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Spatial Coupling - Essential Technology for High Throughput Coding?

Laurent Schmalen

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2019 Oberpfaffenhofen Workshop on High Throughput Coding (OWHTC)

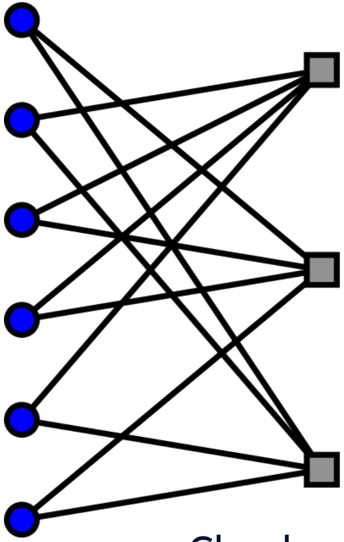
Outline

1. Spatially coupled (SC) LDPC Codes
2. Non-uniformly coupled SC-LDPC codes
3. Problems with windowed decoding of SC-LDPC codes (and first solutions)
4. Conclusions and outlook

Spatially Coupled LDPC Code Ensemble

Start with LDPC Code

M variable nodes
degree $d_v = 2$



Check nodes
degree d_c

- We start with a regular LDPC code
  - Variable node (code bits) degree d_v
  - Check node (constraints) degree d_c
- Total number of M variable nodes (code bits)

Spatially Coupled LDPC Code Ensemble

Start with LDPC Code

M variable nodes
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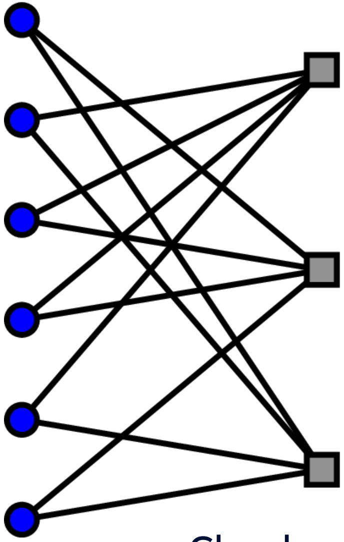
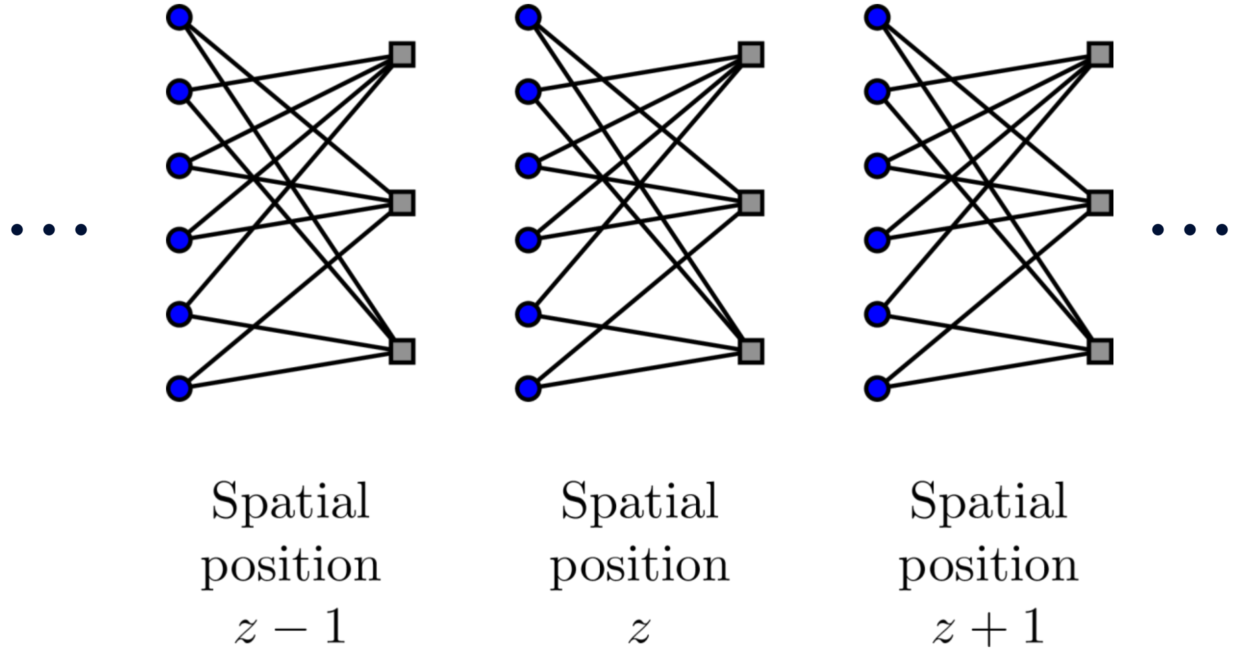


Check nodes
degree d_c

- We start with a regular LDPC code
  - Variable node (code bits) degree d_v
  - Check node (constraints) degree d_c
- Total number of M variable nodes (code bits)
- **Spatially coupled code**: replicate L copies of this code along a new, spatial dimension
- L denotes the *replication factor* of the code

Spatially Coupled LDPC Code Ensemble

L Disjoint LDPC Codes



[KRU11] S. Kudekar, T. Richardson, R. Urbanke, “Threshold saturation via spatial coupling: Why convolutional LDPC ensembles perform so well over the BEC,” *IEEE Trans. Inf. Theory*, 2011

