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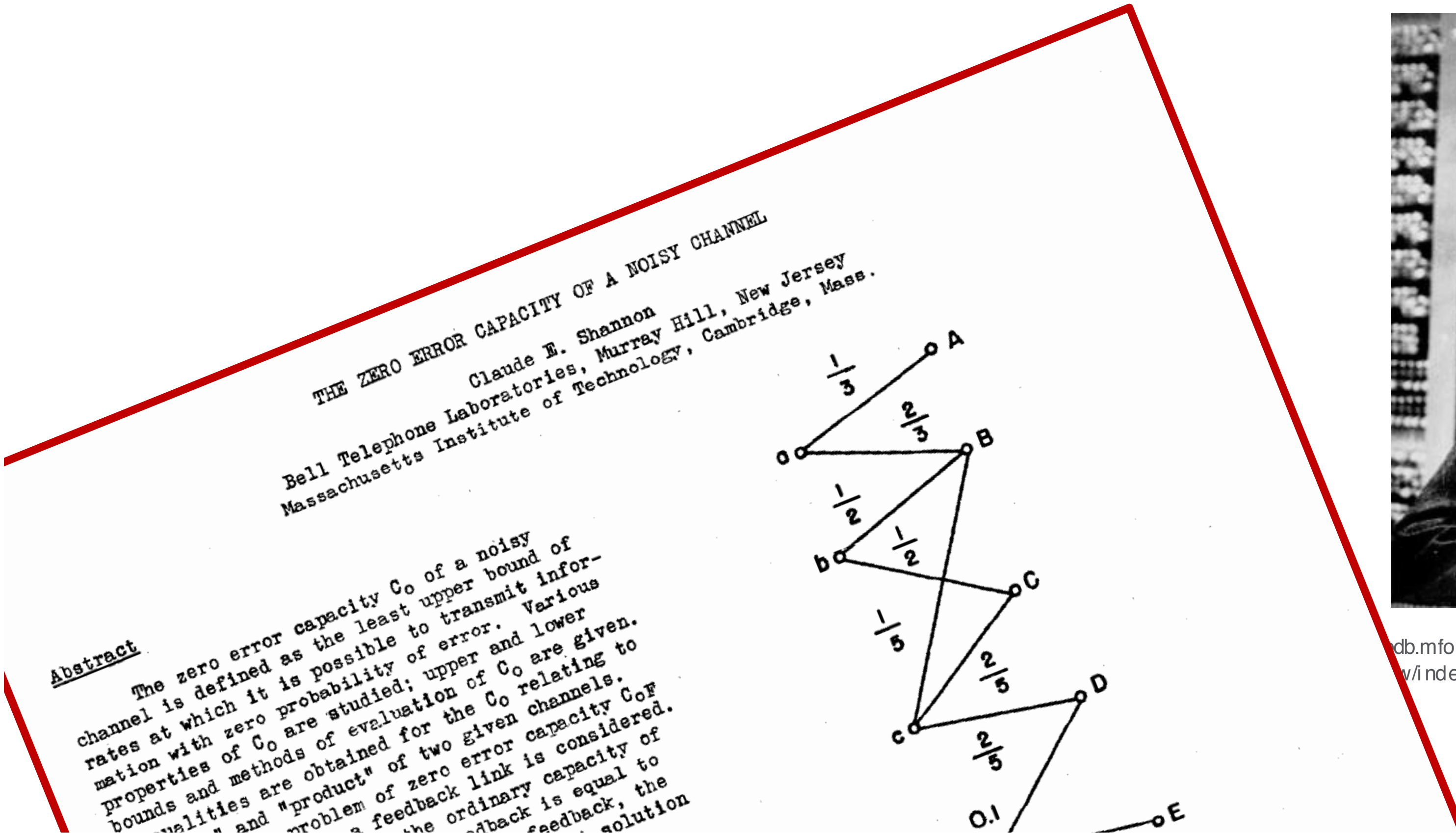
# Short Variable-Length Codes with Shared Incremental Redundancy: A New Architecture for High Throughput

2019 Oberpfaffenhofen Workshop on High Throughput Coding

Richard Wesel, Haobo Wang, Sudarsan Ranganathan

UCLA Electrical and Computer Engineering Department

# Shannon: Feedback does not increase capacity



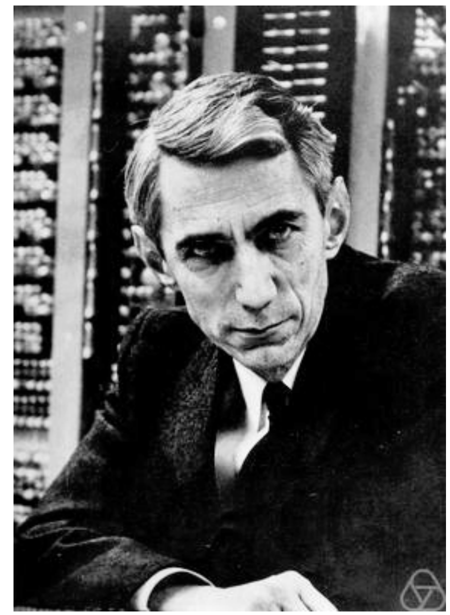
THE ZERO ERROR CAPACITY OF A NOISY CHANNEL

Claude E. Shannon
Bell Telephone Laboratories, Murray Hill, New Jersey
Massachusetts Institute of Technology, Cambridge, Mass.

Abstract

The zero error capacity $C_o$ of a noisy channel is defined as the least upper bound of rates at which it is possible to transmit information with zero probability of error. Various properties of $C_o$ are studied; upper and lower bounds and methods of evaluation of $C_o$ are given. ...ualities are obtained for the $C_o$ relating to ... and "product" of two given channels. ... problem of zero error capacity $C_{oF}$ ... a feedback link is considered. ... the ordinary capacity of ... feedback is equal to ... feedback, the ... solution





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# Shannon: Feedback does not increase capacity

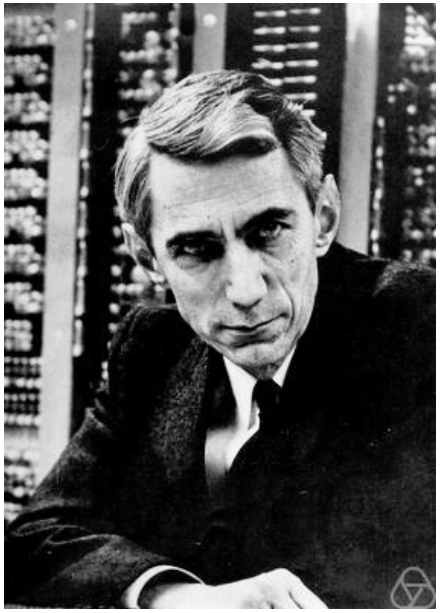
**Mutual information between message $W$ and receiver's knowledge**

**Before we send $X_j = f\left(W, Y_1 \cdots Y_{j-1}\right)$:**

$$I(W; Y_1^{j-1})$$

**After we send $X_j = f\left(W, Y_1 \cdots Y_{j-1}\right)$:**

$$I(W; Y_1^{j-1}, Y_j)$$





$$I\left(W; Y_1^{j-1}, Y_j\right) - I\left(W; Y_1^{j-1}\right) \le C$$

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# An Operational Interpretation and a surprise.

**THEOREM (SHANNON)**

Any rate that can be achieved with feedback can be achieved without feedback.

**COROLLARY (Further constraining the feedback cannot help.)**

Any rate that can be achieved with ACK/NACK feedback and a Variable-Length Code can be achieved without feedback.

**SURPRISE**

Any rate that can be achieved with ACK/NACK feedback and a Variable-Length Code can be closely approached without feedback using the same Variable-Length Code.

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# Polyanskiy, Poor, and Verdu

**FINITE-BLOCKLENTH ANALYSIS OF VARIABLE-LENGTH CODES WITH STOP FEEDBACK**

The feedback is limited to ACK/NACK.


