Non-Binary LDPC Codes for Quantum Key Distribution

Lara Dolecek

Based on joint work with:
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MSCT, TUM, Germany
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Outline

1. Background and Motivation

2. System Model

3. NB-LDPC codes for Quantum Key Distribution
   - Preliminaries
   - Channel code design
   - Sequential Decoding and Interactive Communication
   - Privacy Amplification-Aware LDPC Code Design

4. Concluding Remarks and Future Outlook

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Quantum Key Distribution (QKD)

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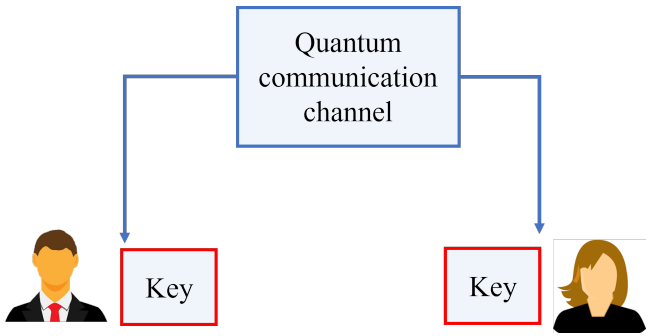

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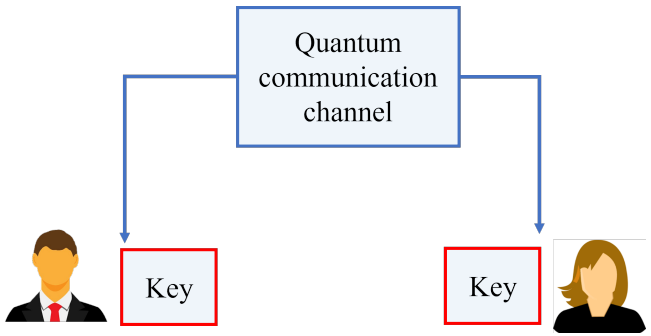

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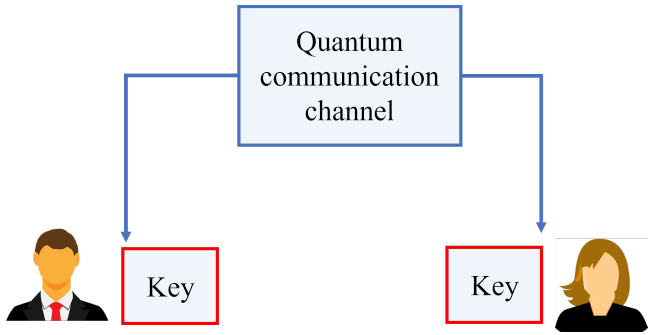

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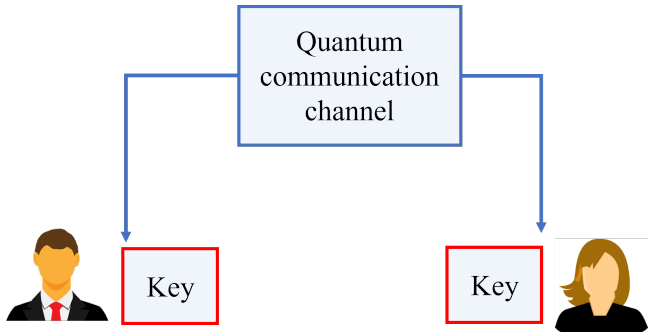

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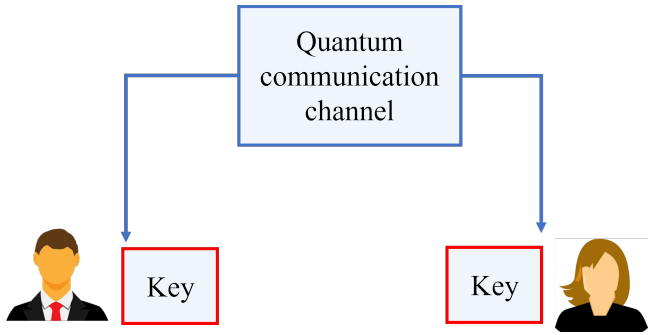

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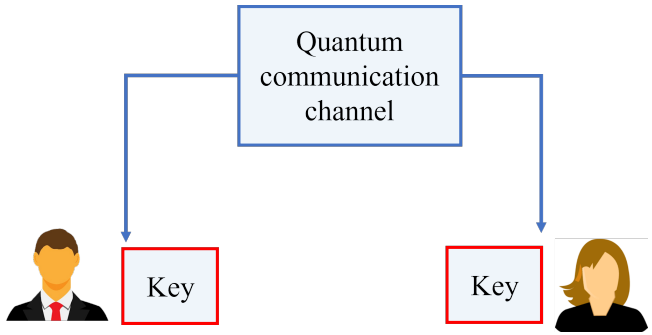

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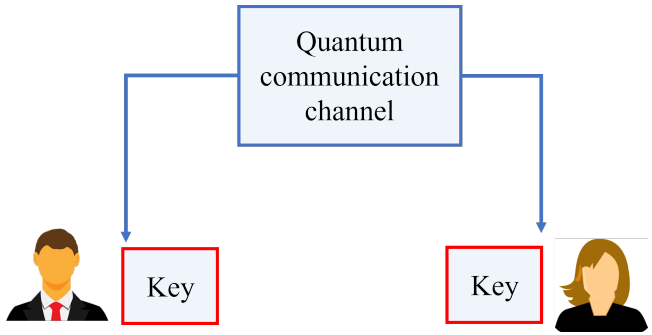

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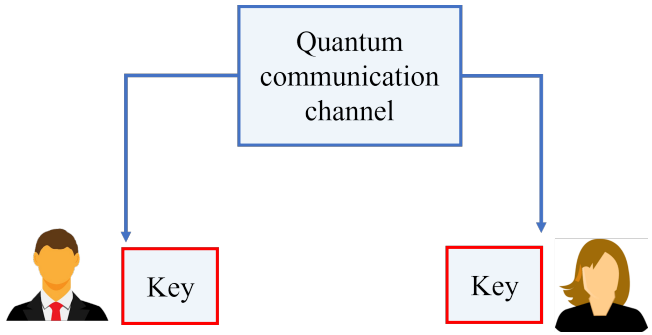

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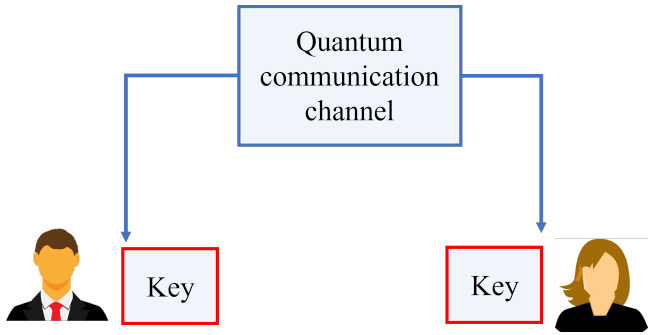

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Error Correction Coding is applied to the information reconciliation phase.

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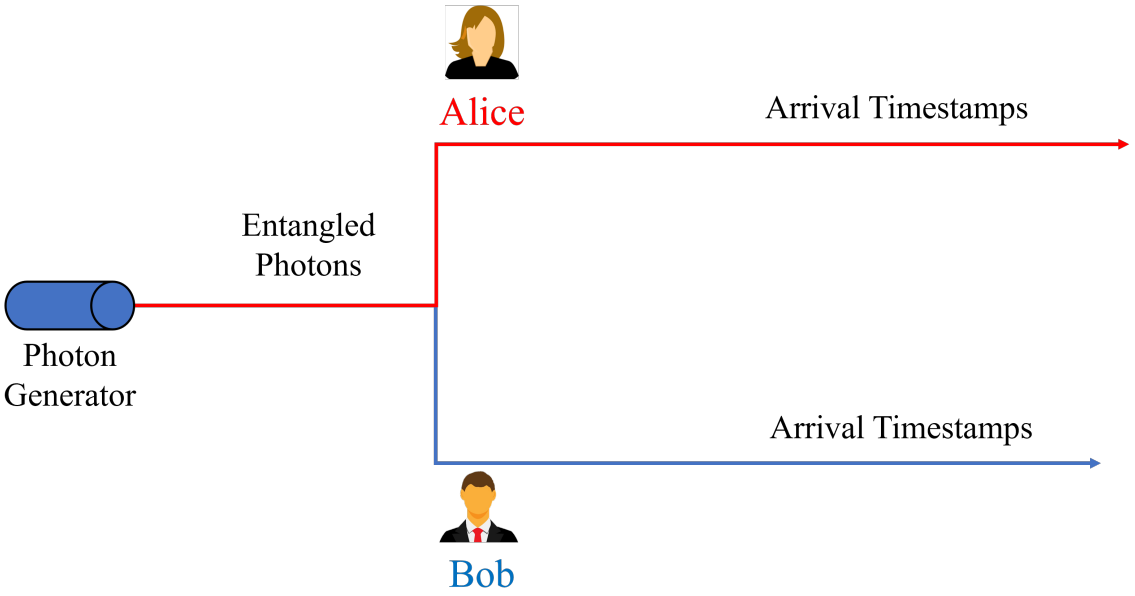

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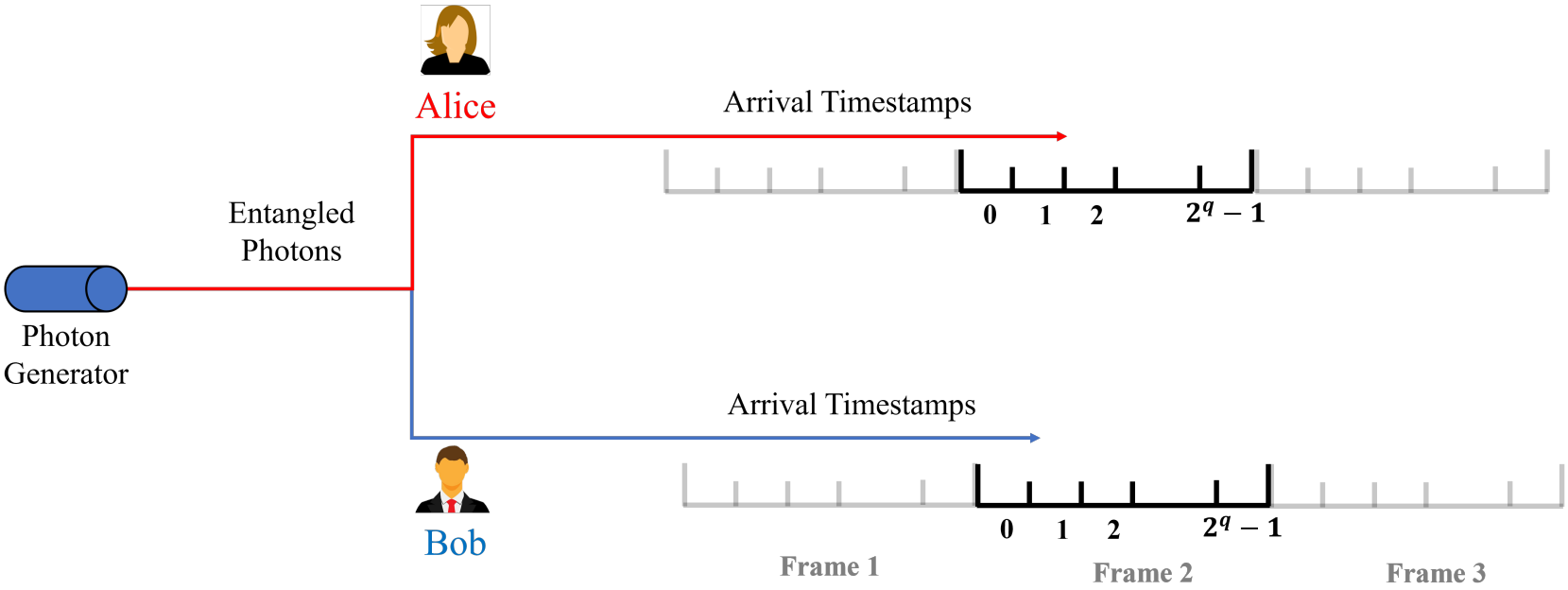

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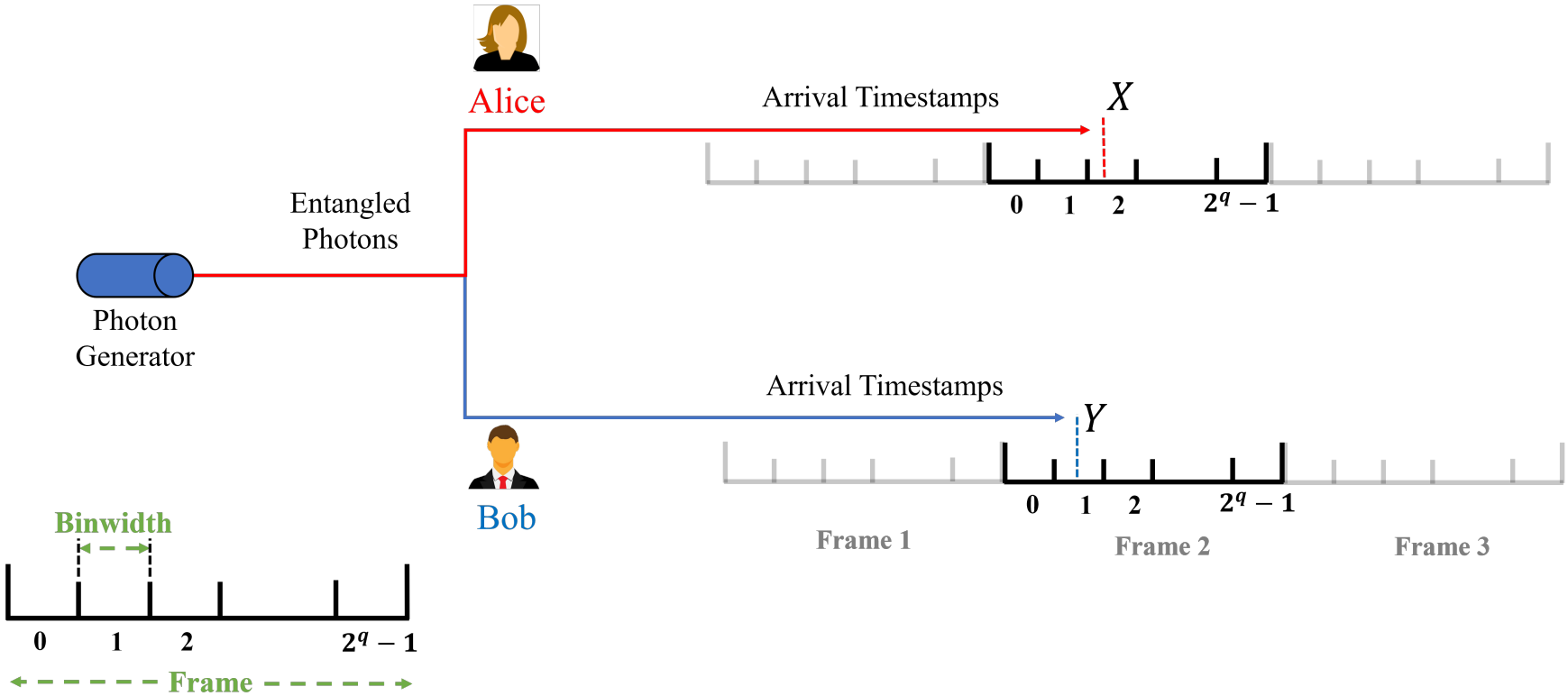

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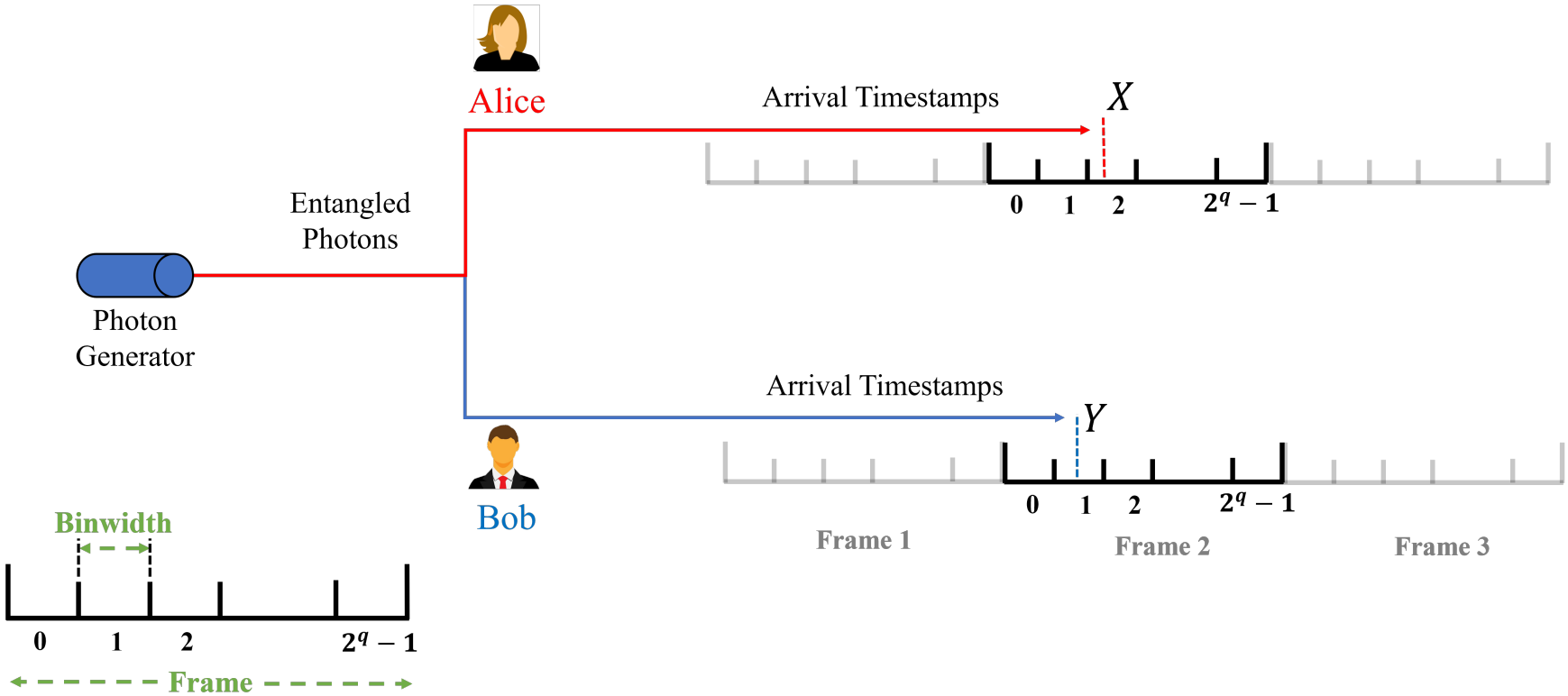

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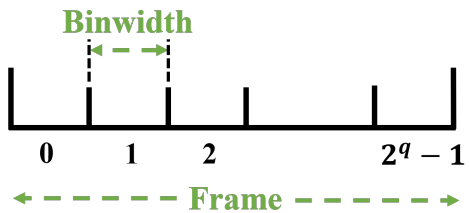

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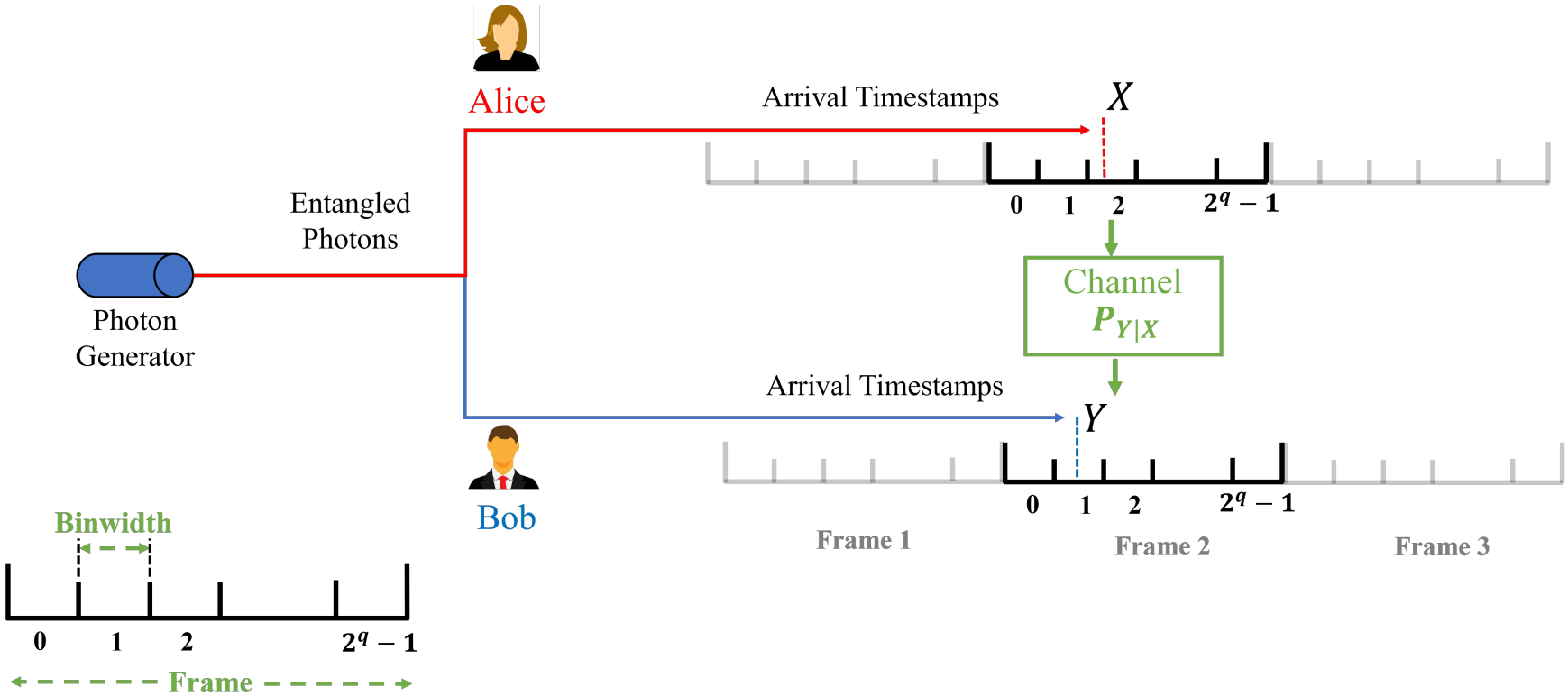

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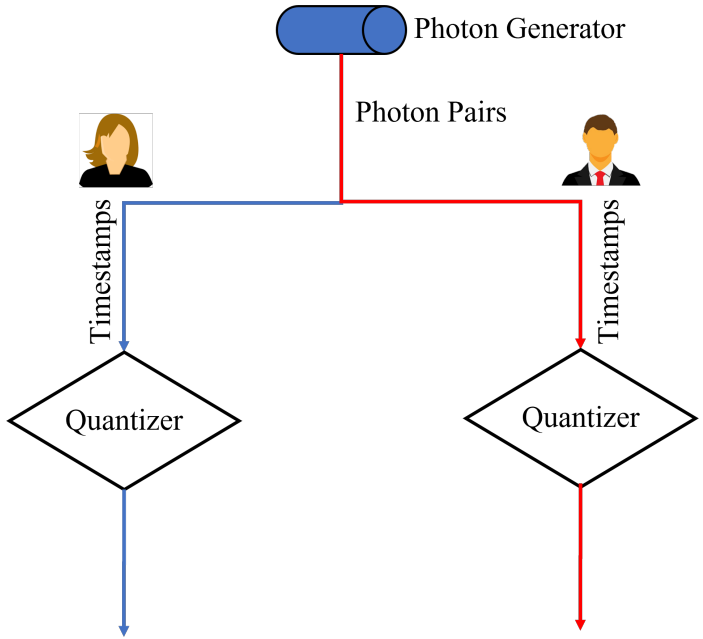

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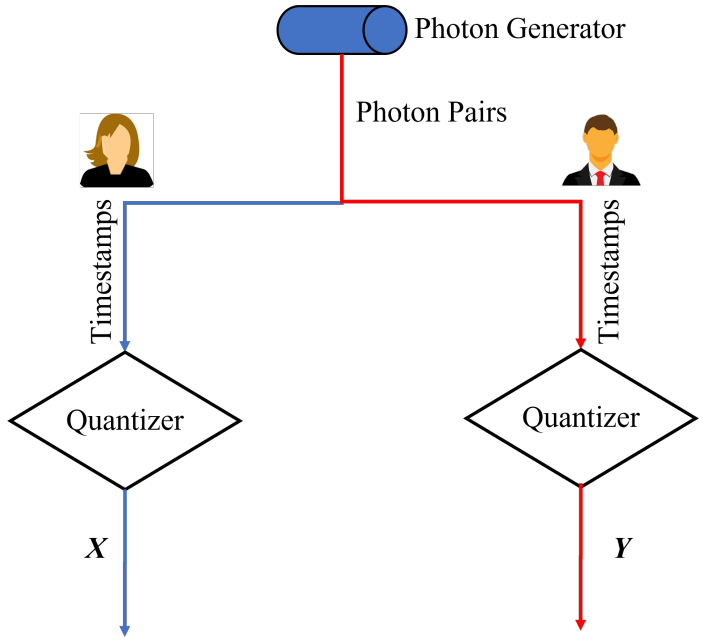

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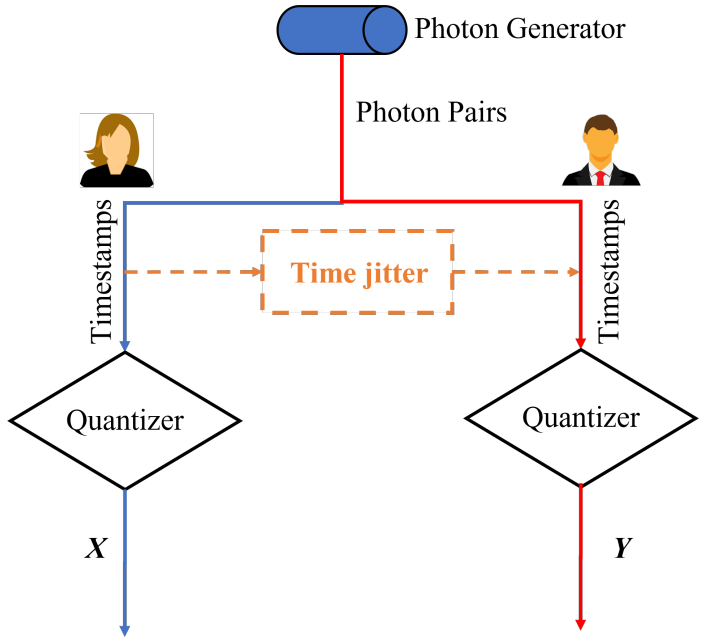

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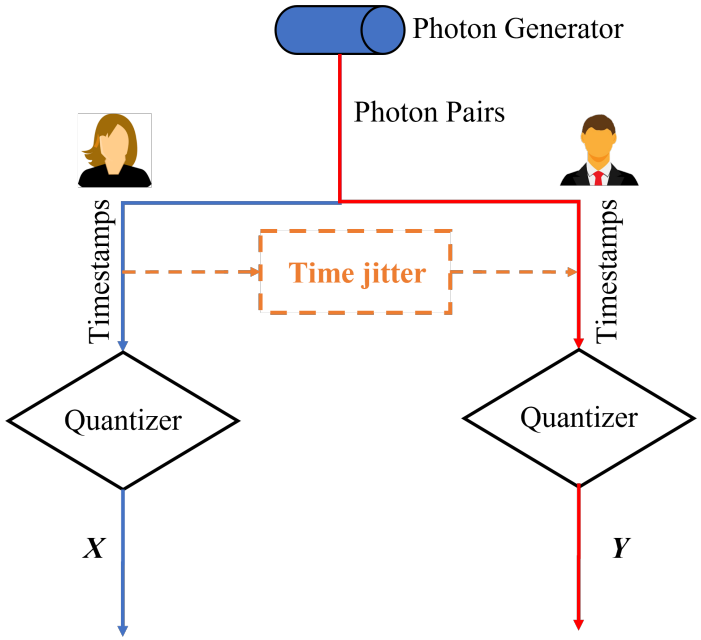

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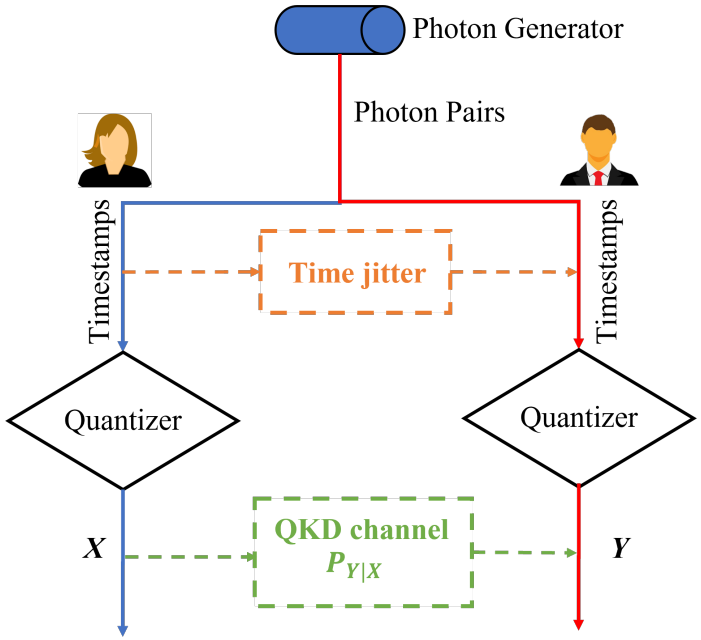

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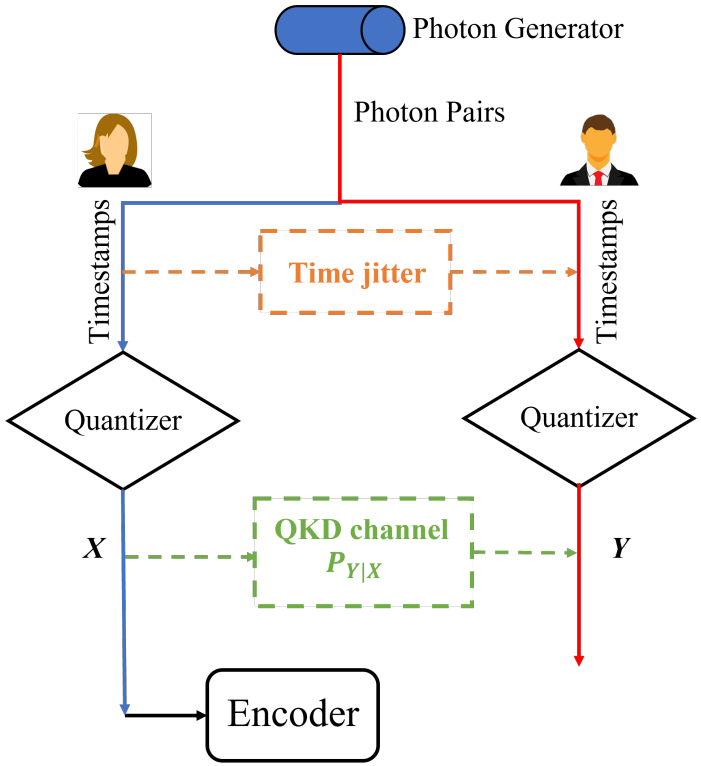

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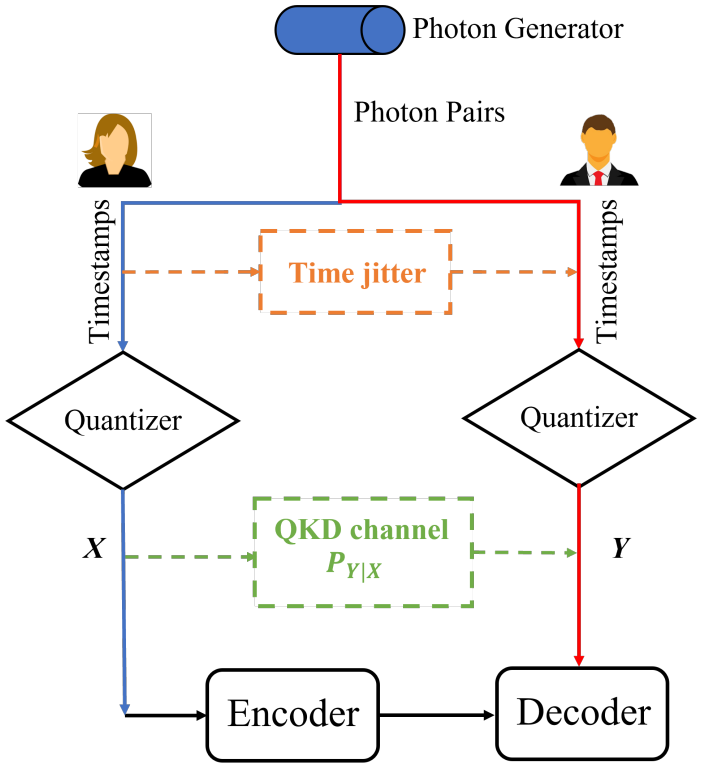

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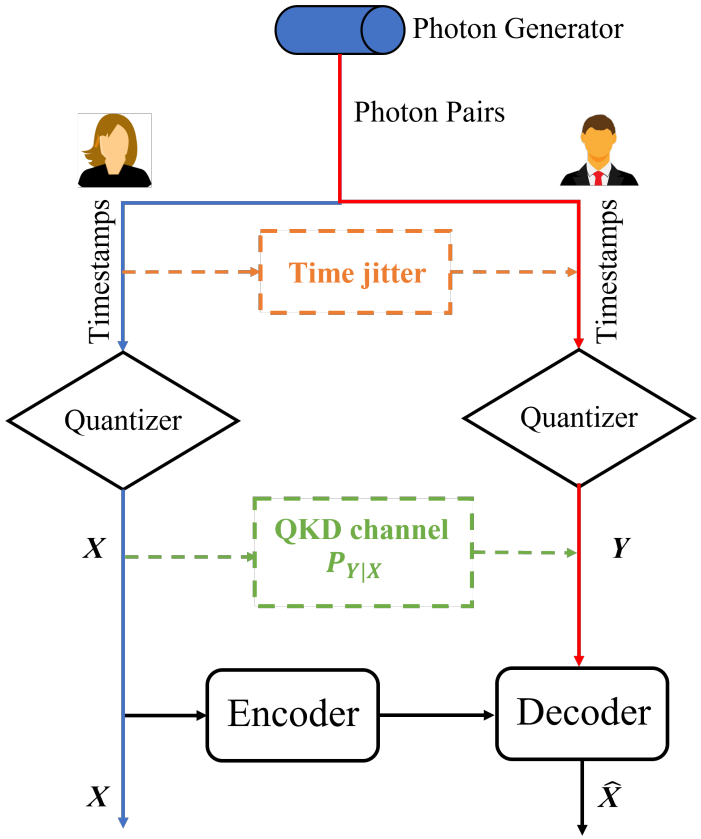

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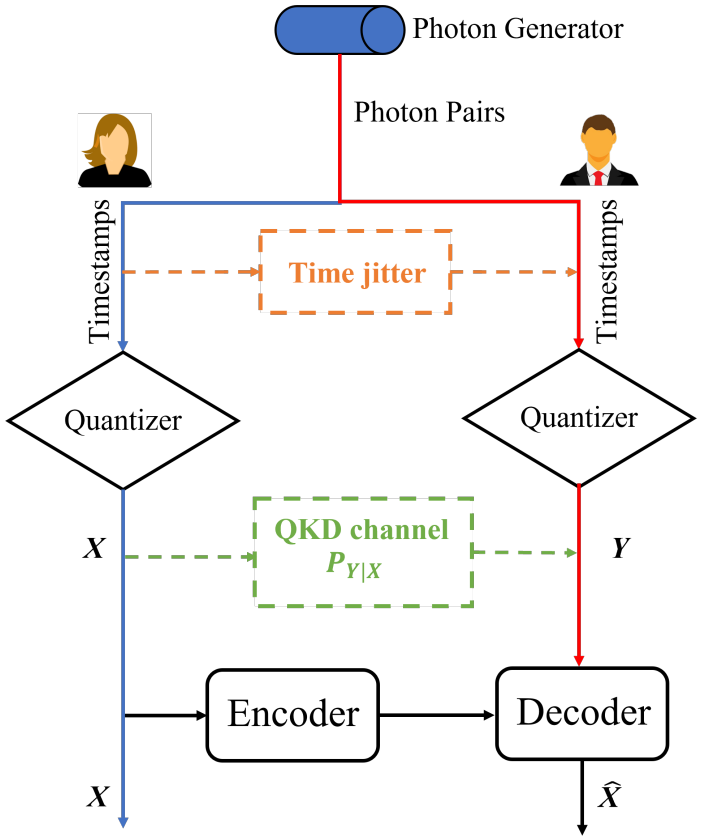

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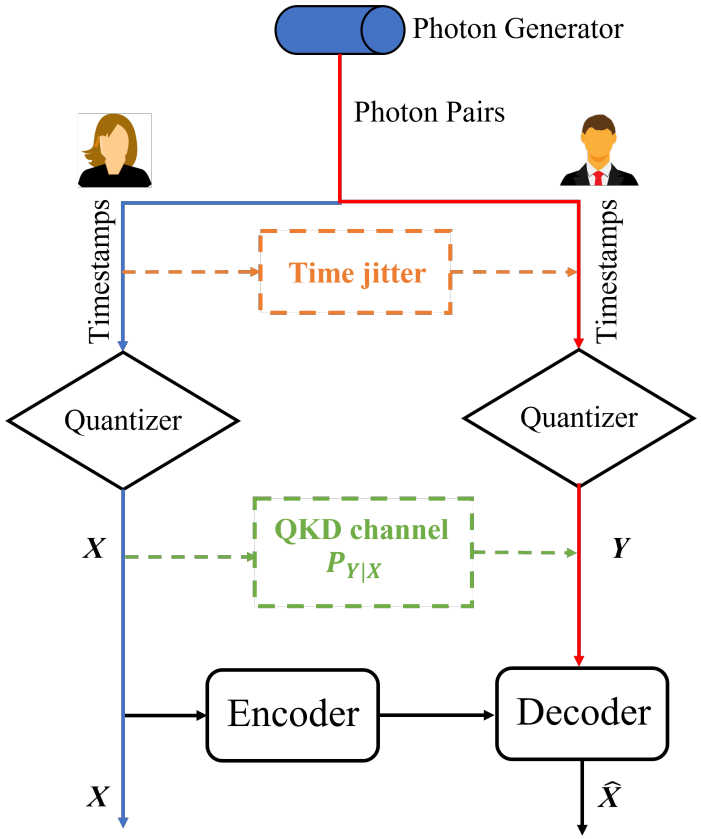

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- Any information sent over the public channel is seen by Eve and will reduce the final key length (due to privacy amplification)

Earliest Reconciliation Protocol: Cascade

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Figure: https://cascade-python.readthedocs.io/en/latest/protocol.html

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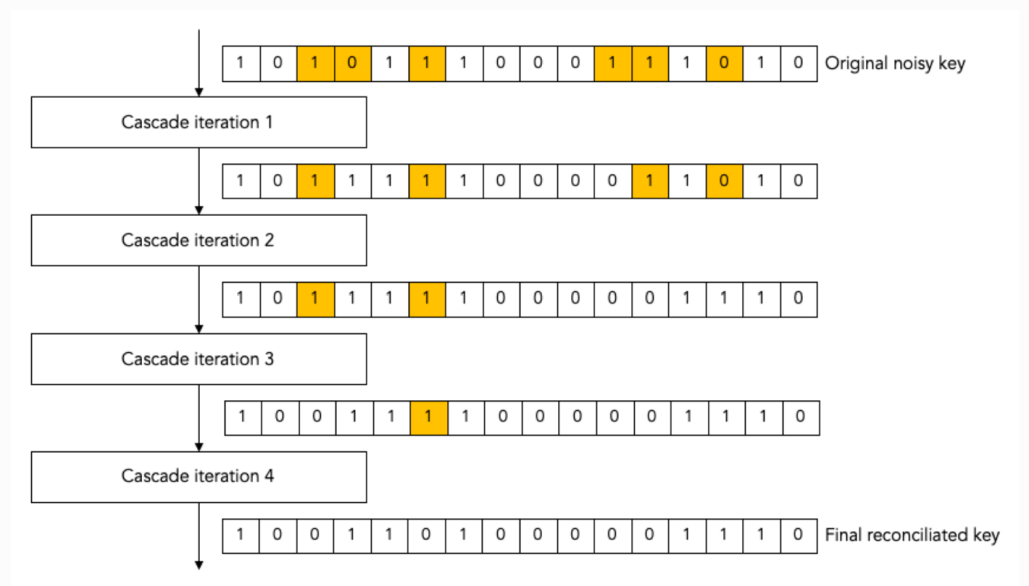

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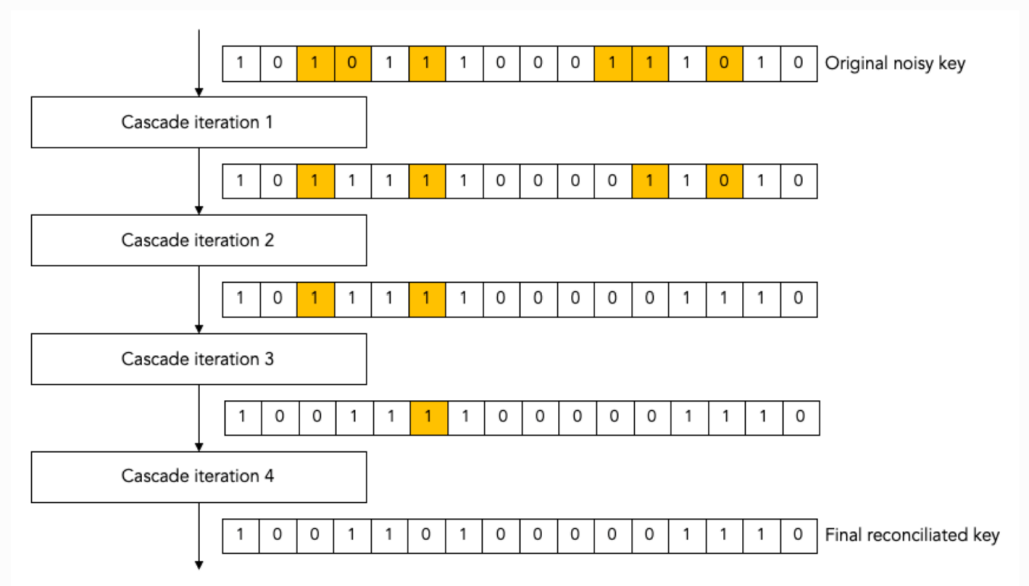

- During each iteration, the raw key is shuffled and divided into top-level blocks
  - Size of the top-level block depends on the iteration number and the estimated quantum bit-error rate

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Can LDPC codes cognizant of the properties of the QKD channel improve key rates ?