Spatially Coupled LDPC Code Ensemble

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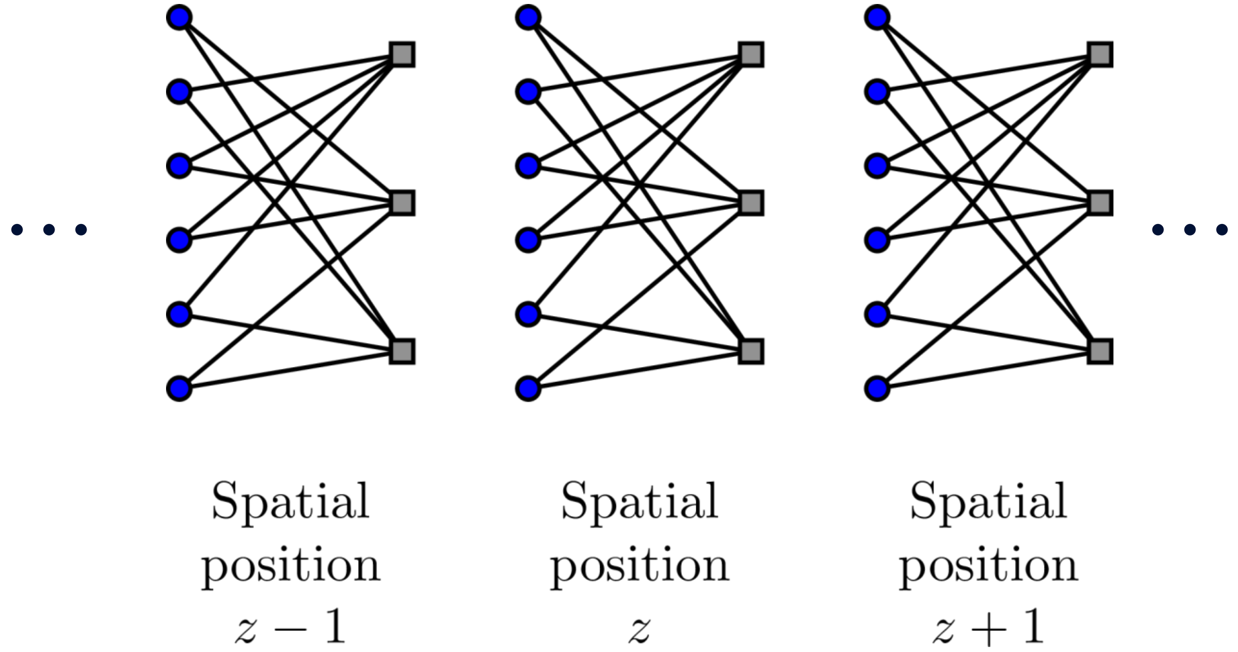

Spatial coupling: connect **uniformly at random** each edge from variable node at SP z to check node at position $\{z, z+1, \ldots, z+w-1\}$

w: coupling factor

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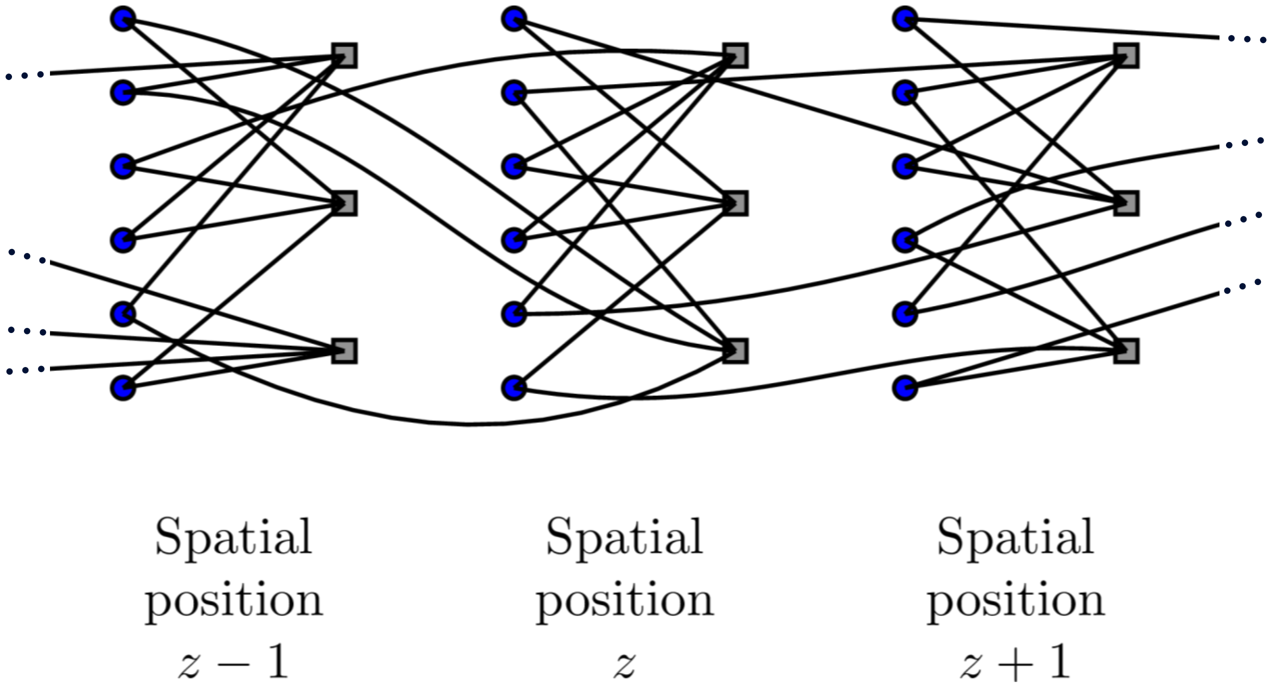
Spatially Coupled LDPC Code Ensemble