IEEE TRANSACTIONS ON INFORMATION THEORY, VOL. 57, NO. 8, AUGUST 2011 4903

Feedback in the Non-Asymptotic Regime

Yury Polyanskiy, *Member, IEEE*, H. Vincent Poor, *Fellow, IEEE*, and Sergio Verdú, *Fellow, IEEE*

***Abstract*—Without feedback, the backoff from capacity due to non-asymptotic blocklength can be quite substantial for blocklengths and error probabilities of interest in many practical applications. In this paper, novel achievability bounds are used to demonstrate that in the non-asymptotic regime, the maximal achievable rate improves dramatically thanks to variable-length coding and feedback. For example, for the binary symmetric channel with capacity 1/2 the blocklength required to achieve 90% of the capacity is smaller than 200, compared to at least 3100 for the best fixed-blocklength code (even with noiseless feedback). Virtually all the advantages of noiseless feedback are shown to be achievable, even if the feedback link is used only to send a single signal informing the encoder to terminate the transmission (stop-feedback). It is demonstrated that the non-asymptotic behavior of the fundamental limit depends crucially on the particular model chosen for the "end-of-packet" control signal. Fixed-blocklength codes and related questions concerning communicating with a guaranteed delay are discussed, in which situation feedback is demonstrated to be almost useless even non-asymptotically.**

***Index Terms*—Achievability bounds, channel capacity, converse bounds, feedback, memoryless channels, non-asymptotic analysis, Shannon theory, stop-feedback.**

I. INTRODUCTION

# ITA 2019 Presentation Final Slide

$$\tau^* = \min_n\{n \geq 0 \colon i(X^n(\theta), Y^n) > \gamma\}$$

Abraham Wald

$$E\tau^* < \frac{\gamma}{C} + 1$$

*Theorem 3:* Fix a scalar $\gamma > 0$, a channel $\{P_{Y_i|X^i Y^{i-1}}\}_{i=1}^{\infty}$ and an arbitrary process $X = (X_1, X_2, \ldots, X_n, \ldots)$ taking values in $\mathcal{A}$. Define a probability space with finite-dimensional distributions given by (23), shown at the bottom of the page, i.e., $X$ and $\bar{X}$ are independent copies of the same process and $Y$ is the output of the channel when $X$ is its input. For the joint distribution (23) define a sequence of information density functions $\mathcal{A}^n \times \mathcal{B}^n \to \mathbb{R}$

$$\imath(a^n; b^n) = \log \frac{dP_{Y^n|X^n}(b^n|a^n)}{dP_{Y^n}(b^n)} \qquad (24)$$

and a pair of hitting times

$$\tau = \inf\{n \geq 0 : \imath(X^n; Y^n) \geq \gamma\} \qquad 25)$$

$$\bar{\tau} = \inf\{n \geq 0 : \imath(\bar{X}^n; Y^n) \geq \gamma\}. \qquad 26$$

Then for any $M$ there exists an $(\ell, M, \epsilon)$ VLF code with

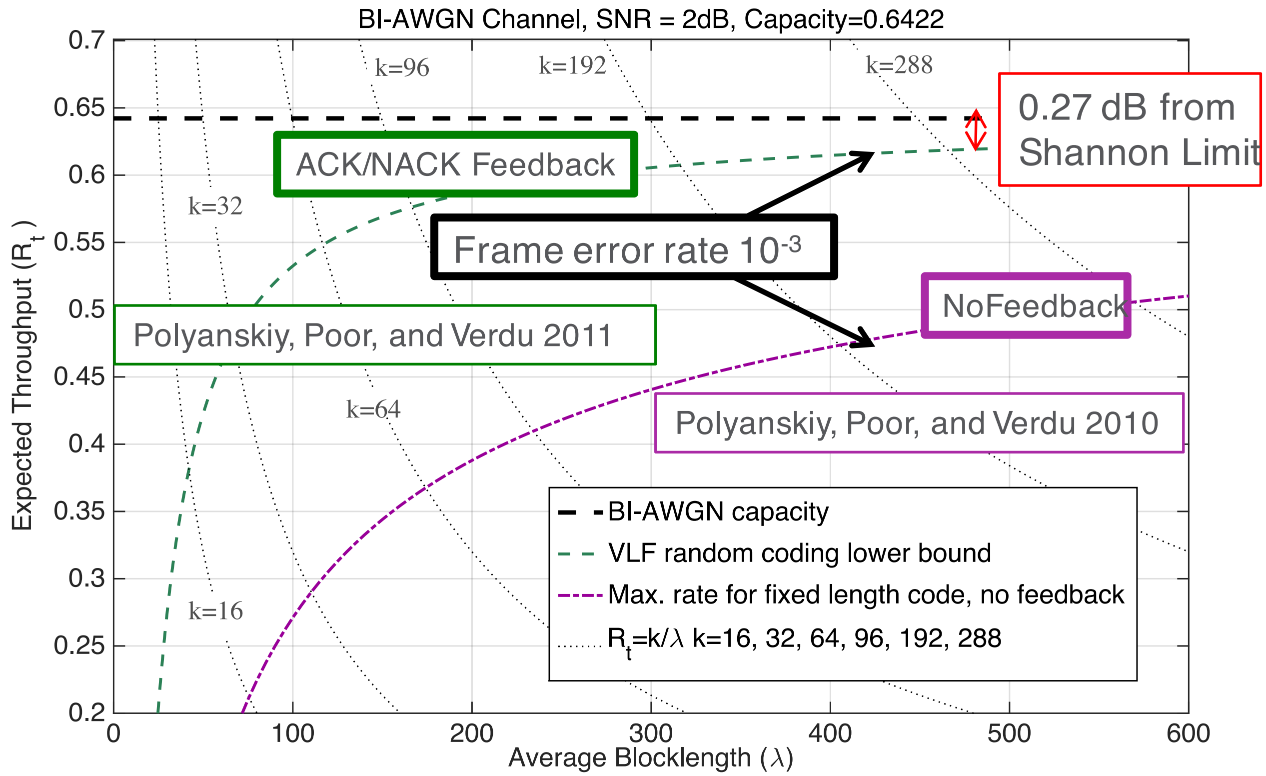
$$\ell \leq \mathbb{E}[\tau]$$

$$\epsilon \leq (M-1)\mathbb{P}[\bar{\tau} \leq \tau].$$

Furthermore, for any $M$ there exists a determinis VLF code with $\epsilon$ satisfying (28) and

$$\ell' \leq \operatorname{esssup} \mathbb{E}[\tau|X].$$

# PPV Achievable Rates for BI-AWGN Channel



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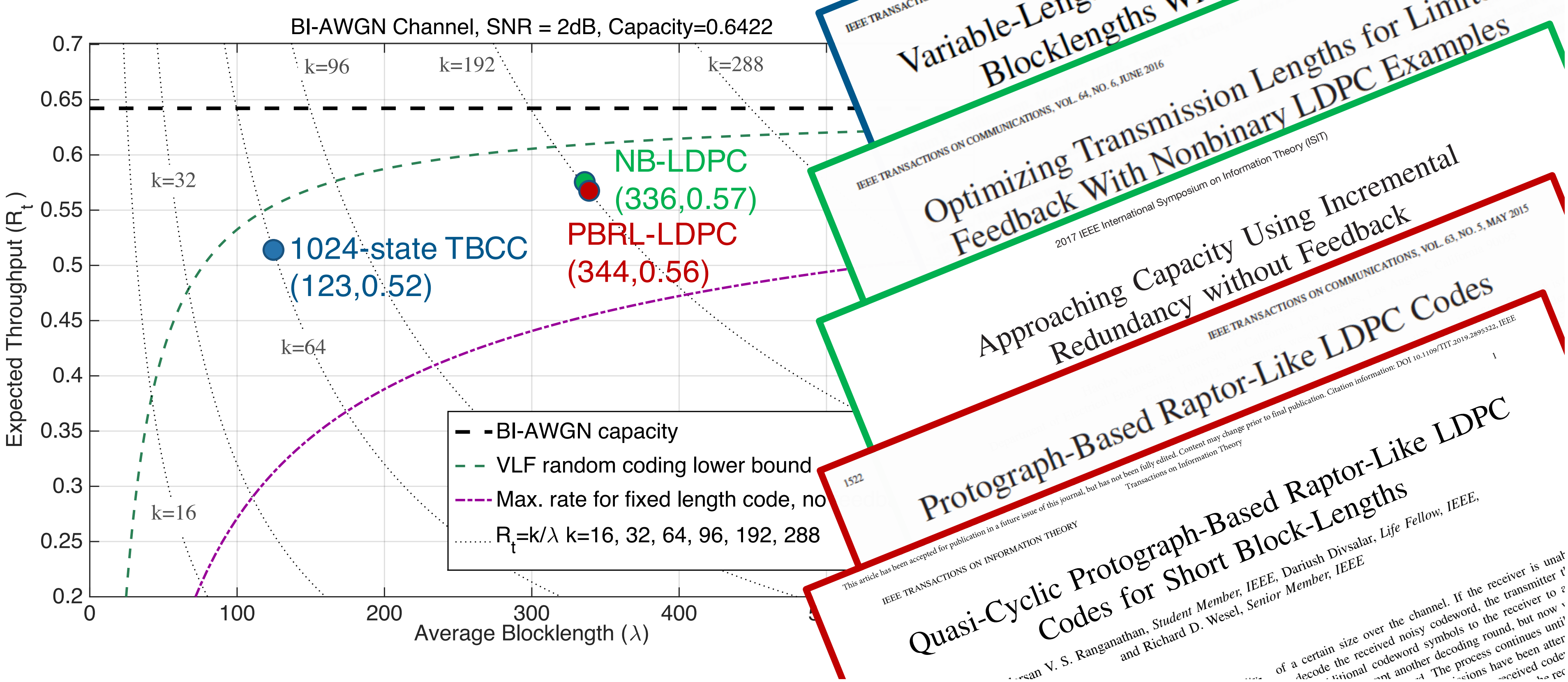
# Real codes, 5 Transmissions, Constant