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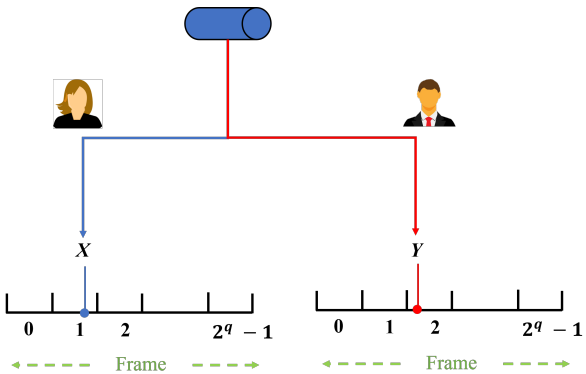

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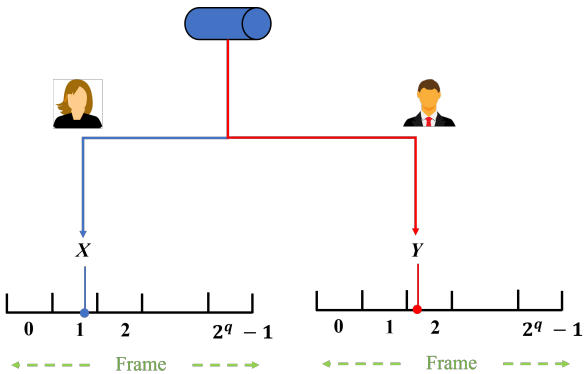

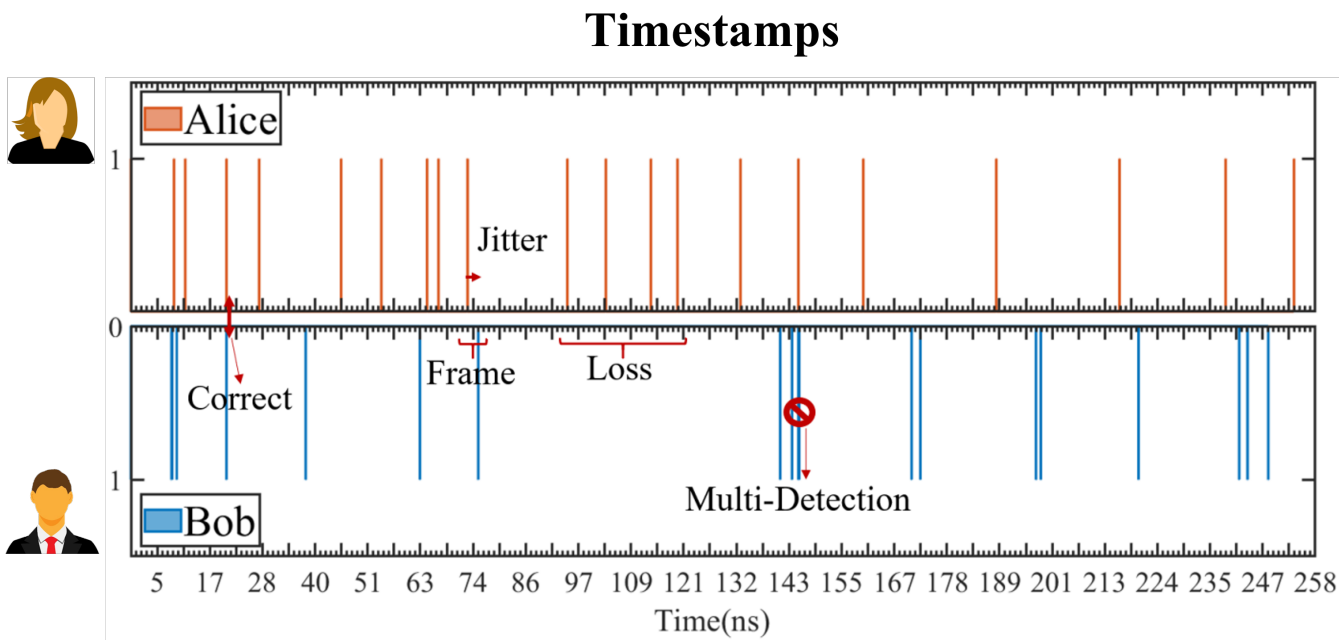

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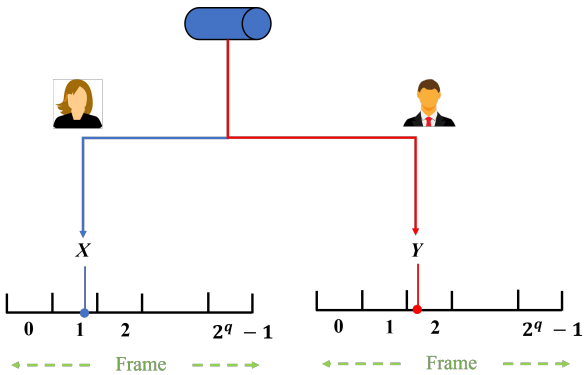

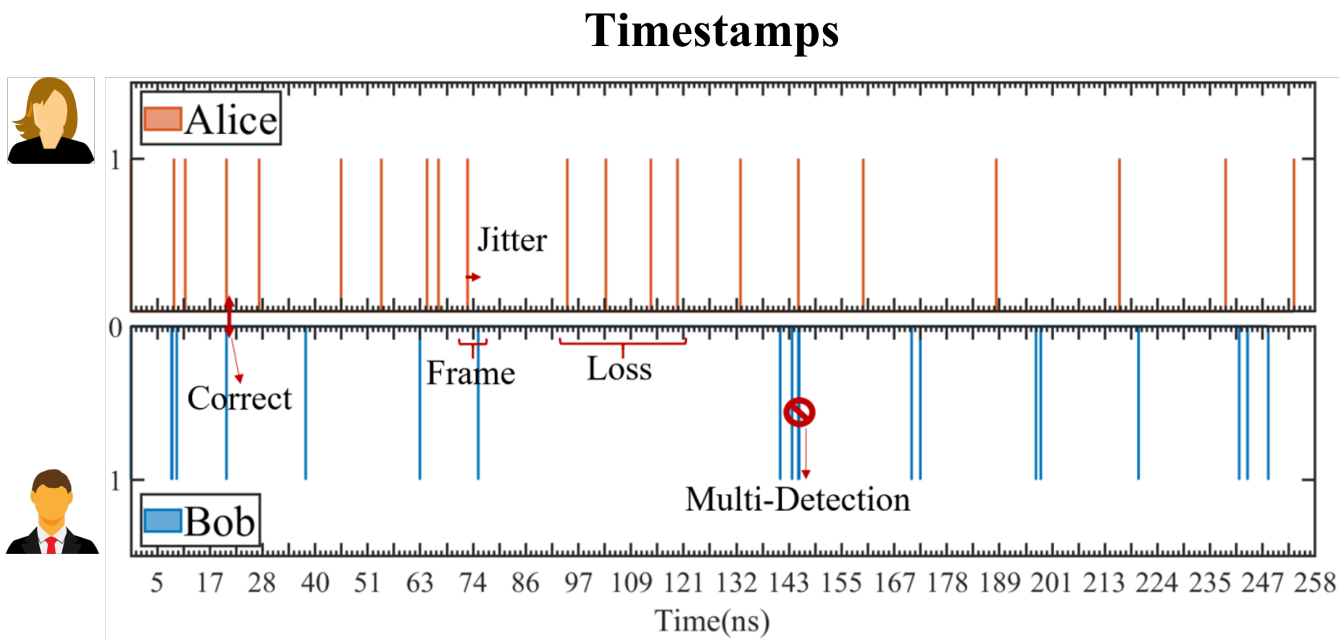

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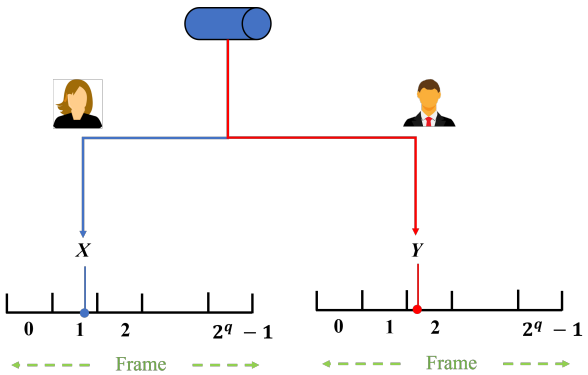
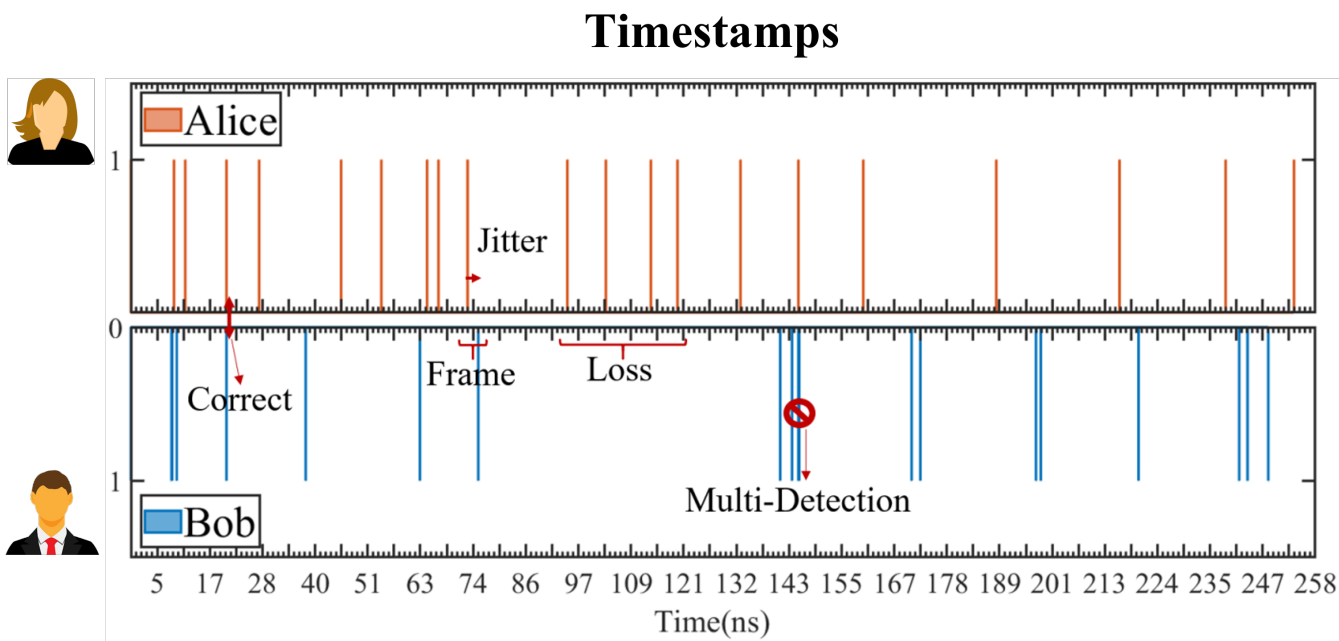
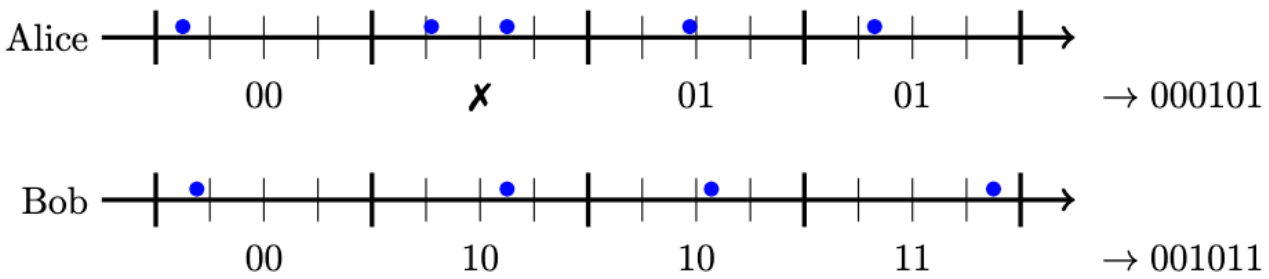
- Complex error sources: timing jitter errors, detector downtime, dark counts, photon loss





- Experimental Set-up: Wong Group at UCLA.

An Example



- **In the leftmost frame**, both Alice and Bob map their result to '00' (the leftmost bin is occupied in both). Bit extraction is successful despite a slight arrival jitter.

Figure credit: E. Soljanin.

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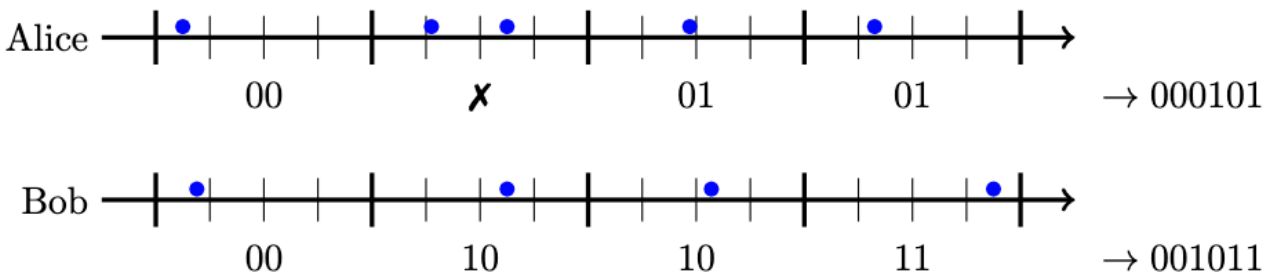

- **In the second frame**, Alice detects two arrivals, with the spurious one due to dark counts. While Bob initially can map his result to '01' for his second frame, upon receiving information from Alice that this frame is invalid, he too discards it.

Figure credit: E. Soljanin.

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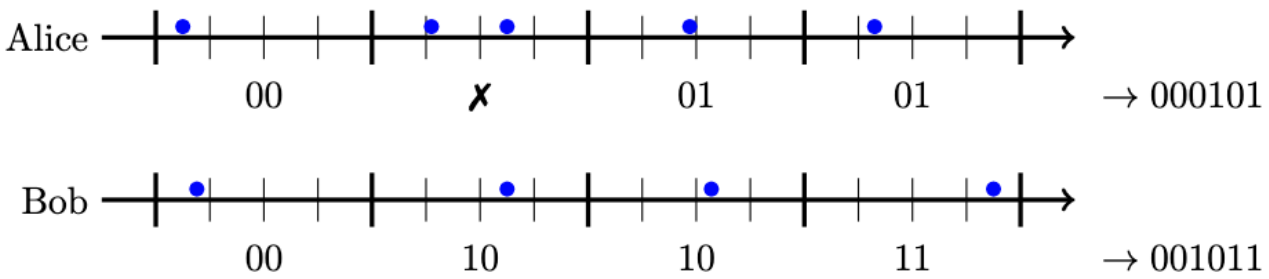
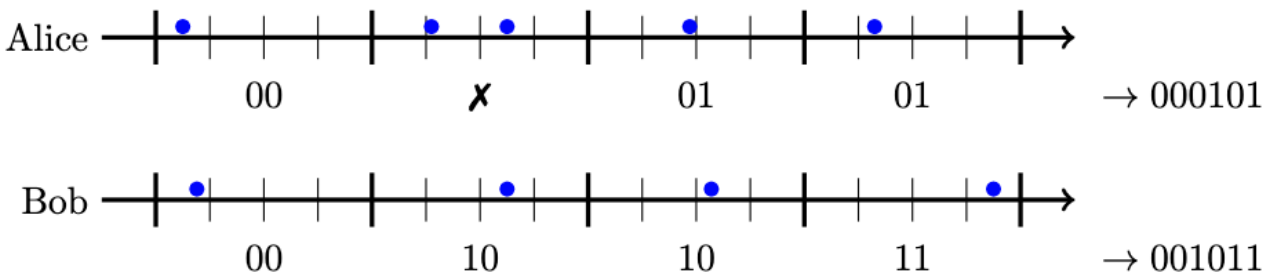


- **In the third frame**, both Alice and Bob observe a single arrival. However, due to timing jitter, the two arrivals fall into adjacent bins. Alice maps her result to '01' whereas Bob maps his to '10', resulting in a 2-bit discrepancy.

Figure credit: E. Soljanin.

An Example

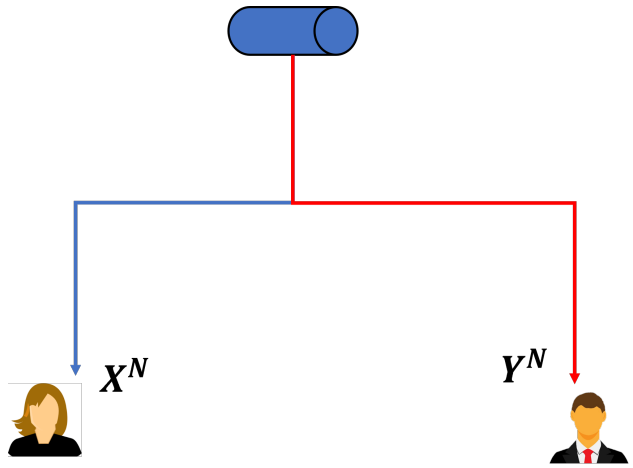


- **In the rightmost frame**, Alice and Bob each again detect a single arrival. In this example, two arrivals are due to dark counts and are uncorrelated (unbeknownst to Alice and Bob at this point). Alice maps her result to '01' and Bob his to '11'.

Figure credit: E. Soljanin.

Information Reconciliation: Source Coding with Side Information (Elkouss '09)

- Encoding
  - Sequences received by Alice and Bob are X^N and Y^N respectively



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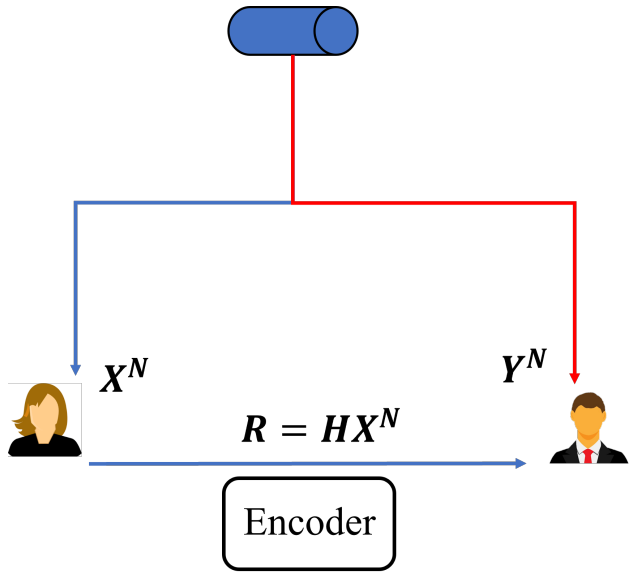

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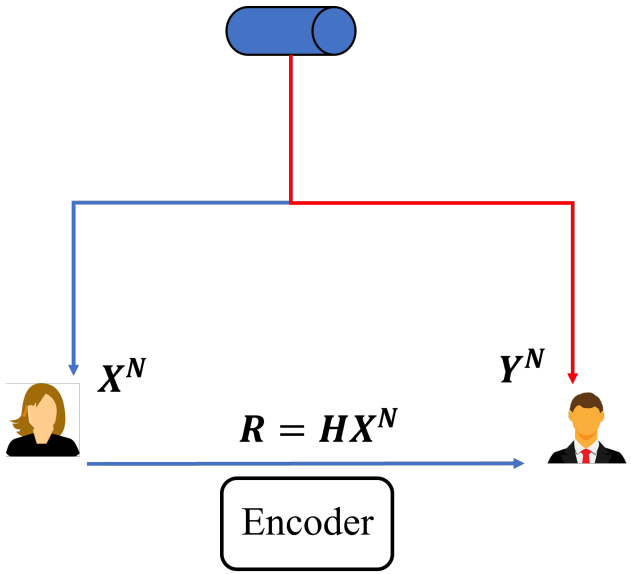

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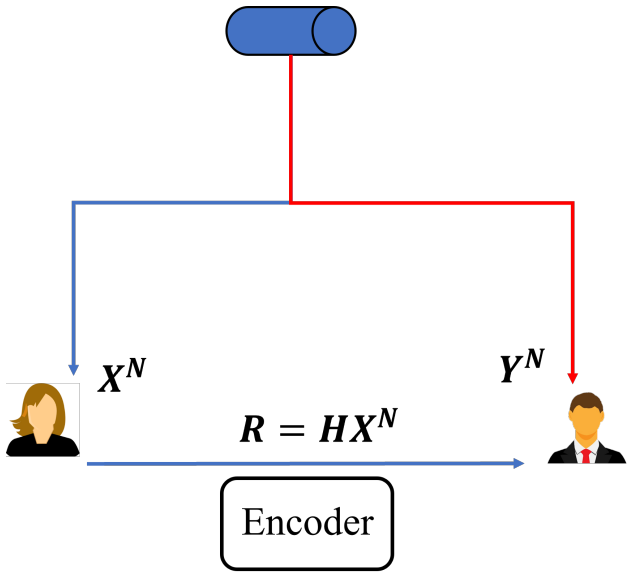

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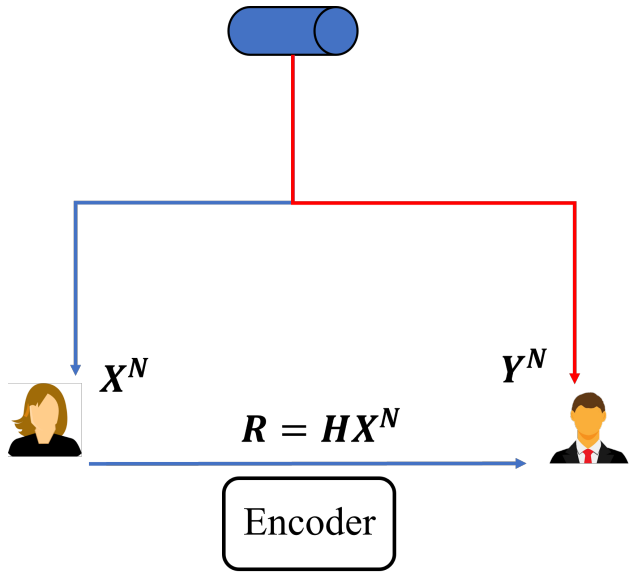

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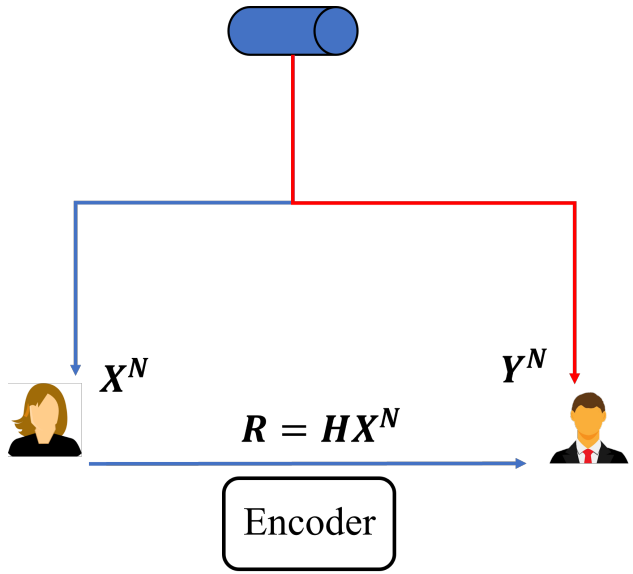

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Parity bits are visible to Eve.
Need to balance code rate and its error correction capability.

Prior related work

Classical design approaches

- LDPC coding (Kasai '10, Mao '19, Milicevic '17, 18, Muller '23), Polar coding (Jouguet '14, Fang '22)
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Next: Information Reconciliation using LDPC codes designed based on the properties of the QKD channel

ET-QKD Channel: Details

QKD Channel:

- $P_{Y|X}(y|x): \ x, y \in \{0, 1, \ldots q-1\}$

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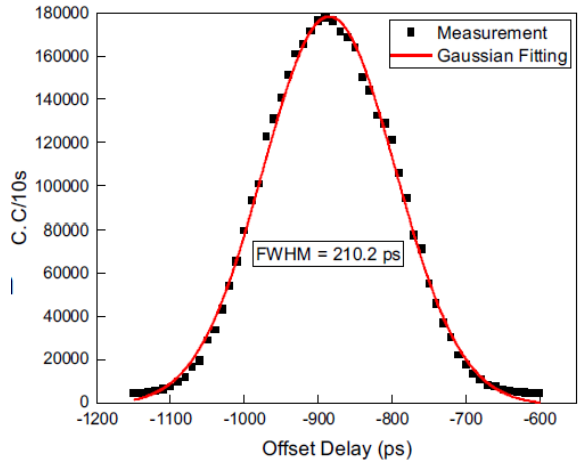

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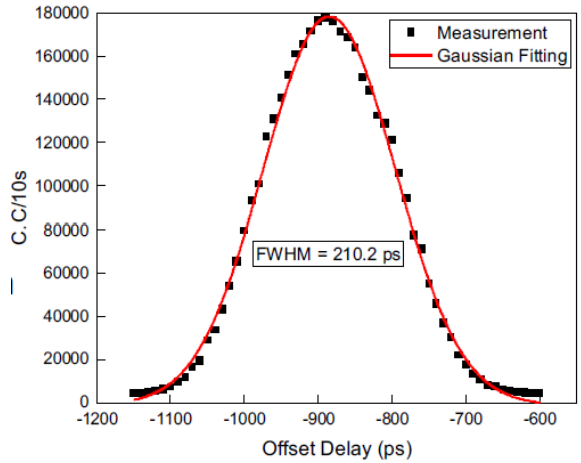

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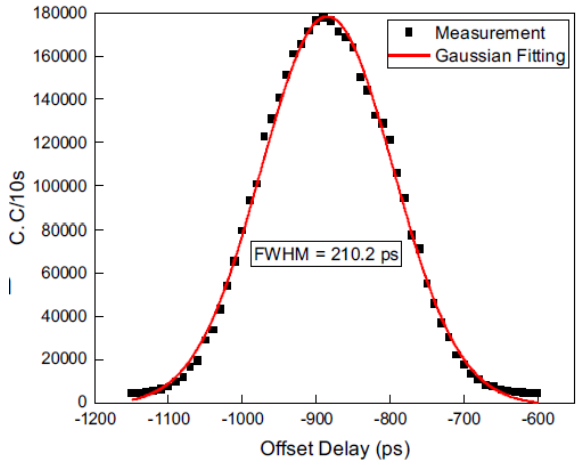

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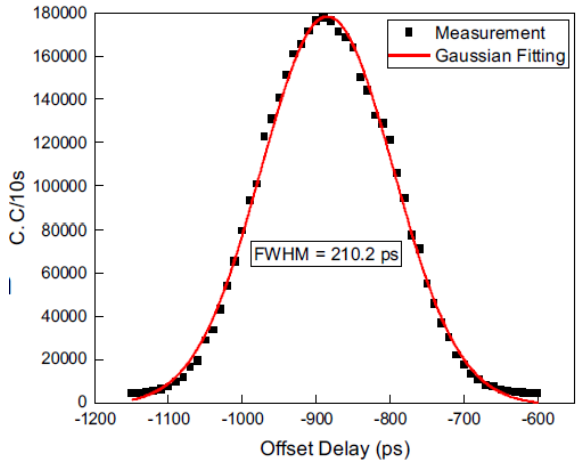

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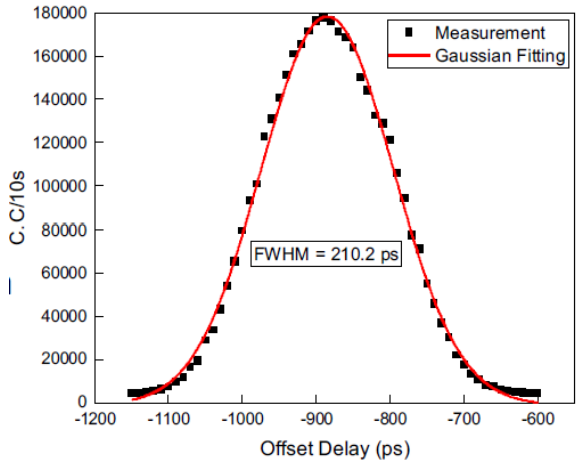

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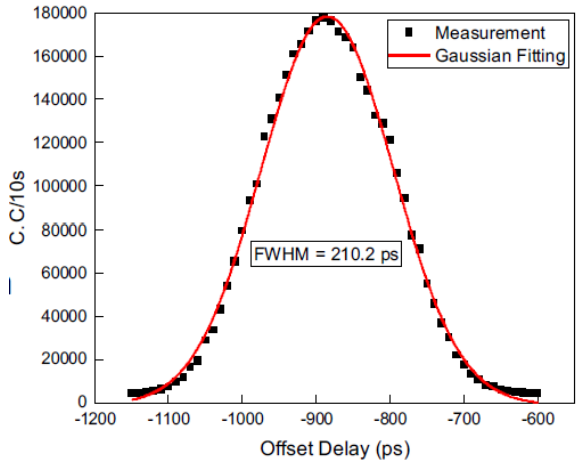

Outline

1. Background and Motivation

2. System Model

3. NB-LDPC codes for Quantum Key Distribution
   - Preliminaries
   - Channel code design
   - Sequential Decoding and Interactive Communication
   - Privacy Amplification-Aware LDPC Code Design

4. Concluding Remarks and Future Outlook

Towards a New Solution: Fully Non-Binary (FNB) Protocol for IR

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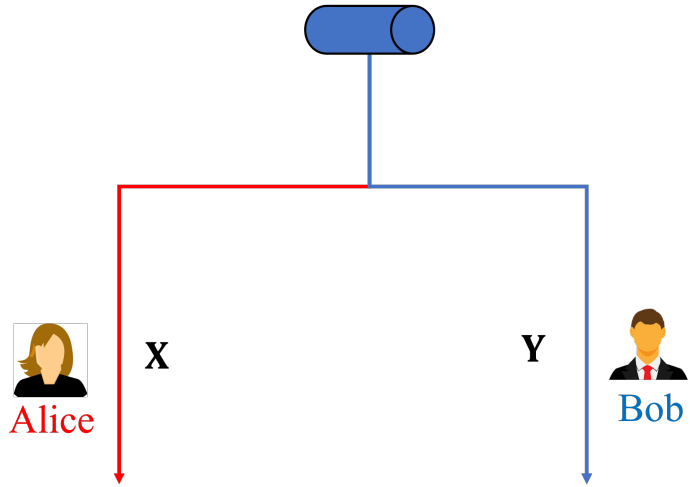

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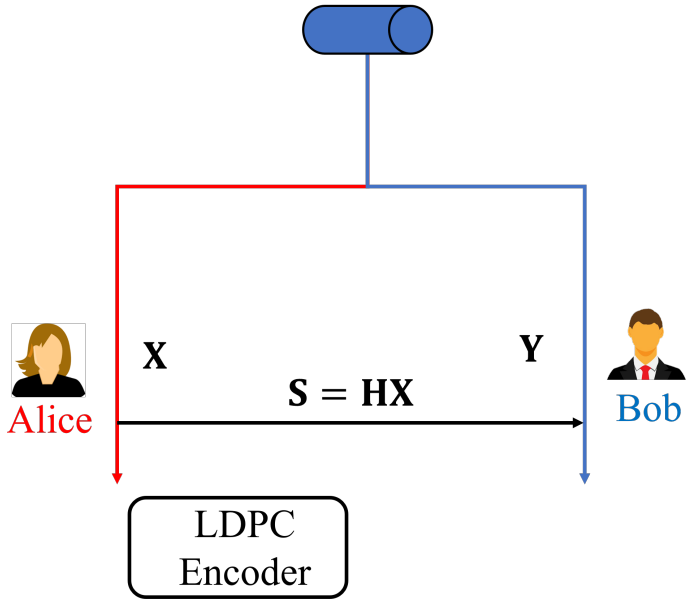

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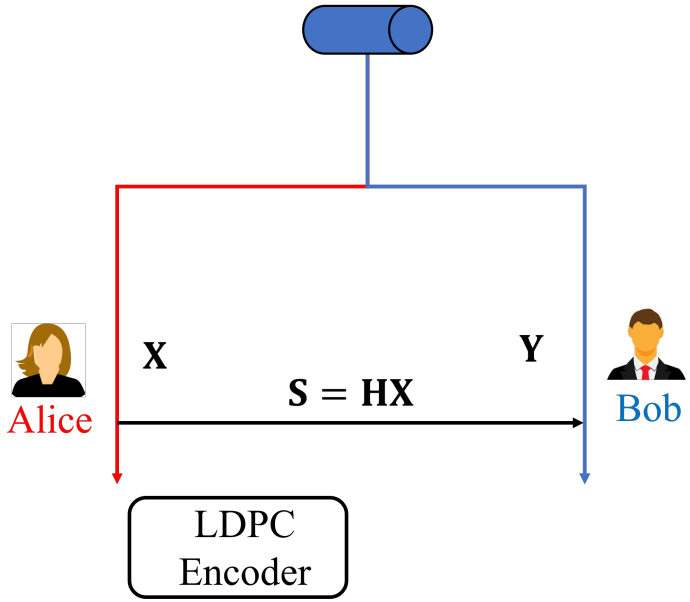

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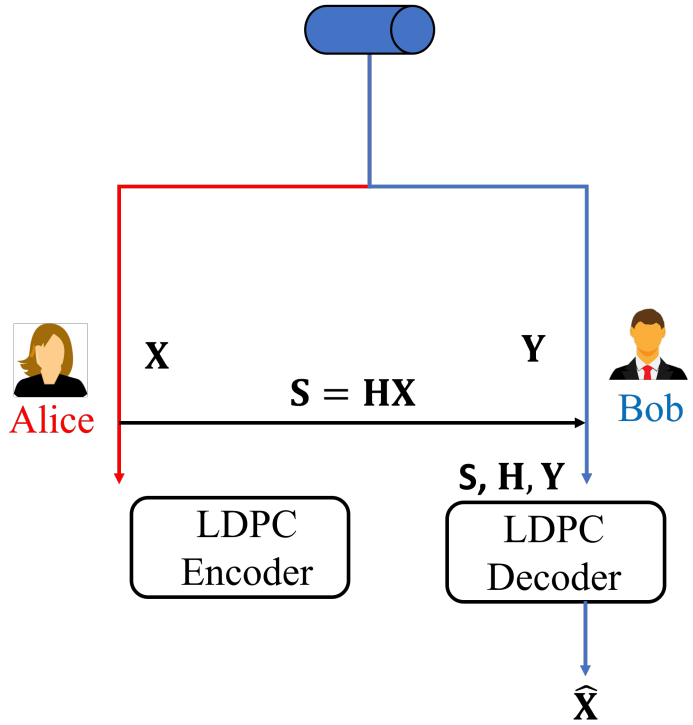