Spatially Coupled LDPC Code with $w = 2$



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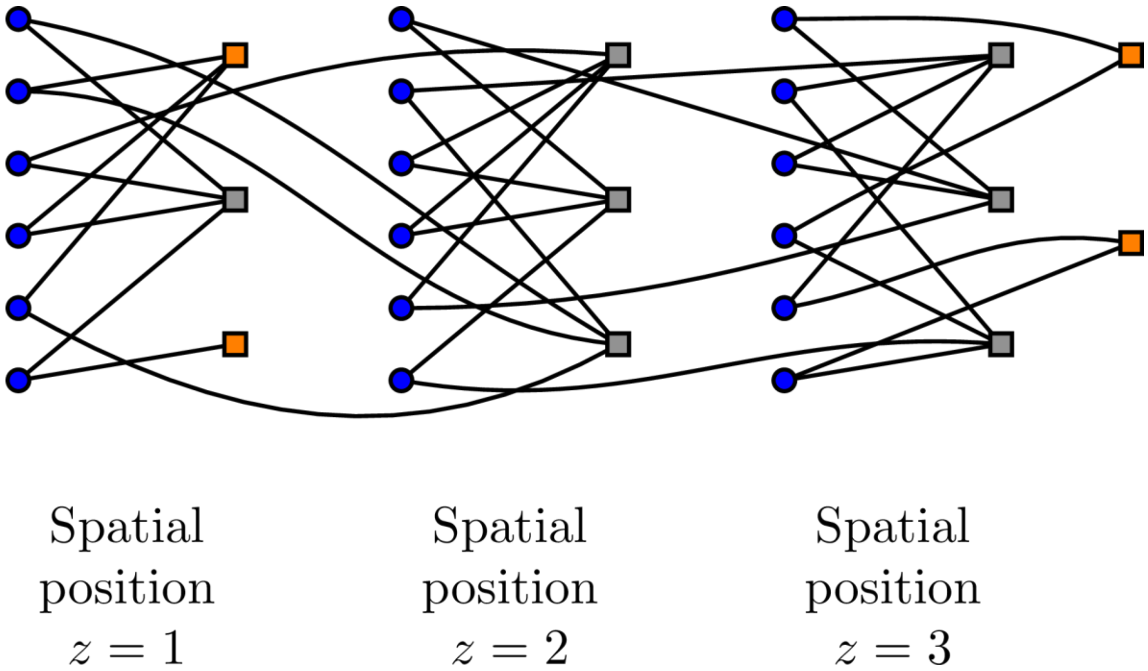
Spatially Coupled LDPC Code Ensemble

Terminated Spatially Coupled LDPC Code with $w = 2$ and $L = 3$



- Two extra check nodes lead to rate loss (**negligible if L large enough**)
- Check nodes at boundary have lower degree, hence better correction capabilities

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Spatially Coupled LDPC Codes are Capacity-Achieving

- Under some conditions, SC-LDPC codes are capacity-achieving **[KRU11]**, in particular, for the decoding threshold on the binary erasure channel (BEC),

$$\lim_{w\to\infty}\lim_{L\to\infty}\varepsilon_{\mathrm{BP}}(d_v,d_c,L,W)=\lim_{w\to\infty}\lim_{L\to\infty}\varepsilon_{\mathrm{MAP}}(d_v,d_c,L,W)=\varepsilon_{\mathrm{MAP,uncoupl.}}(d_v,d_c)$$

[OU15] P. Olmos, R. Urbanke, “A scaling law to predict the finite-length performance of spatially-coupled LDPC codes,” *IEEE Trans. Inf. Theory*, 2015

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- Rate of the SC-LDPC code ensemble: $R=\left(1-\frac{d_v}{d_c}\right)-O(w/L)$

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Practical code constructions:

- Keep L small, as large L can worsen finite length performance **[OU15]**
- For small, fixed L, keep w small to keep rate loss and decoder complexity small
- Performance for small w not necessarily good
- Modified, generalized ensemble for small w required

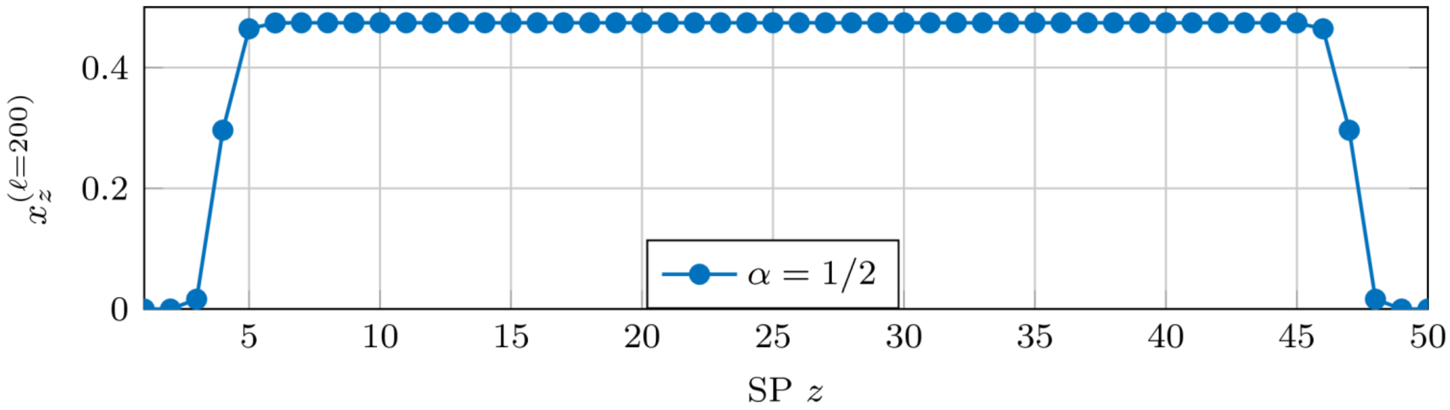
[OU15] P. Olmos, R. Urbanke, “A scaling law to predict the finite-length performance of spatially-coupled LDPC codes,” *IEEE Trans. Inf. Theory*, 2015

Density Evolution for $\varepsilon = 0.48,\ L = 50$

Conventional Spatially Coupled LDPC Code

$d_v = 5,\ d_c = 10$

$I = 200$ iter.