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# Variable Length Codes without feedback.

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Any rate that can be achieved with ACK/NACK feedback and a Variable-Length Code can be closely approached without feedback using the same Variable-Length Code.

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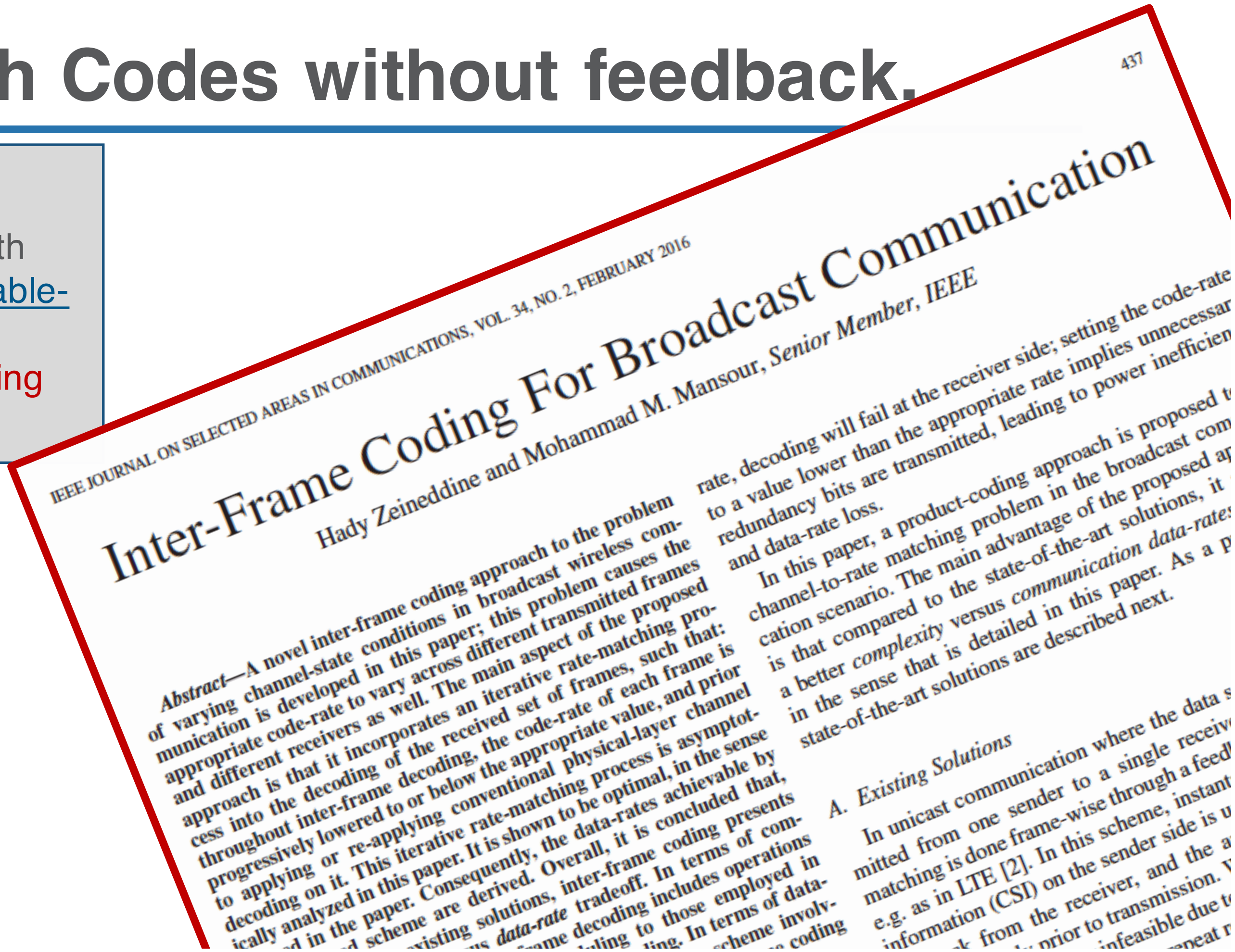
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IEEE JOURNAL ON SELECTED AREAS IN COMMUNICATIONS, VOL. 34, NO. 2, FEBRUARY 2016

Inter-Frame Coding For Broadcast Communication

Hady Zeineddine and Mohammad M. Mansour, *Senior Member, IEEE*

*Abstract*—A novel inter-frame coding approach to the problem of varying channel-state conditions in broadcast wireless communication is developed in this paper; this problem causes the appropriate code-rate to vary across different transmitted frames and different receivers as well. The main aspect of the proposed approach is that it incorporates an iterative rate-matching process into the decoding of the received set of frames, such that: throughout inter-frame decoding, the code-rate of each frame is progressively lowered to or below the appropriate value, and prior to applying or re-applying conventional physical-layer channel decoding on it. This iterative rate-matching process is asymptotically analyzed in this paper. It is shown to be optimal, in the sense

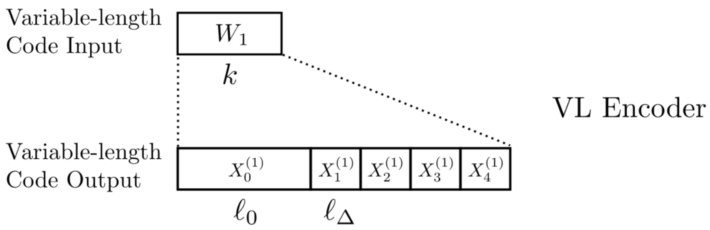
rate, decoding will fail at the receiver side; setting the code-rate to a value lower than the appropriate rate implies unnecessar redundancy bits are transmitted, leading to power inefficien and data-rate loss.

In this paper, a product-coding approach is proposed t channel-to-rate matching problem in the broadcast com cation scenario. The main advantage of the proposed ap is that compared to the state-of-the-art solutions, it a better *complexity* versus *communication data-rate* in the sense that is detailed in this paper. As a p state-of-the-art solutions are described next.