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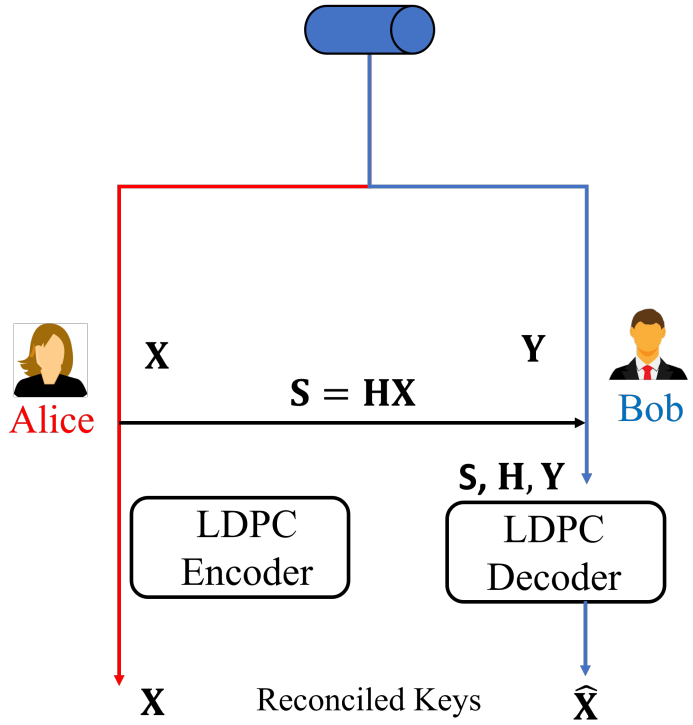

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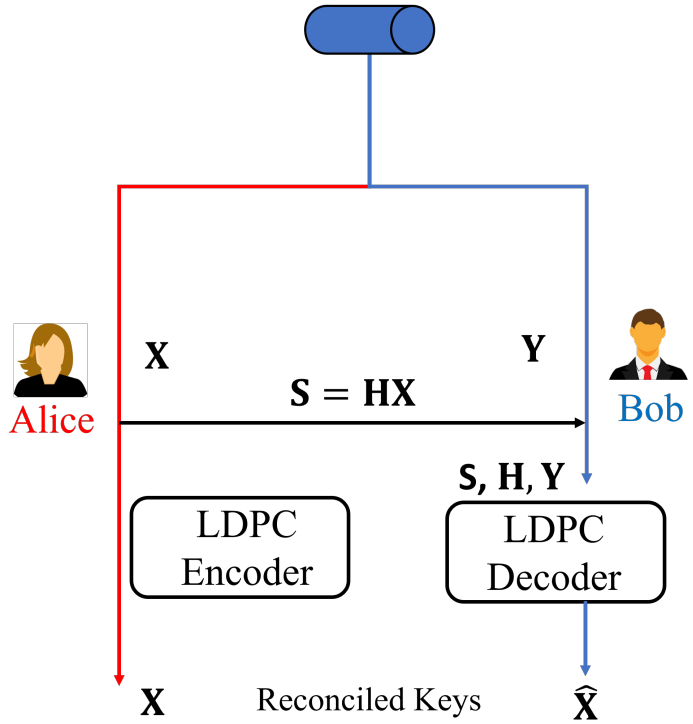

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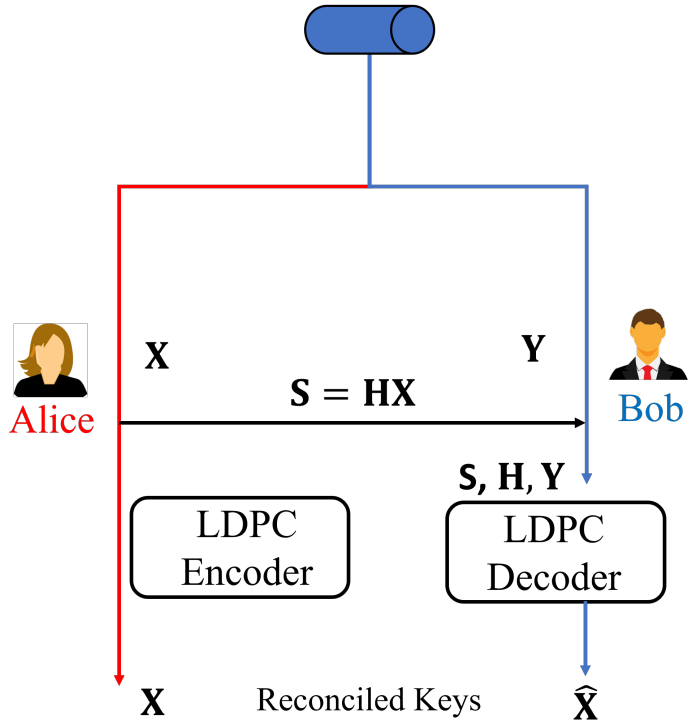

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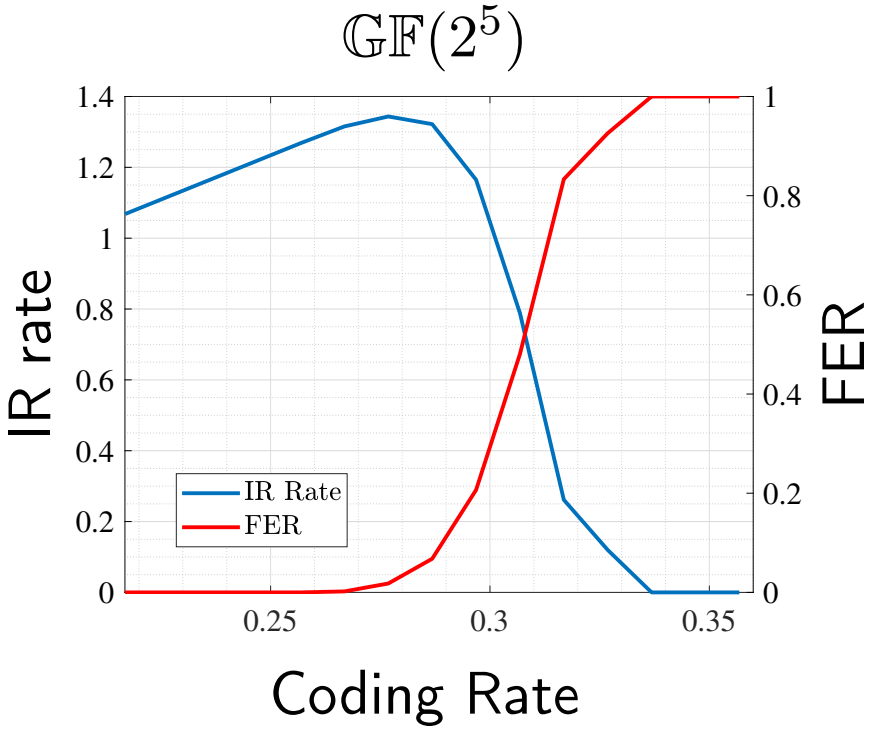

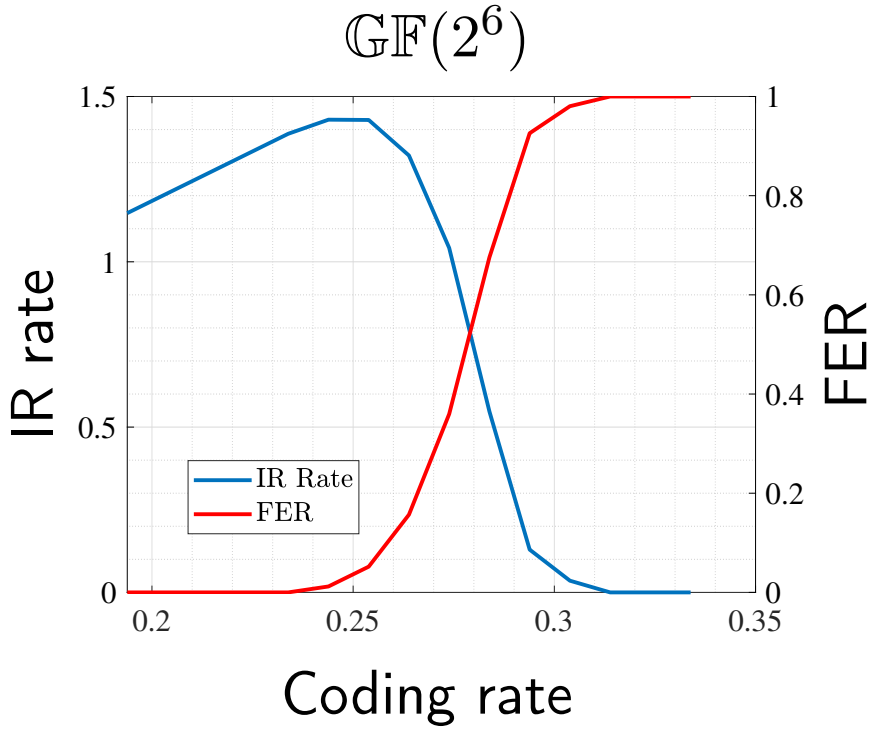

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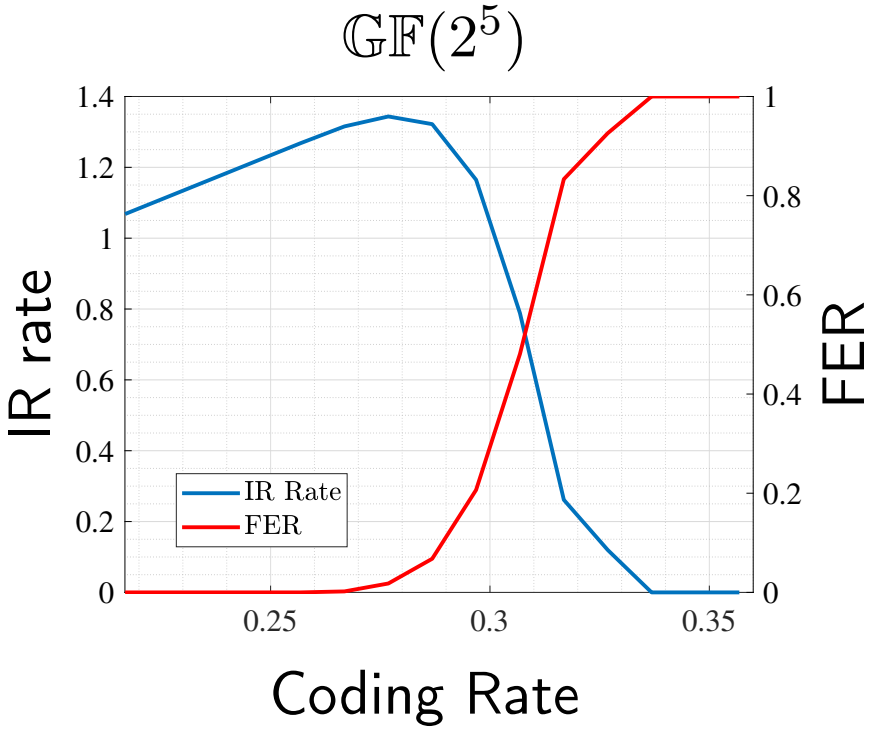

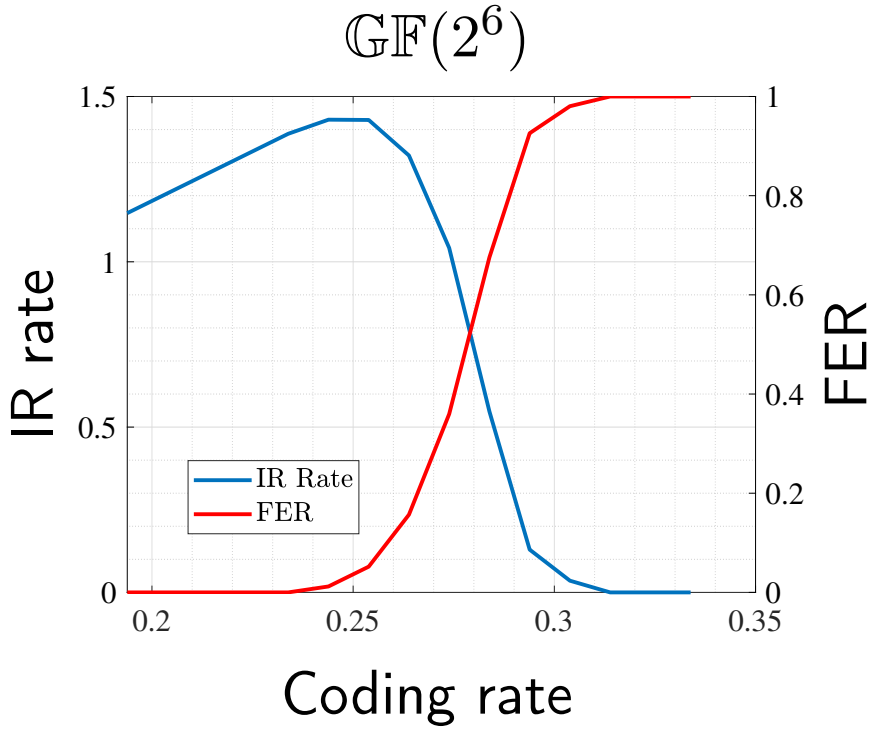

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Binary LDPC codes:

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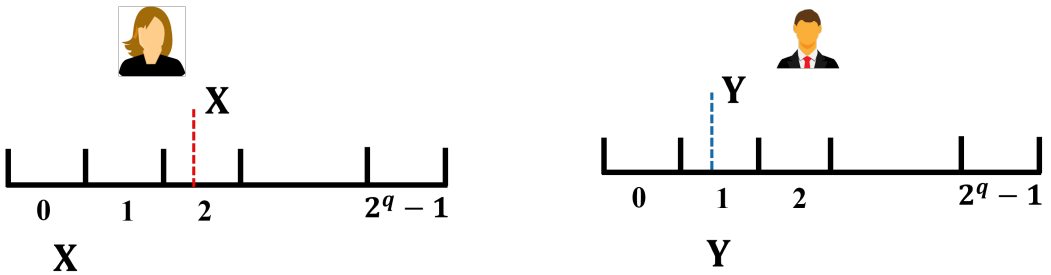

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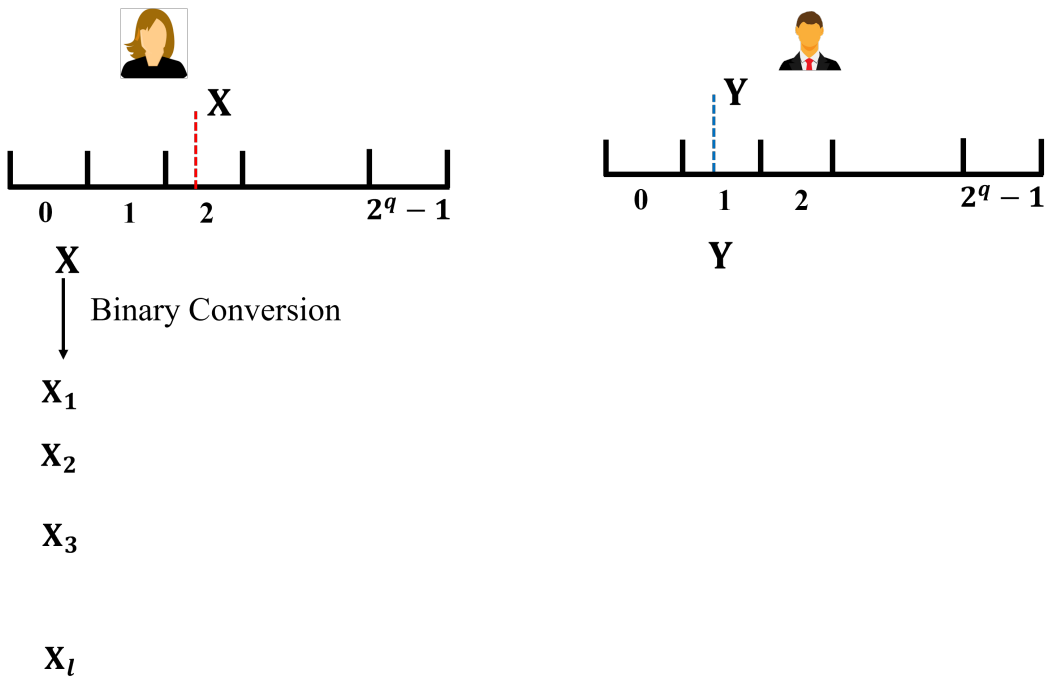

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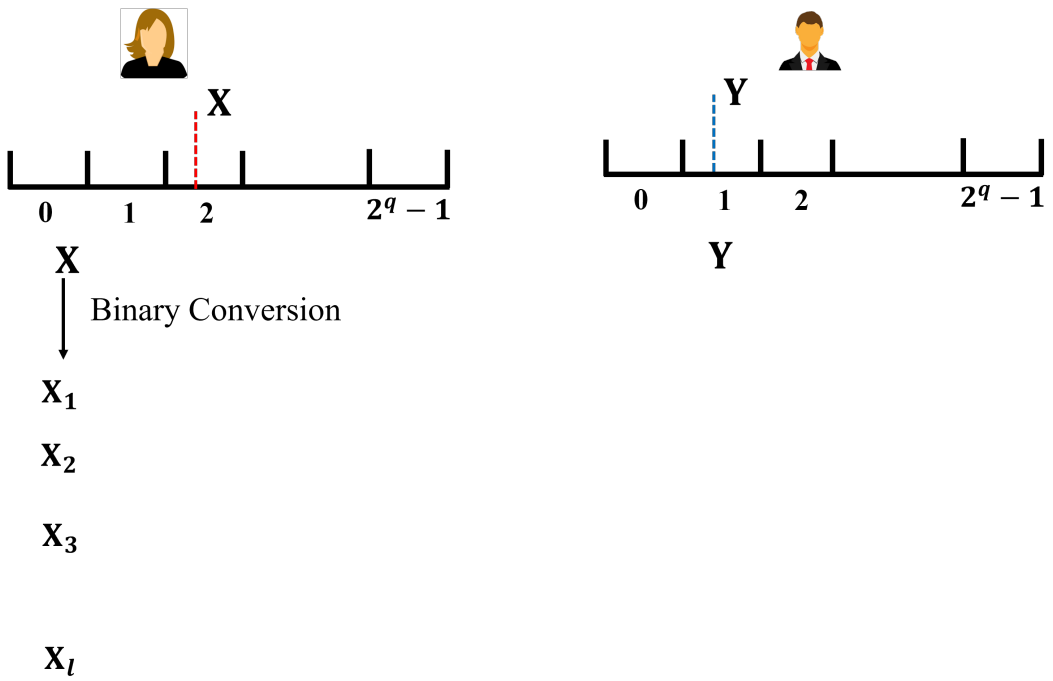

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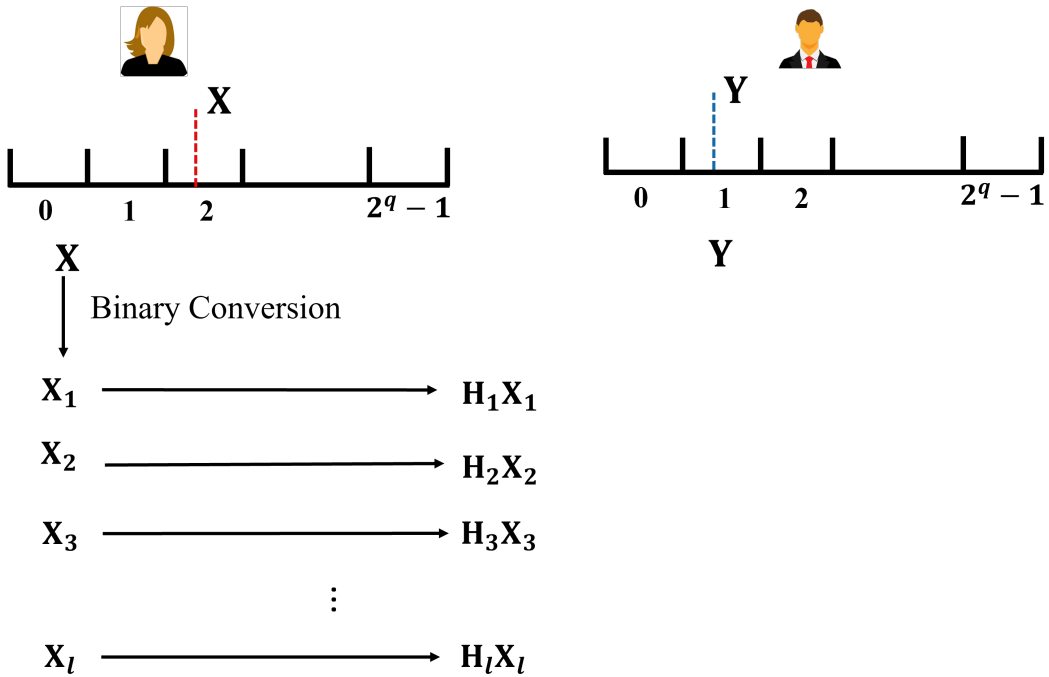

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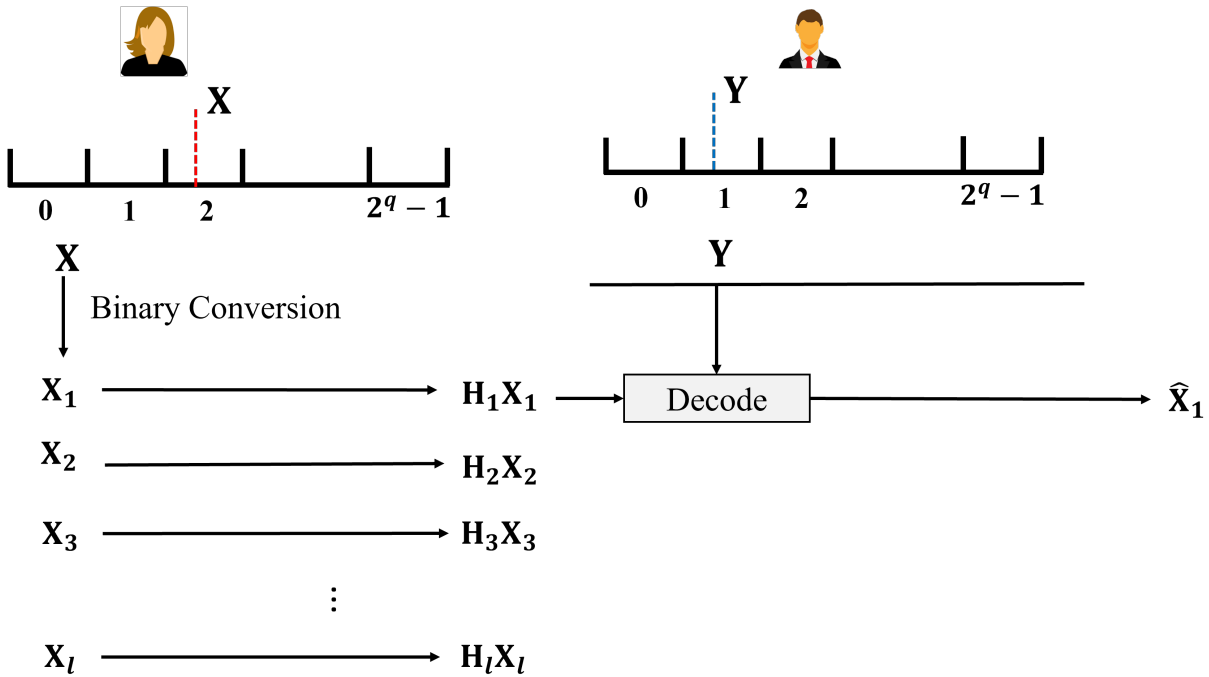

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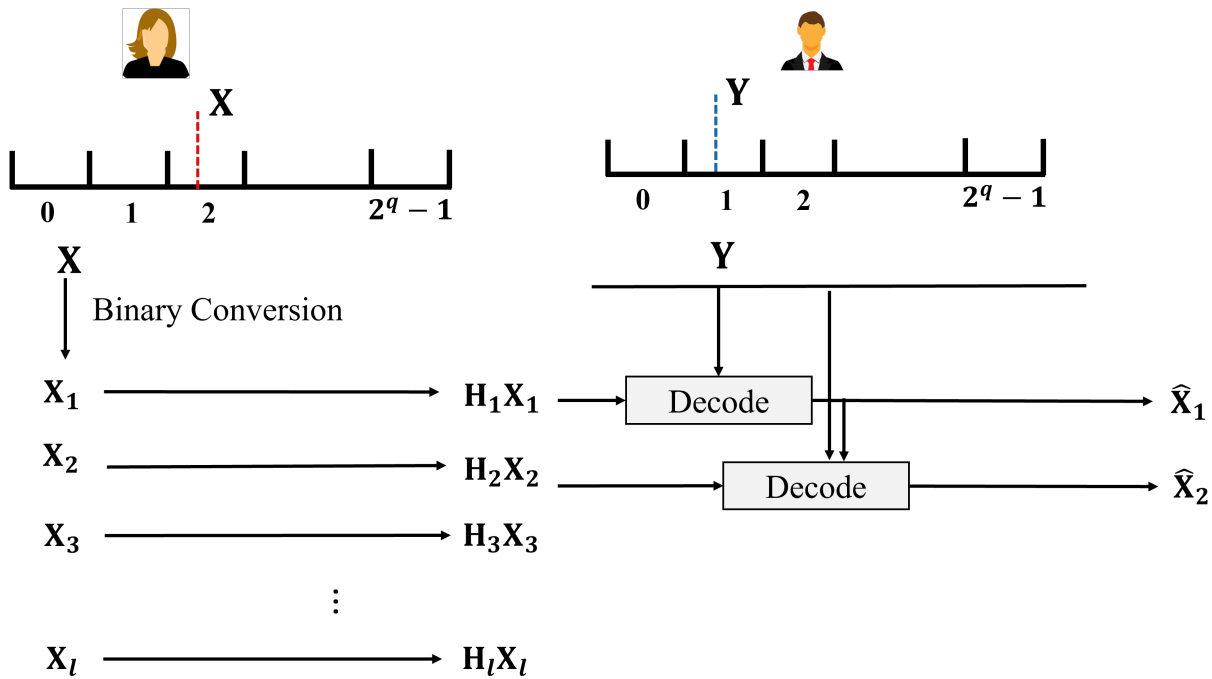

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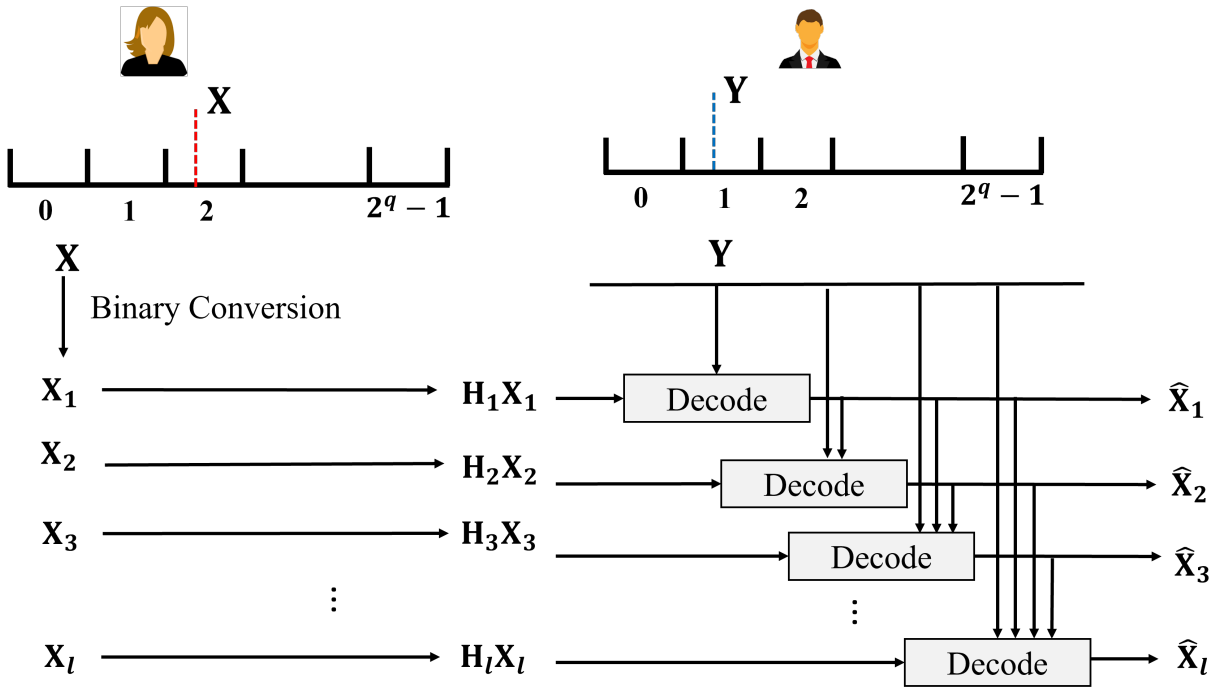

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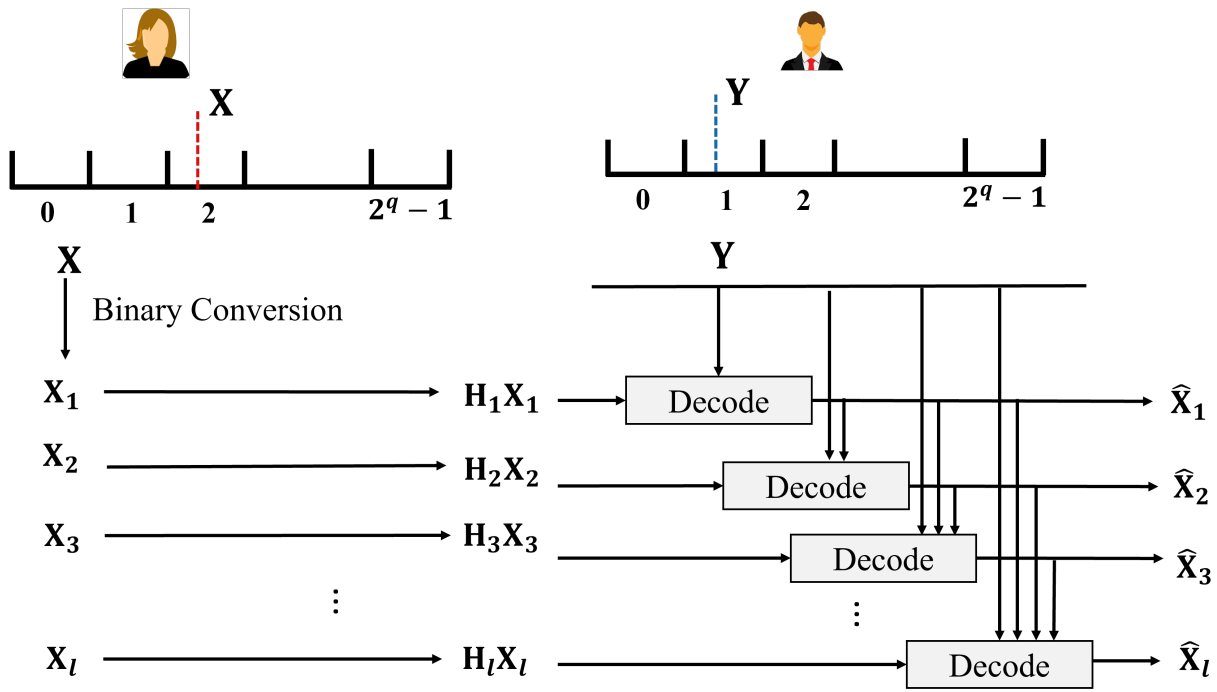

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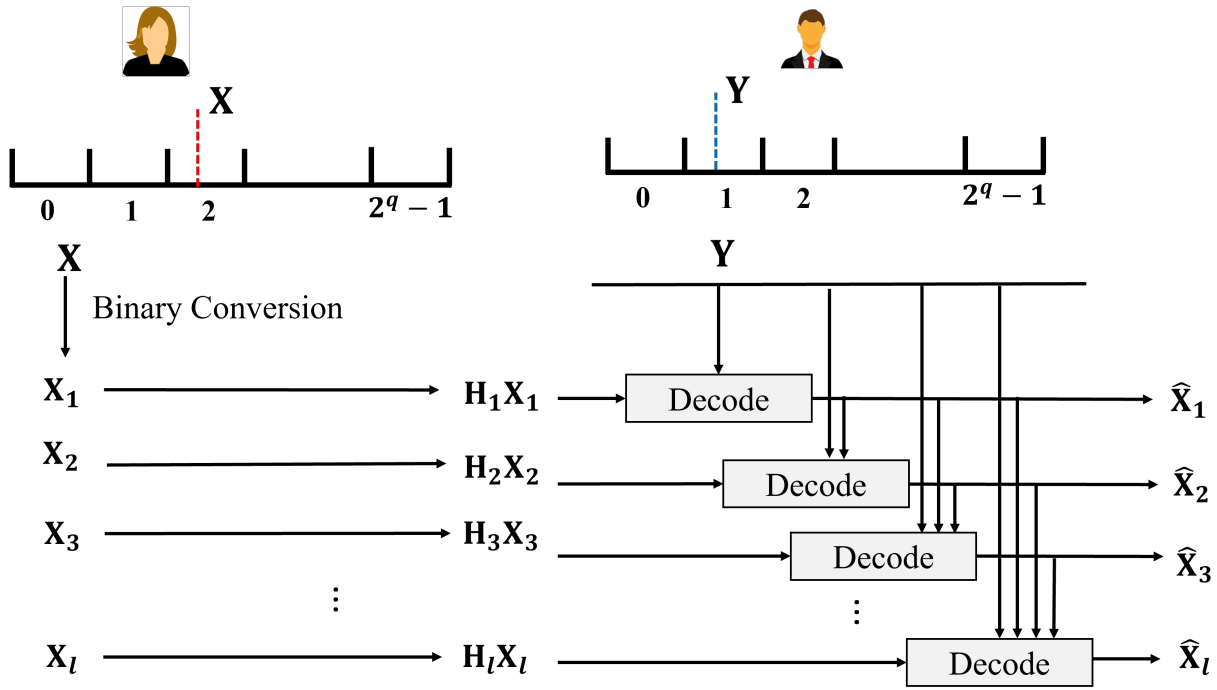

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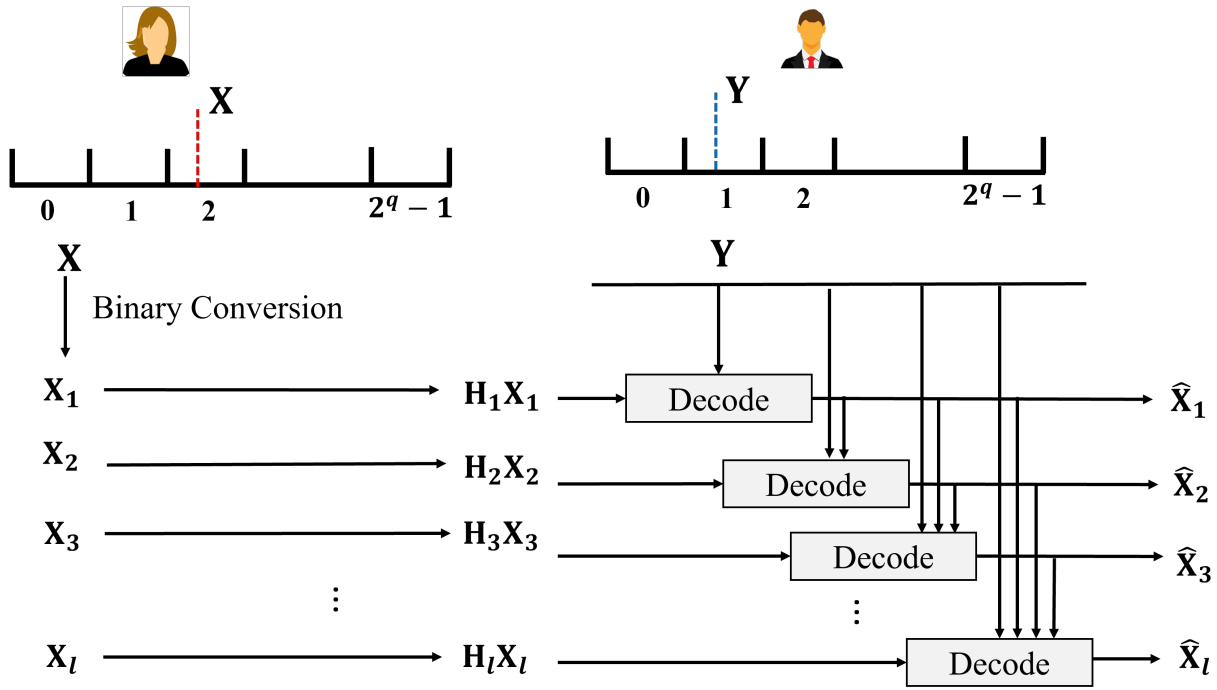

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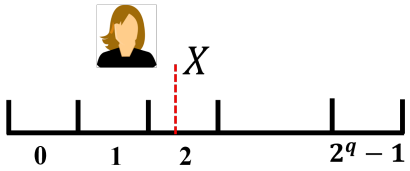

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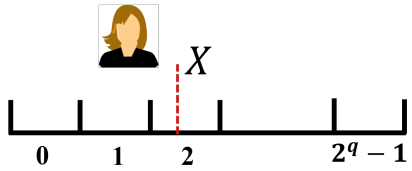

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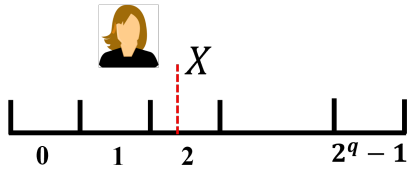

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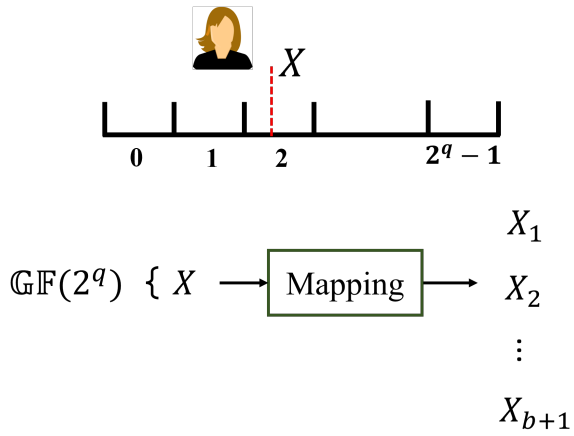

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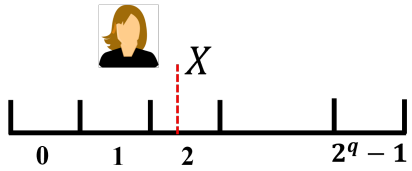

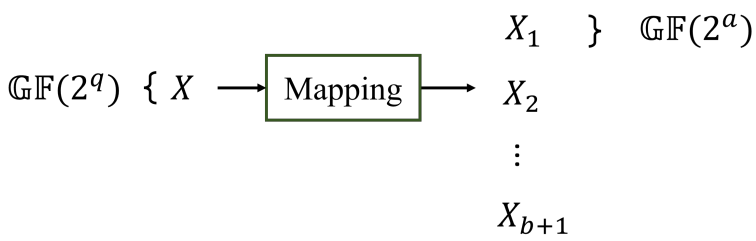

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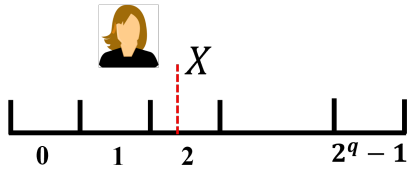

