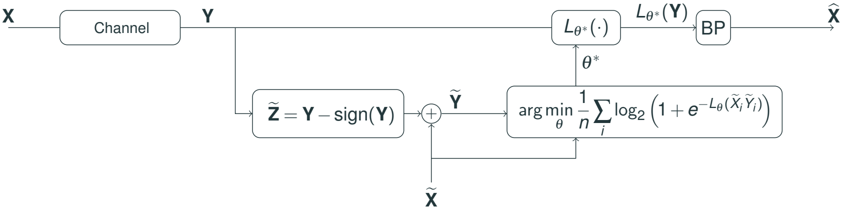
Supervised estimation [Mestrah'20]

- + Used as benchmark
- − Requires pilot samples ⇒ decrease of the useful rate

Unsupervised estimation [Mestrah'20]

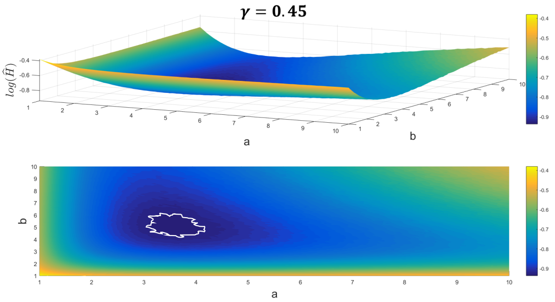
- + No pilot samples needed
- + Sign detector and all-zero codeword to simulate a transmission at the receiver side

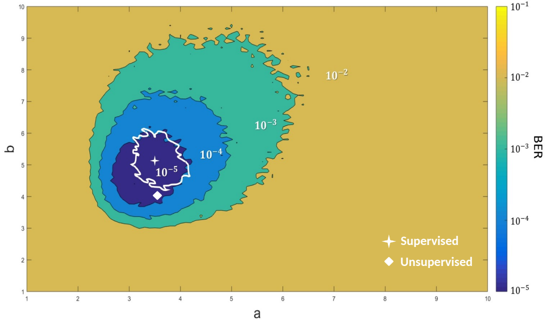




(3,6) LDPC length $N = 20000$ in highly impulsive noise

Empirical link between objective function and BER





Take away

- Same (a,b) parameter range minimizing the objective function and the BER
- Both supervised and unsupervised estimation achieve BER of 10^{-5}

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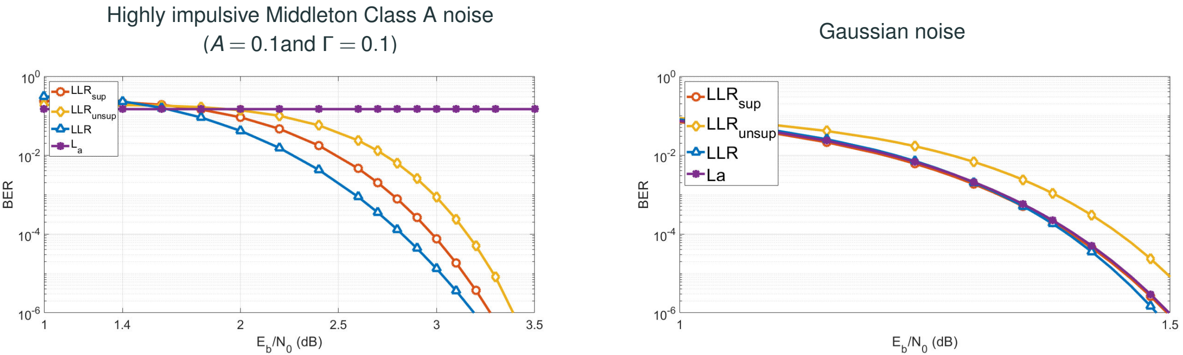
(3,6) LDPC length $N = 20000$ in different noise models



Take away

- Supervised estimation close to LLR: Validation of proposed approach
- Approximated LLR outperforms linear (La) in impulsive noise
- Works for both Gaussian and non Gaussian noises

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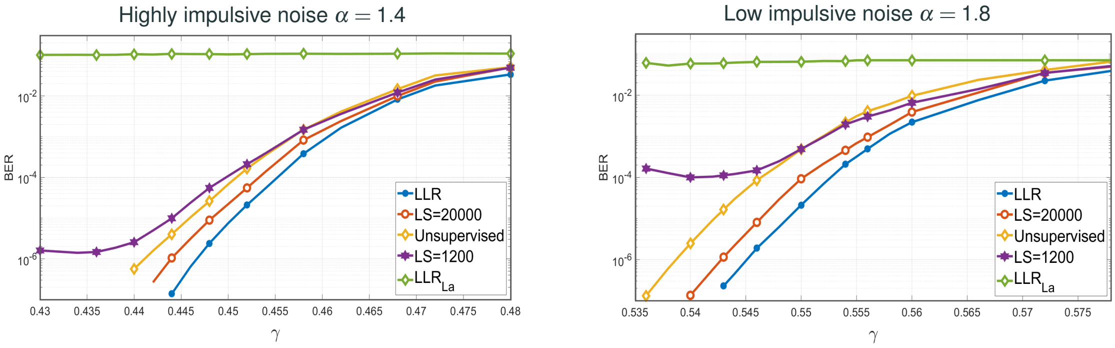
(3,6) LDPC of length $N = 20000$ in impulsive $S\alpha S$ noise: BER