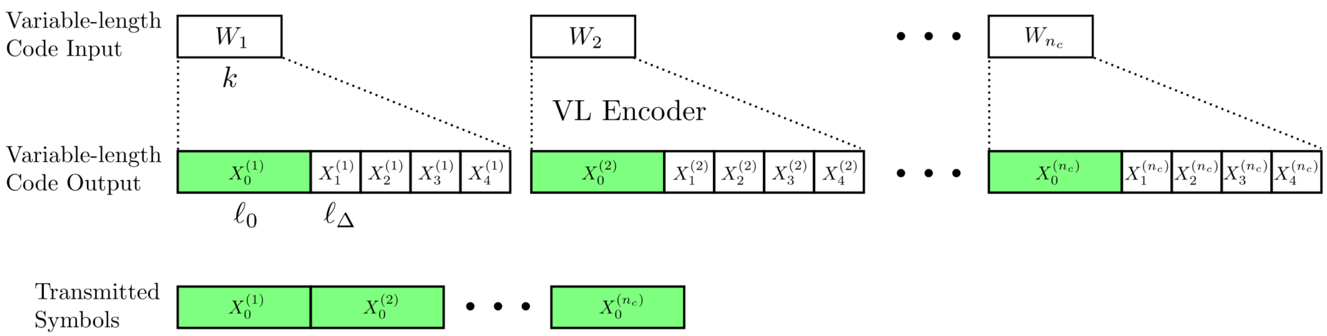
*A. Existing Solutions*

In unicast communication where the data s mitted from one sender to a single receiv matching is done frame-wise through a feed e.g. as in LTE [2]. In this scheme, instant information (CSI) on the sender side is u

# Inter-frame Coding [Zeineddine & Mansour]



# Inter-frame Coding [Zeineddine & Mansour]



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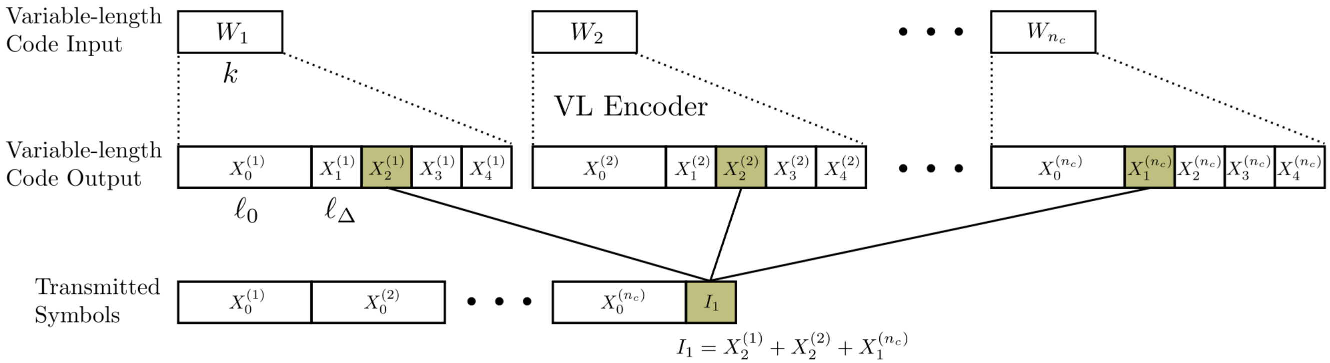

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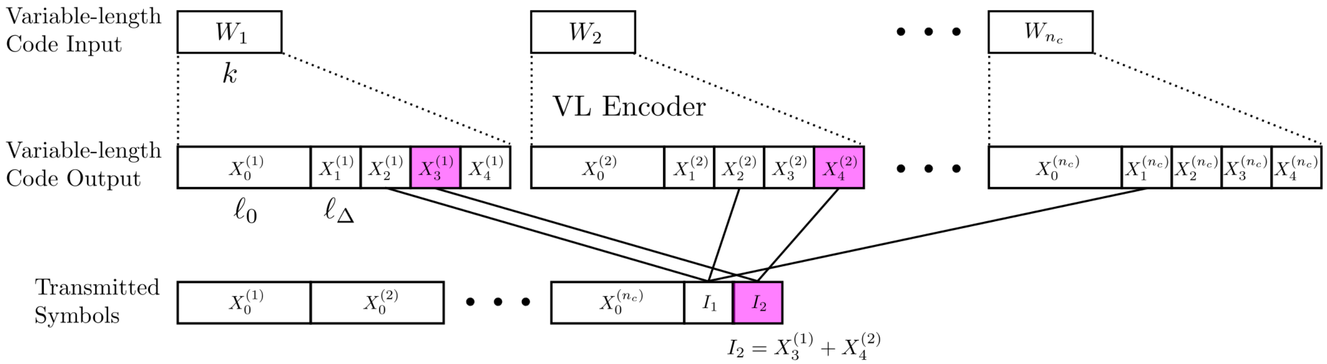
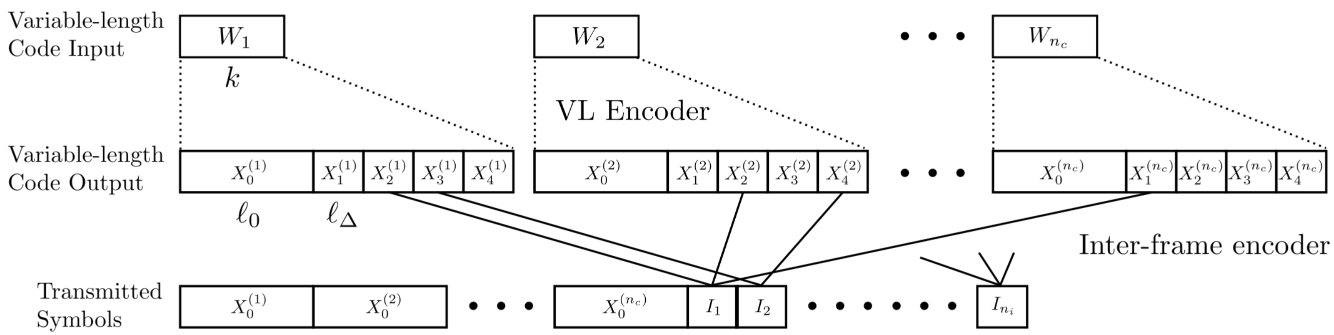
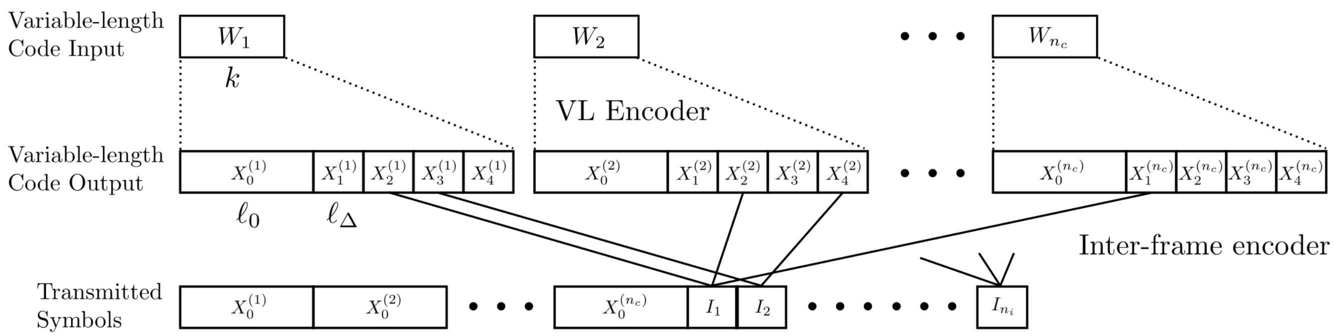


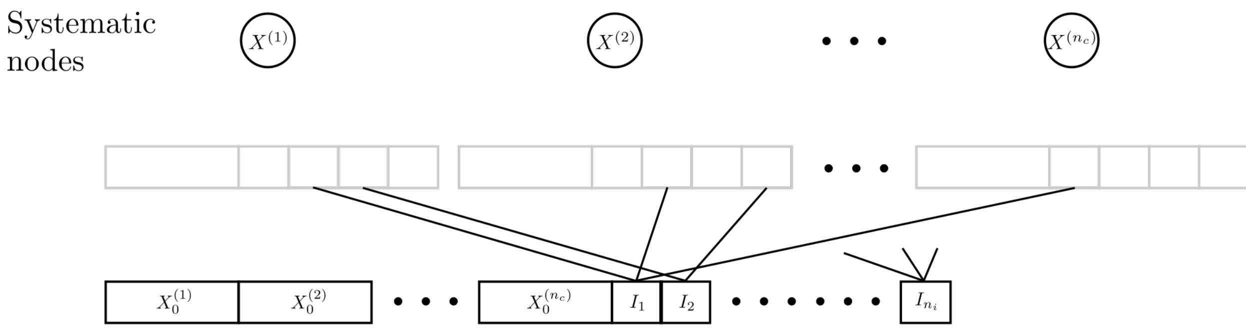
# Inter-frame Coding [Zeineddine & Mansour]