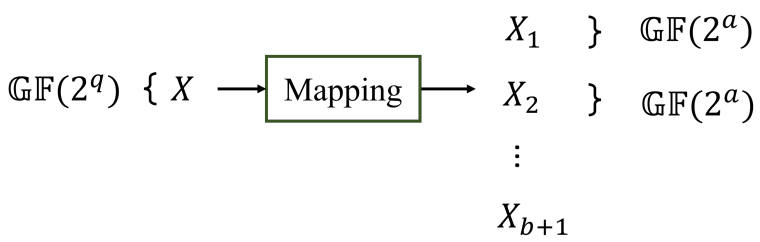

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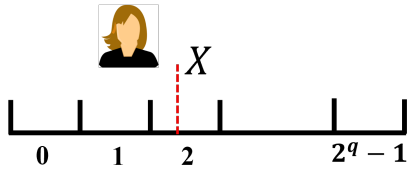

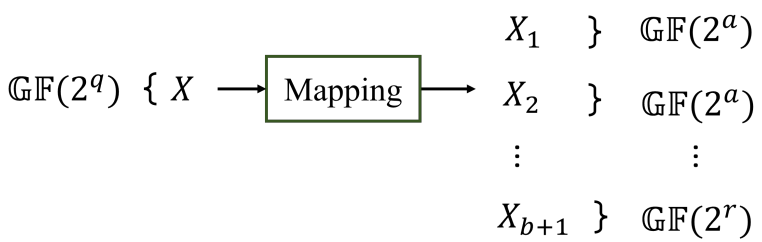

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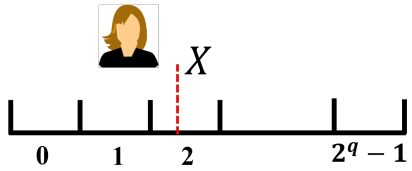

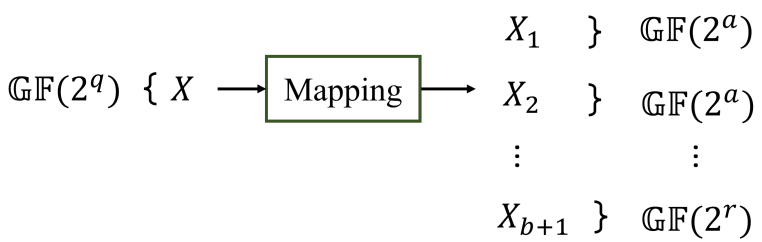

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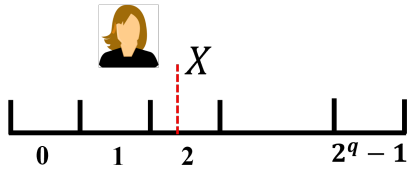

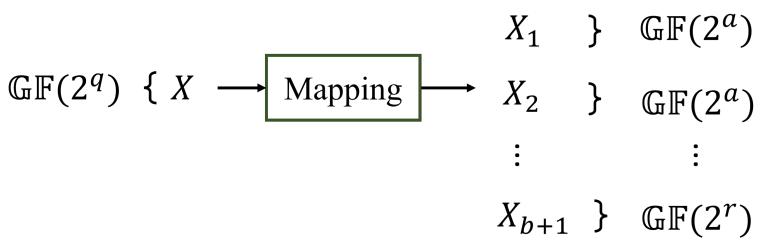

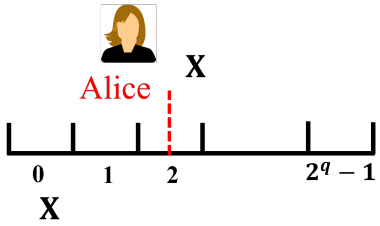

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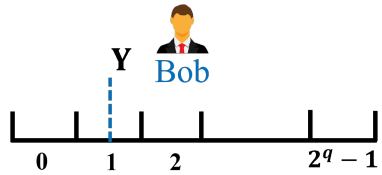

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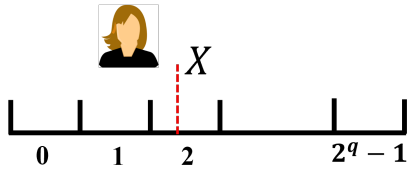

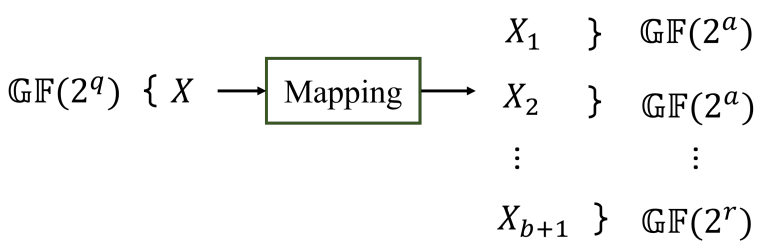

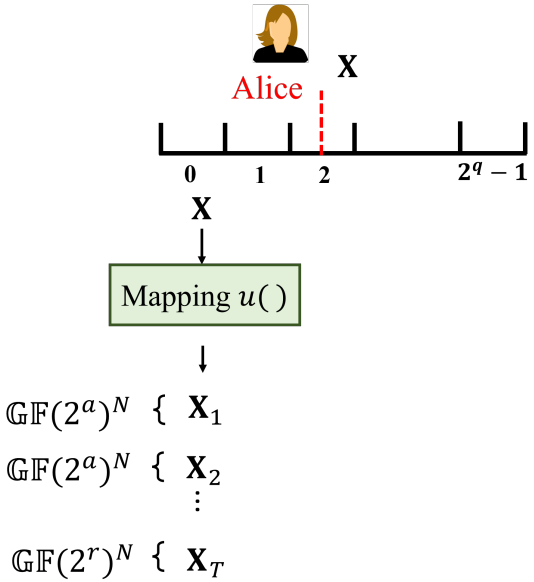

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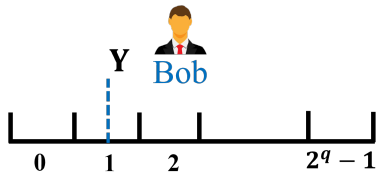

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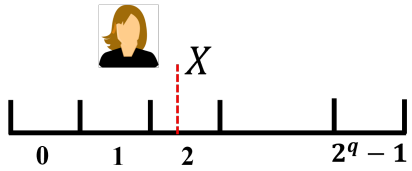

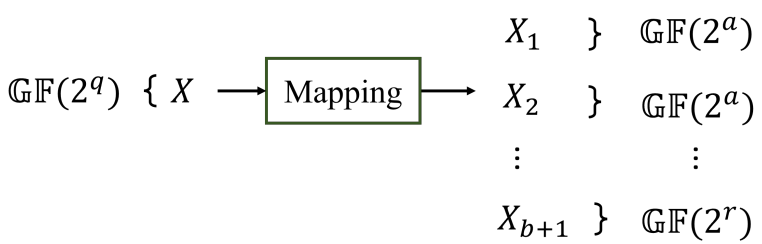

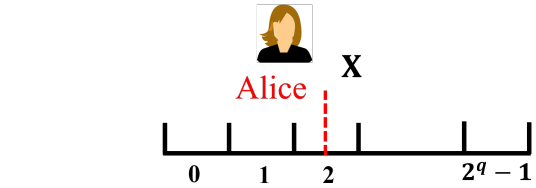

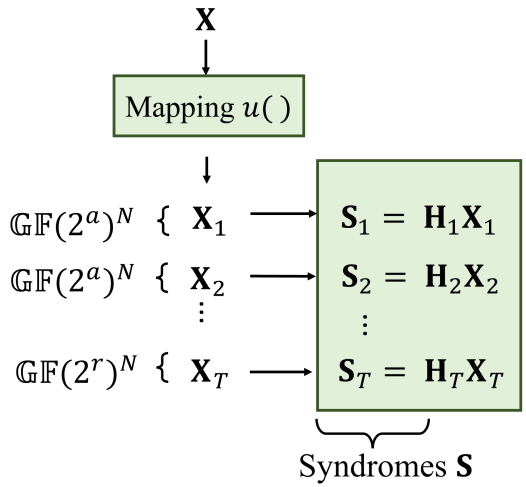

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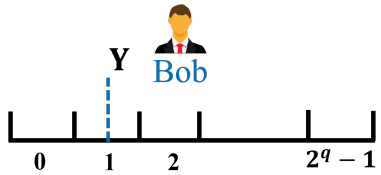

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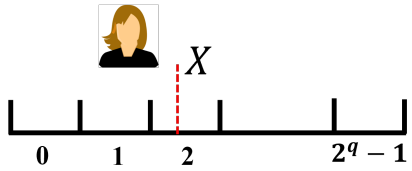

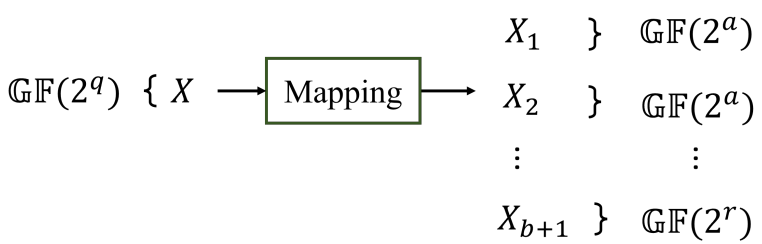

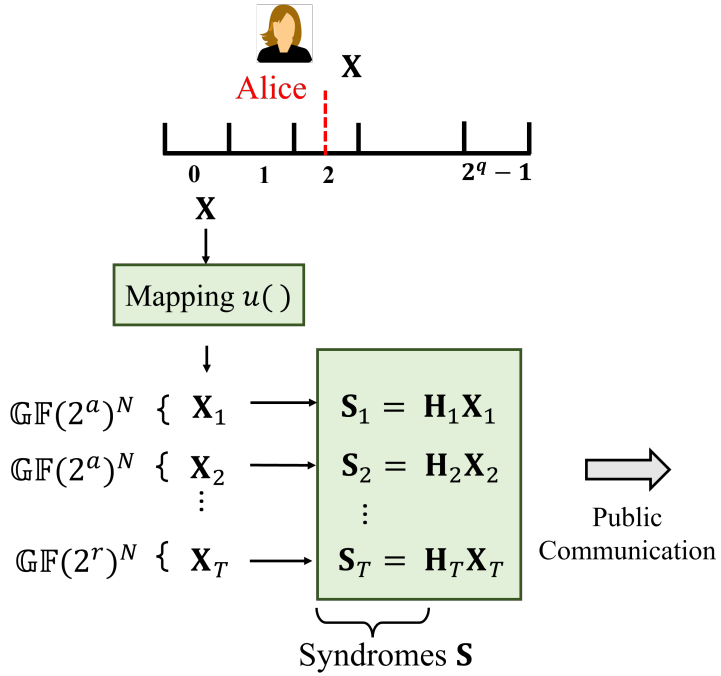

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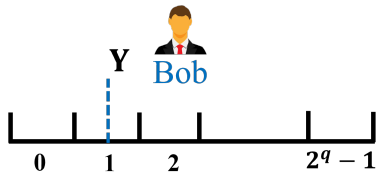

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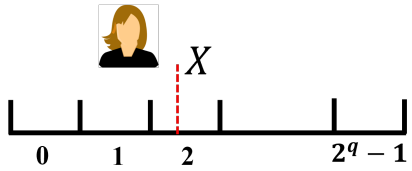

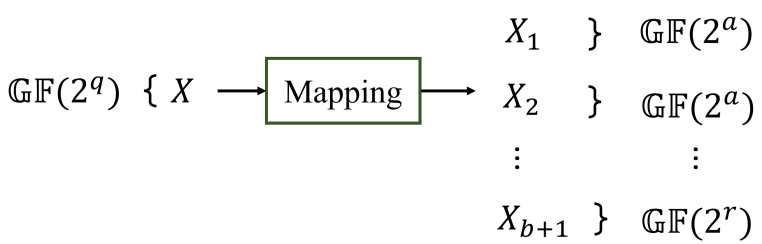

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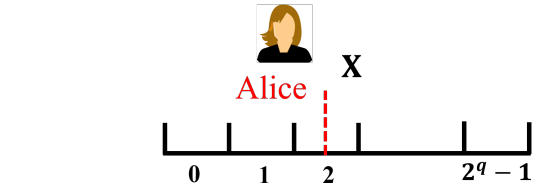

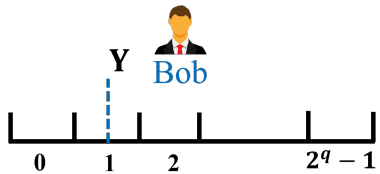

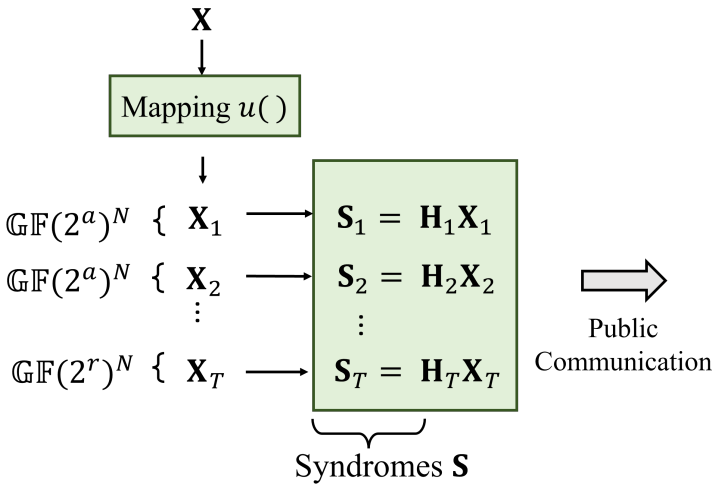

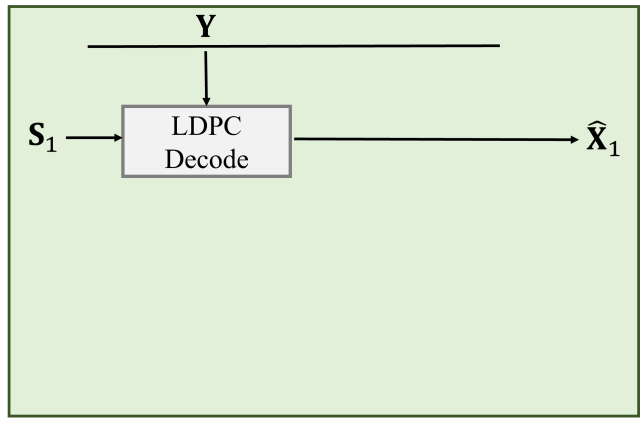

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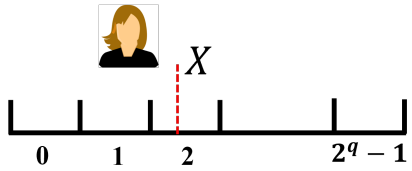

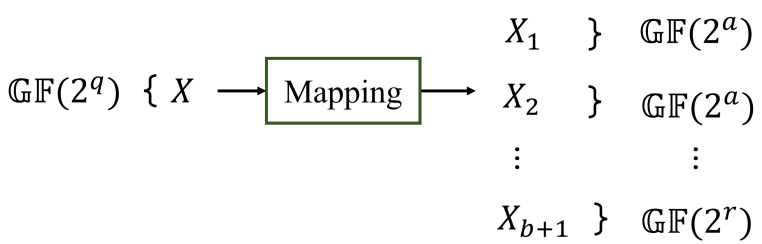

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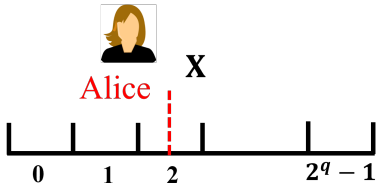
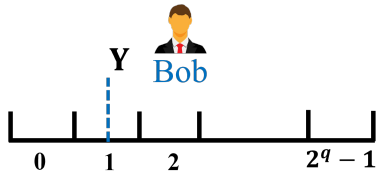
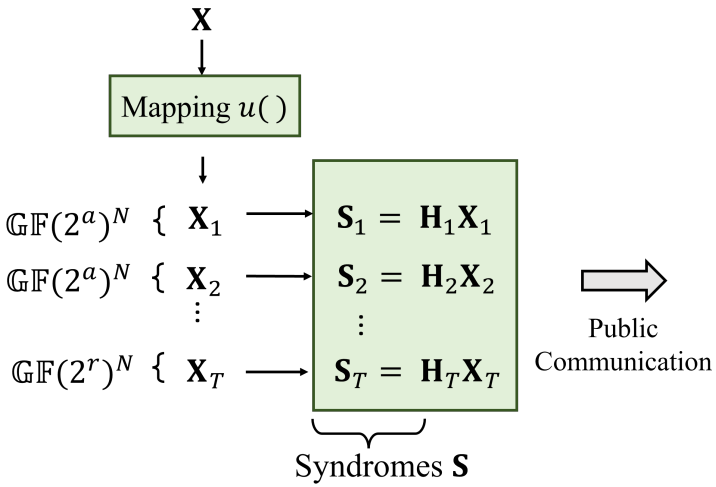
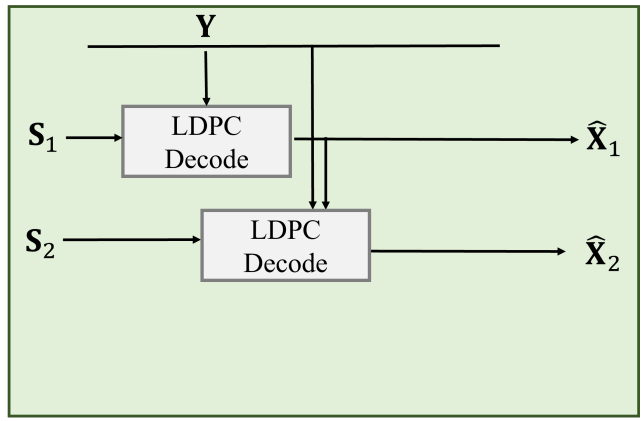
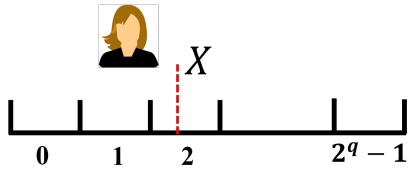


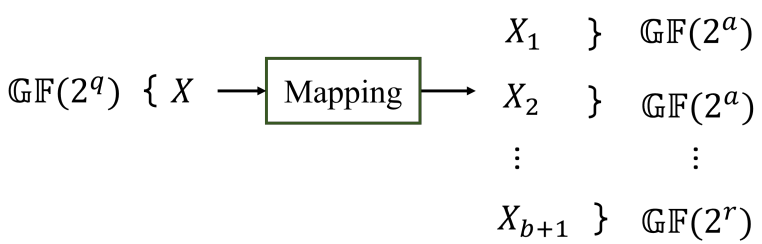






NB–MLC(a) Protocol





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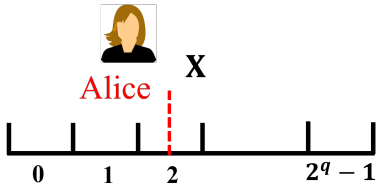

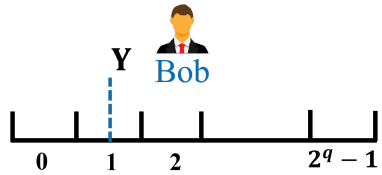

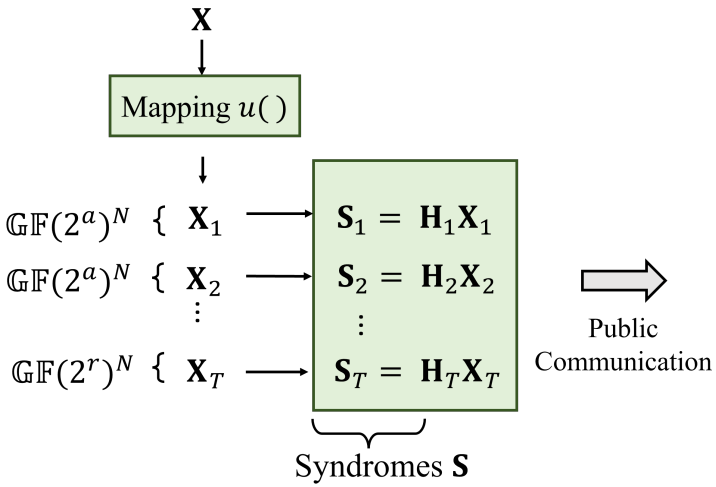

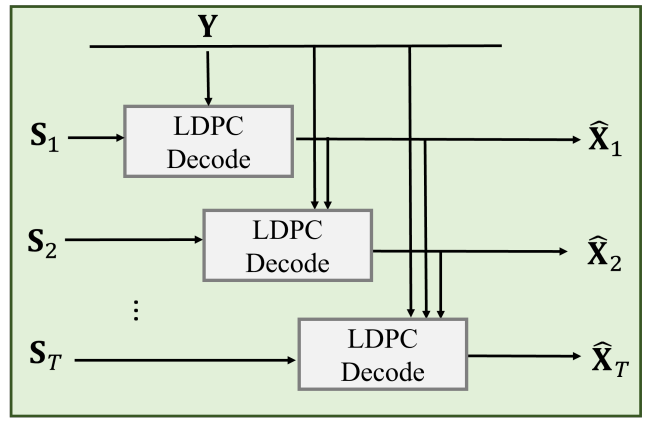

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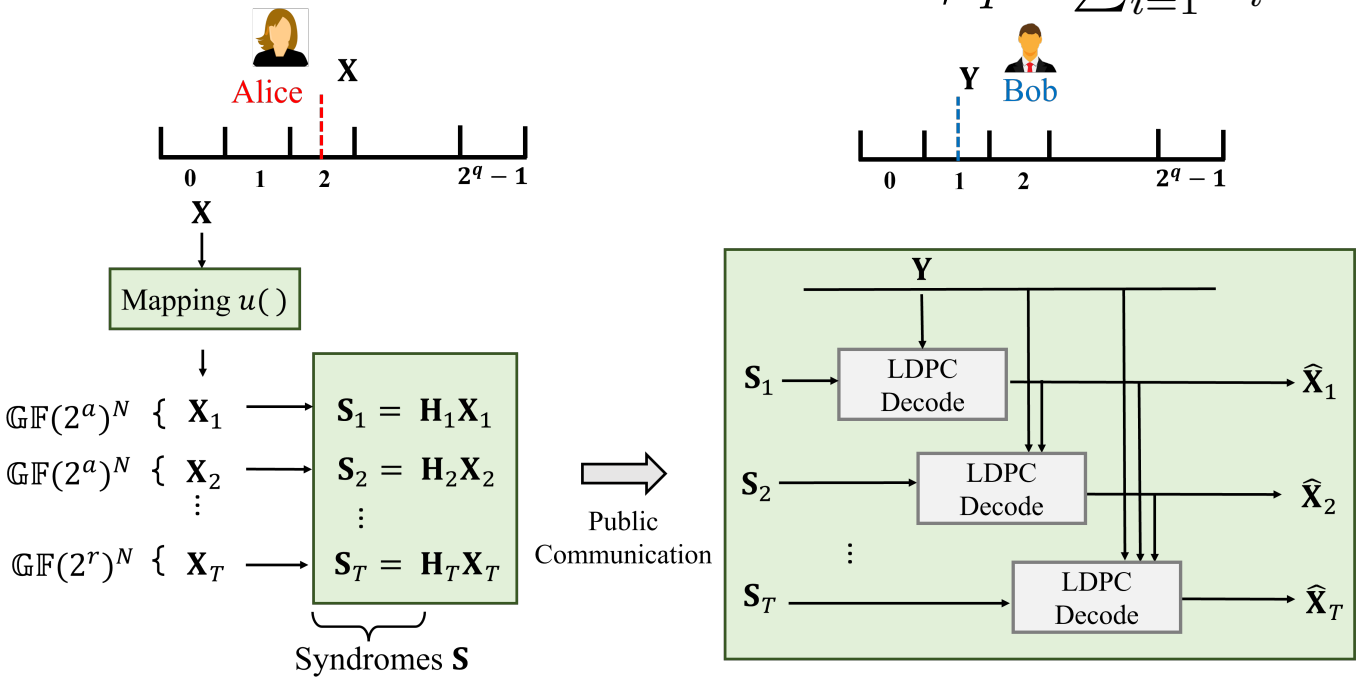

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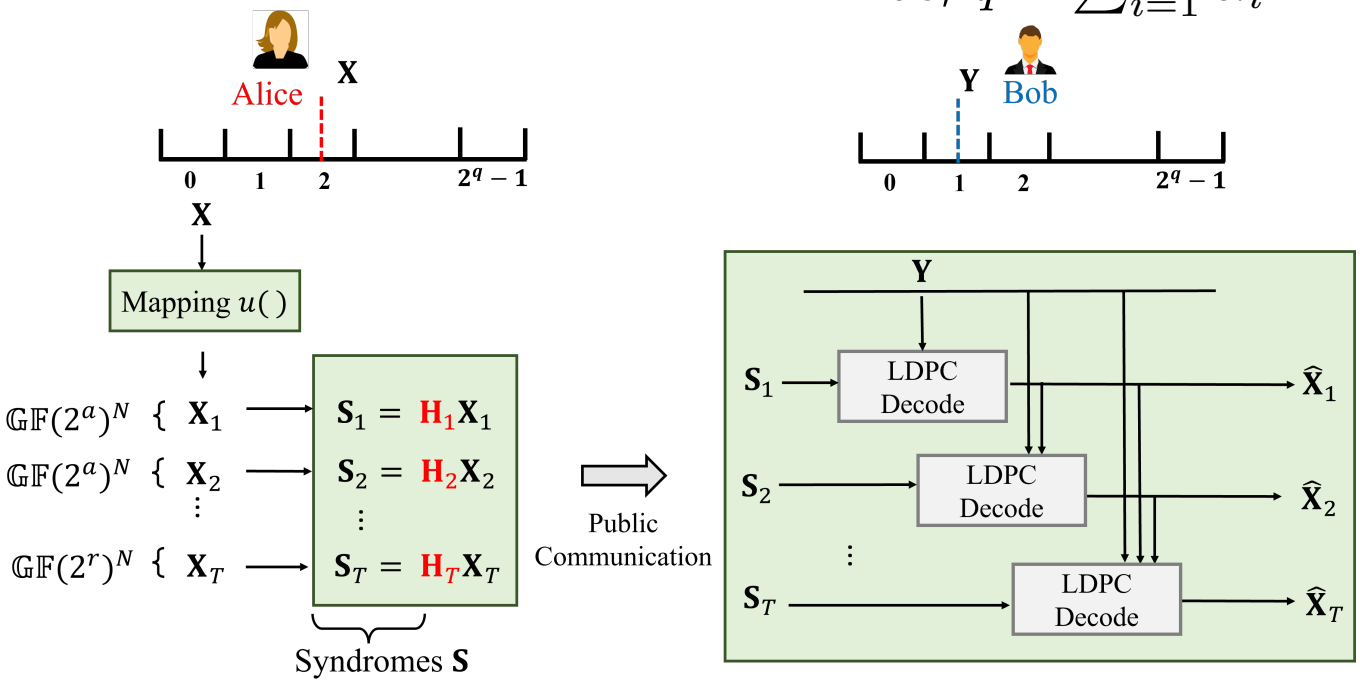

NB-LDPC Code design

Irregular LDPC codes

- Improves the FER performance of the code
- Can be characterized using their VN and CN degree distributions $L(x)$ and $P(x)$, respectively

$$L(x) = \sum_d L_d x^d \qquad P(x) = \sum_d P_d x^d$$

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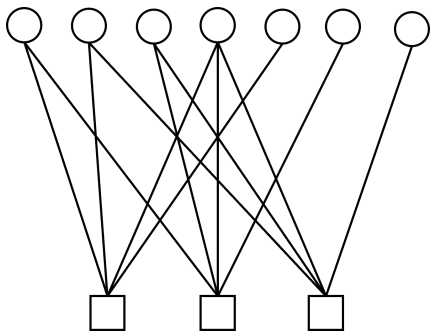

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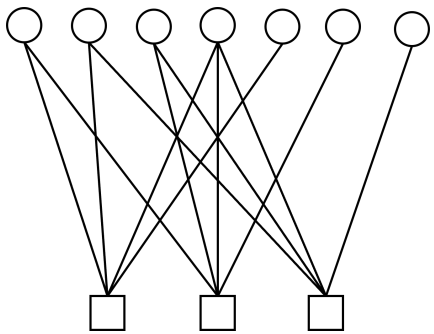

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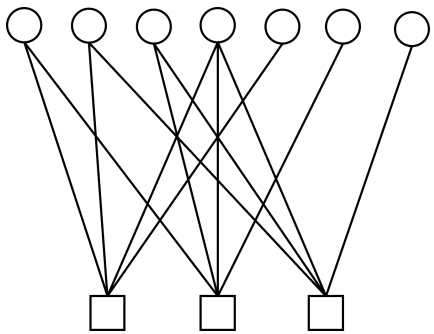

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Our work: we optimize degree distribution $L(x), P(x)$ for the QKD channel to result in large IR rates

Degree Distribution Optimization using Differential Evolution

- General procedure to optimize degree distribution (Shokrollahi '00):

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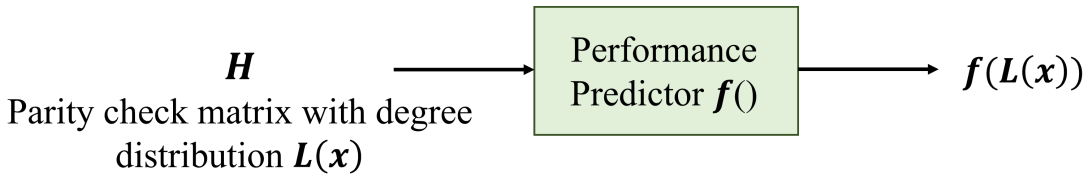

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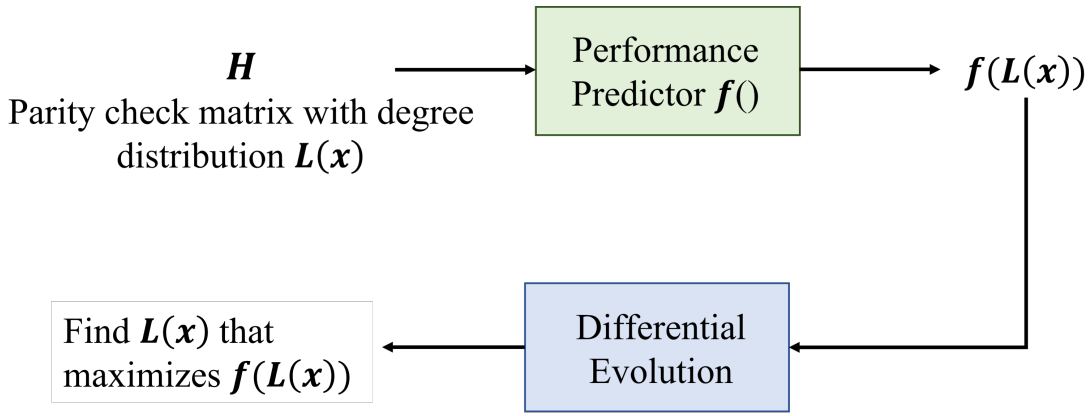

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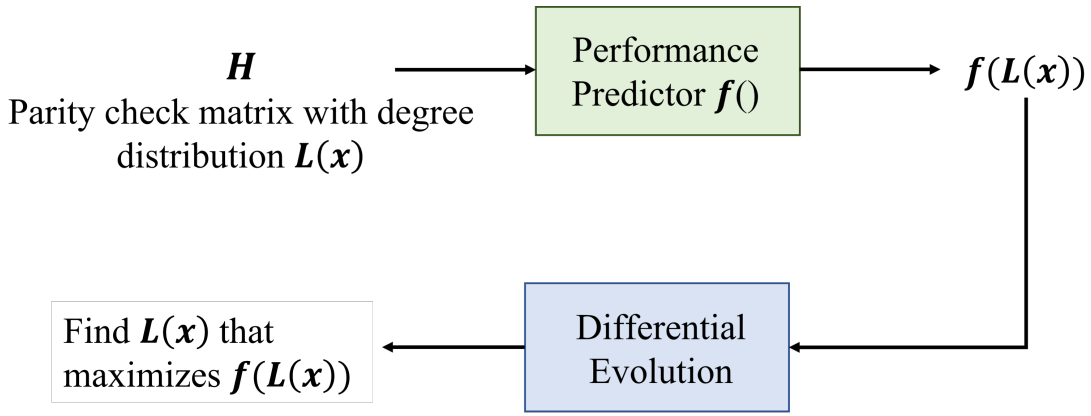

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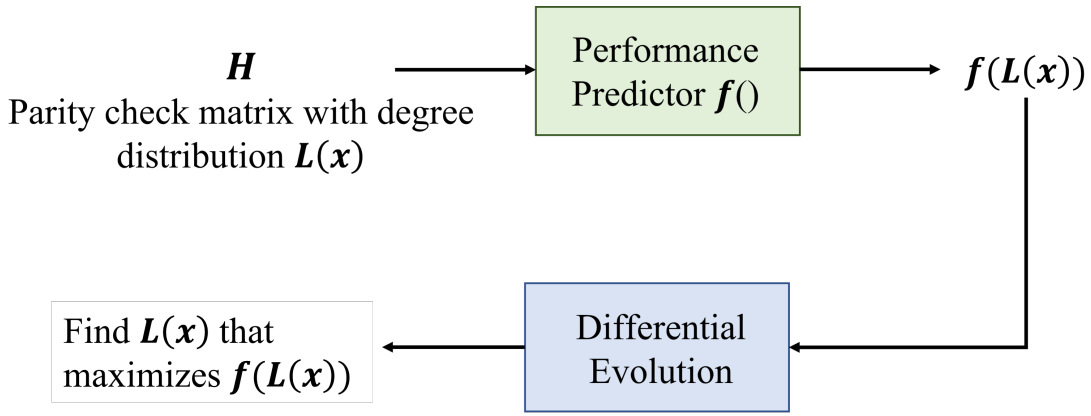
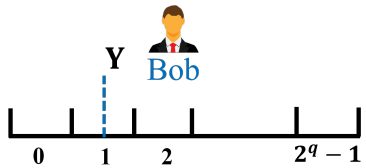


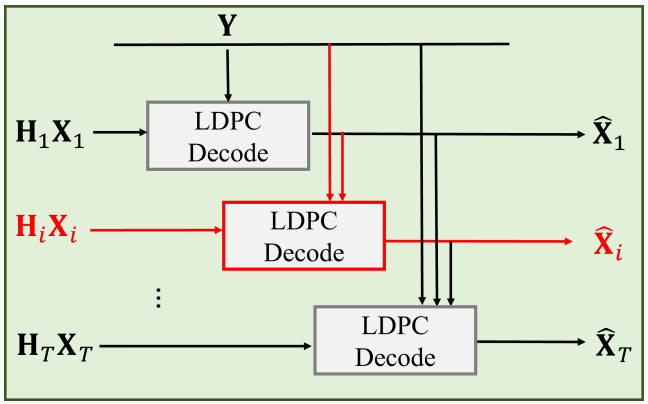
Generally

- The performance predictor is the code threshold (low complexity to compute)
- $L(x)$ is optimized for a fixed code rate R

Joint Rate and Degree Distribution Optimization (JRDO)

$$\text{IR rate} = \sum_{i=1}^{T}(1 - FER_i) * Rate_i * \alpha_i$$





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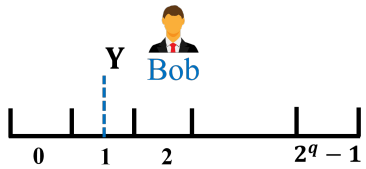

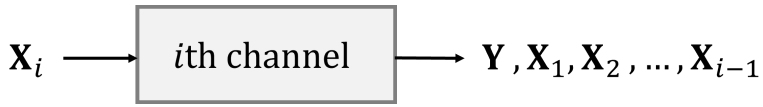

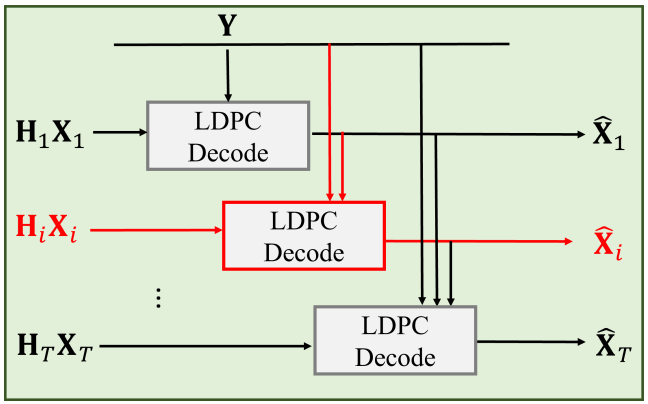

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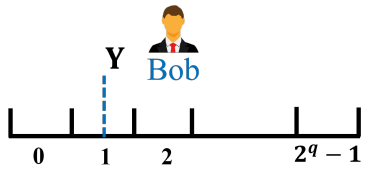

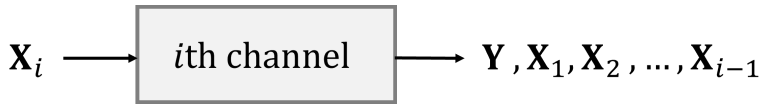

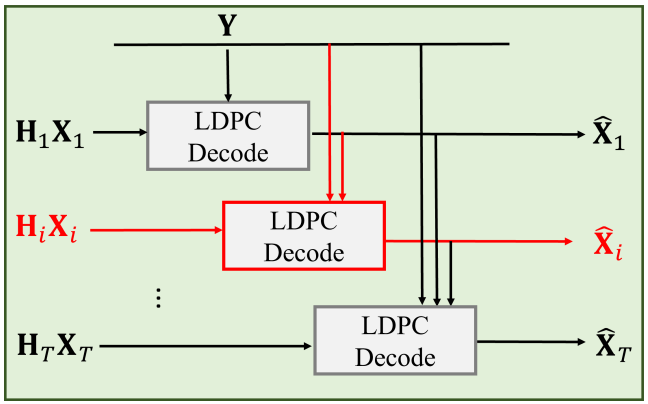

- $\gamma_i = P(Y = y, X_1 = x_1, \ldots, X_{i-1} = x_{i-1} | X_i = x_i)$

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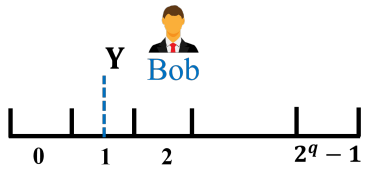