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Density Evolution for $\varepsilon = 0.48,\ L = 50$

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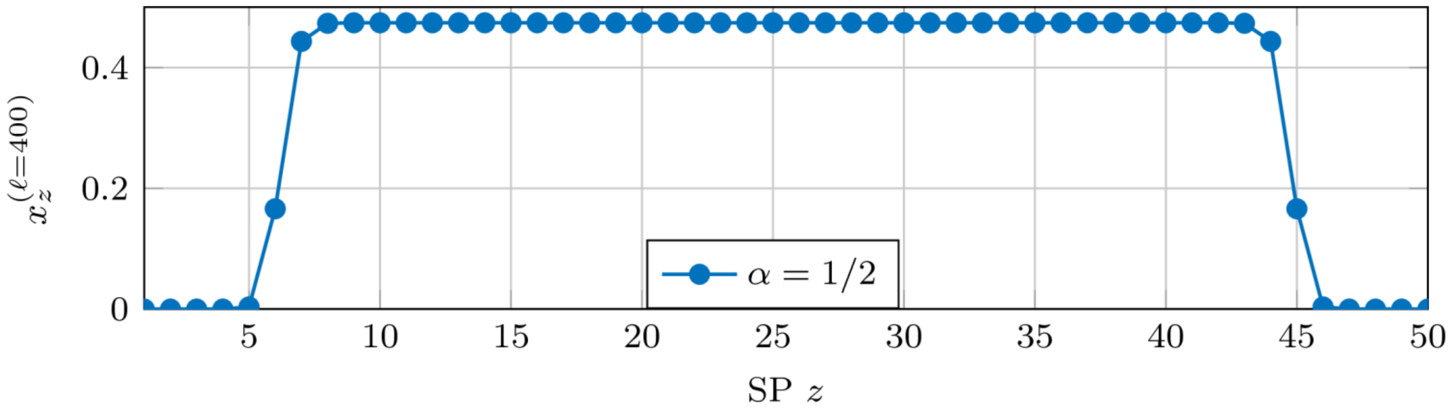
$d_v = 5,\ d_c = 10$

$I = 400$ iter.



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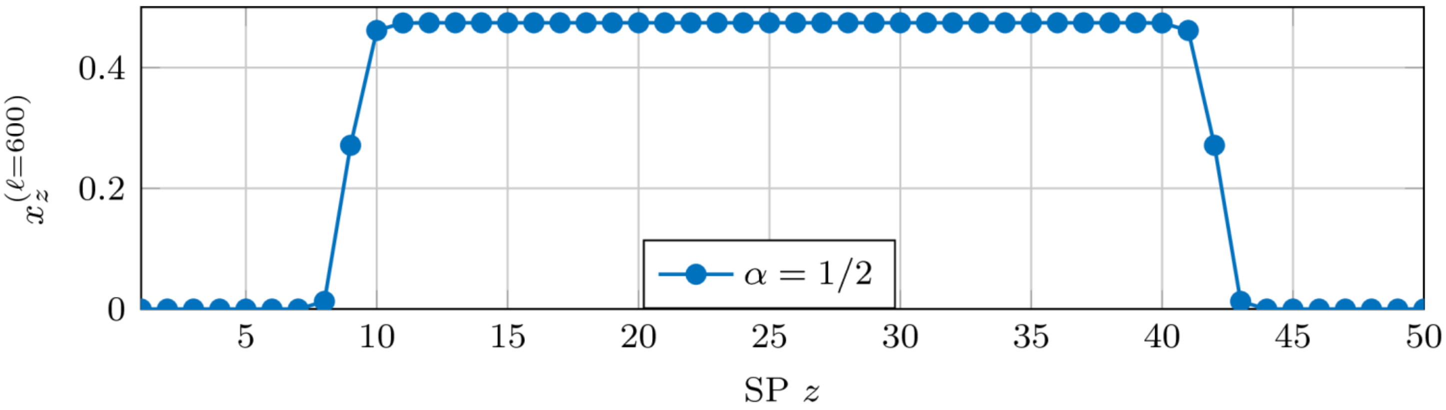
$I = 600$ iter.



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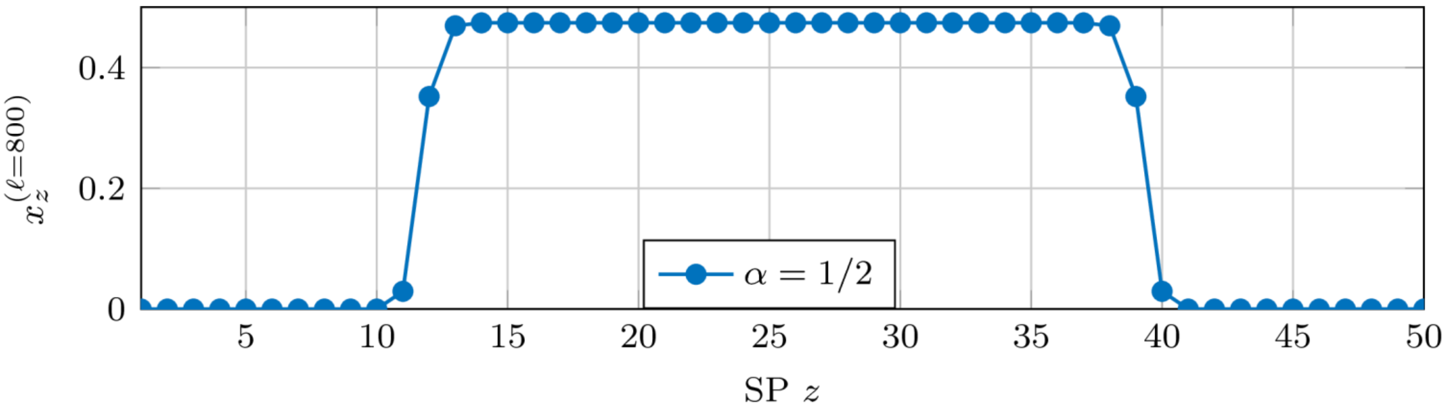
$I = 800$ iter.