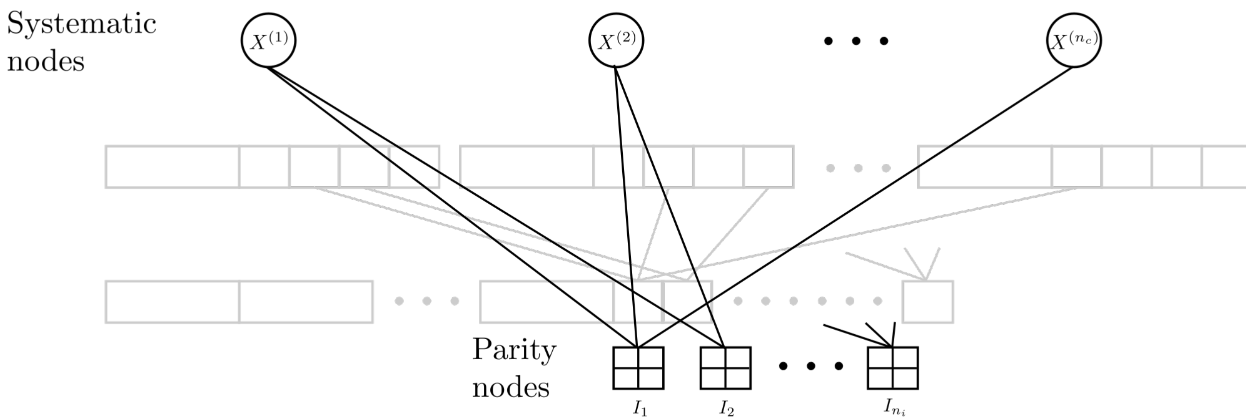
# Inter-frame Coding (IFC) [Zeineddine & Mansour]



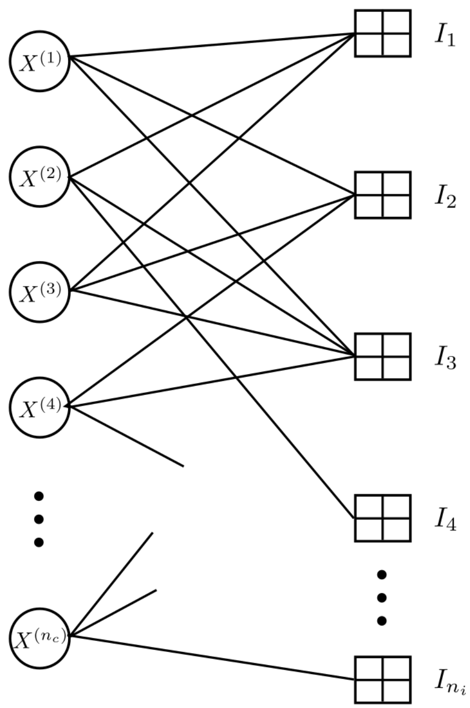
# IFC as a Low-Density Generator Matrix



# IFC as a Low-Density Generator Matrix



# Generalized Peeling Decoder for IFC



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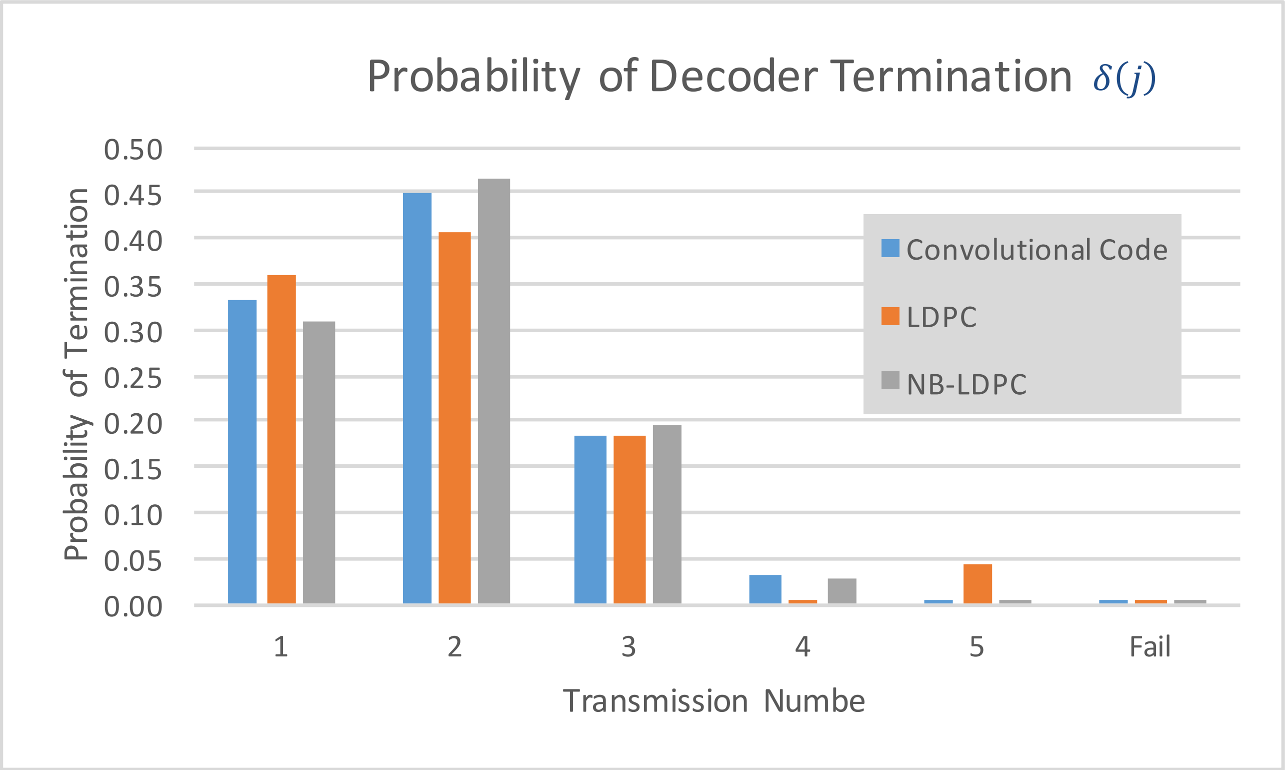
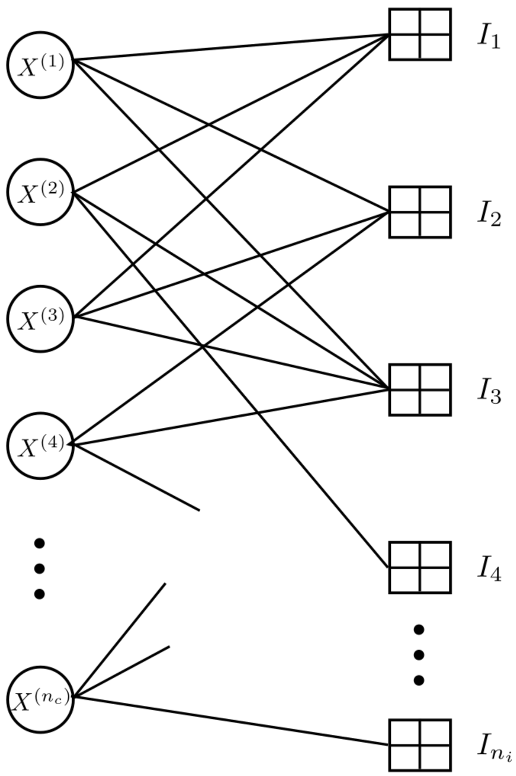
# Probability of Termination for our codes