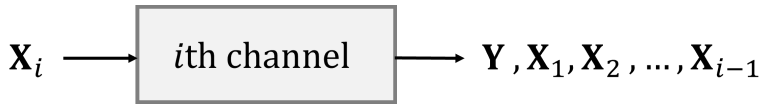

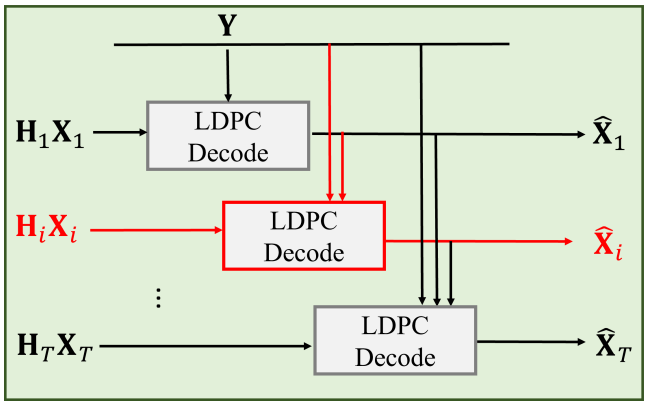

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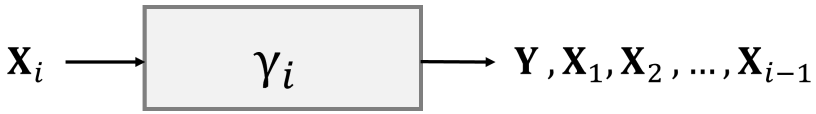

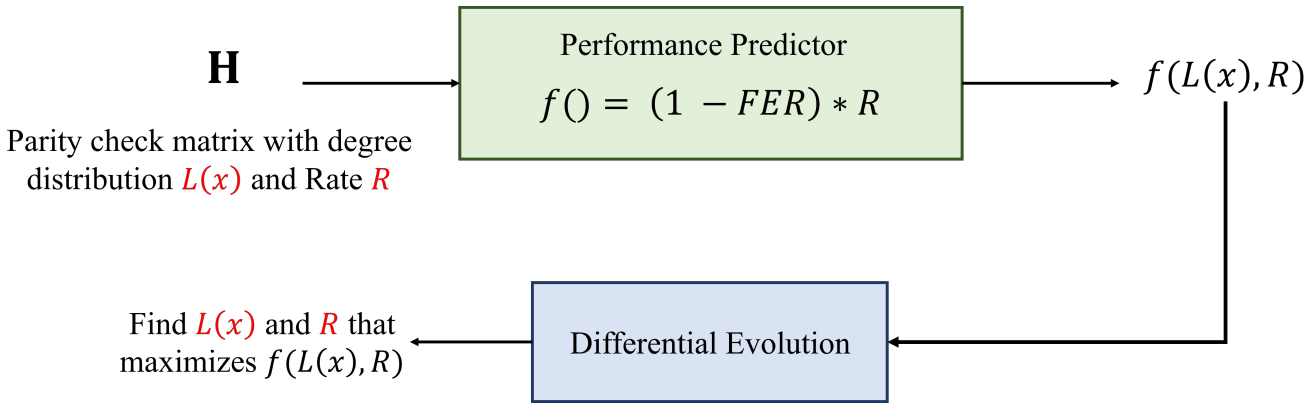

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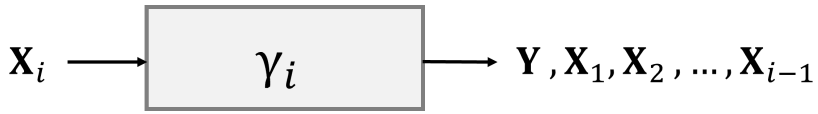

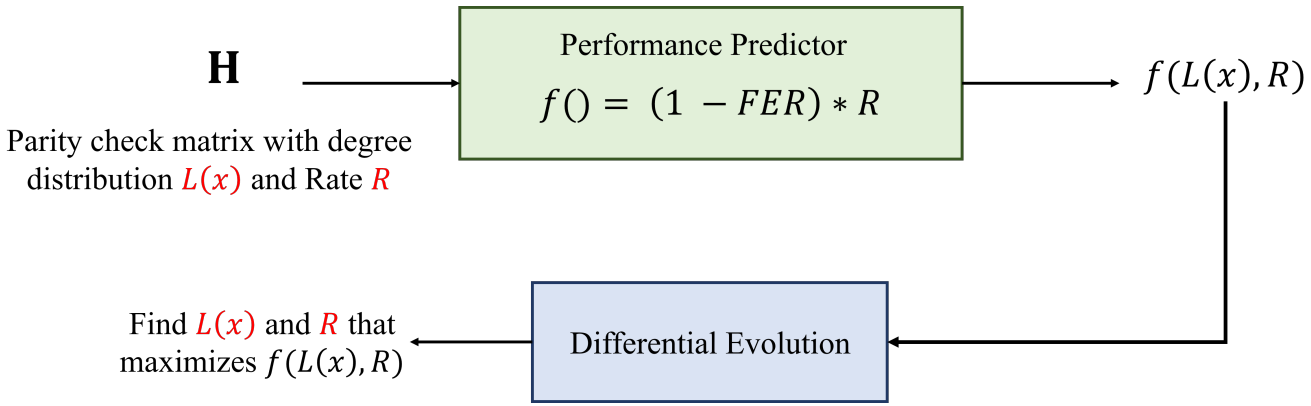

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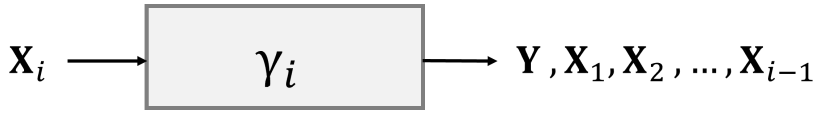

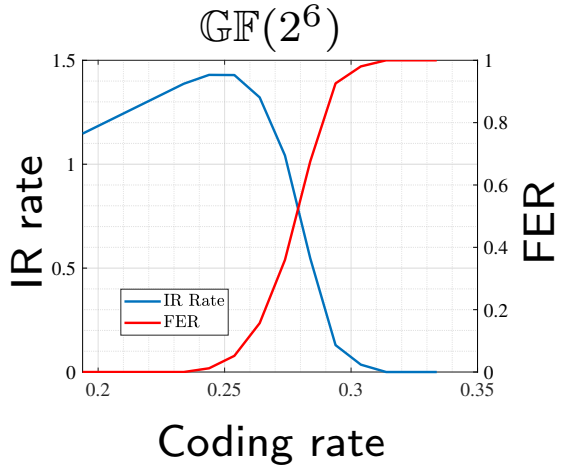

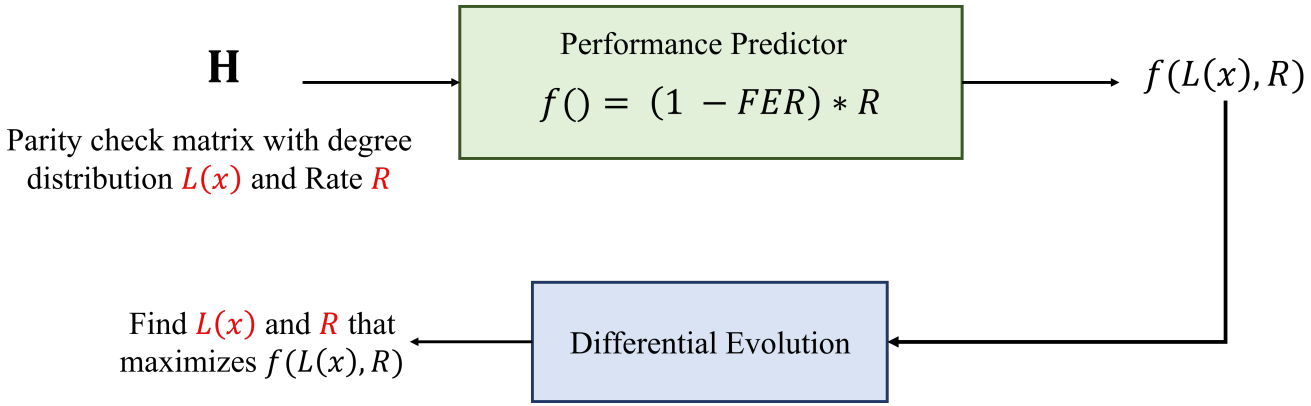

- FER is the frame error rate encountered on channel γ_i
- FER can be obtained with low complexity by Monte-Carlo simulation of a small number of codewords

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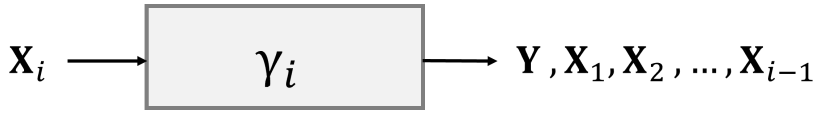
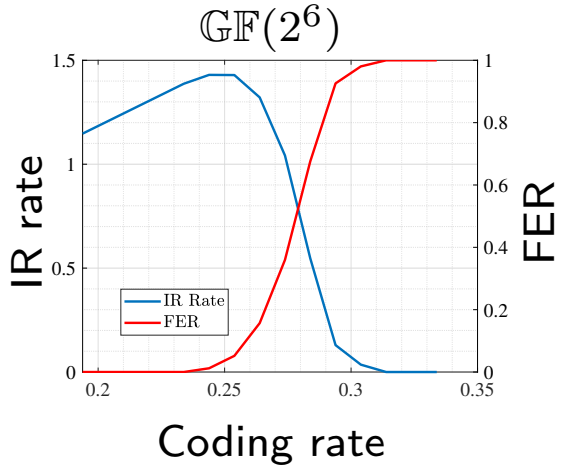
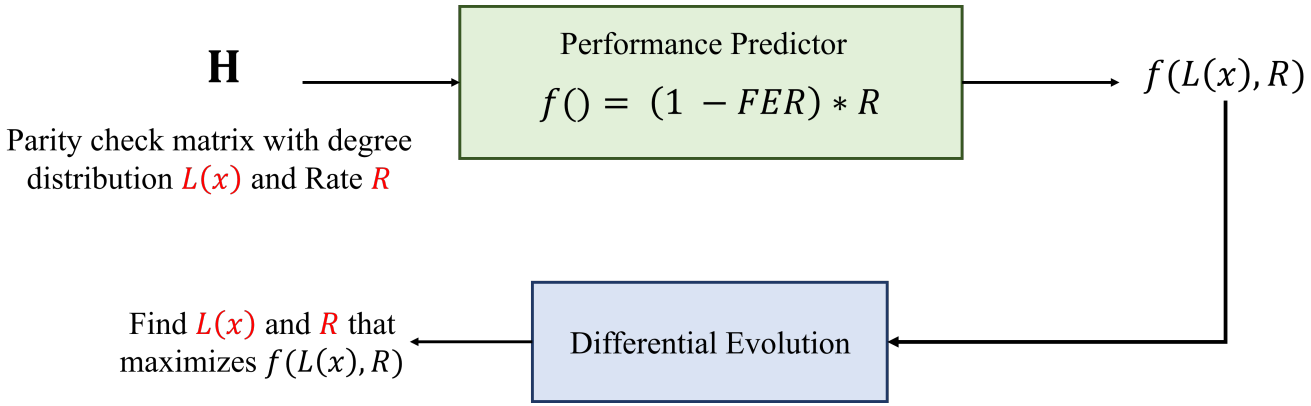
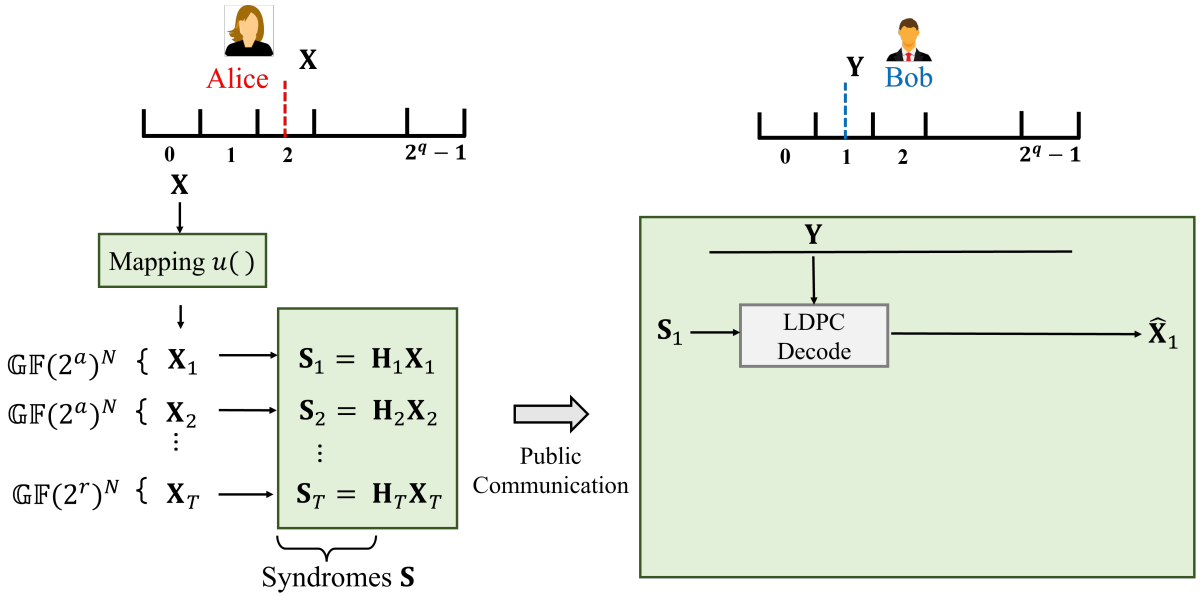






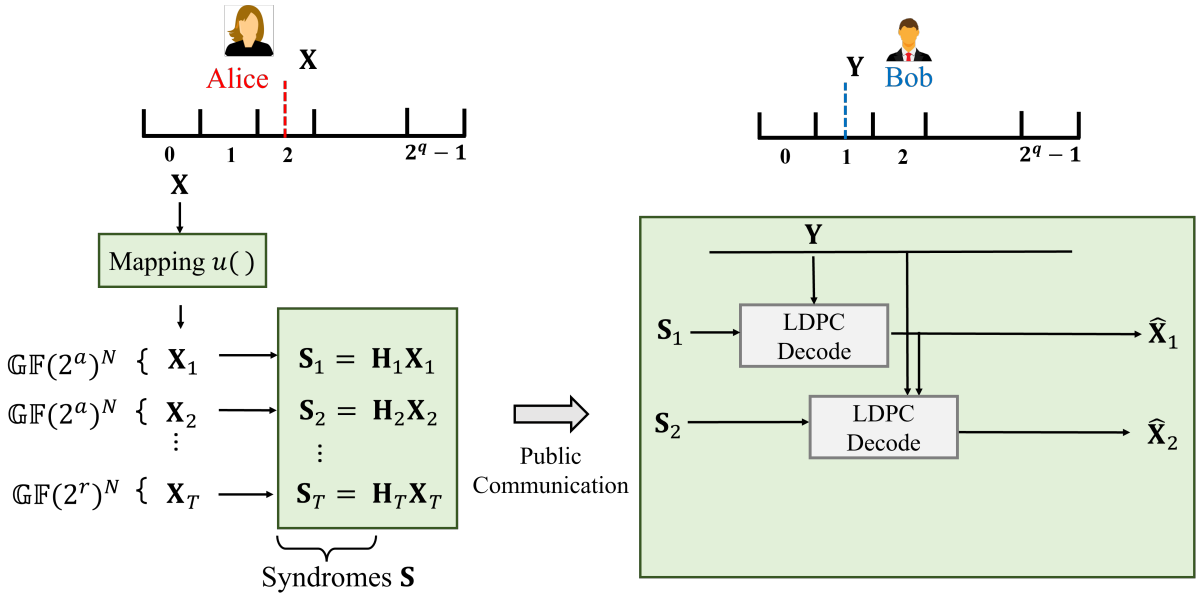
- FER is the frame error rate encountered on channel γ_i
- Can jointly optimize the degree distribution $L(x)$ and rate R to get high IR rates

Sequential Decoding and Interactive Communication



Error propagation and current remedy:

Sequential Decoding and Interactive Communication



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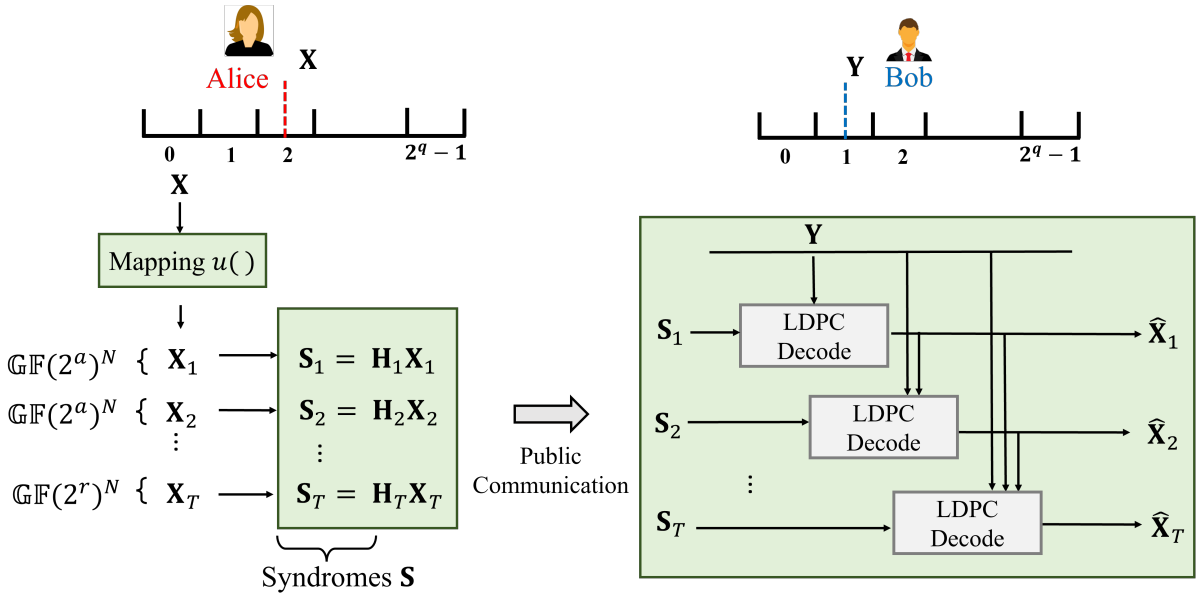

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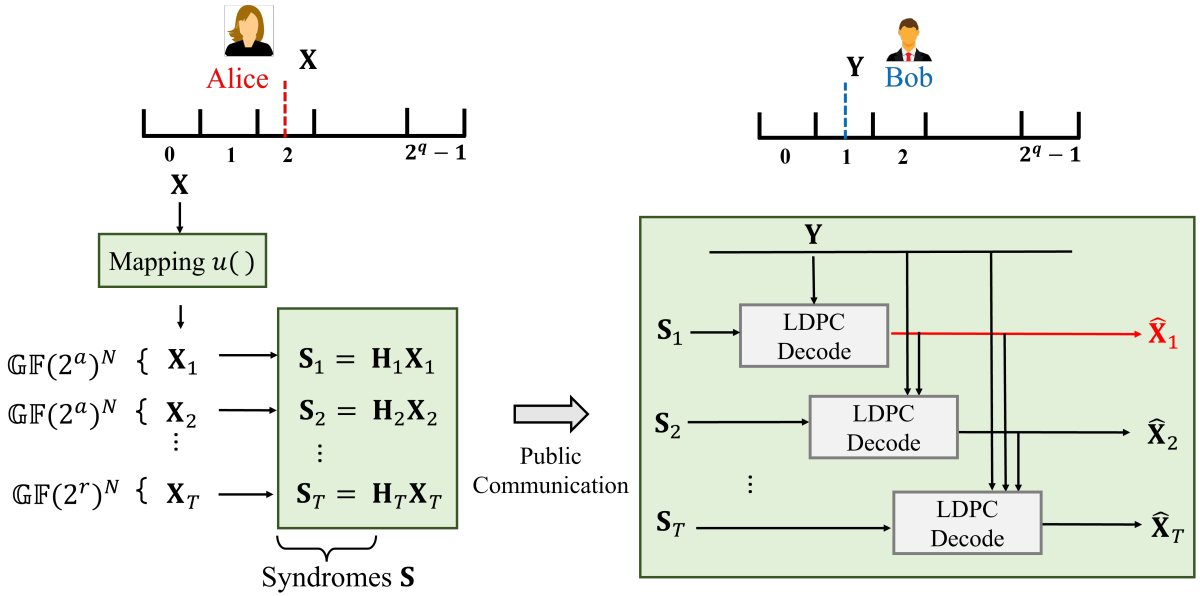
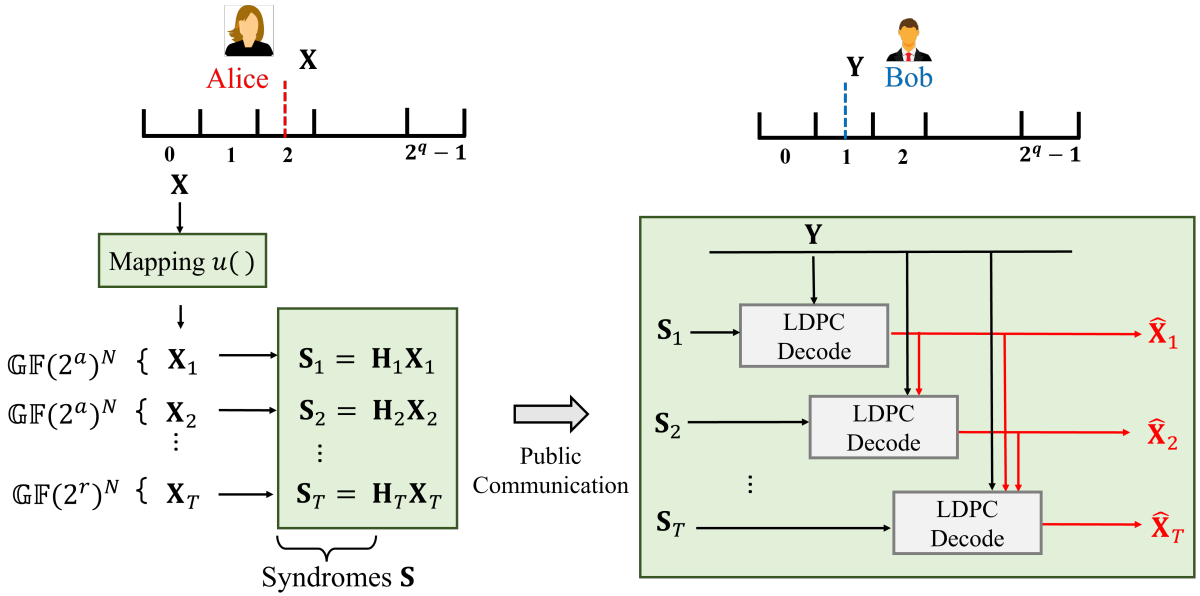


Error propagation and current remedy:

- A decoding error in one layer $\rightarrow$ decoding error in subsequent layers

Sequential Decoding and Interactive Communication



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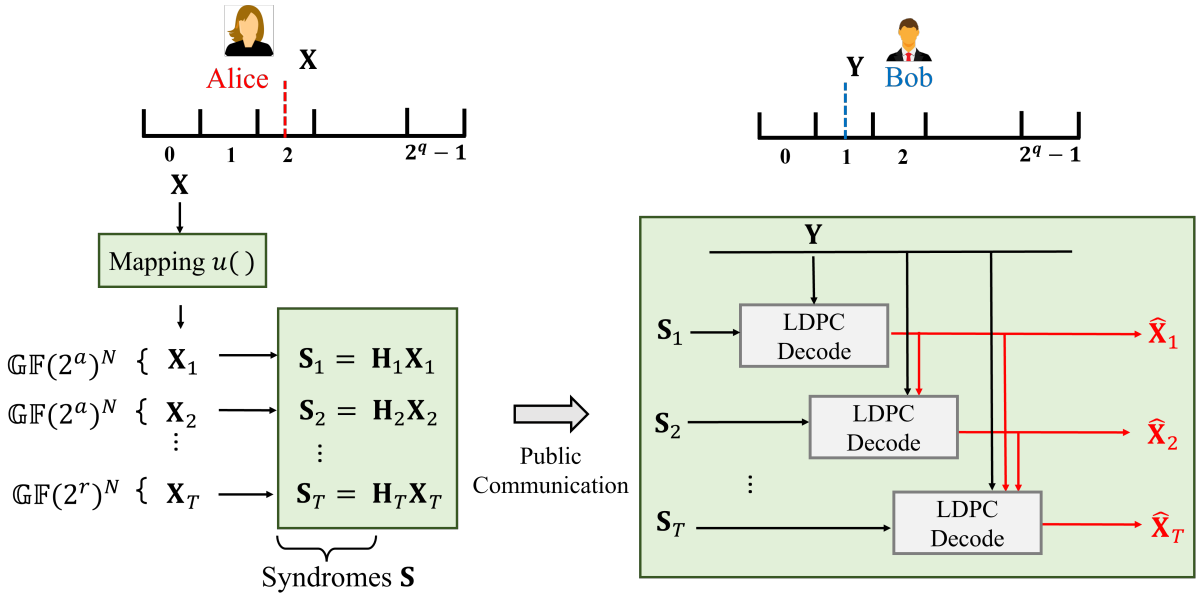

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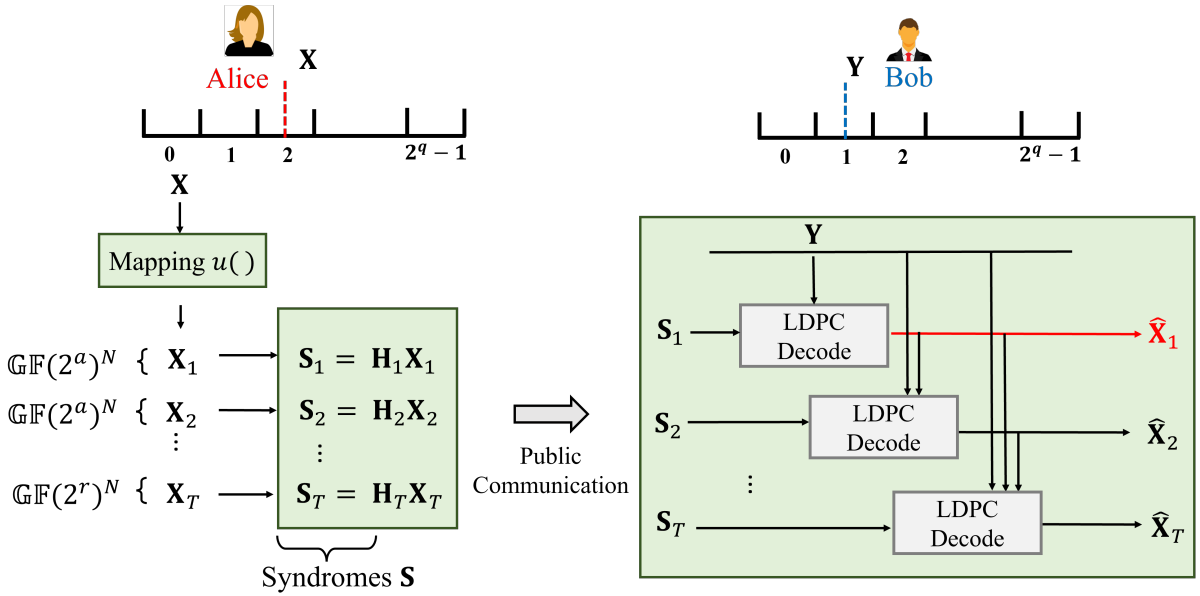

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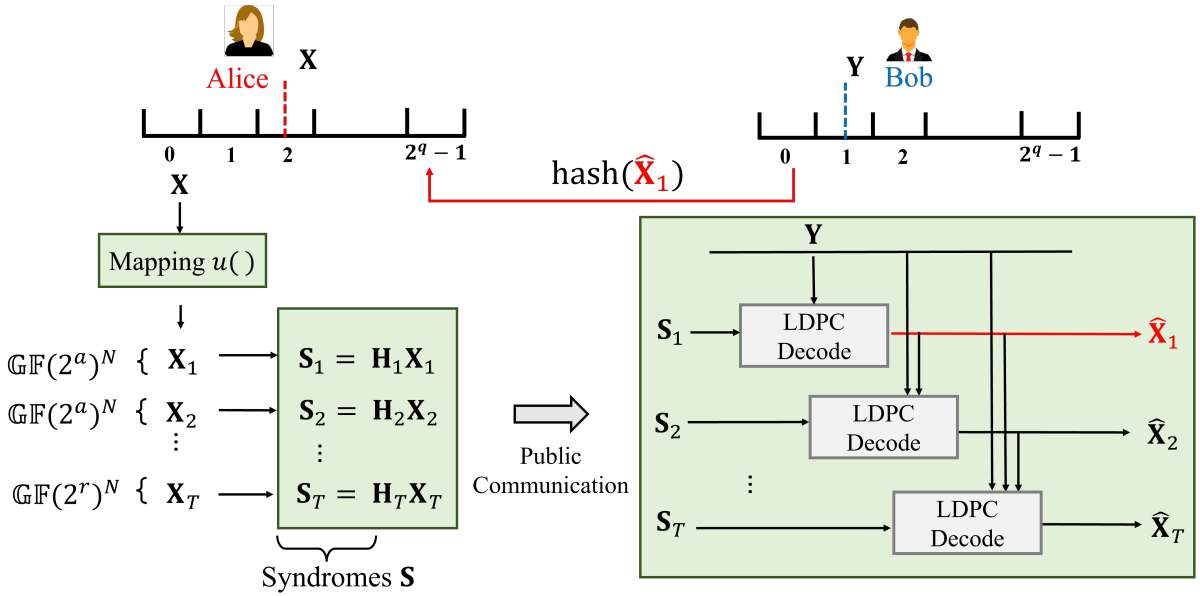

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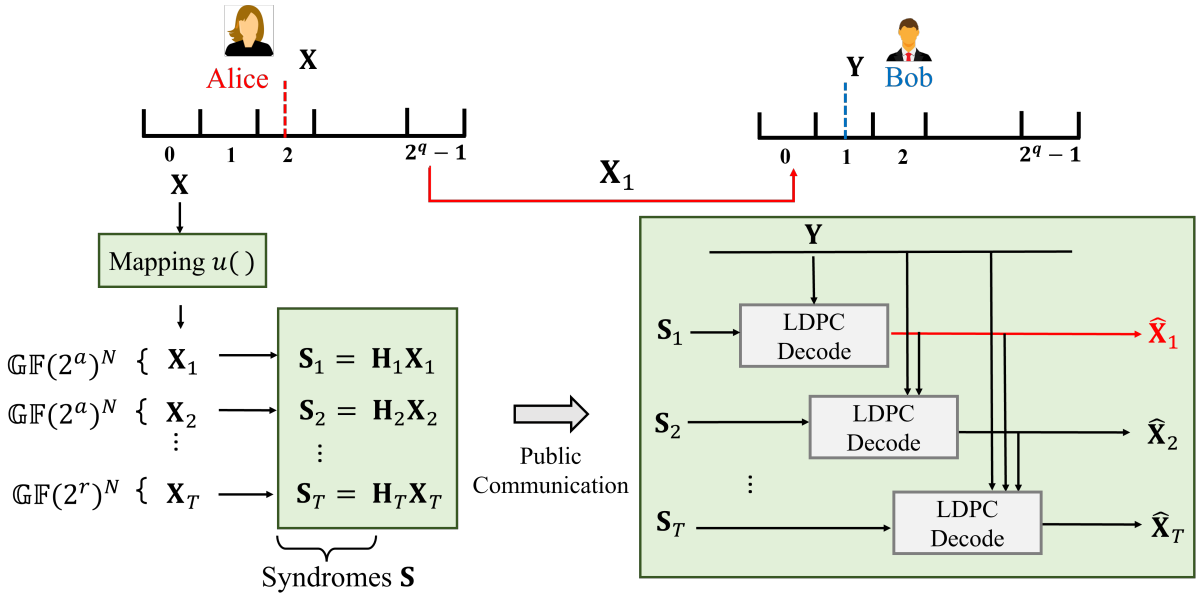

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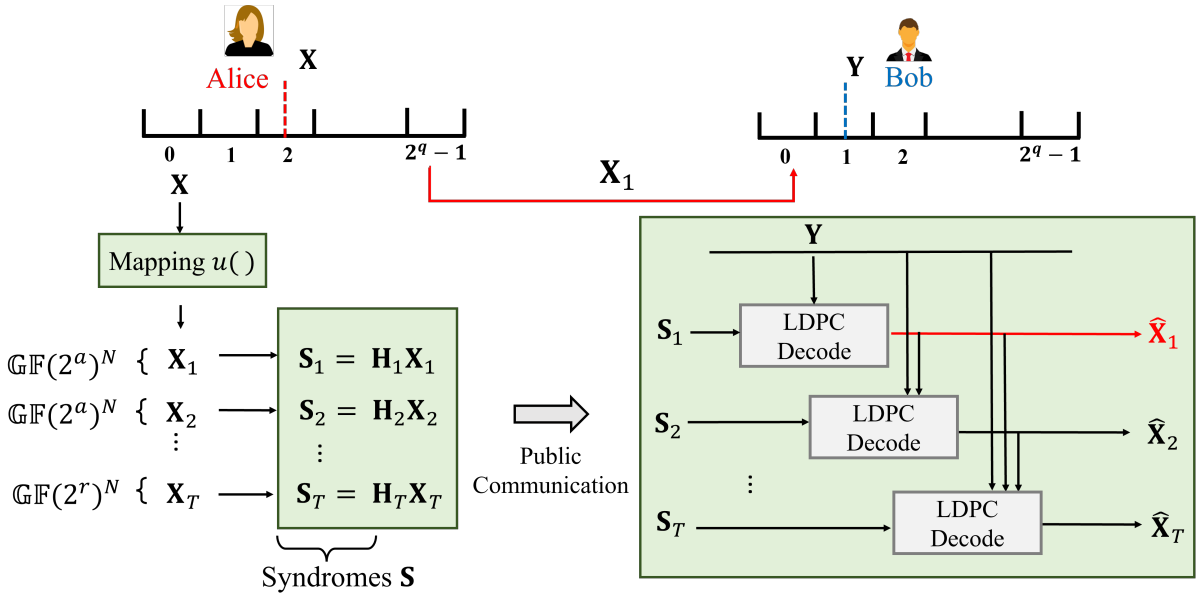

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Decoding and interactive communication is sequential

Interleaved Decoding and Communication (IDC)

Idea: Use interactive communication of the later layers to decode previous layers

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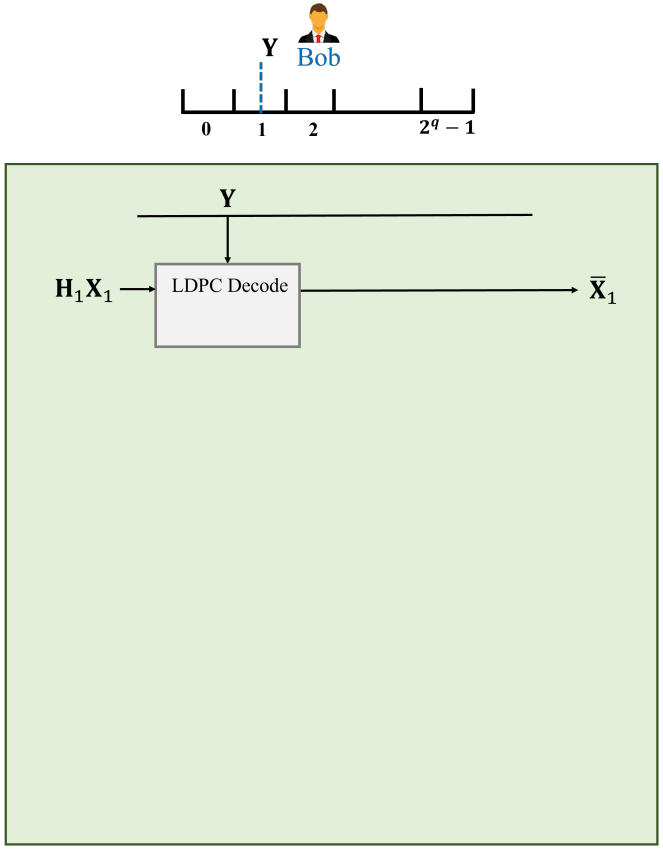

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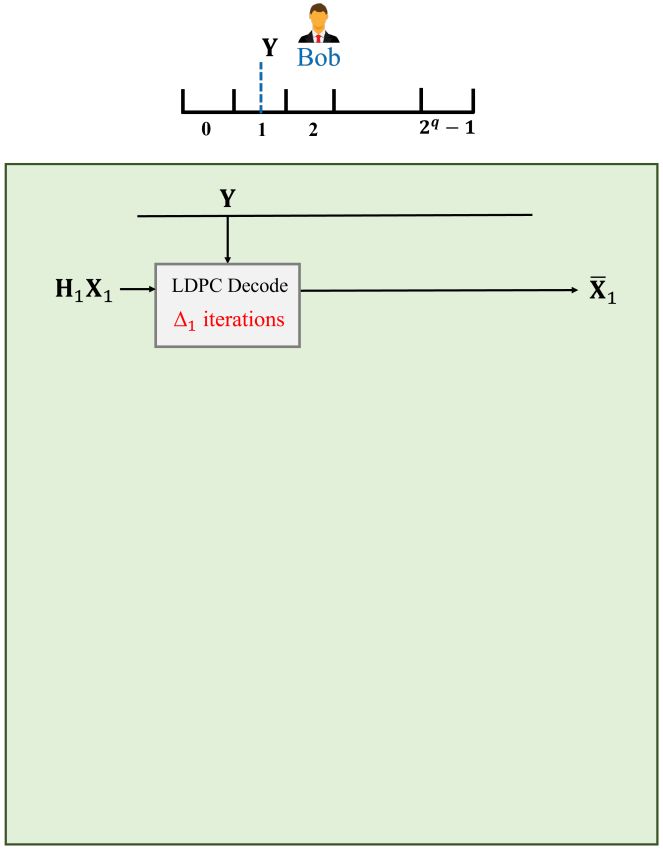

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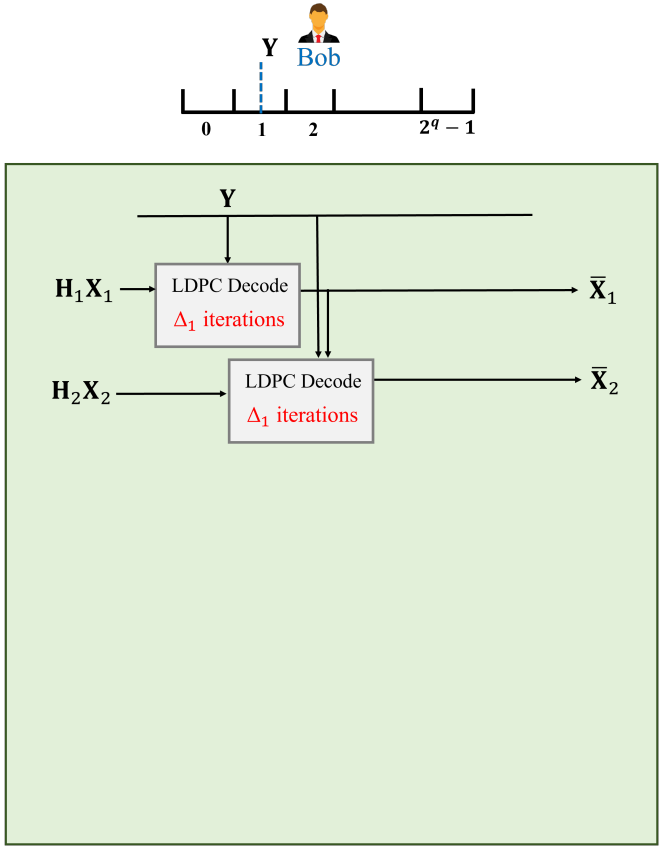