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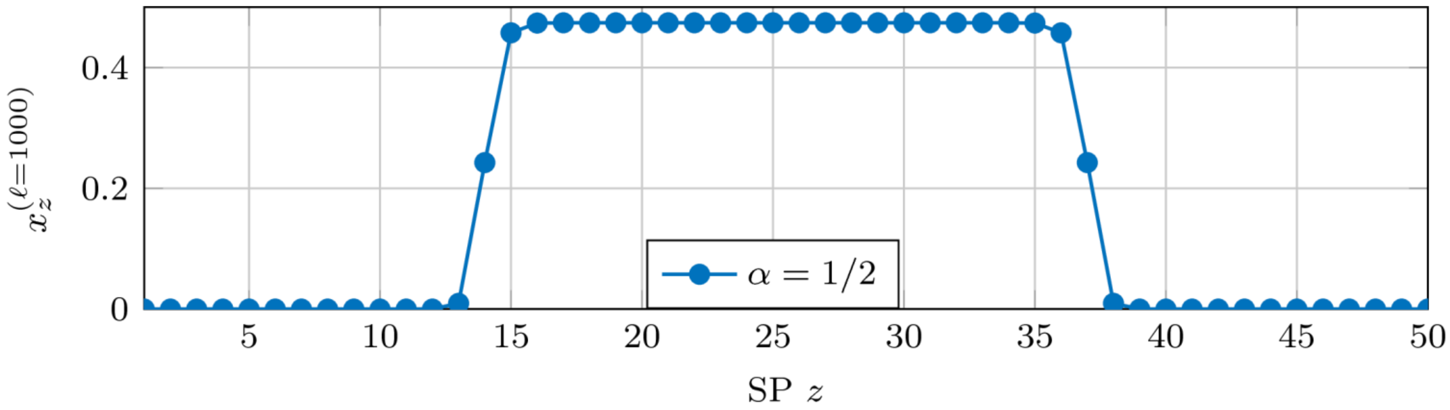

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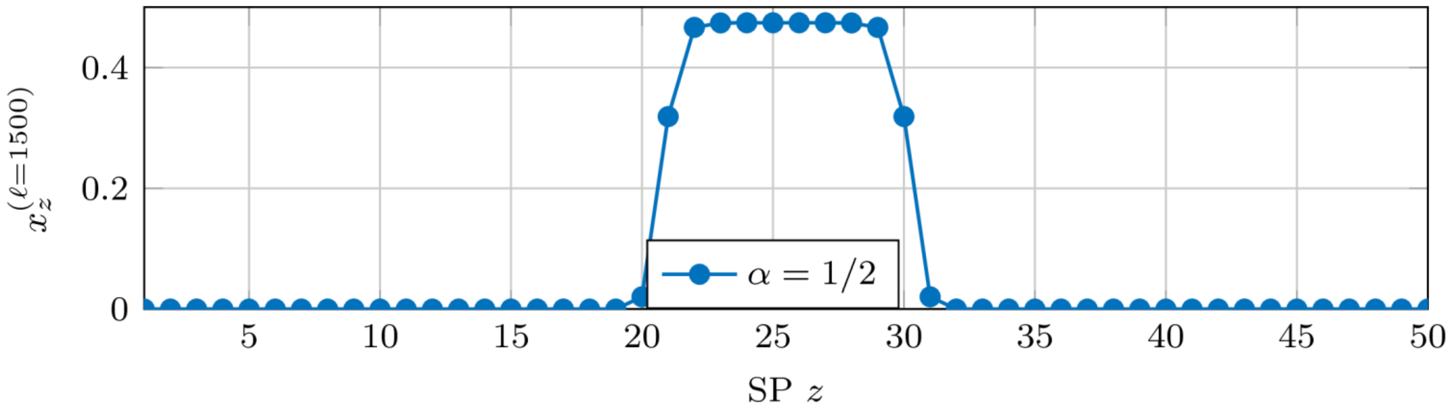

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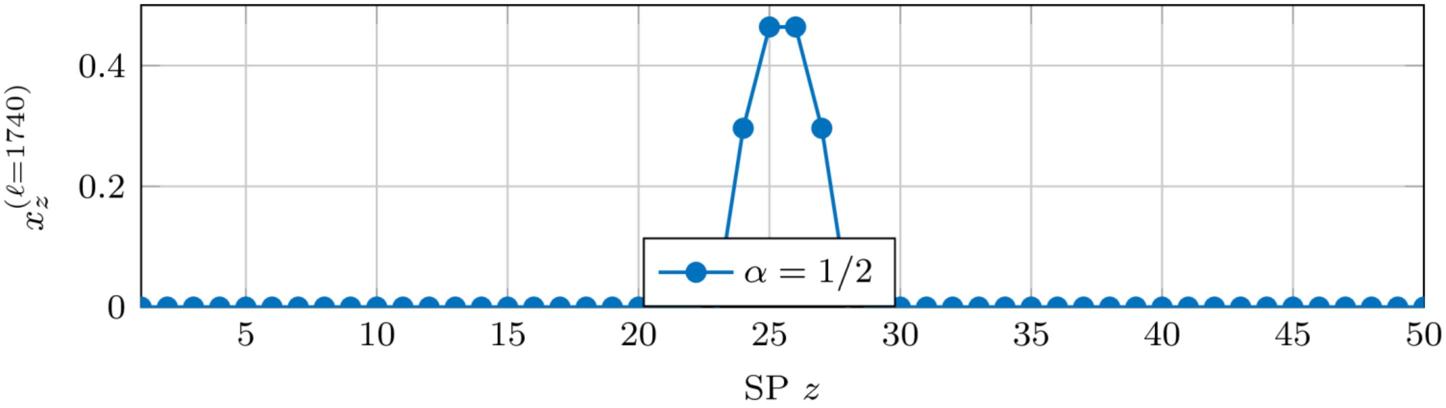
Conventional Spatially Coupled LDPC Code
$d_v = 5,\ d_c = 10$

$I = 1740$ iter.