# What degree distributions allow GPD to converge?

$$\lambda(x) = \sum_{i=1}^{d_L} \lambda_i x^{i-1}$$

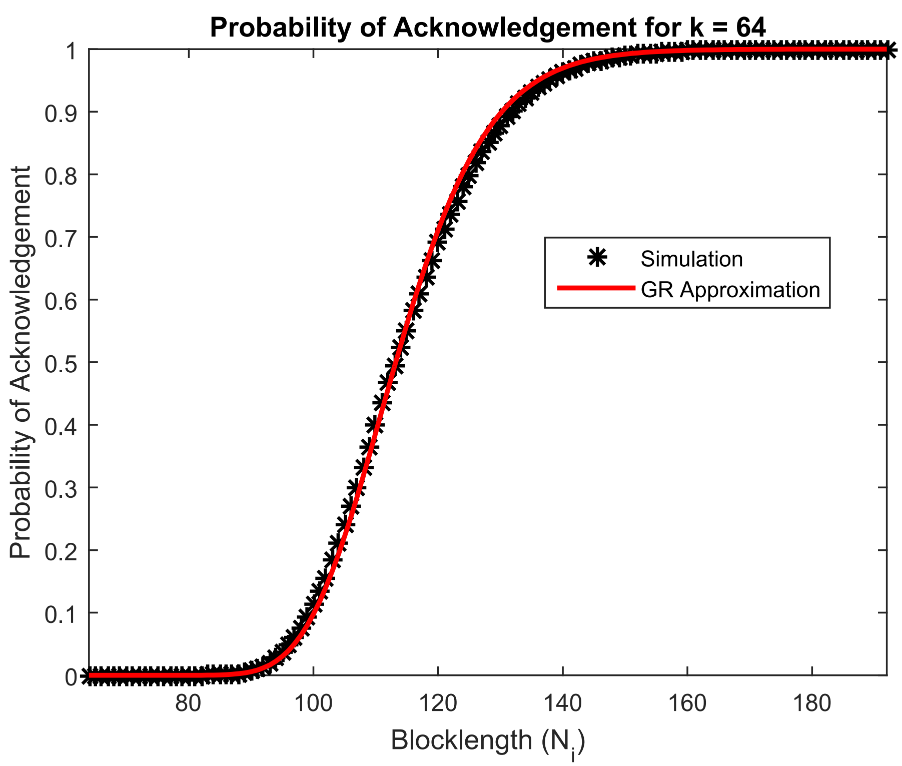



$$\rho(x) = \sum_{i=1}^{d_R} \rho_i x^{i-1}$$

# What degree distributions allow GPD to converge?

- Following [Luby *et al.* Trans. IT 2001] and assuming a *geometric* PMF for $\delta(j)$ [Zeineddine & Mansour] found a sequence of $\lambda(x)$ and $\rho(x)$ distributions as parameters $d \rightarrow \infty$ and $j \rightarrow \infty$.
- But these distributions have unbounded support and anyway $\delta(j)$ turns out to follow a Gaussian model.

# Rate of first decoding is well-modeled by Gaussian.



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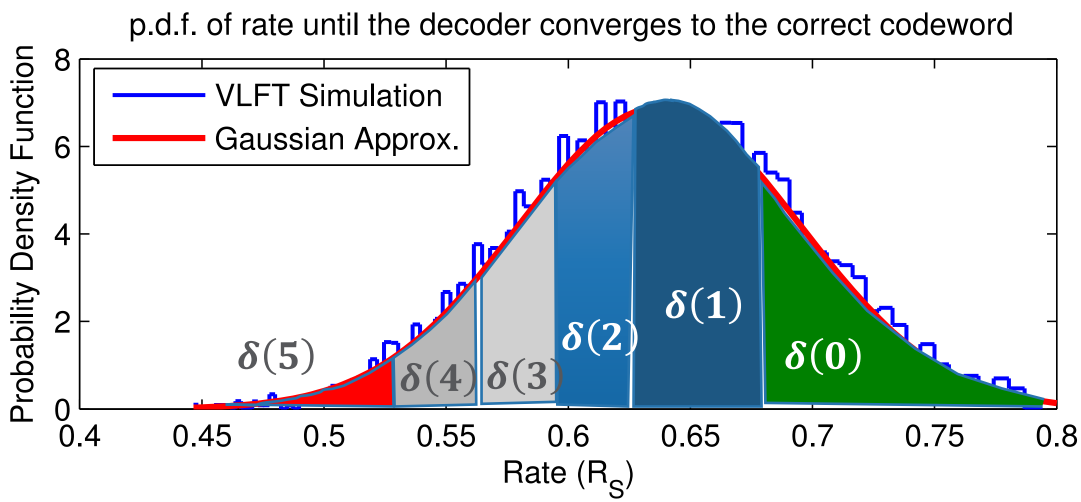
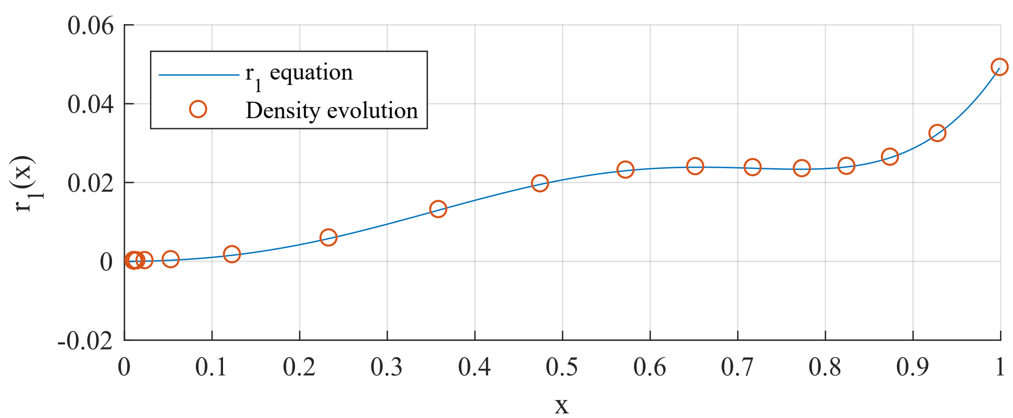
# $\delta(j)$ PMF as slices from the Gaussian



# Generalizing Luby's equation

$$\ell(x) = \sum_{\omega=1}^{m} \delta_\omega \sum_{i=1}^{d_L} \lambda_i \sum_{j=0}^{\min(\omega,i)-1} \binom{i-1}{j} (1-x)^j x^{i-1-j} .$$

$$r_1(x) = \ell(x)[\rho(1-\ell(x)) - (1-x)] ,$$



# The probability of VL code not decoding

$$\epsilon_{GPD}(i,\omega) = \sum_{j=0}^{\min(\omega-1,i)} \binom{i}{j} (1-x_0)^j {x_0}^{i-j} \,. \qquad (12)$$

As a result, the probability of failure can be calculated as

$$\epsilon_{GPD} = \sum_{\omega=1}^{m} \delta_\omega \sum_{i=1}^{d_L} \Lambda_i \sum_{j=0}^{\min(\omega-1,i)} \binom{i}{j} (1-x_0)^j {x_0}^{i-j} \qquad (13)$$

$$= \sum_{i=1}^{d_L} \Lambda_i \sum_{j=0}^{\min(m-1,i)} \gamma_j \binom{i}{j} (1-x_0)^j {x_0}^{i-j} \,, \qquad (14)$$

where $\gamma_j = \sum_{j+1}^{m} \delta_\omega$ as in $\ell(x)$, and $\Lambda_i = \frac{\lambda_i/i}{\sum_{j=1}^{d_L} \lambda_i/i}$ is the left *node* degree distribution.