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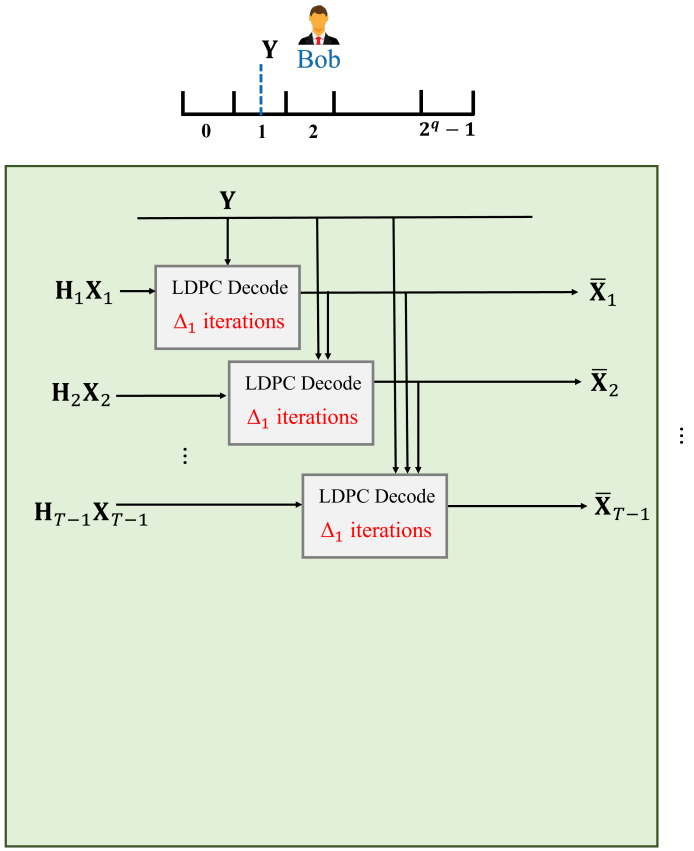

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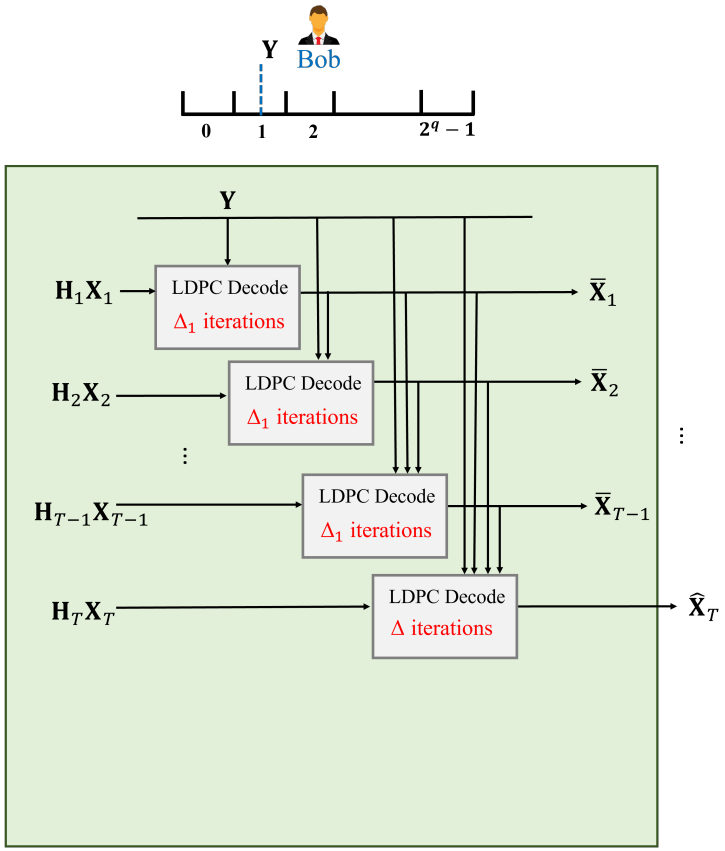

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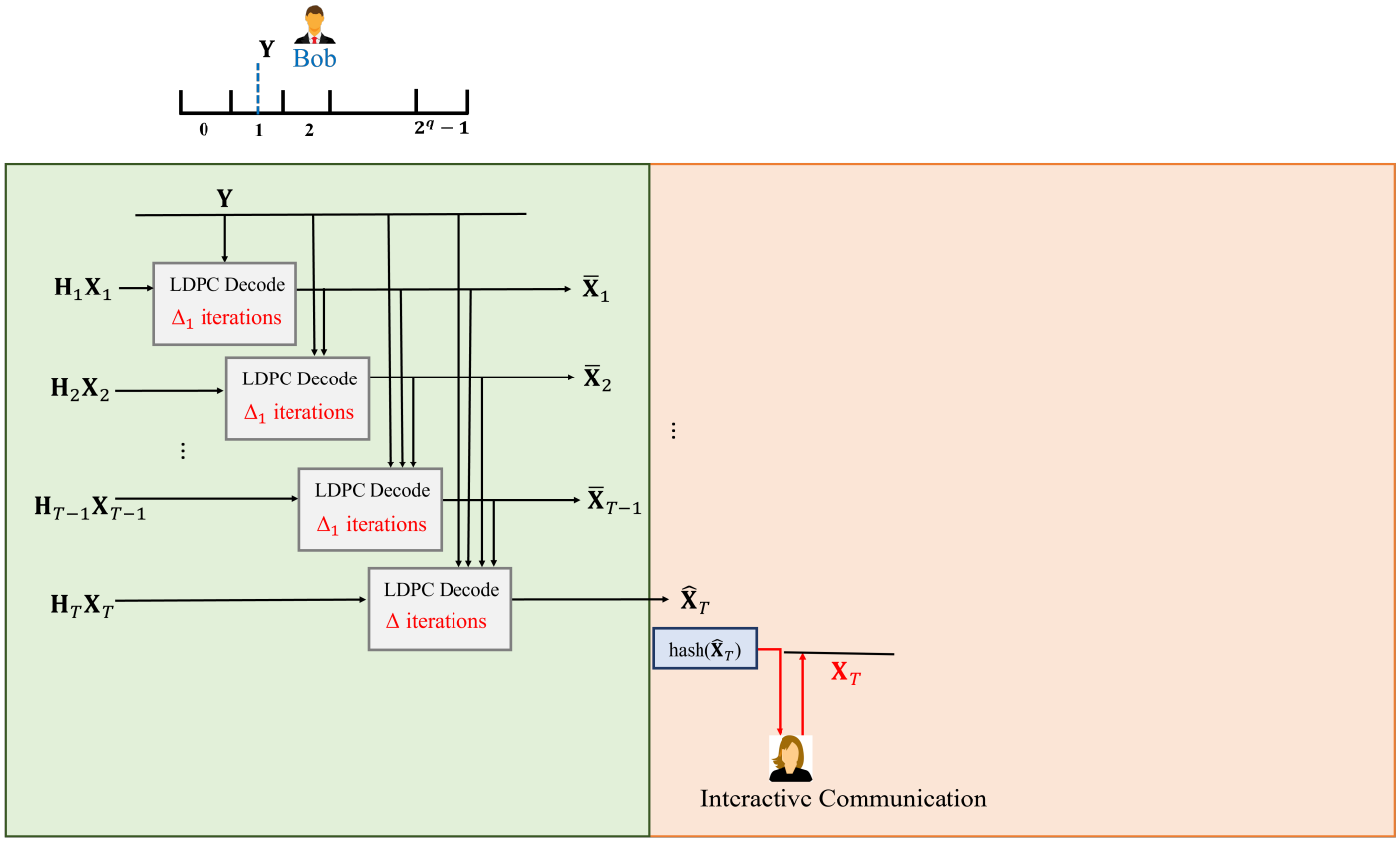

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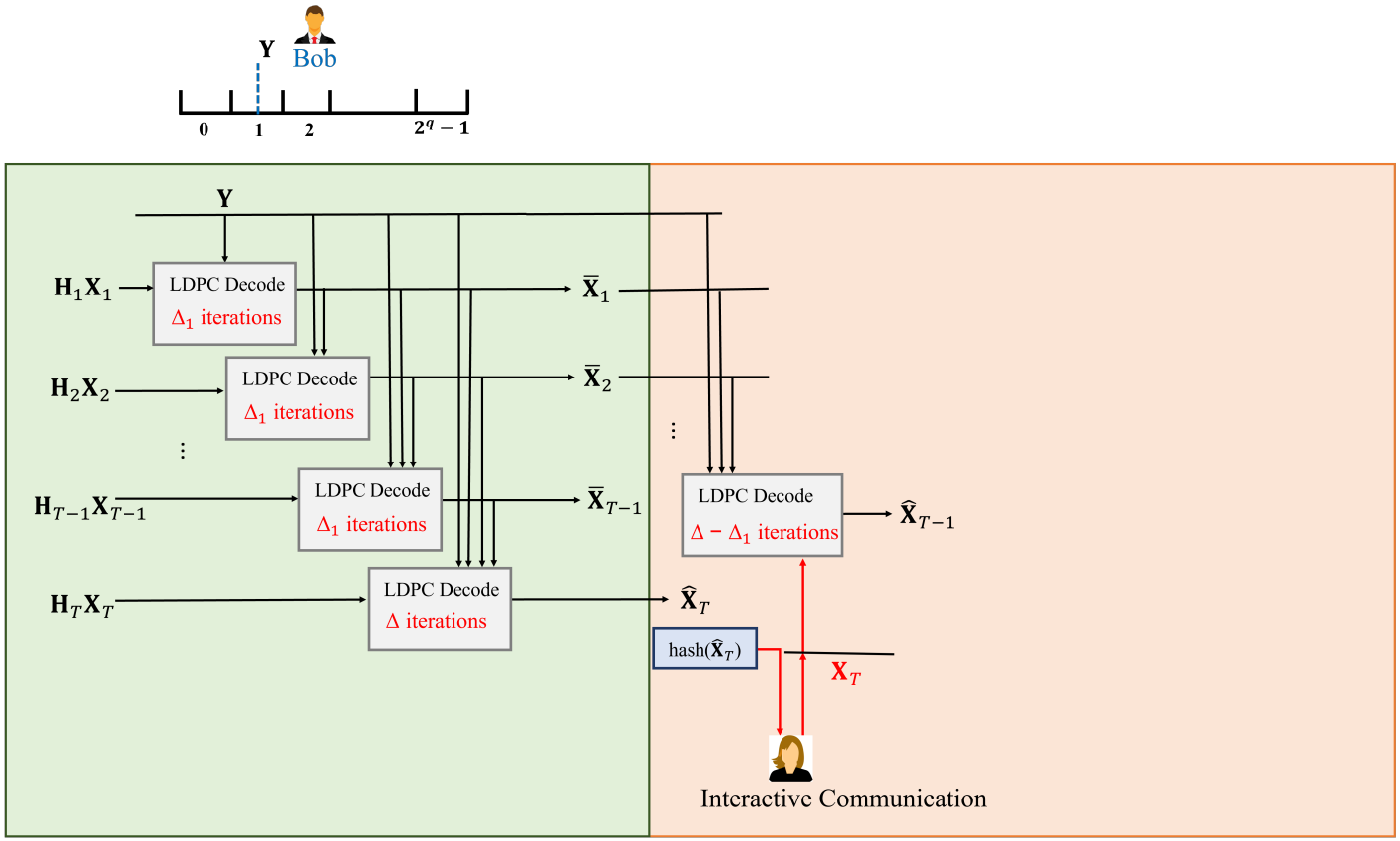

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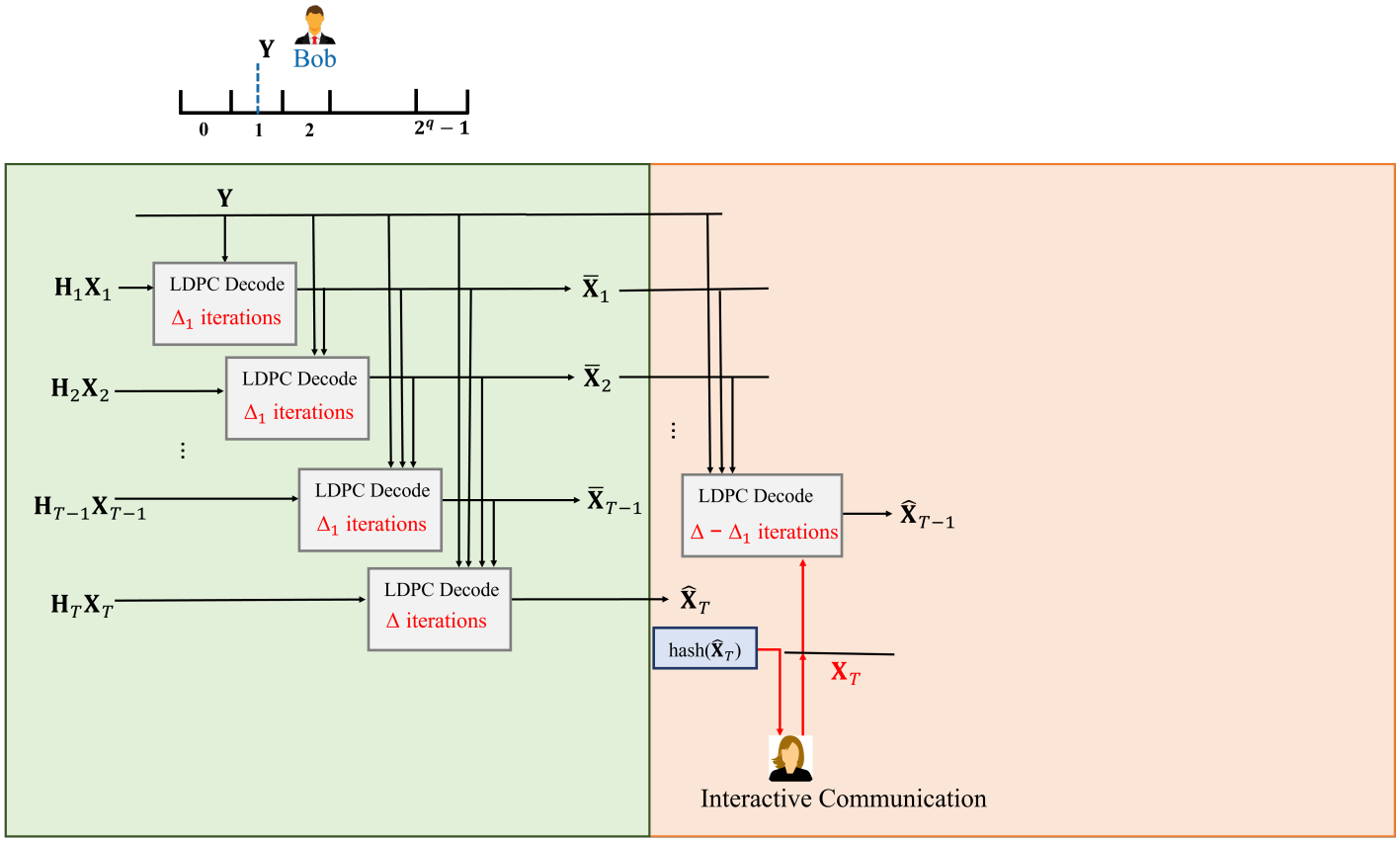

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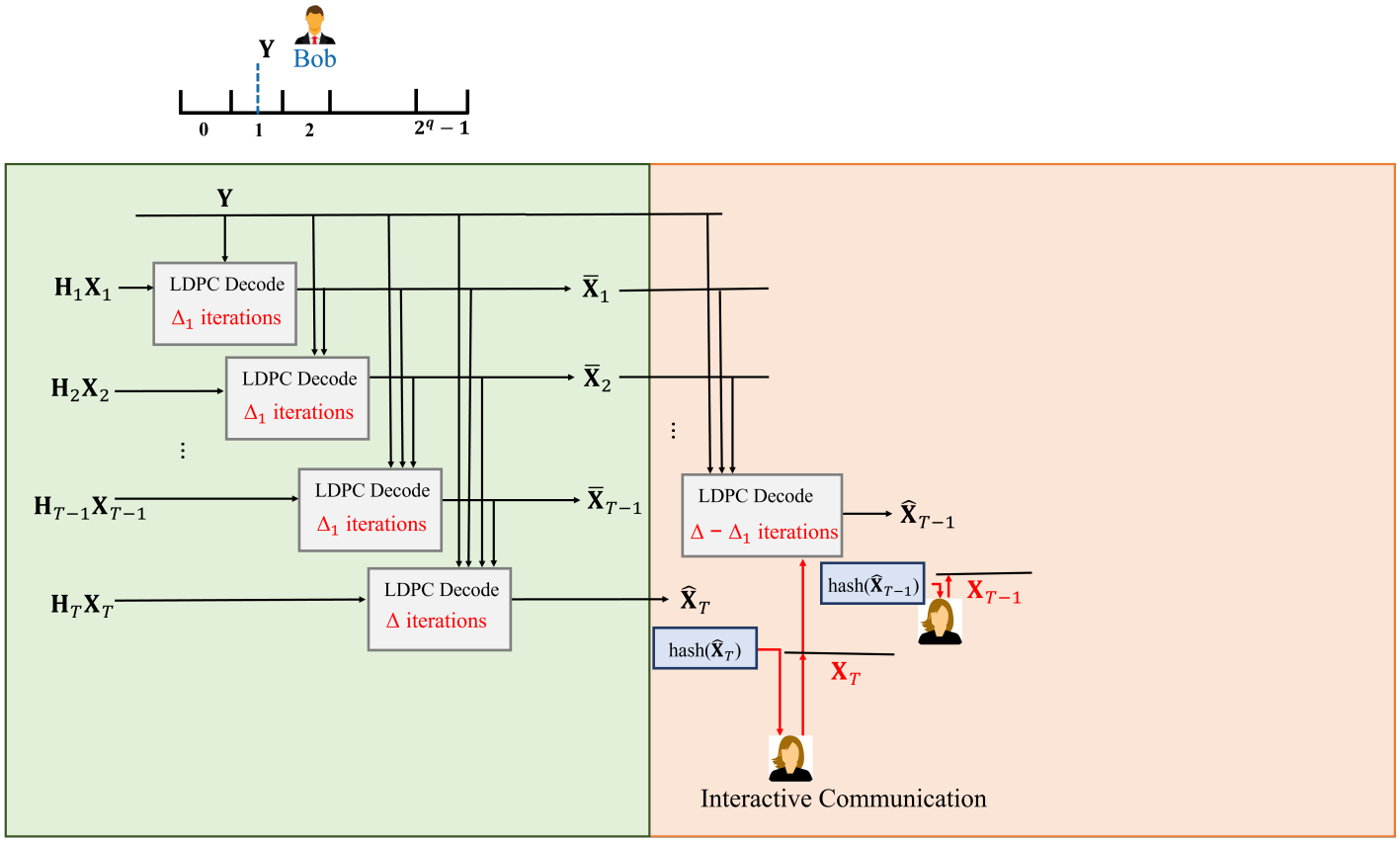

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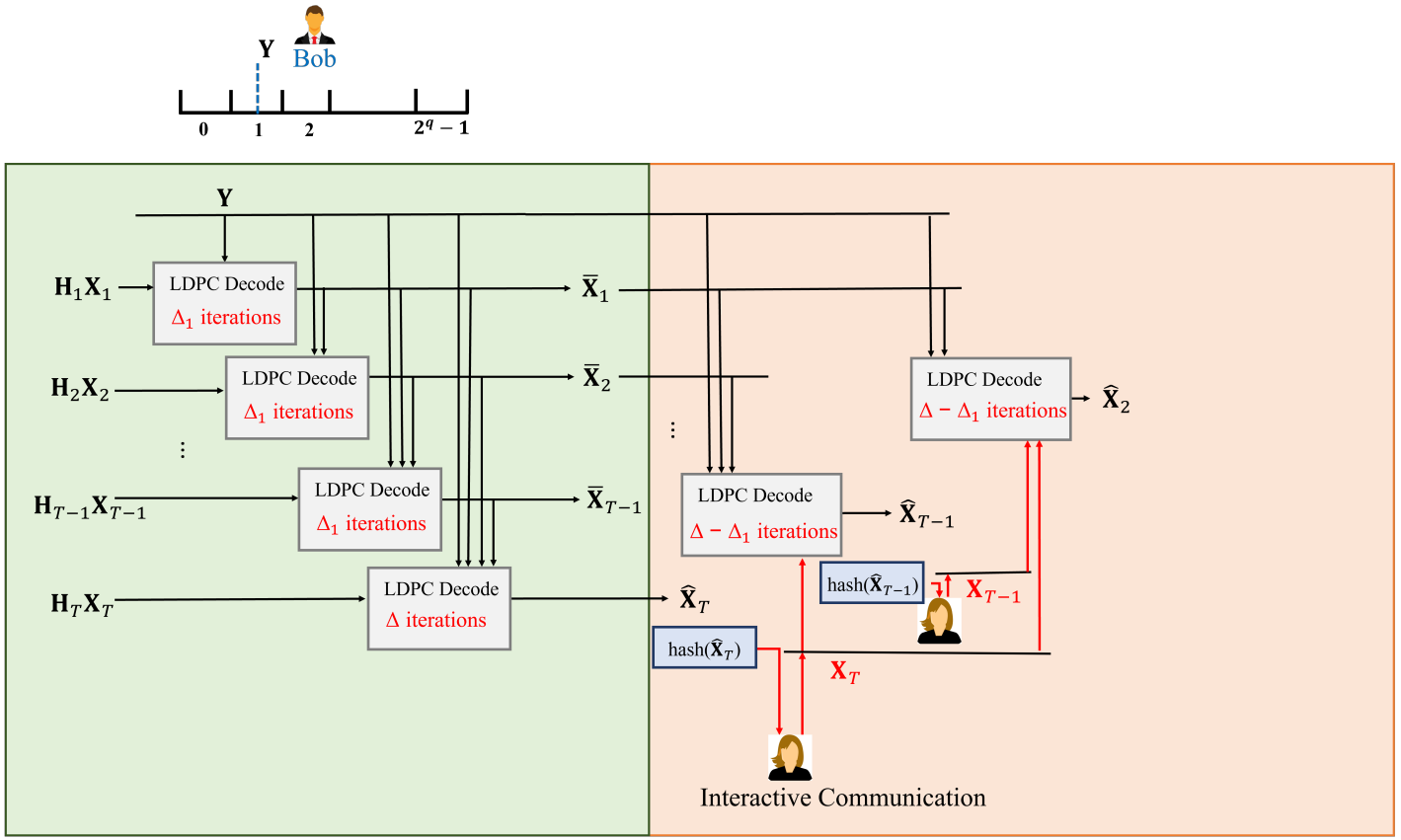

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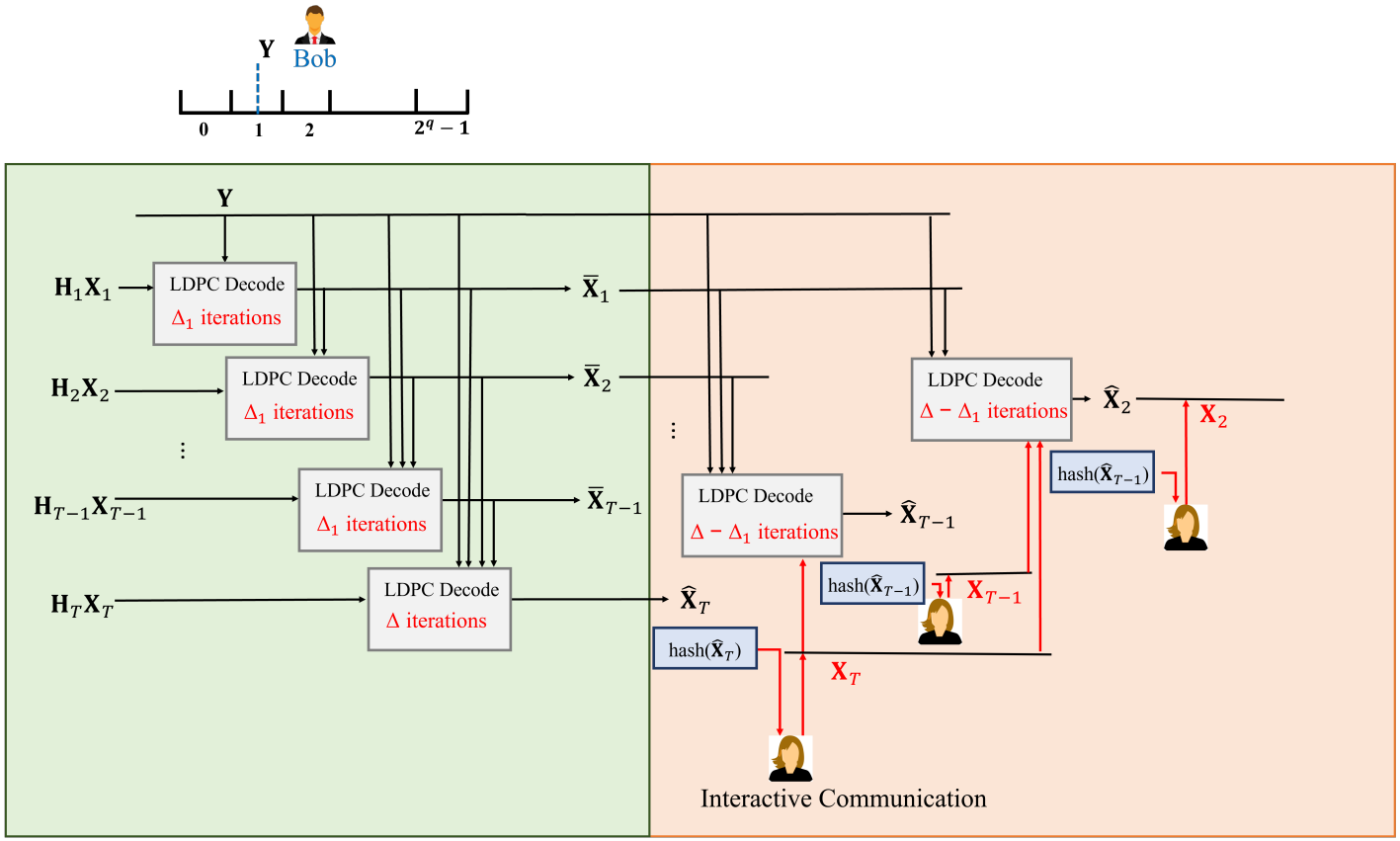

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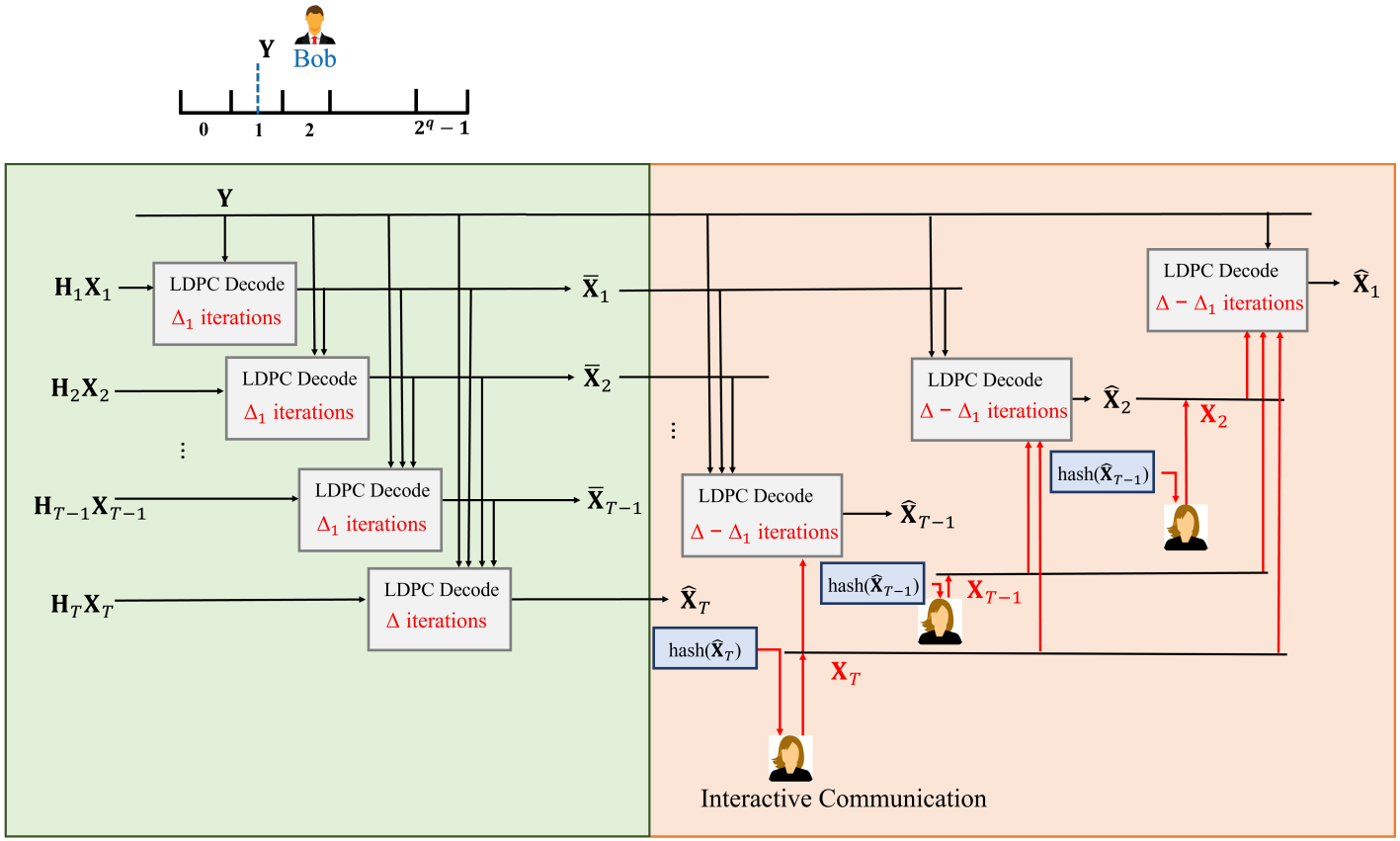

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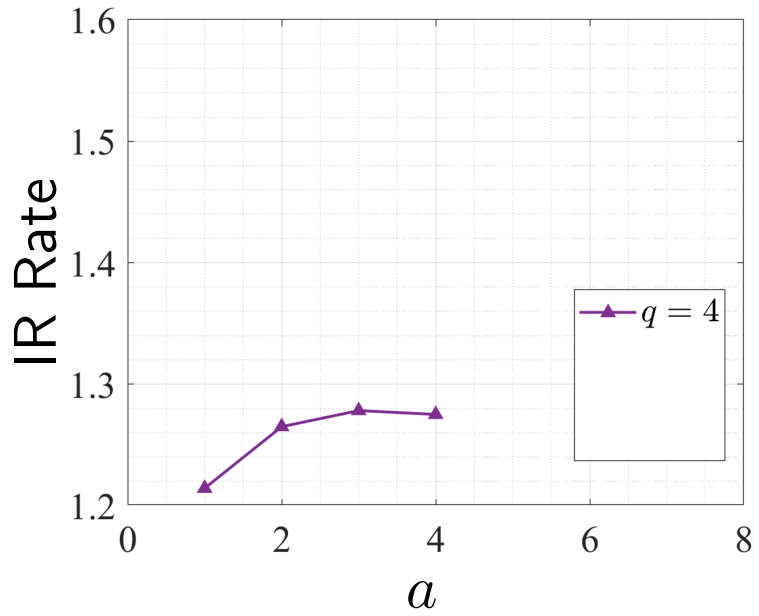

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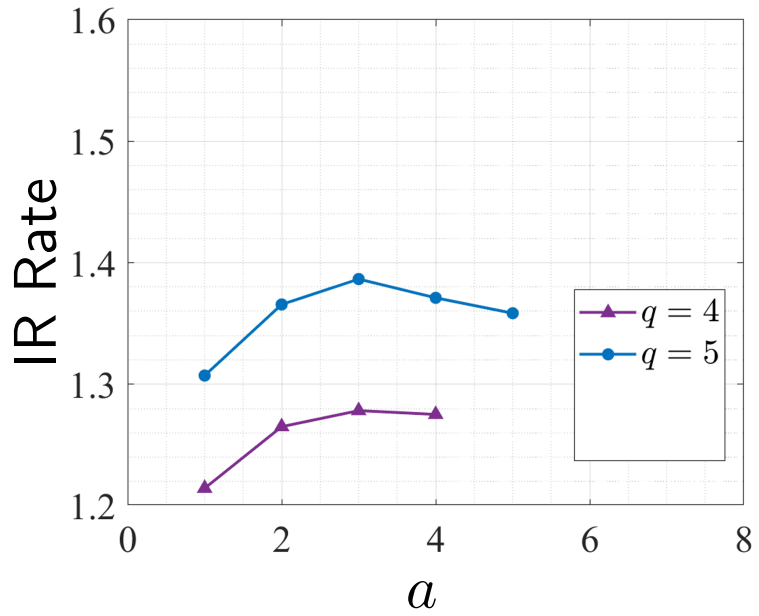

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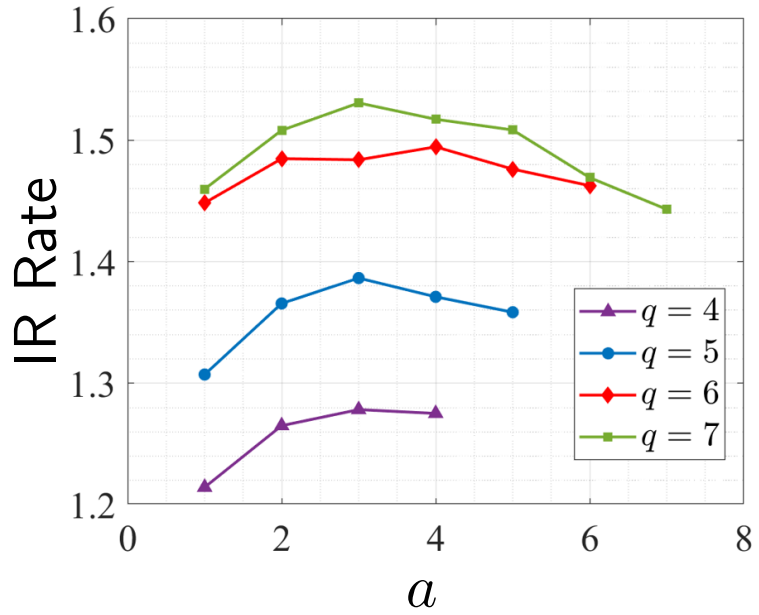

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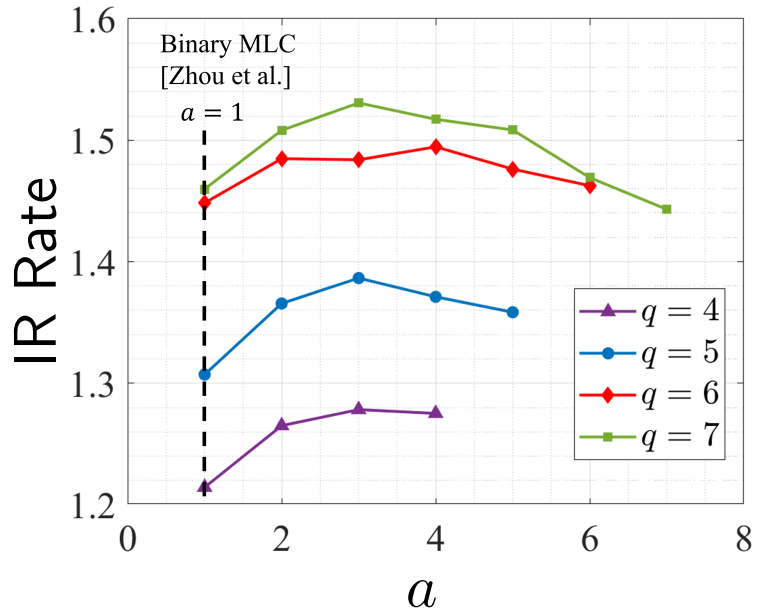

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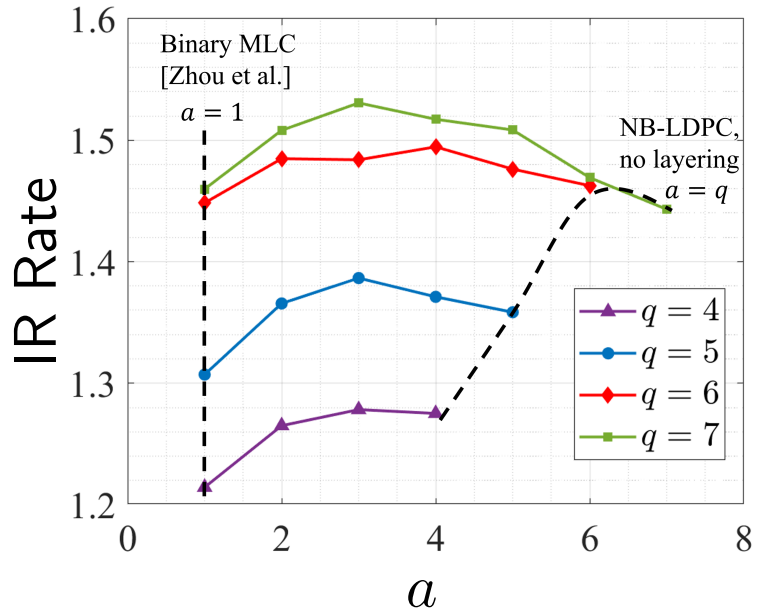

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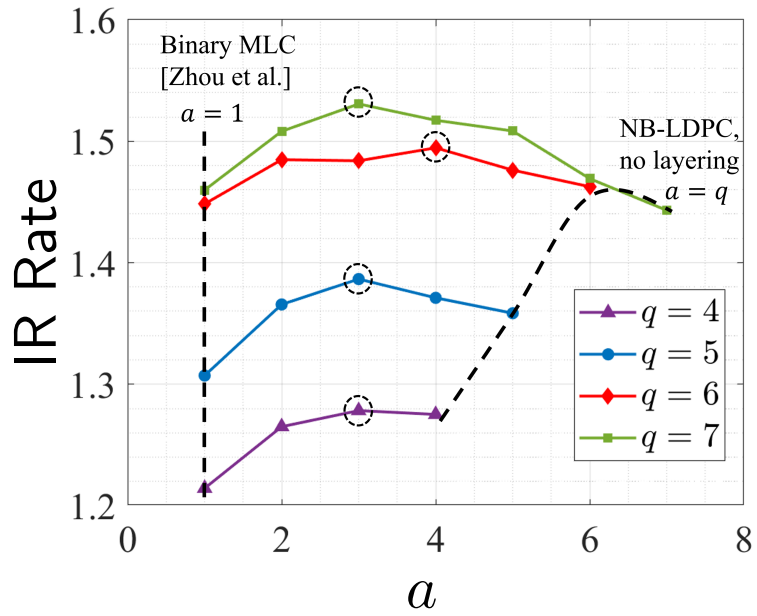

q	best a
4	3
5	3
6	4
7	3

- The IR rate is non-monotonic in a and has a maximum when a is strictly between 1 and q.