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Windowed Decoding of Spatially Coupled LDPC Codes

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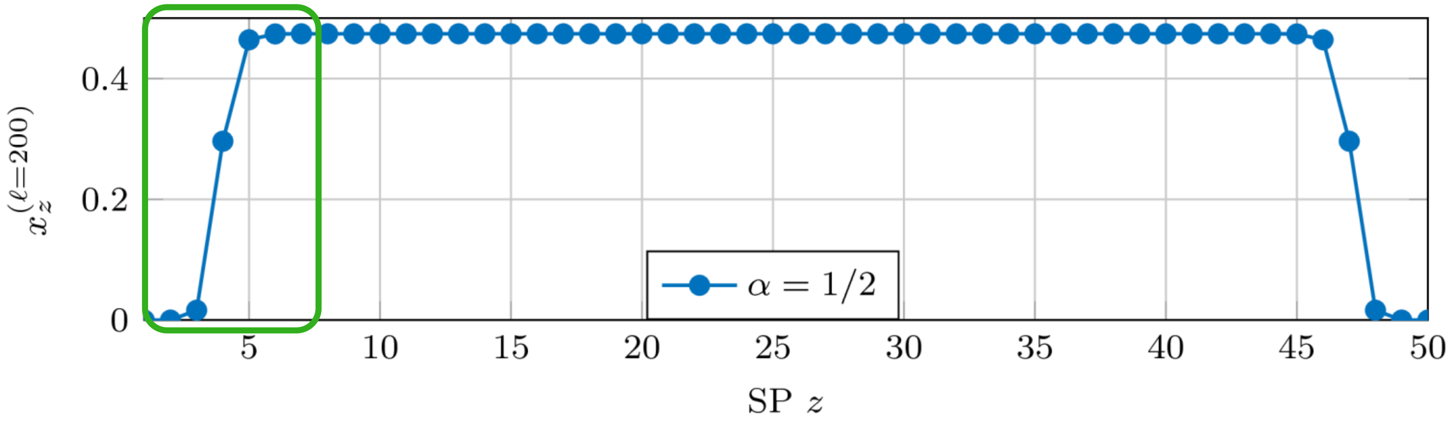

- Windowed decoding sufficient to achieve capacity [ISU+13]
- To save latency, we are only interested in **left-most portion** of wave and use windowed decoder of size W_D for this part (decode while receive)
- Window latency of order $W_\mathrm{D} + w$ ($W_\mathrm{D} + w - 1$ SPs in window)
- Decoding complexity of order $(W_\mathrm{D} + w) \cdot I$ (I: number iterations per window)

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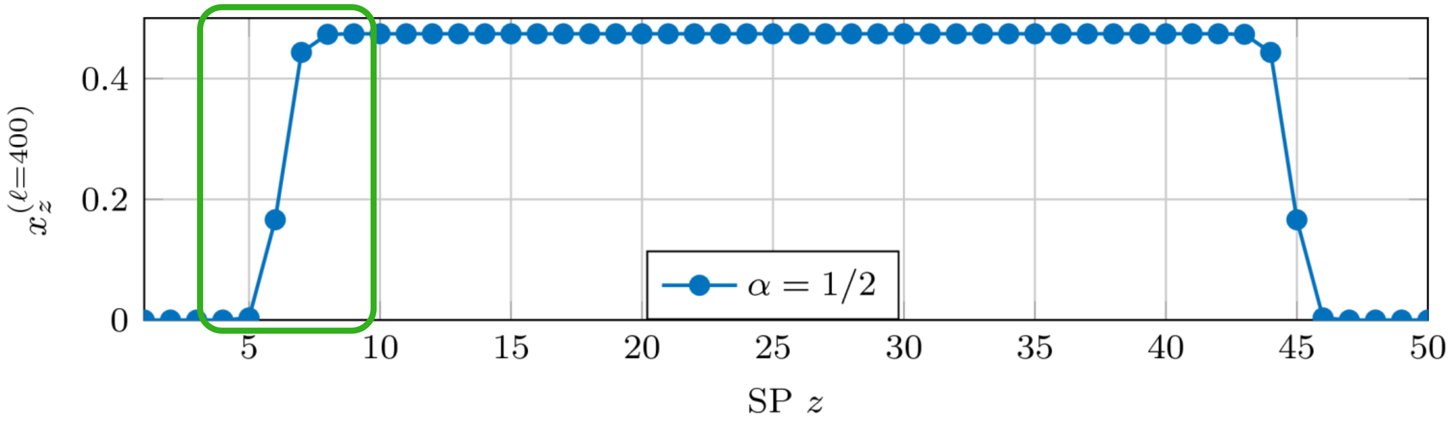

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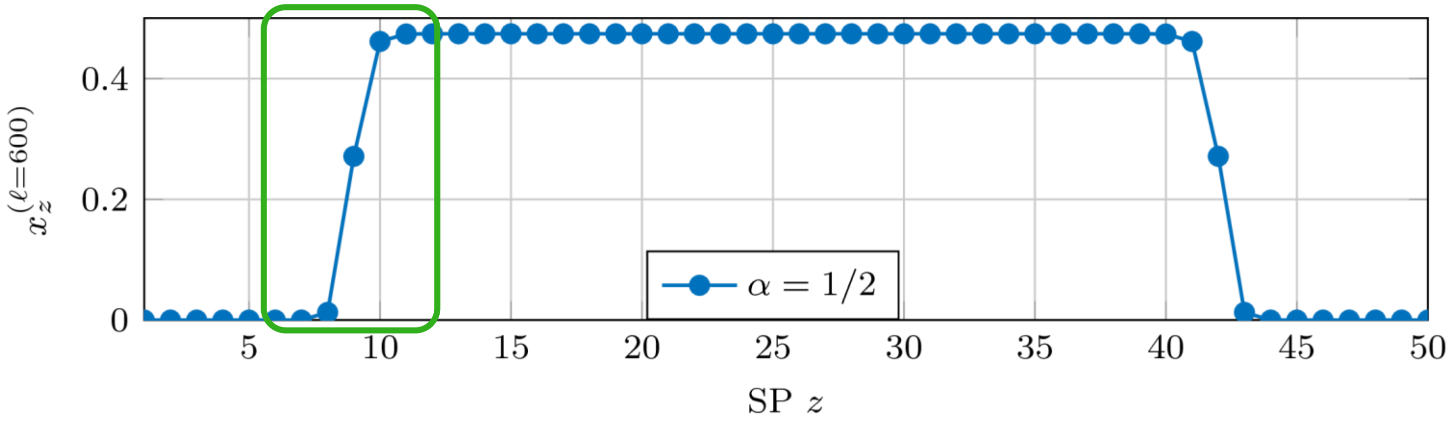