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# Differential Evolution

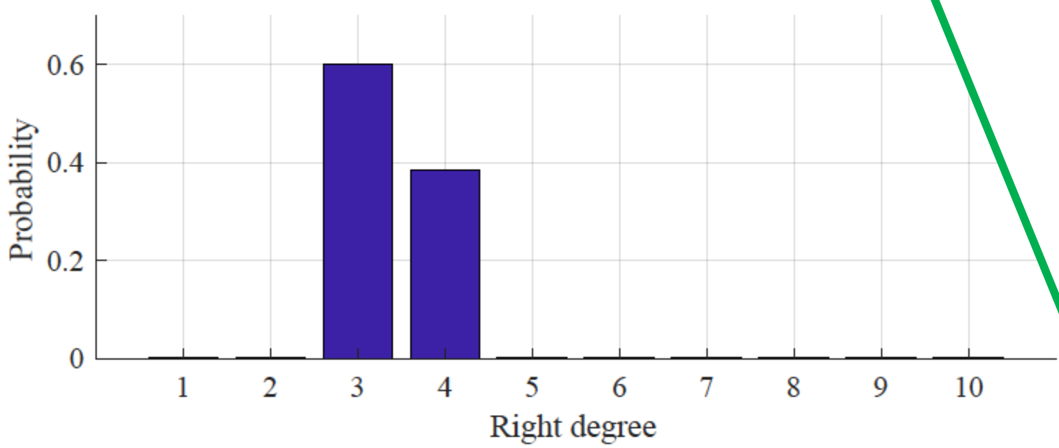
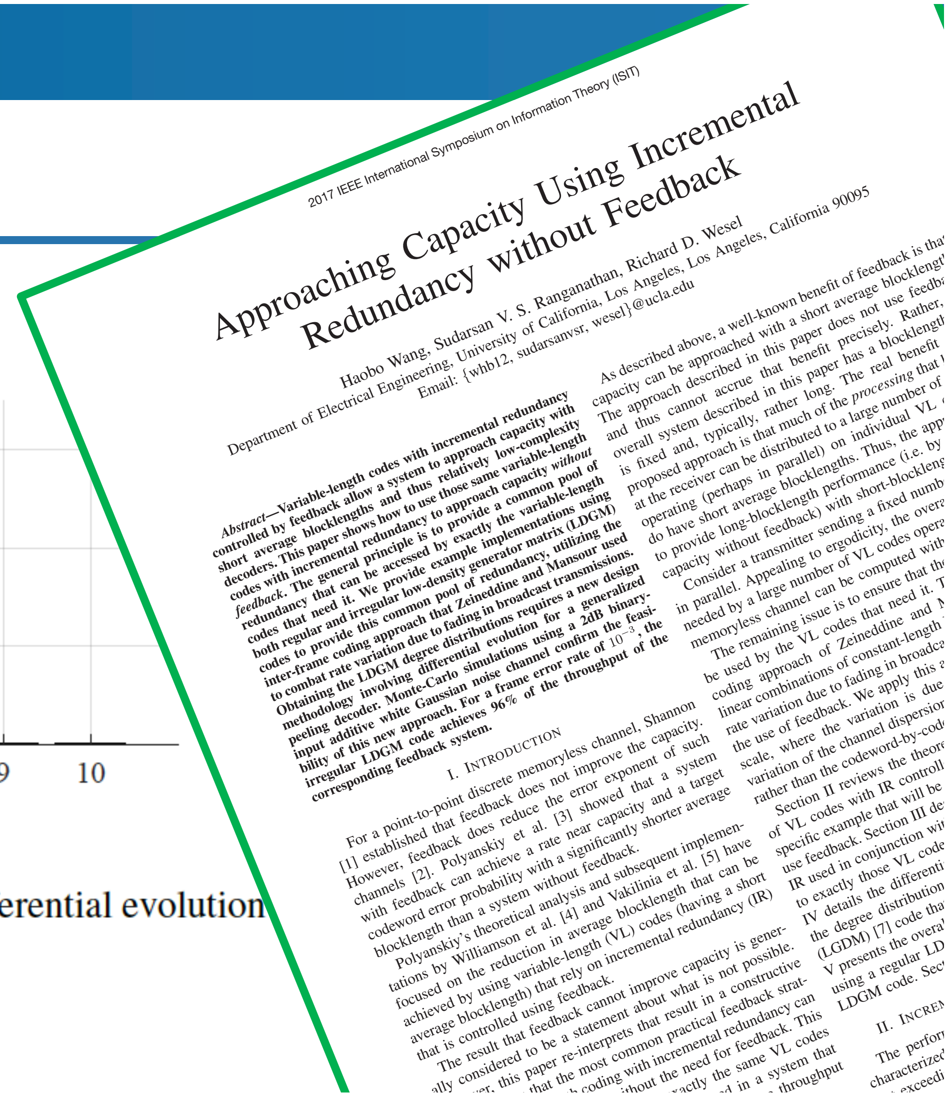



Fig. 5. Example right degree distribution $\rho_{DE}(x)$ from differential evolution as described in Sec. V-A1.



2017 IEEE International Symposium on Information Theory (ISIT)

## Approaching Capacity Using Incremental Redundancy without Feedback

Haobo Wang, Sudarsan V. S. Ranganathan, Richard D. Wesel
Department of Electrical Engineering, University of California, Los Angeles, Los Angeles, California 90095
Email: {whb12, sudarsanvsr, wesel}@ucla.edu

***Abstract*—Variable-length codes with incremental redundancy controlled by feedback allow a system to approach capacity with short average blocklengths and thus relatively low-complexity decoders. This paper shows how to use those same variable-length codes with incremental redundancy to approach capacity *without feedback*. The general principle is to provide a common pool of redundancy that can be accessed by exactly the variable-length codes that need it. We provide example implementations using both regular and irregular low-density generator matrix (LDGM) codes to provide this common pool of redundancy, utilizing the inter-frame coding approach that Zeineddine and Mansour used to combat rate variation due to fading in broadcast transmissions. Obtaining the LDGM degree distributions requires a new design methodology involving differential evolution for a generalized peeling decoder. Monte-Carlo simulations using a 2dB binary-input additive white Gaussian noise channel confirm the feasibility of this new approach. For a frame error rate of $10^{-3}$, the irregular LDGM code achieves 96% of the throughput of the corresponding feedback system.**

### I. Introduction

For a point-to-point discrete memoryless channel, Shannon [1] established that feedback does not improve the capacity. However, feedback does reduce the error exponent of such channels [2]. Polyanskiy et al. [3] showed that a system with feedback can achieve a rate near capacity and a target codeword error probability with a significantly shorter average blocklength than a system without feedback.

Polyanskiy's theoretical analysis and subsequent implementations by Williamson et al. [4] and Vakilinia et al. [5] have focused on the reduction in average blocklength that can be achieved by using variable-length (VL) codes (having a short average blocklength) that rely on incremental redundancy (IR) that is controlled using feedback.

# Great performance with almost-regular $\rho(x)$

TABLE II
PERFORMANCE CHARACTERISTICS OF CONCENTRATED $\rho(x) = \alpha x^2 + (1-\alpha)x^3$. $\lambda(x) = x^3$ IN ALL CASES.

| $\alpha$ | $a_R$ | $\beta$ | iter. | % $R_t^{(FB)}$ | $\epsilon_{FF}$ |
|---|---|---|---|---|---|
| 1 | 3 | 1.33333 | 15 | 93.52% | $7.09\times10^{-4}$ |
| 0.531 | 3.39847 | 1.17700 | 20 | 95.36% | $7.82\times10^{-4}$ |
| 0.244 | 3.69914 | 1.08133 | 30 | 96.52% | $8.35\times10^{-4}$ |
| 0.168 | 3.78788 | 1.05600 | 40 | 96.83% | $8.50\times10^{-4}$ |
| 0.139 | 3.82287 | 1.04633 | 50 | 96.94% | $8.56\times10^{-4}$ |
| 0.108 | 3.86100 | 1.03600 | 100 | 97.08% | $8.63\times10^{-4}$ |


Channel Code Analysis and Design using Multiple Variable-Length Codes in Parallel without Feedbac

Haobo Wang and Richard D. Wesel
UCLA Communications Systems Laboratory, Electrical and Computer Engineering Department
University of California, Los Angeles (UCLA), Los Angeles, CA 90095, USA
Email: {whb12, wesel}@ucla.edu

***Abstract*—This paper considers a channel coding paradigm that enables high throughput by using many variable-length codes in parallel, where each of the parallel codes has a short average blocklength. The inter-frame coding of Zeineddine and Mansour provides variable-length codes with incremental redundancy from a common pool of redundancy in a way that does not require feedback. A probability-based derivation of a generalized peeling decoder extends the results of Luby *et al.* to the inter-frame scenario. A new expression characterizes the probability that a variable-length decoder in the inter-frame system will fail. Additionally, the three causes for throughput loss as compared to the original feedback system are identified, yielding a new, and far simpler, quasi-regular design methodology for the right degree distribution of the inter-frame code. The inter-frame paradigm can apply to any communication channel, but this paper uses the additive white Gaussian noise channel to demonstrate the concepts.**

I. INTRODUCTION

Practical systems and theoretical analysis [1]–[3] show that using a variable-length (VL) code with incremental transmissions controlled by ACK/NACK feedback can approach capacity with short average blocklengths on the order of 200-500 symbols. This paper studies the analysis and design of systems that use many variable-length codes in parallel and *without feedback* to approach capacity for point-to-point communication.