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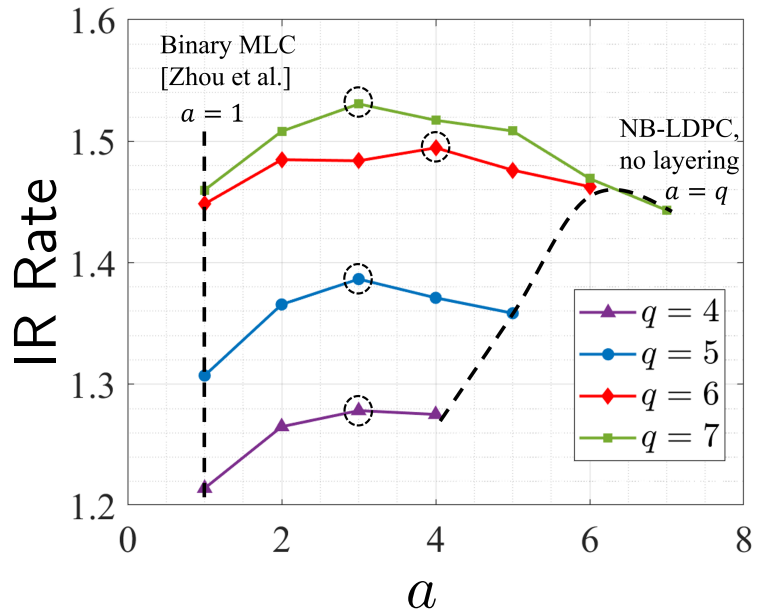

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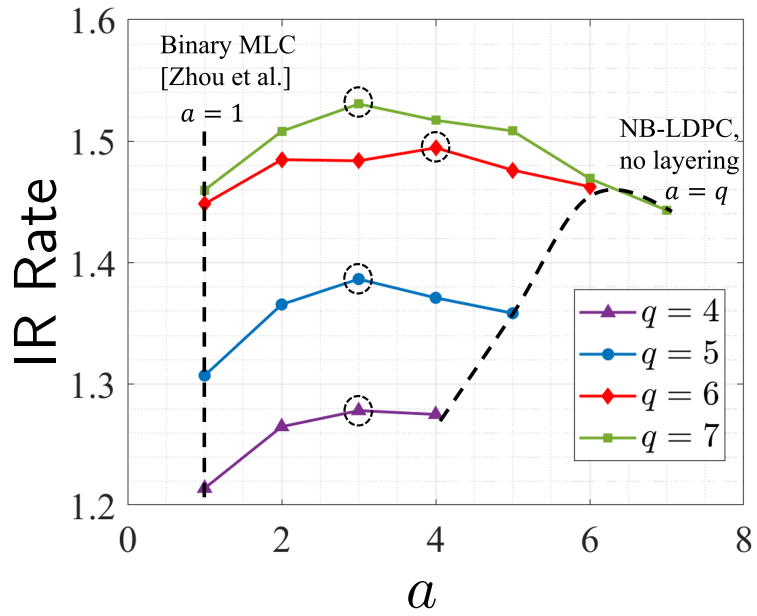

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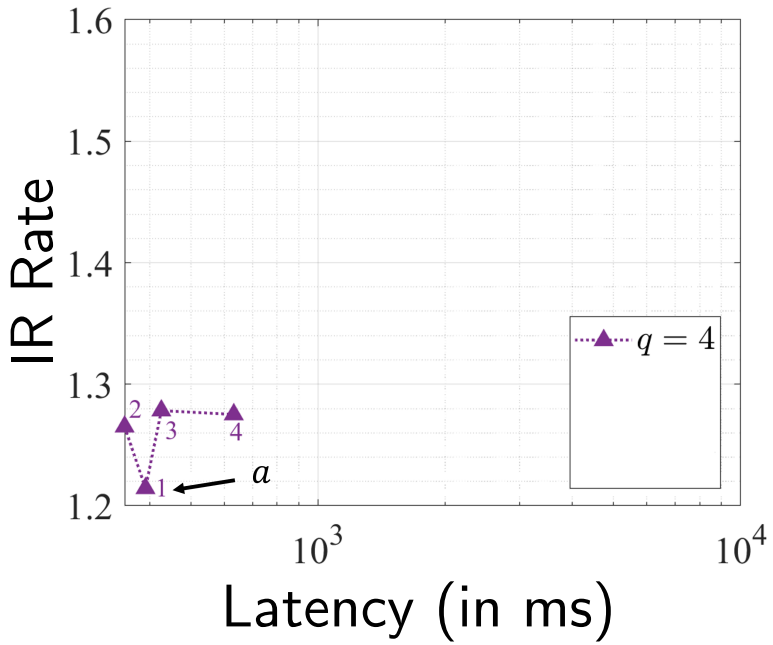

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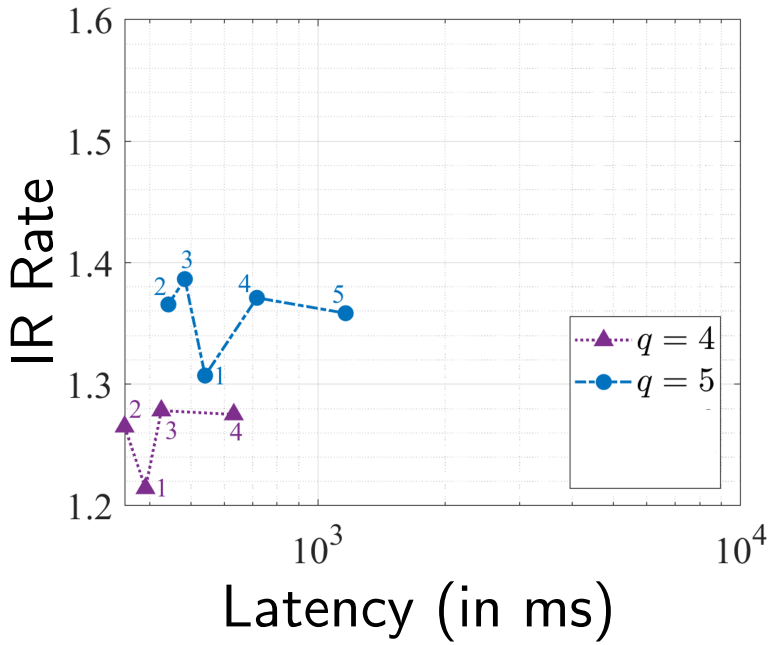

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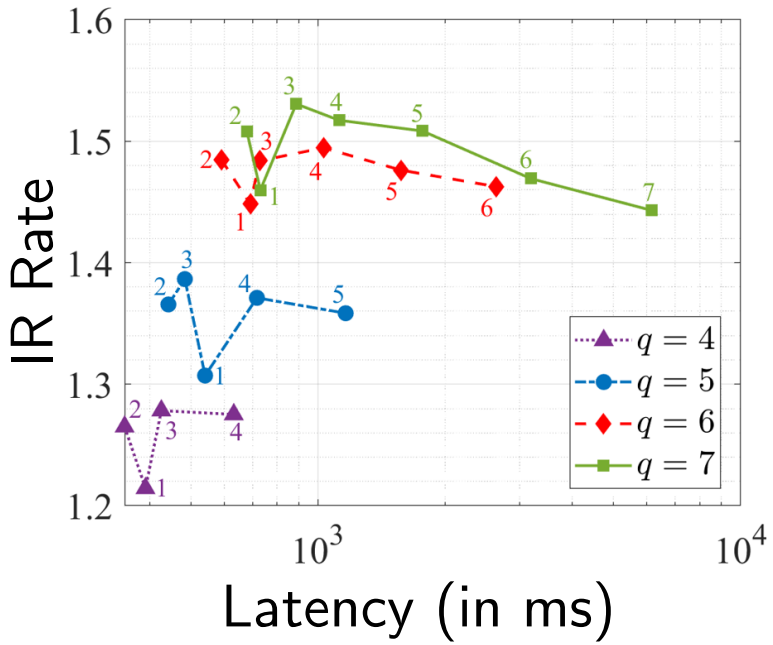

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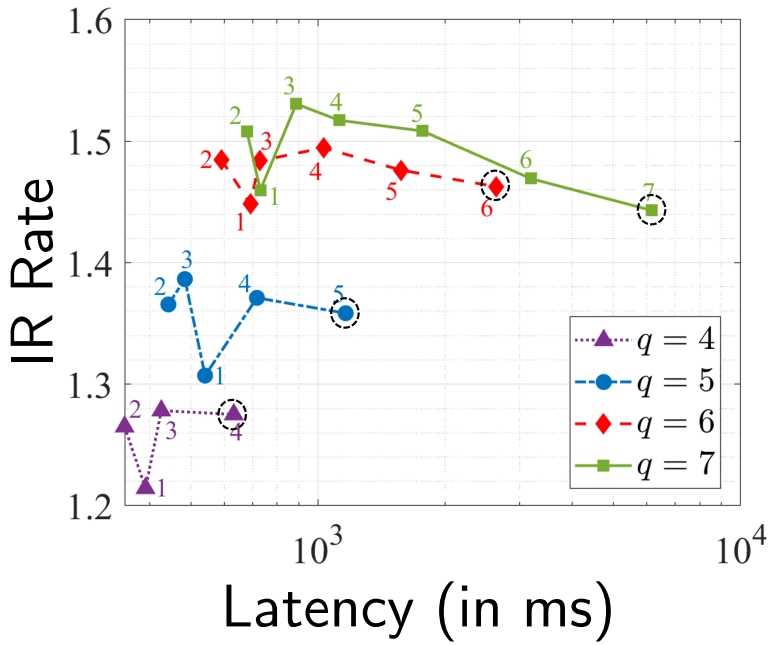

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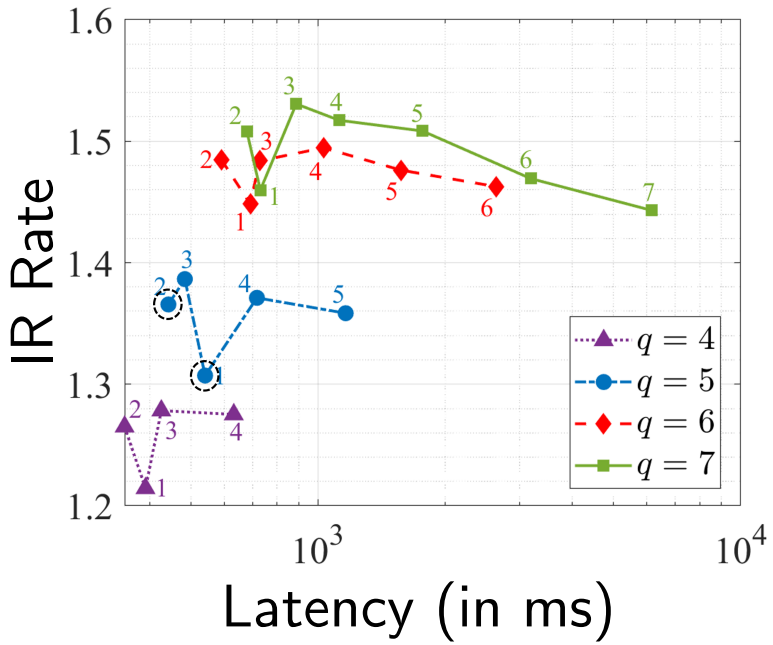

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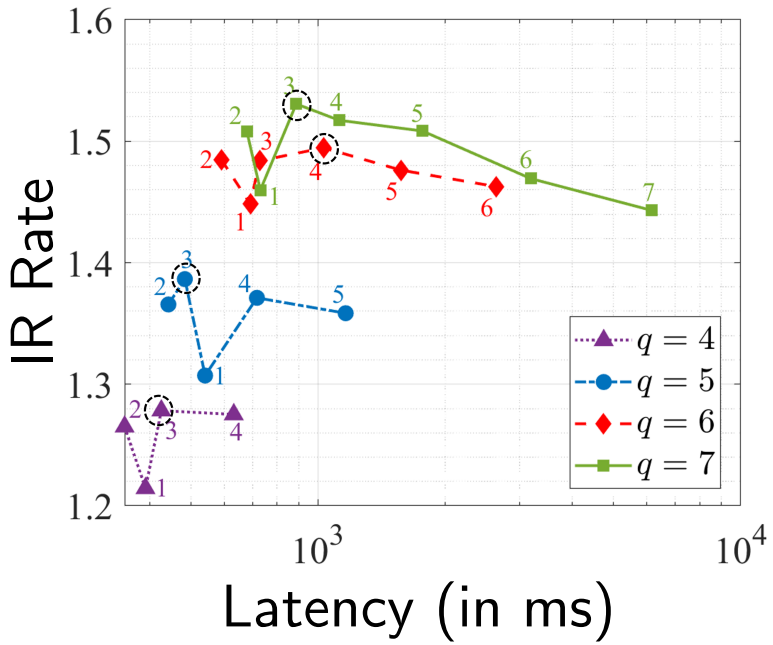
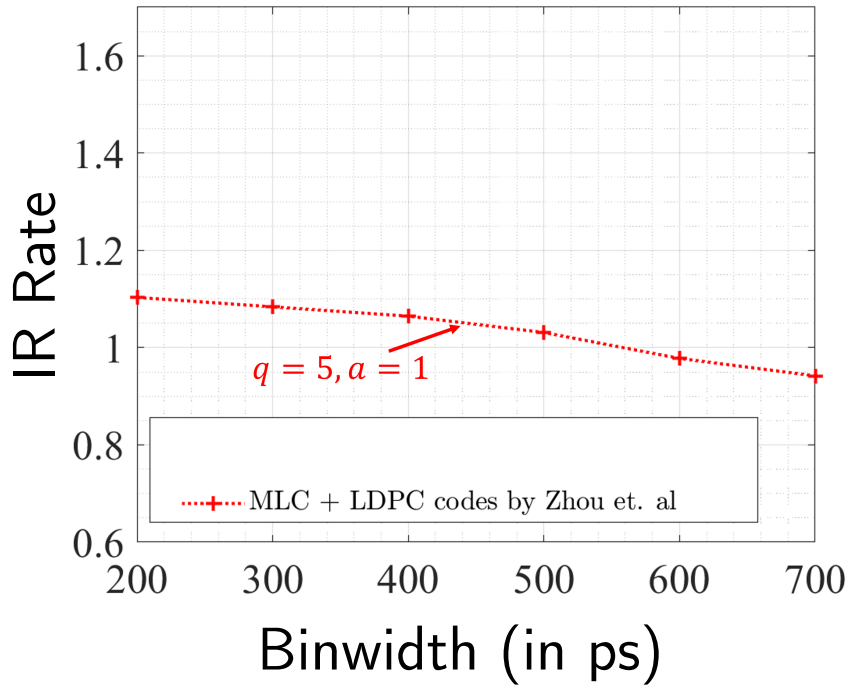


- Latency becomes significantly large as a becomes large (close to q)
- Latency is non-monotonic in a: since as a increases, the increase in decoding complexity is offset by the decrease in the number of layers
- Best trade-off between IR rate and latency is obtained for a small value of a (3 or 4)

Simulation Results: Comparison with Classical Schemes

- 32 bins per frame $\rightarrow q = 5$.



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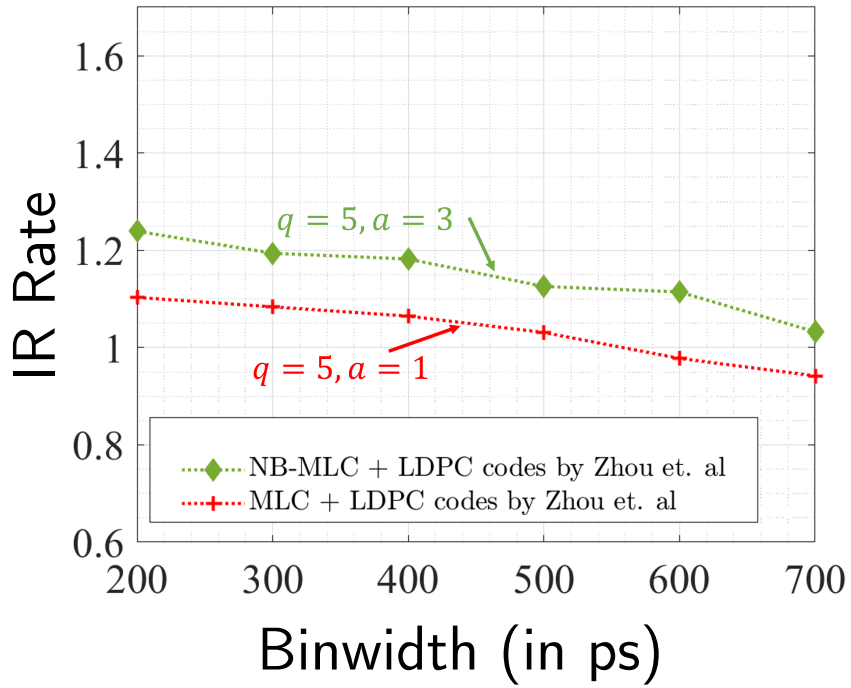

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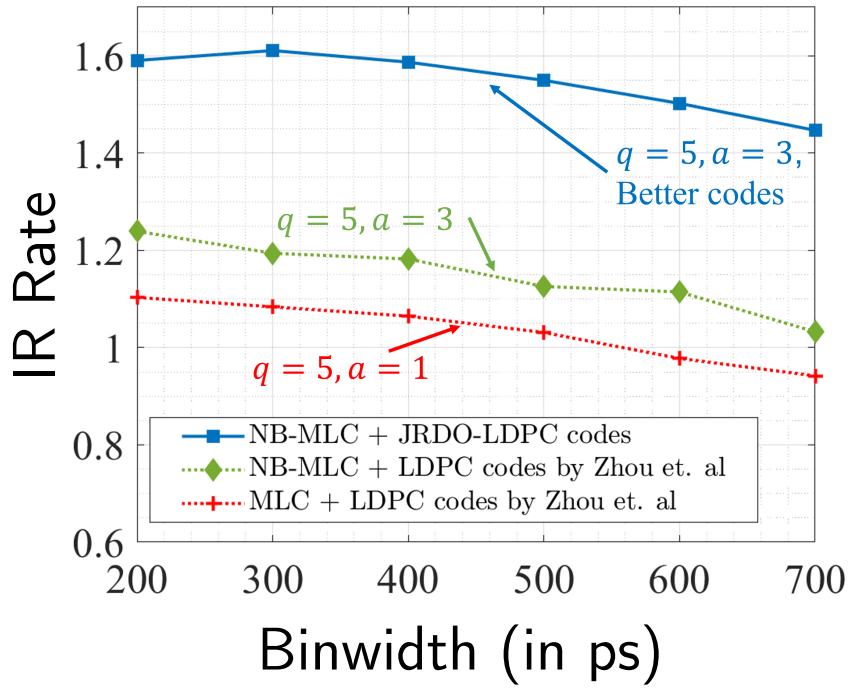

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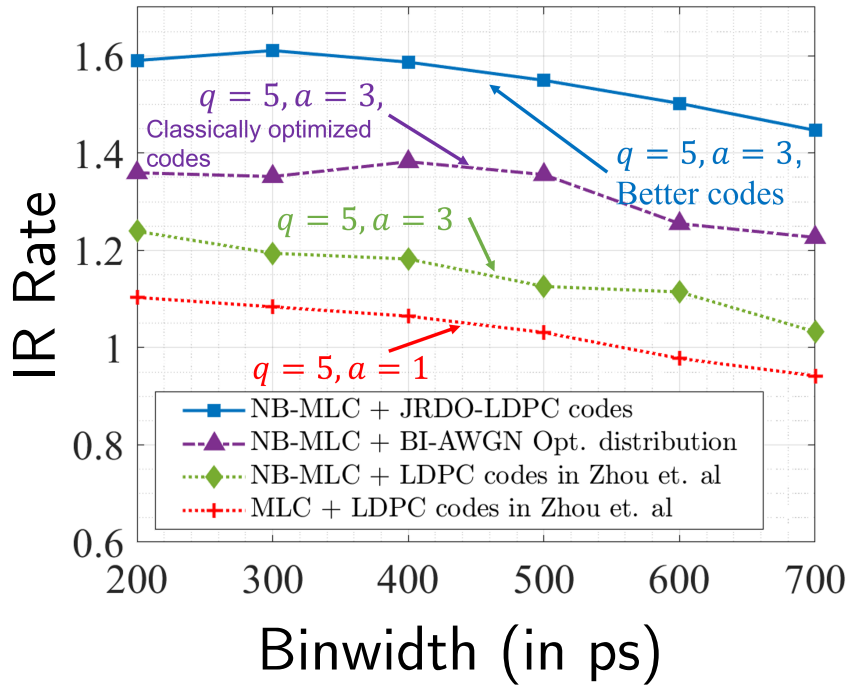

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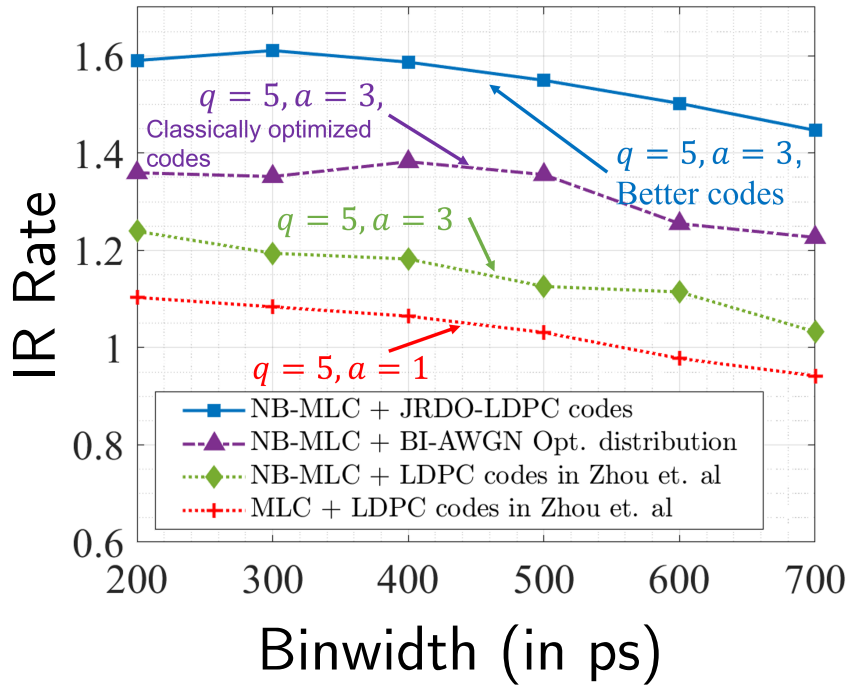
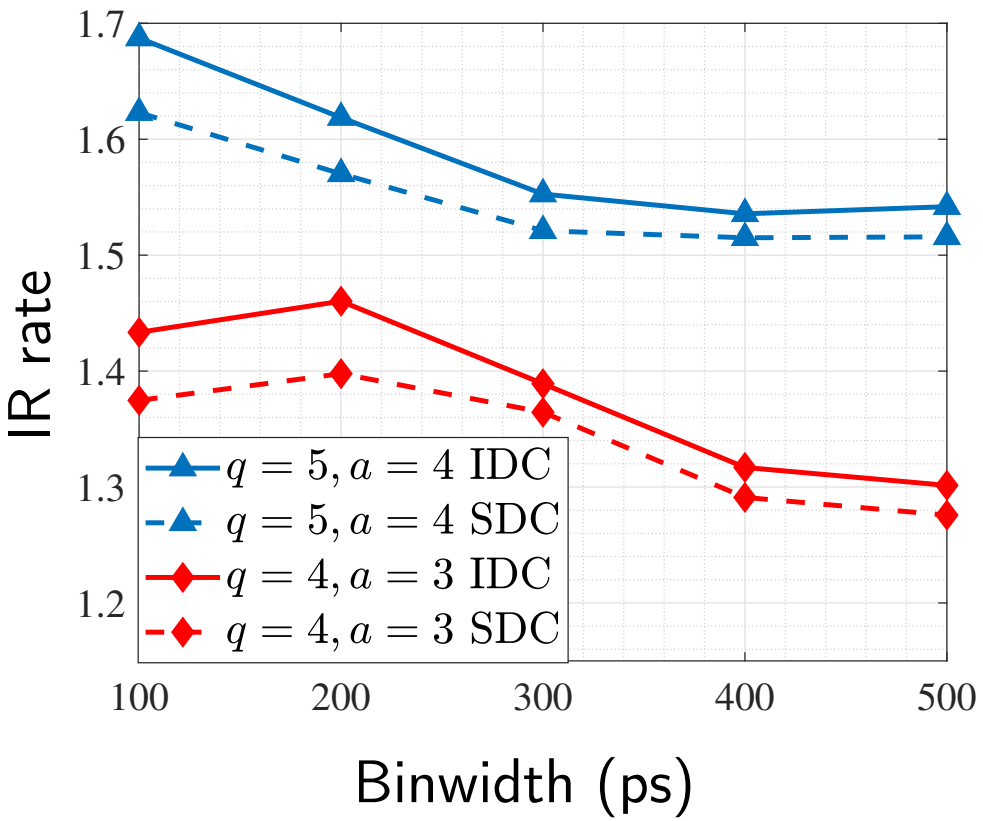


- Improvement due to $a = 1 \rightarrow a = 3$
- Improvement due to using JRDO-LDPC codes
- Overall, JRDO-LDPC codes combined with NB-MLC with a small a result in the largest IR rates (effectively uses one code over $GF(8)$ and one code over $GF(4)$)

Performance Comparison: Benefits of Interleaved Decoding



- IDC: Interleaved Decoding and Communication
- SDC: Sequential Decoding and Communication

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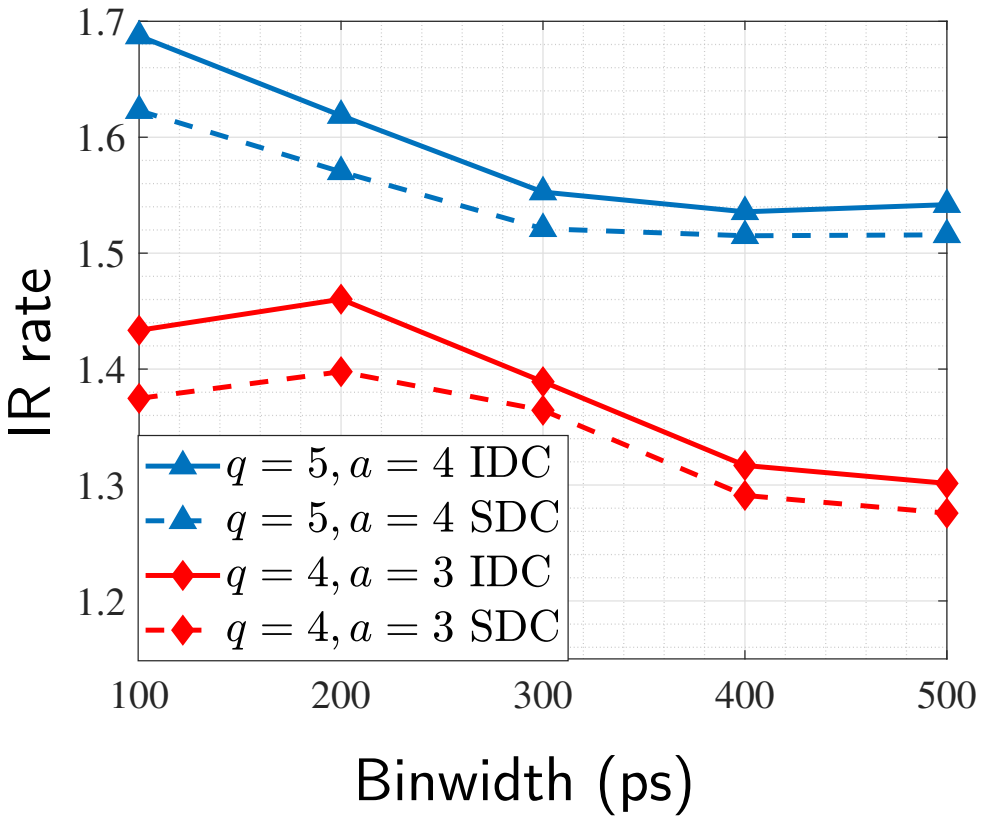

- IDC: Interleaved Decoding and Communication
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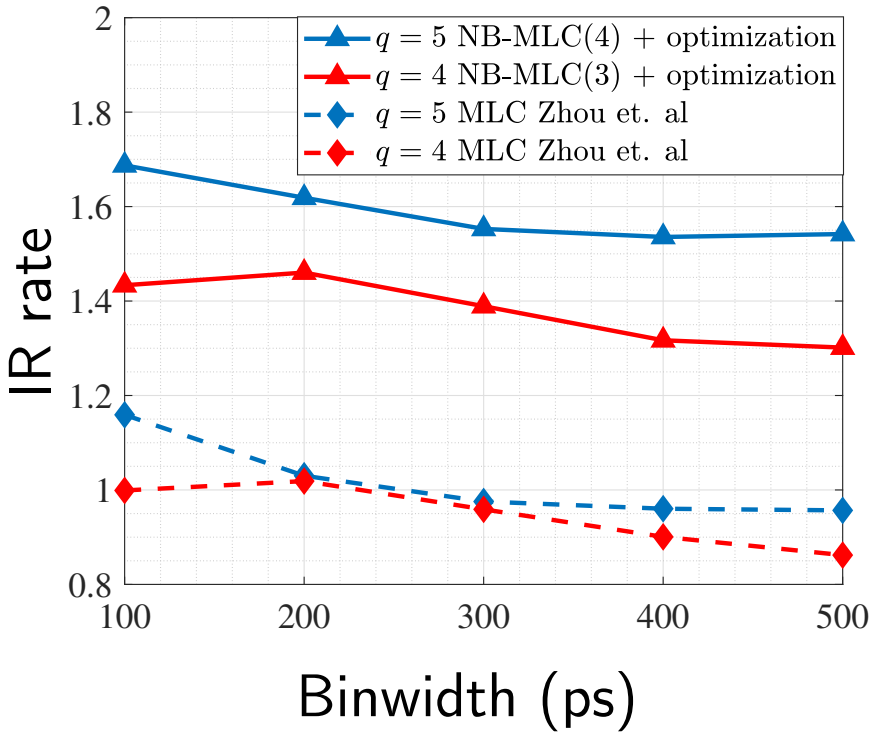
- IDC protocol results in higher IR rates compared to the conventional SDC protocol

The Overall Comparison



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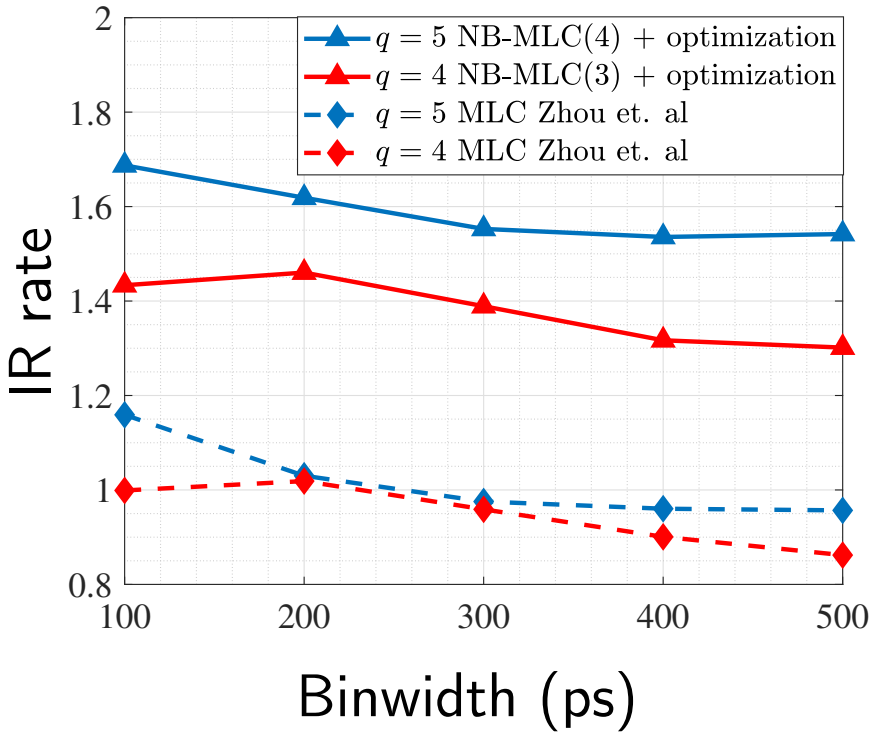
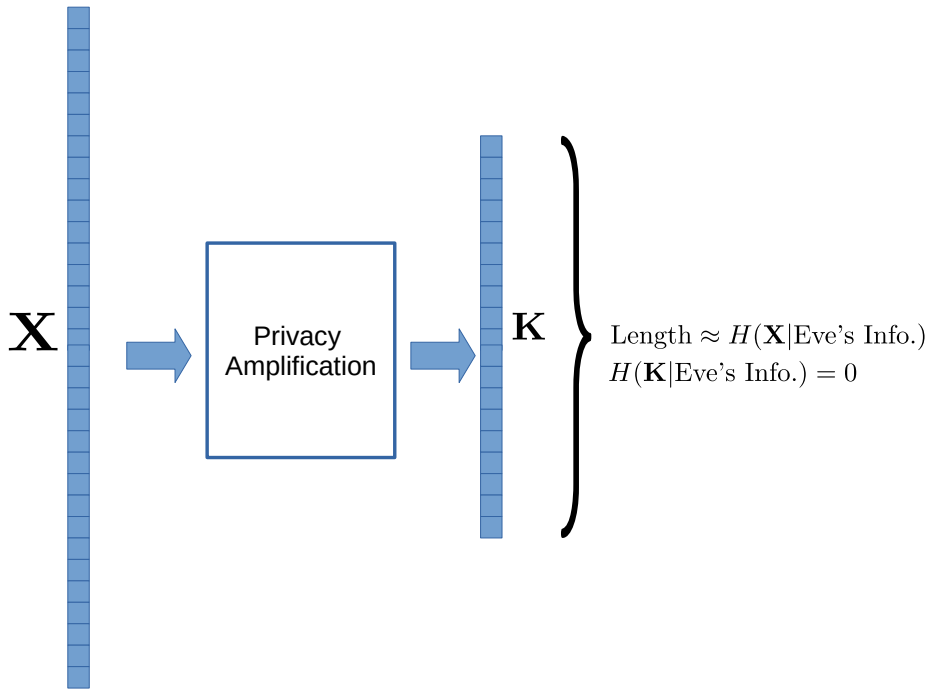


- Overall, our techniques result in around $40 - 60\%$ improvement in IR rates compared to the MLC scheme (prior state of the art).
- Asymptotic limit of a simplified (better) channel is around 2.1 for $q = 5$ and 1.8 for $q = 4$.

Privacy Amplification-Aware LDPC Code Design

- Privacy Amplification compresses the common string between Alice and Bob whose length is dependent on the information Eve has (Bennet '88)



Privacy Amplification-Aware LDPC Code Design

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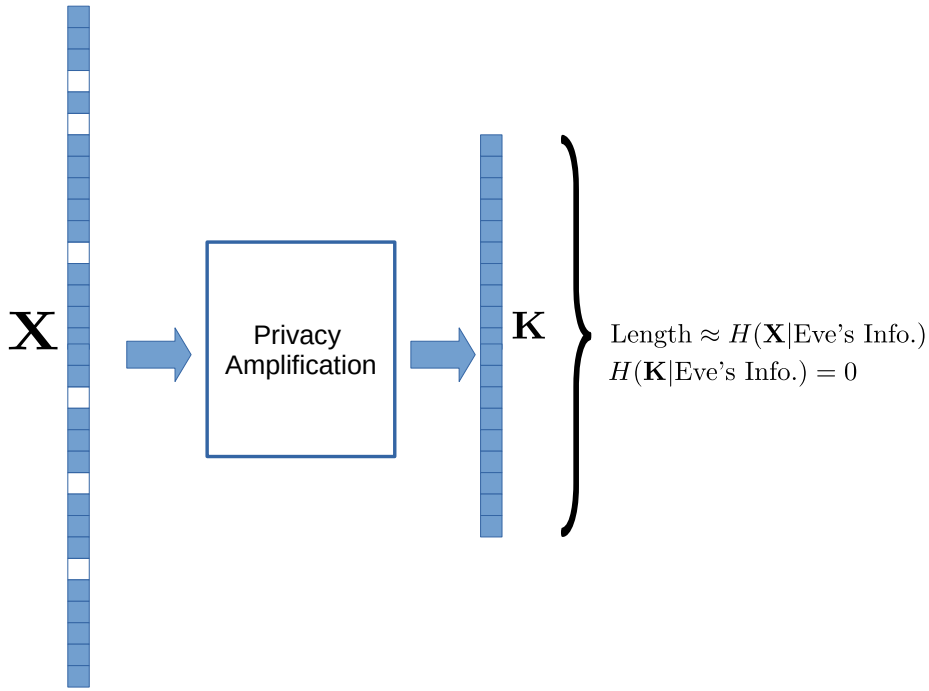
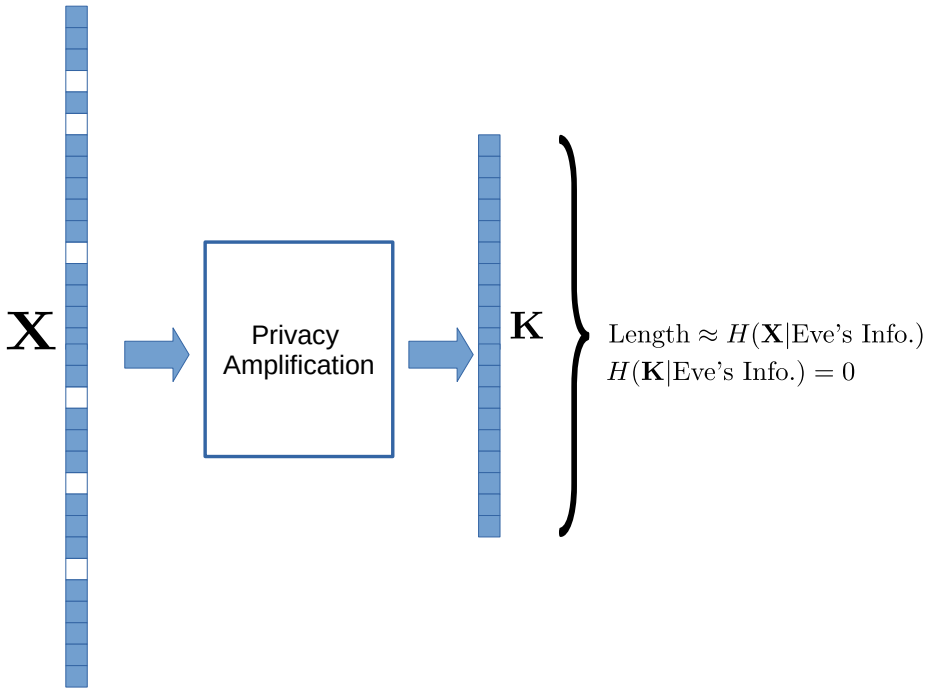
- Question: Depending on the information Eve has, do we need all of $\mathbf{X}$ in order to extract $\mathbf{K}$?