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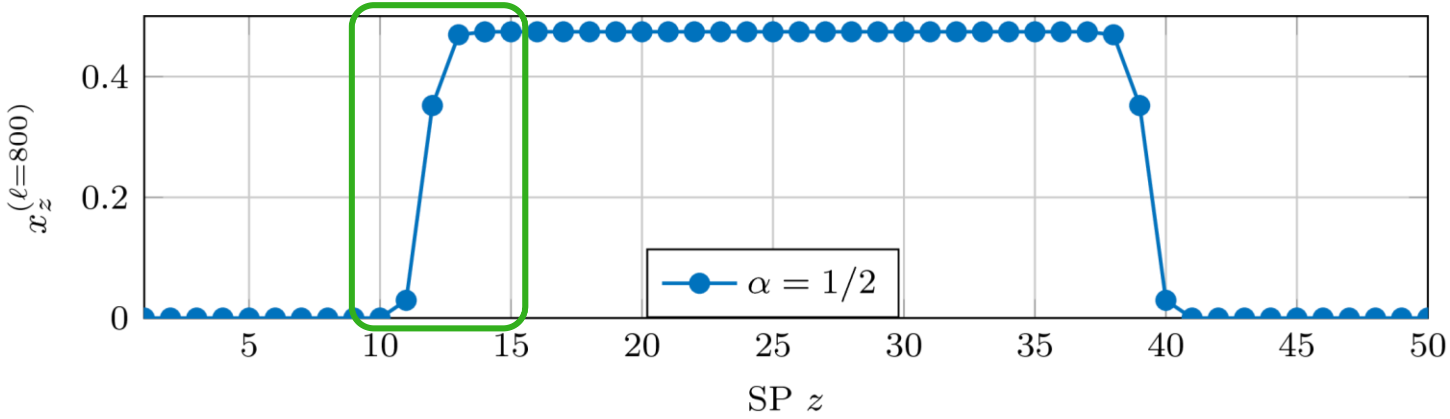

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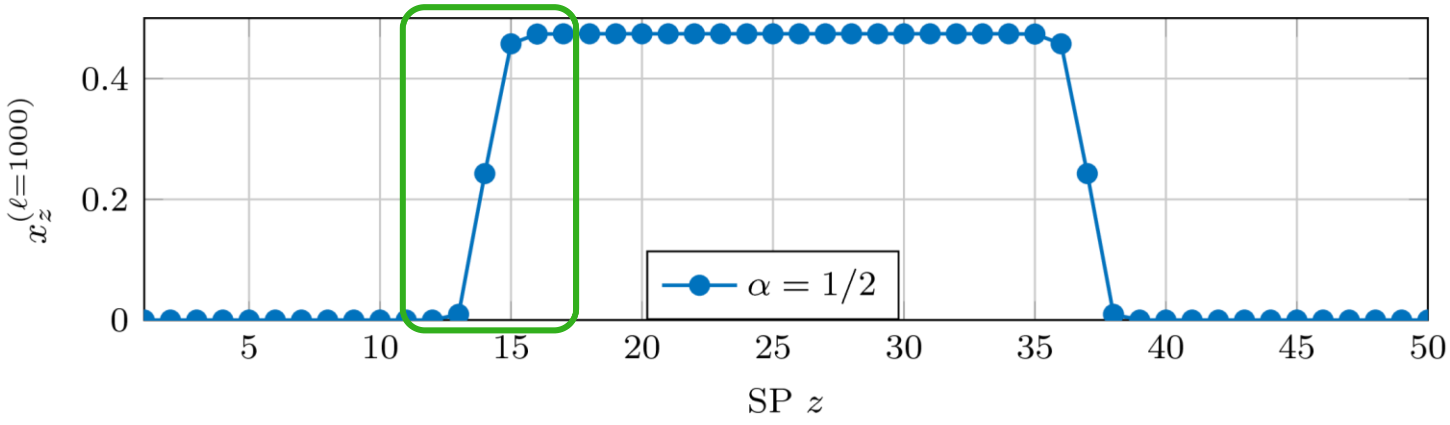

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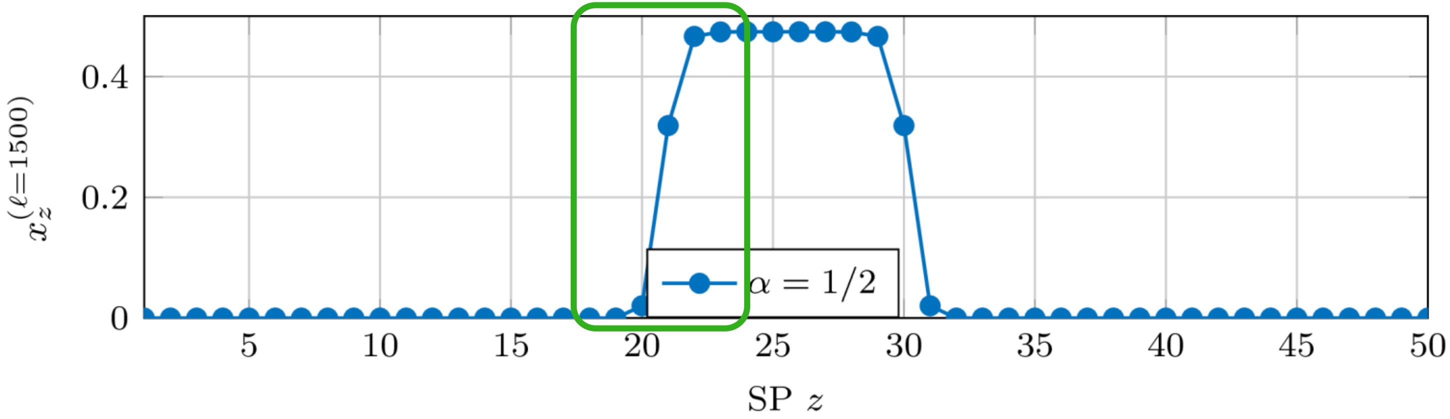

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Decoding Velocity and Windowed Decoding