# Great performance with almost-regular $\rho(x)$

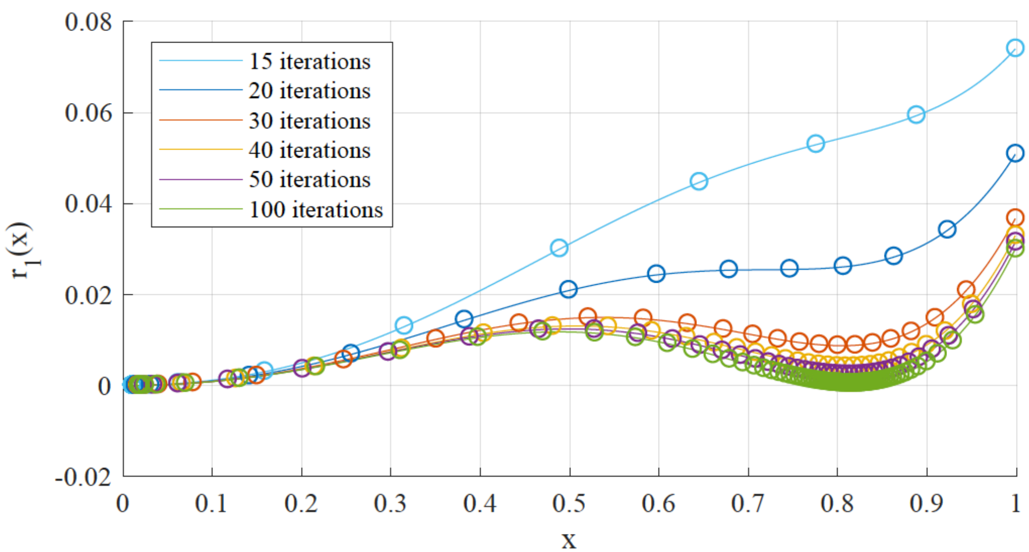



Fig. 5. $r_1(x)$ vs. $x$ for Table II. The curves are generated using (10). Circles indicate iteration points determined through density evolution. The circles at $x = 1$ represent the first iteration.

Haobo Wang

# Simulations compared to Density Evolution

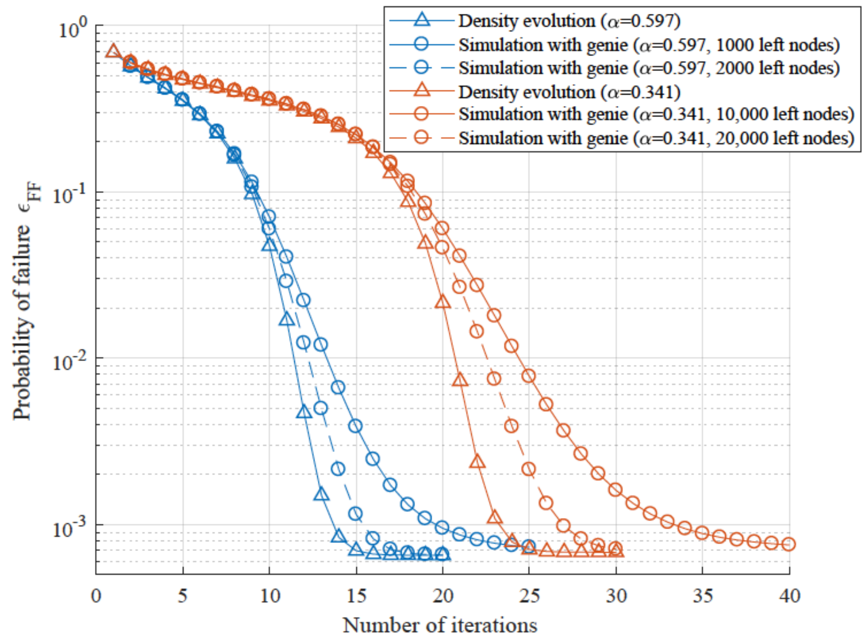



Fig. 6. Probability of failure vs. the number of iterations for the designs in Table III from density evolution and genie-aided simulations.

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# Rate loss due to extra right nodes

$$R_t^{(FB)} = \frac{k(1-\epsilon_{FB})}{\ell_0 + \beta_{FB}\ell_\Delta},$$

$$R_t^{(FF)} = \frac{k(1-\epsilon_{FF})}{\ell_0 + \beta_{FF}\ell_\Delta},$$

$$\beta_{FF} = \frac{d_L}{a_r} = \frac{4}{3+(1-\alpha)}$$

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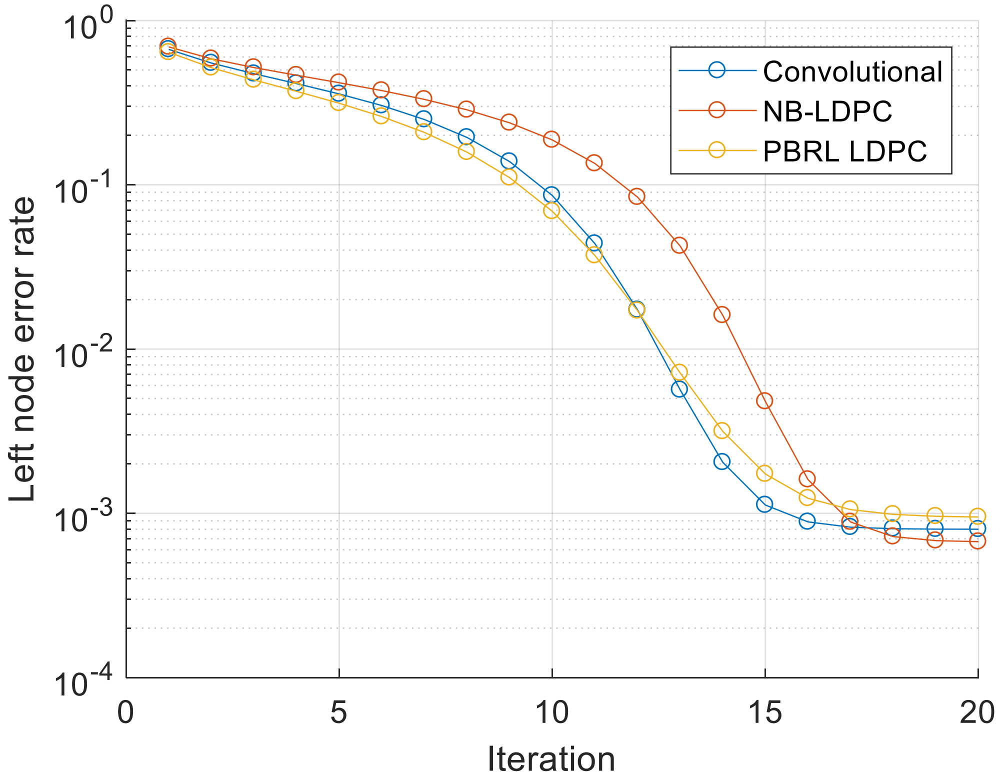
# Density Evolution for Peeling Decoders




| | Convolutional Code | LDPC | NB-LDPC |
|---|---|---|---|
| alpha | 0.432 | 0.43 | 0.466 |
| ar | 3.568 | 3.57 | 3.534 |
| Bff | 1.12 | 1.12 | 1.13 |
| Rff | 0.507681527 | 0.550012498 | 0.559619731 |
| Percent of FB | 97.42% | 98.57% | 98.04% |

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# Engineer Change.