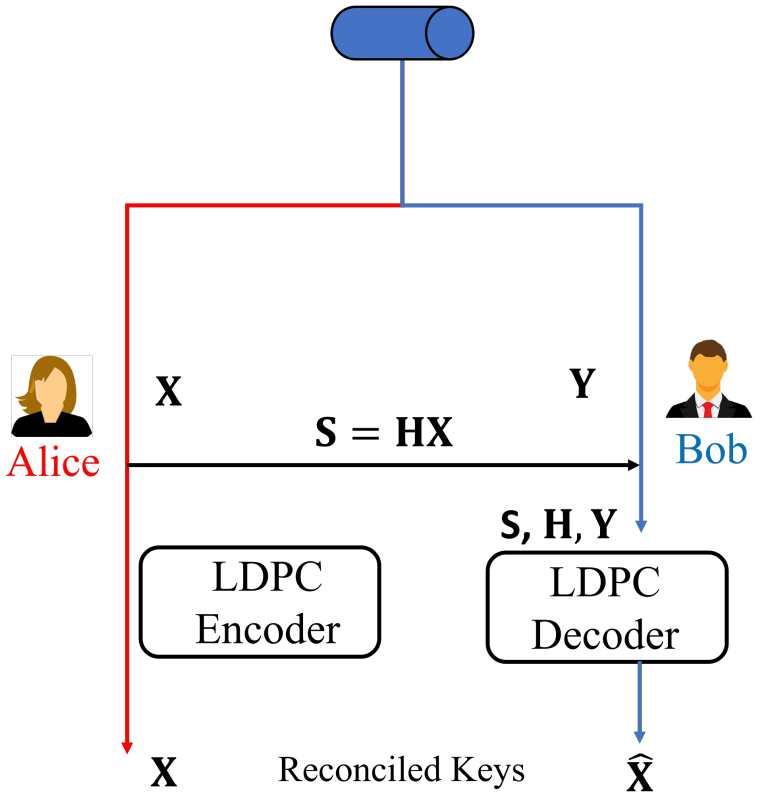
Privacy Amplification-Aware LDPC Code Design

- Privacy Amplification compresses the common string between Alice and Bob whose length is dependent on the information Eve has (Bennet '88)
- Question: Depending on the information Eve has, do we need all of $\mathbf{X}$ in order to extract $\mathbf{K}$?
  No



Revisit the Full Non-Binary Example

- Assume that $\mathbf{H}$ can be partitioned into two sub-matrices $\mathbf{H} = [\mathbf{H}_1, \mathbf{H}_2]$ such that $\mathbf{H}_2$ is $m \times m$ and is full rank.



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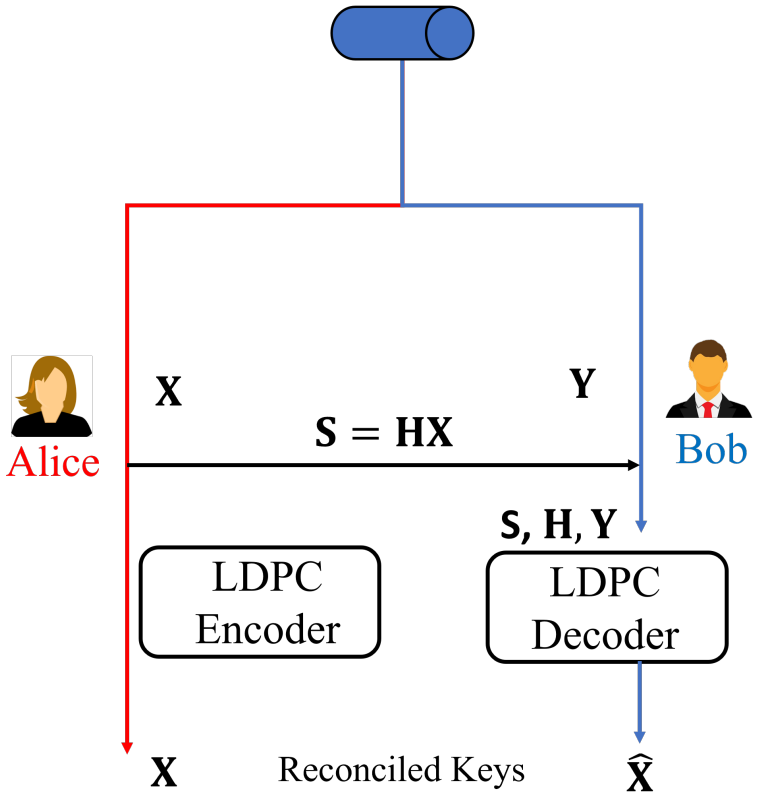

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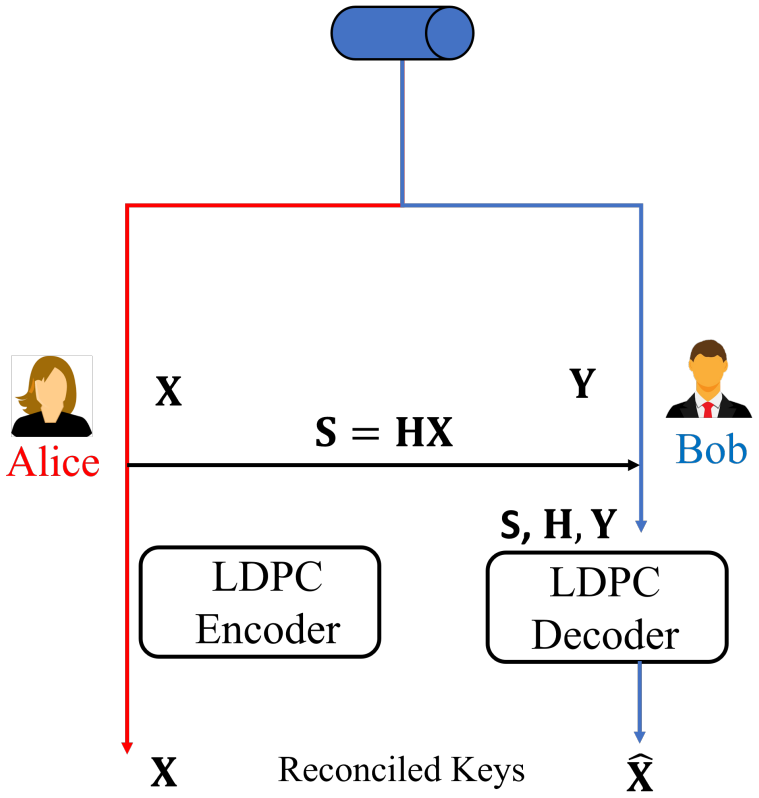

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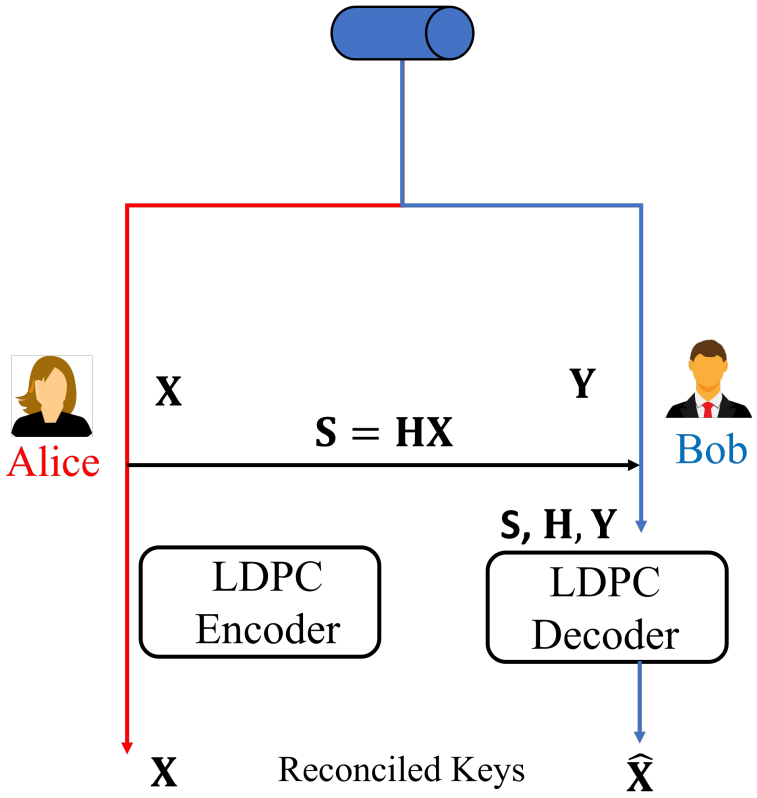

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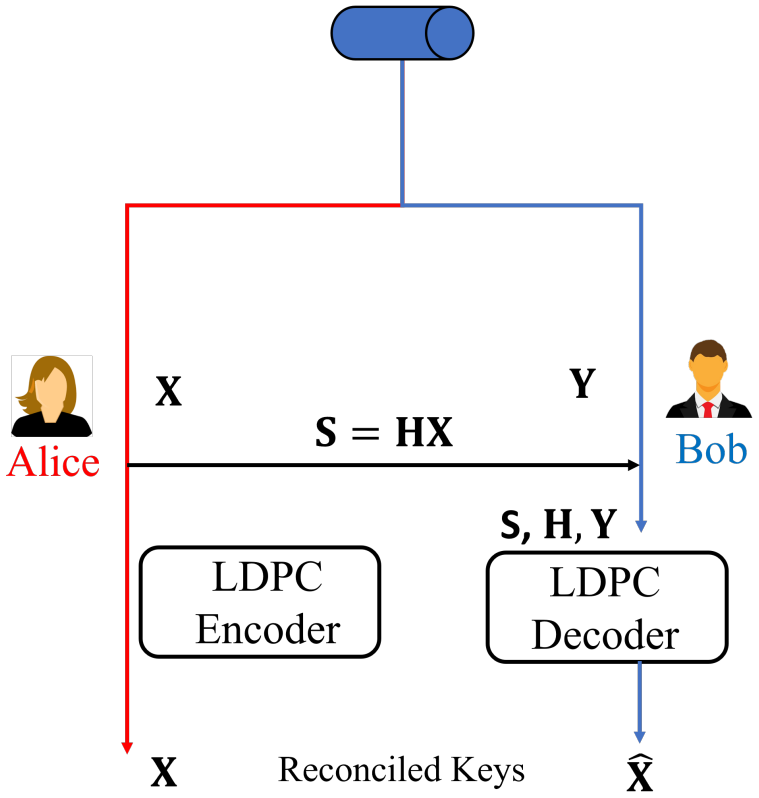

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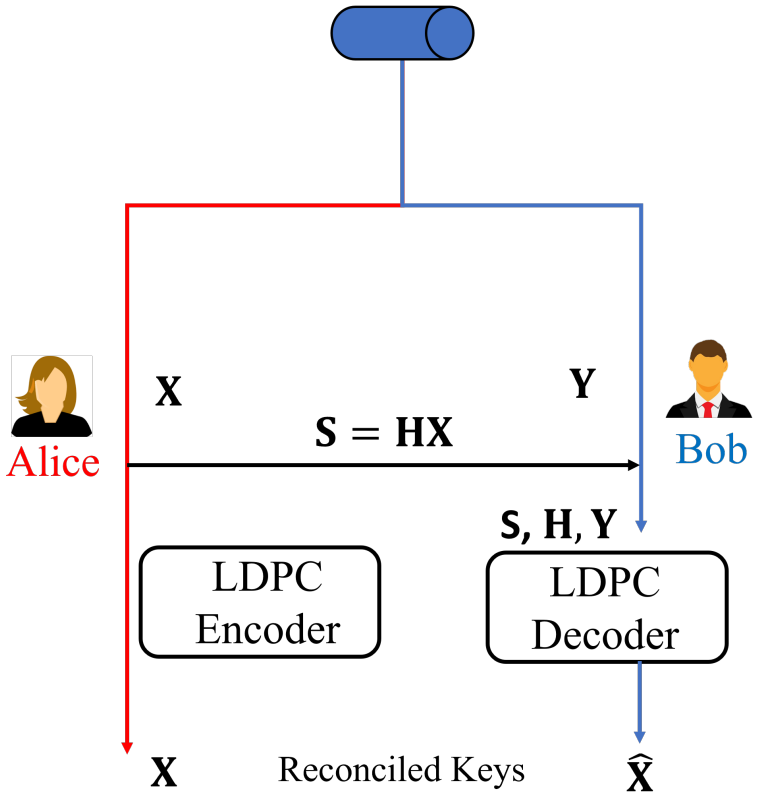

Key Observations

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- There may be multiple subsets that satisfy this property.

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- **MDS codes**:

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  - Structured code construction that guarantees the required subsets on a block matrix level while still having the benefits of LDPC codes

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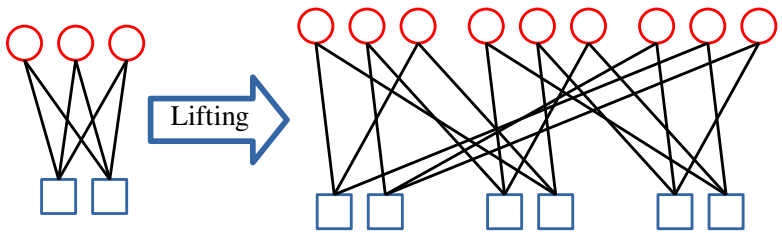

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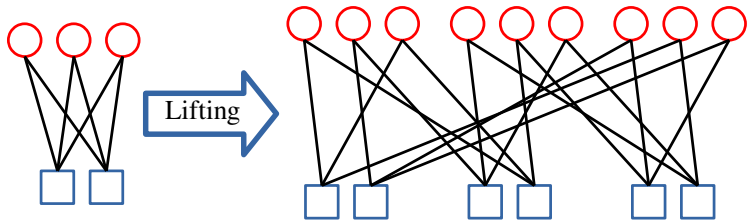

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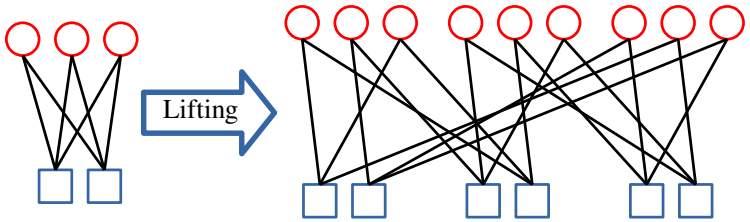

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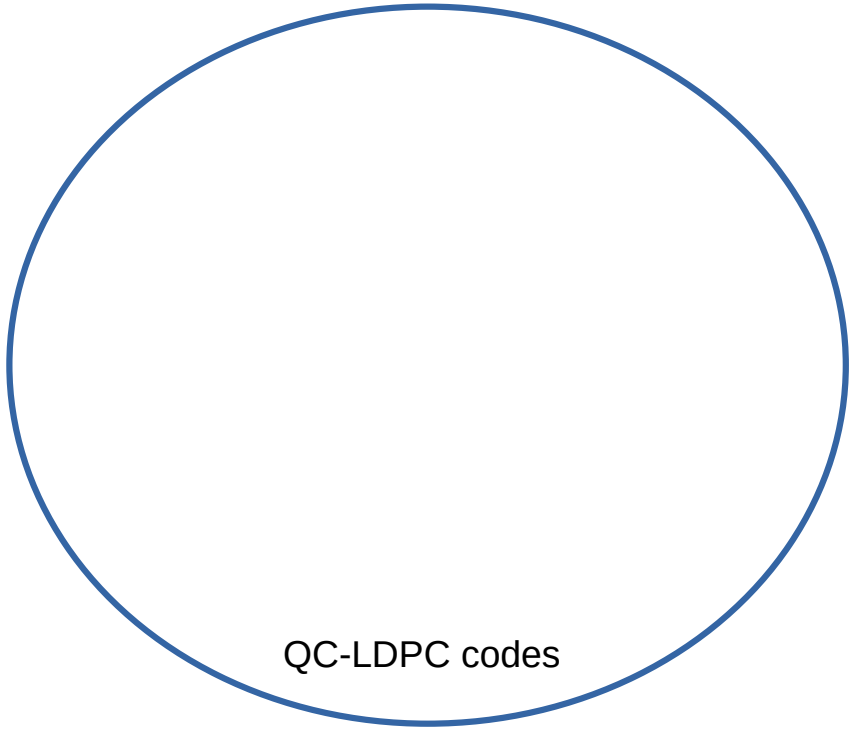

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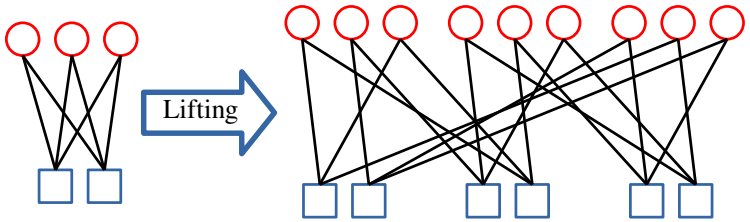

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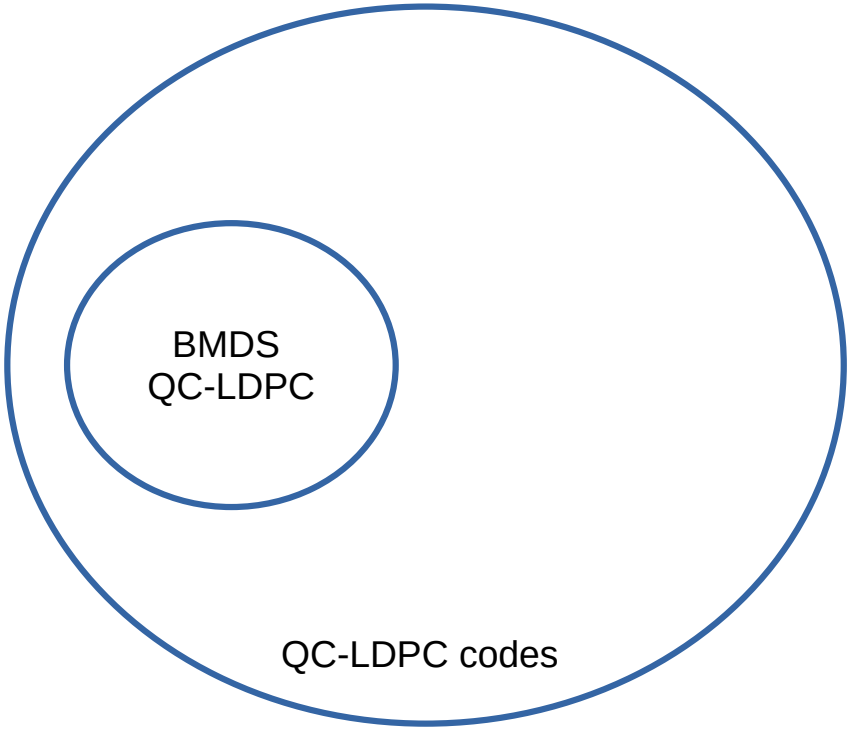

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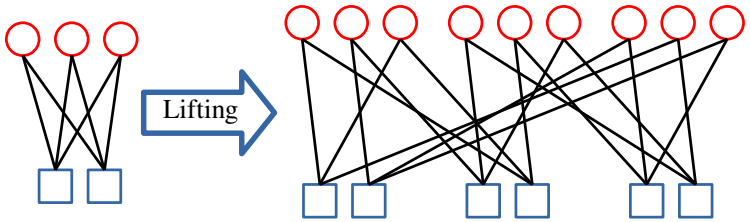

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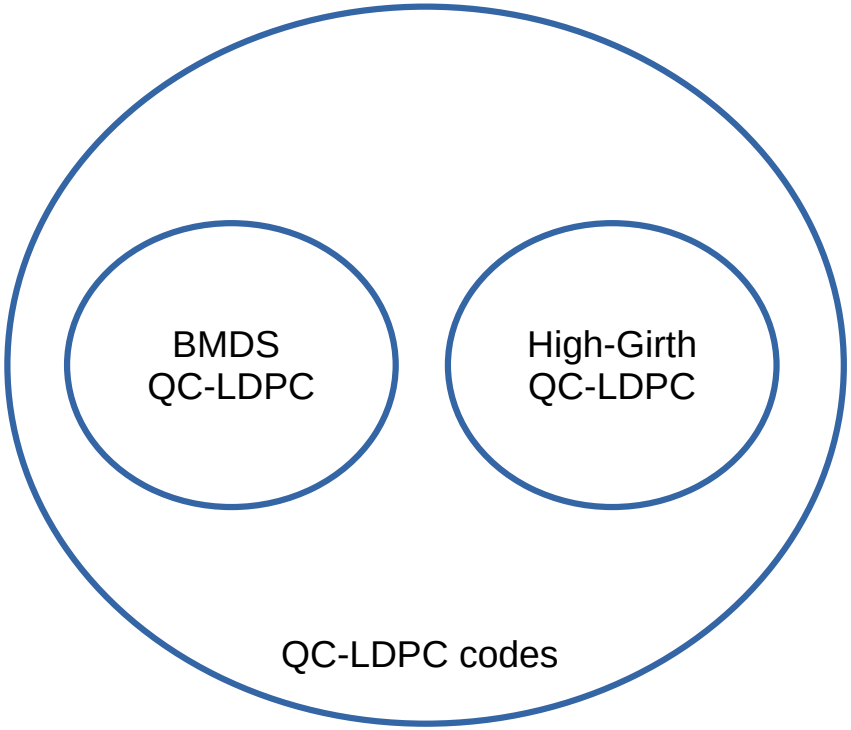

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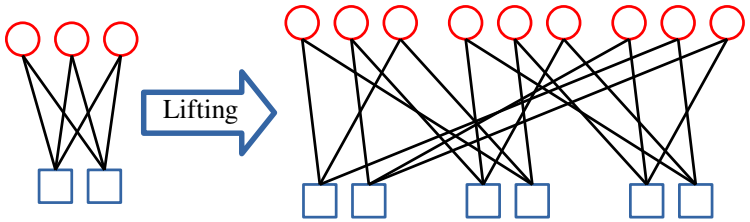

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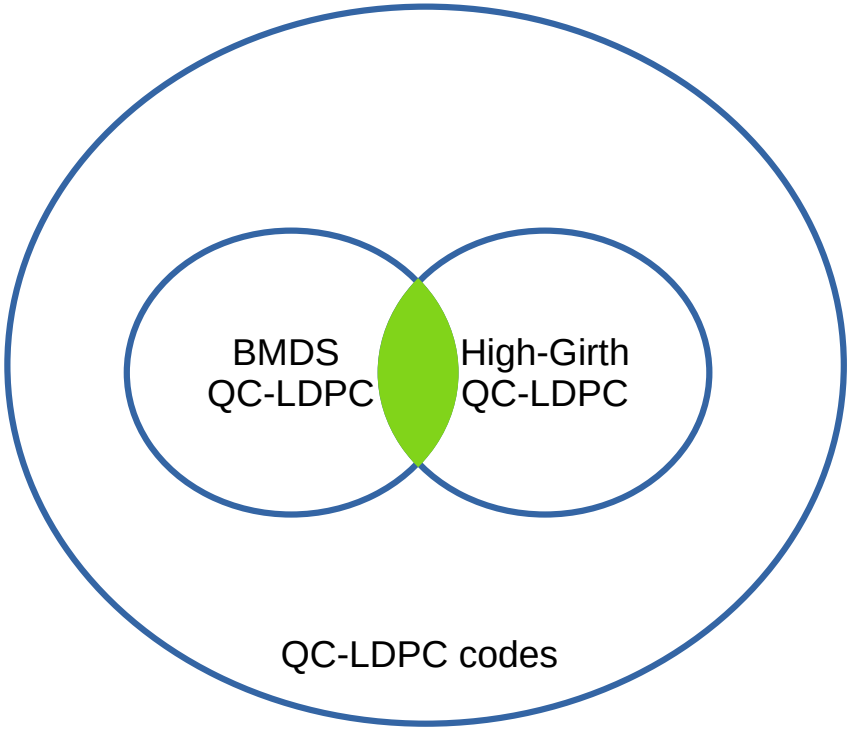

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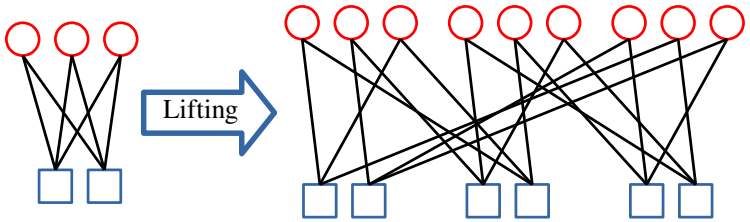

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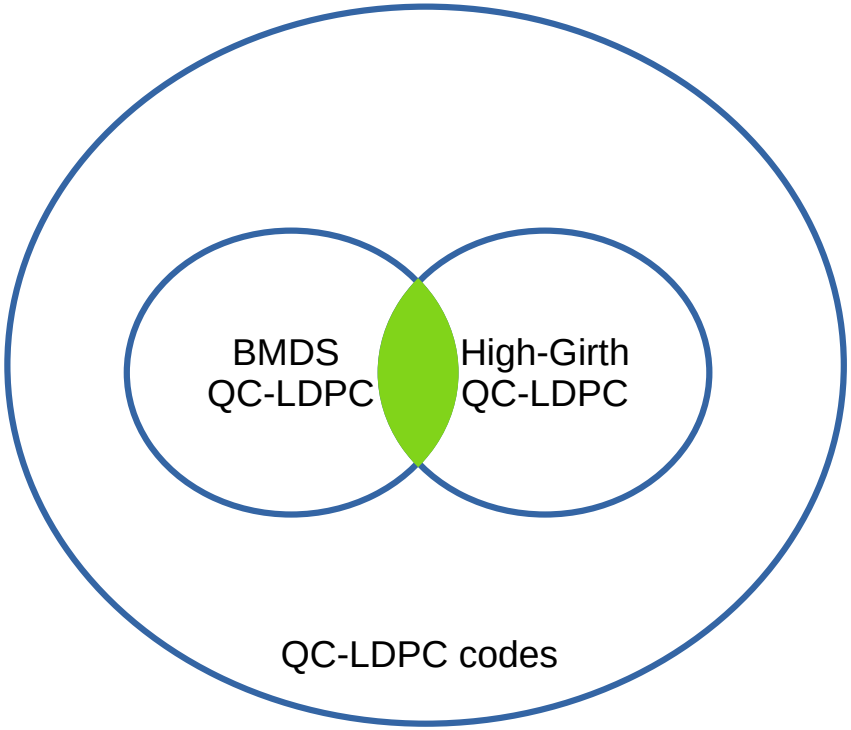
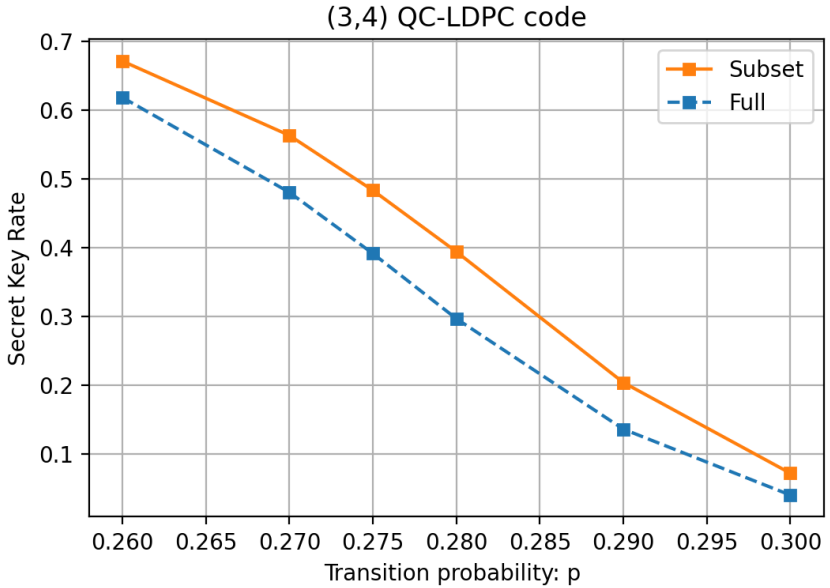
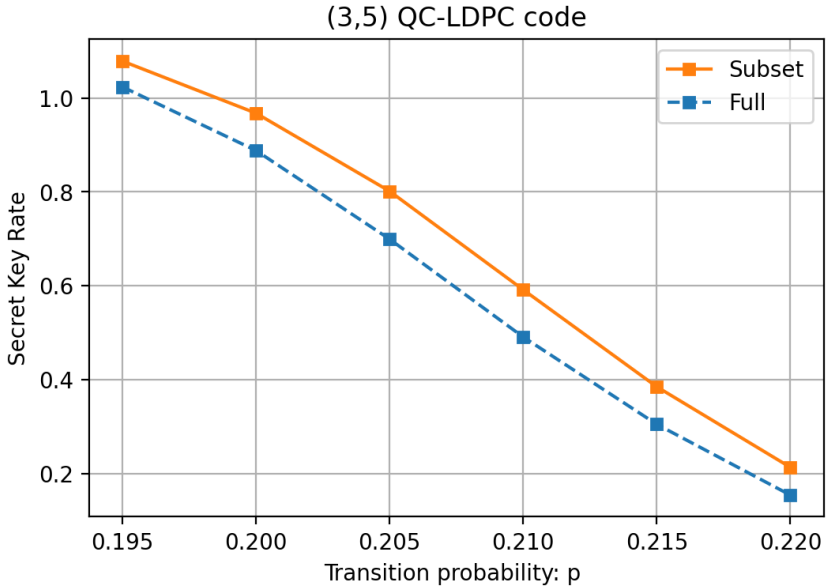


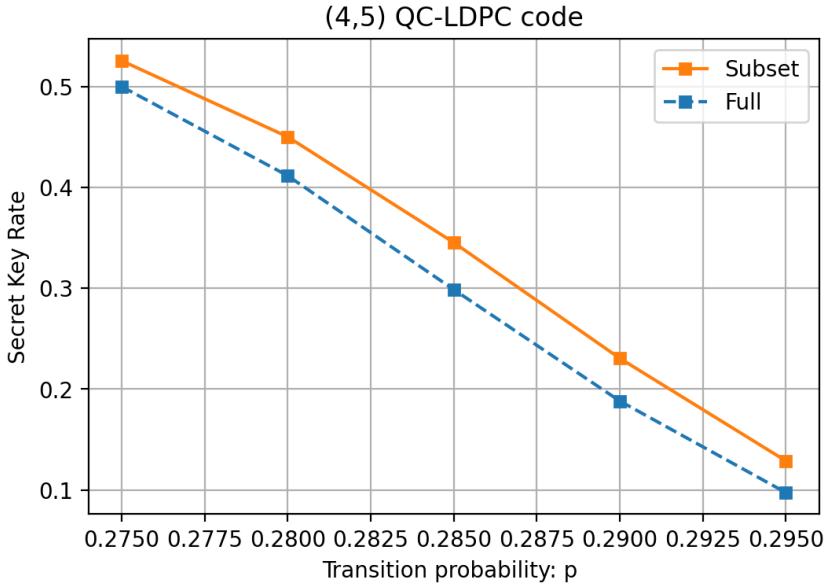
Main Result

A binary regular QC-LDPC code with column weight γ, row weight κ, and girth $\gamma + 2$ can be transformed into a non-binary BMDS code with finite field size at least κ, for a particular prime choice of the lifting factor.

Full Codeword Decoding vs Subset Decoding Simulations







- All codes have girth 10, length of approximately 2000, and perform operations in $\mathbb{GF}(8)$
- Channel is a q-ary symmetric channel with transition probability p

Outline

1. Background and Motivation

2. System Model

3. NB-LDPC codes for Quantum Key Distribution
   - Preliminaries
   - Channel code design
   - Sequential Decoding and Interactive Communication
   - Privacy Amplification-Aware LDPC Code Design

4. Concluding Remarks and Future Outlook

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- Key Result: Novel non-binary codes (Mitra '24, Tauz '24)
  - Research supported by NSF-QuIC-TAQS no. 2137984 and NSF-FET no. 2008728.

Advances in Technology and Suggested Research Directions

- High-dimensional energy-time QKD promises unprecedented key rates. To reach this promise, and in tandem with experimental advances, it will be necessary to investigate:
  - Design of codes tailored to the specifics of the QKD channels (this work)

- Special issue on Quantum Information Science, 2023, including L. Dolecek and E. Soljanin: a primer on time-bin QKD

Advances in Technology and Suggested Research Directions

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  - Relevant attack models and security proofs for them.

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Each of these technologies will benefit from new studies in channel coding and related disciplines.

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Verification after Information Reconciliation

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Example:

$X = 10\ 10\ 01\ 11\ 00\ 10\ 11$

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- Partition X into substrings of length $\lfloor \log_2 p \rfloor$

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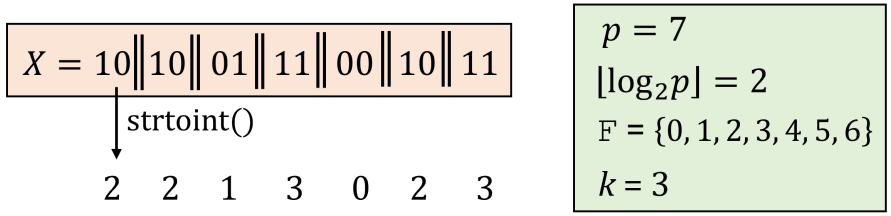

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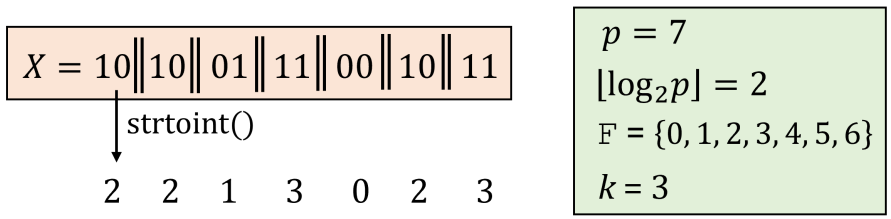

$$2k^0 + 2k^1 + k^2 + 3k^3 + 0k^4 + 2k^5 + 3k^6 \bmod p = 1$$

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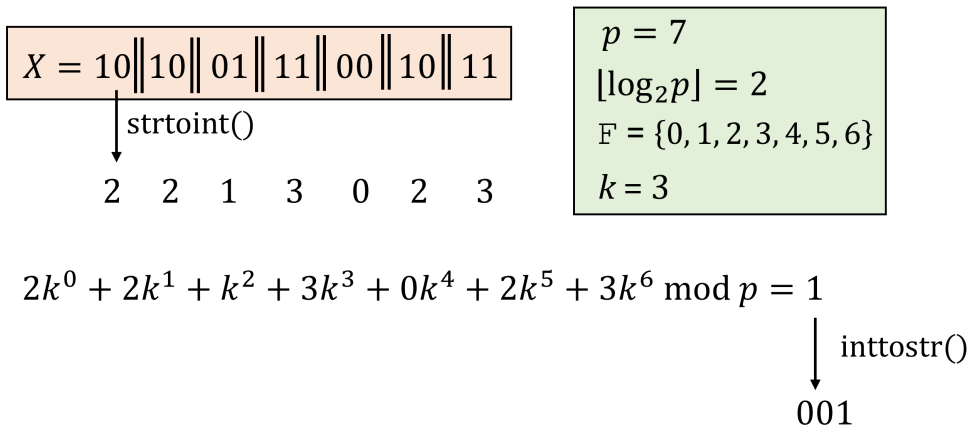

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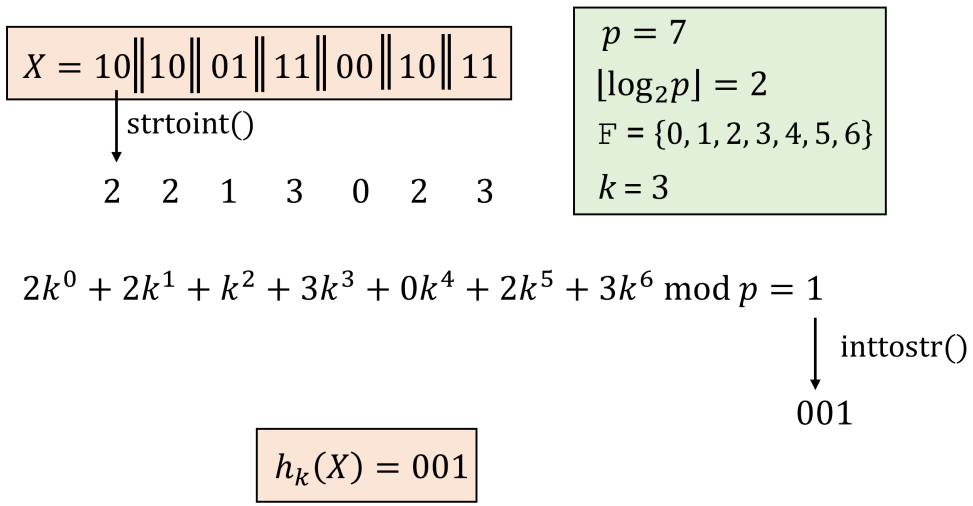

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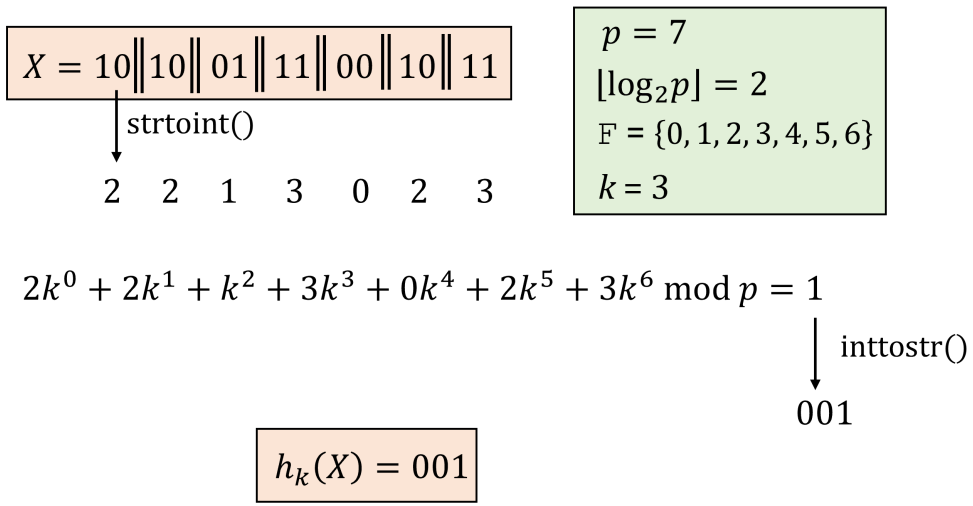

- Verification is achieved by matching the hash of the string X at Alice and decoded (reconciled) string $\widehat{X}$ at Bob