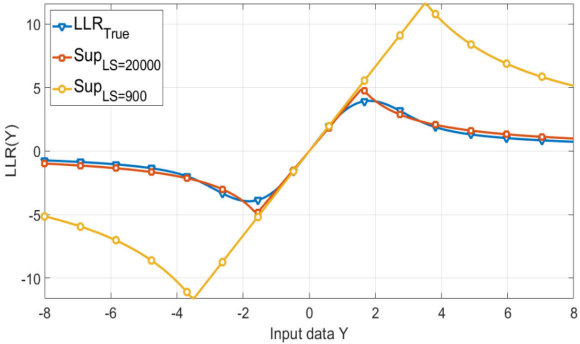


Take away

- Unsupervised approximation outperforms supervised with realistic number of pilots
- Approximated LLR outperforms linear (La) in impulsive noise
- Generalization over various noise models without requiring its knowledge

Plan

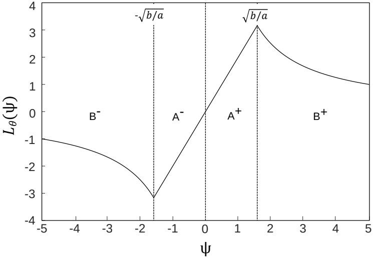
Shortening the block length [Mestrah'22]



$\alpha = 1.8,\ \gamma = 0.55$

— Very bad approximation in the short blocklength regime

Shortening the block length [Mestrah'22]



$B^- = \left[-\infty, -\sqrt{b/a}\right], A^- = \left[-\sqrt{b/a}, 0\right], A^+ = \left[0, \sqrt{b/a}\right], B^+ = \left[\sqrt{b/a}, +\infty\right]$

Objective function

$$\frac{1}{n}\sum_{i=1}^{n}\log_2\left(1+e^{-L_\theta(\widetilde{\Psi}_i)}\right) = \sum_{i:\widetilde{\Psi}_i \in B^- \cup B^+} \log_2(1+e^{-b/\widetilde{\Psi}_i}) + \sum_{i:\widetilde{\Psi}_i \in A^- \cup A^+} \log_2(1+e^{-a\widetilde{\Psi}_i})$$

No samples in A^-: $a^* \to \infty \Rightarrow$ poor LLR approx. around 0, presence of an error floor

Proposed solution: Regularization term to limit the growth of optimization parameter a

New objective function $\frac{1}{n}\sum_{i=1}^{n}\log_2\left(1+e^{-L_\theta(\widetilde{\Psi}_i)}\right) + \log_2(1+e^{a\varepsilon})$

No samples in B^-: $b^* \to \infty$ Not problematic for Gaussian noise ($b^* = \infty$) but for impulsive ones

Proposed solution: New design of the simulated transmission at the receiver side ($\widetilde{X} = \pm 1$)

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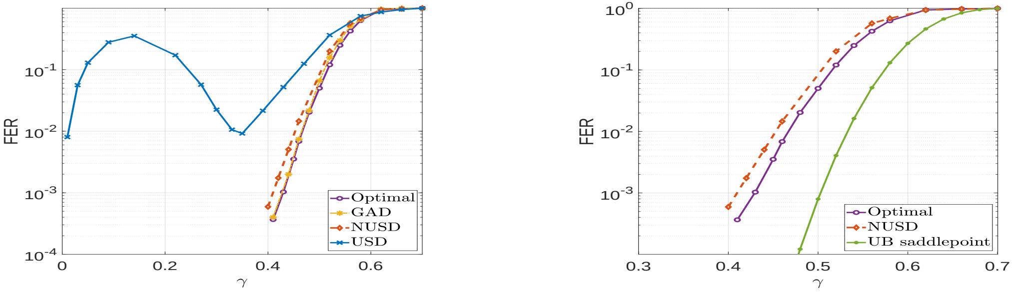
(3,6) LDPC of length $N = 408$ in impulsive noise: FER



GAD: supervised estimation, USD: unsupervised estimation, NUSD: new unsupervised estimation
UB saddlepoint: achievable FER bound [Anade'20]

Take away

- **+** New unsupervised estimation: monotonic FER in short blocklength regime
- **−** Remaining gap to achievable FER bound

$\Rightarrow$ Room for efficient LDPC code design under impulsive noise and short blocklength

Conclusion and future work

Conclusion

- Non-linear LLR approximation with 2 parameters
- Mutual information-based estimated LLR optimization
- Unsupervised estimation able to cope with various noise models without requiring its knowledge
- Unsupervised estimation also for short blocklength regime

Future works

- LDPC design for impulsive noise

Open PhD positions

3 open PhD positions, expected start Oct. 2024

- **Sustainable wireless communications: low-energy, low-cost and zero added electromagnetic waves**
  Supervision: Veronica Belmega, Anne Savard
  Location: ESIEE, Noisy-le-Grand, France

- **AI-enhanced highly mobile and unpredictable IoT networks**
  Supervision: Romain Negrel, Veronica Belmega, Anne Savard
  Location: ESIEE, Noisy-le-Grand, France

- **Resource allocation for energy sobriety in secure wireless communication systems impaired by nonlinearities**
  Supervision: Arthur Louchart, Anne Savard
  Location: IMT Nord Europe, France

Contact: anne.savard@imt-nord-europe.fr

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