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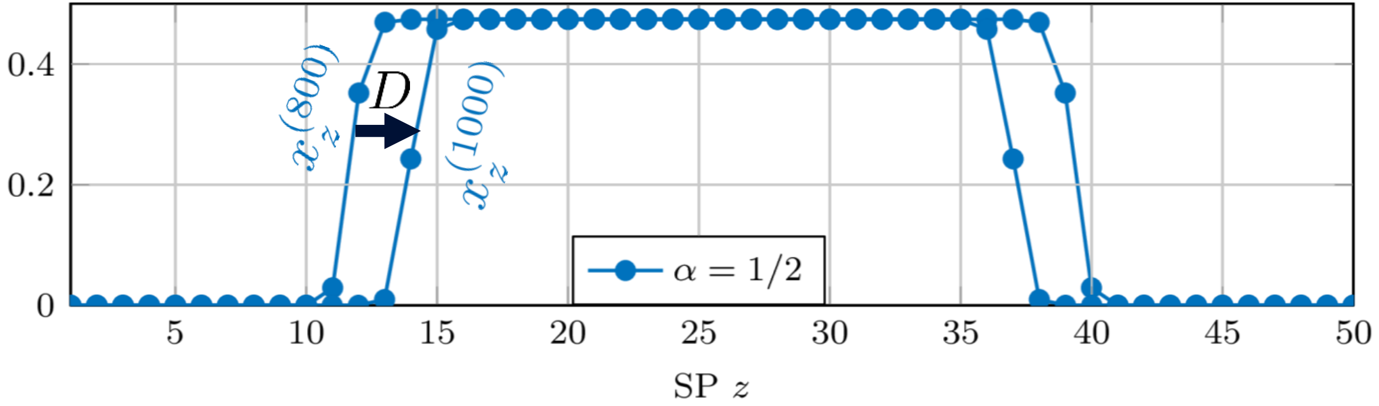

- Decoding velocity as displacement of erasure profile per decoding iteration **[AStB13], [EM16]**
- Decoding velocity v defined as D/I, where I is the number of iterations required to advance the profile by D, i.e., here $v = D/200$

[AStB13] V. Aref, L. Schmalen, S. ten Brink, "On the convergence speed of spatially coupled LDPC ensembles," *Proc. Allerton Conf.*, 2013
[EM16] R. E-Khatib, N. Macris, "The velocity of the decoding wave for spatially coupled codes on BMS channels," *Proc. ISIT*, 2016
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Decoding Velocity and Windowed Decoding

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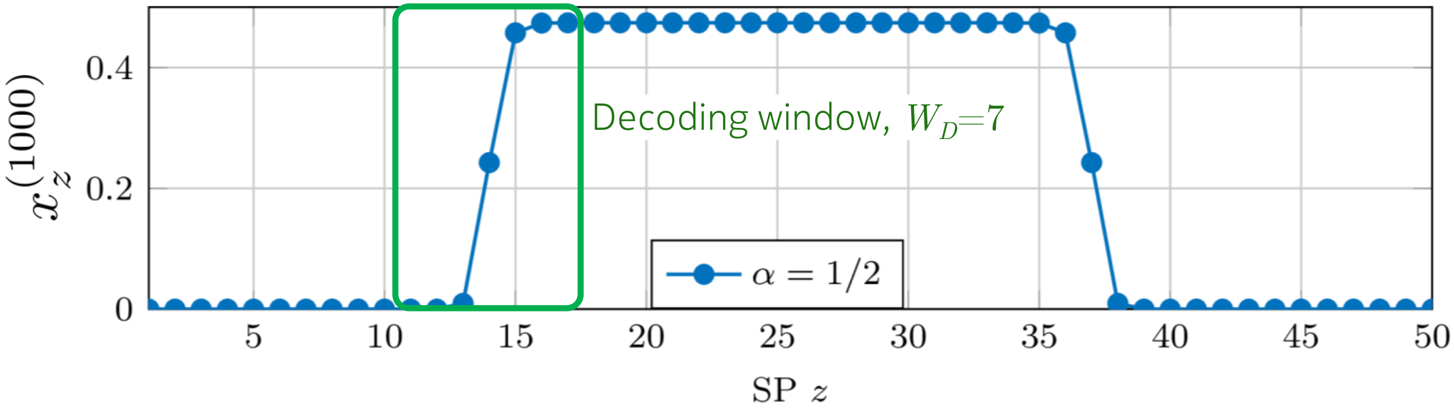

- Windowed decoding only carries out decoding operations on W_D spatial positions that benefit from decoding **[ISU+13]**
- Complexity of windowed decoding directly linked to the velocity of the profile

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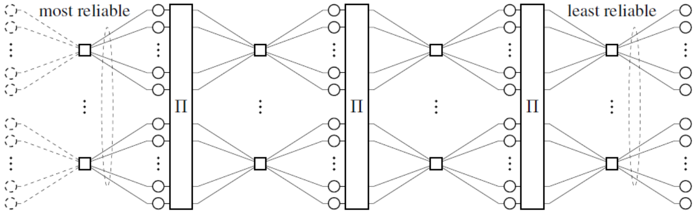
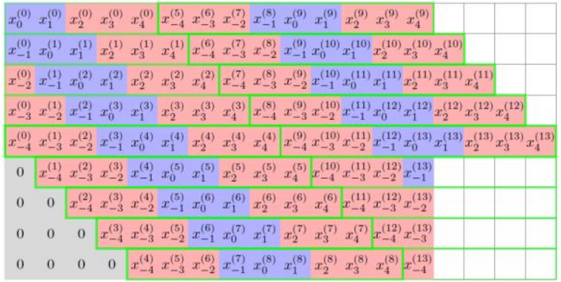
Spatially Coupled Codes for High-Throughput Comms.

- **Staircase codes** [SFH+12] now well established in low-complexity, high-throughput optical communications
- Standardized for interoperable communications
- Very good performance with hard-decision decoding
- Spatially-coupled generalized LDPC codes



- Other high-performing spatially coupled codes have been proposed as well
- Example: **Braided BCH codes** presented in [JPN+13]
- Similar performance than staircase codes
- Extra performance gains by using extrinsic decoder requiring more memory

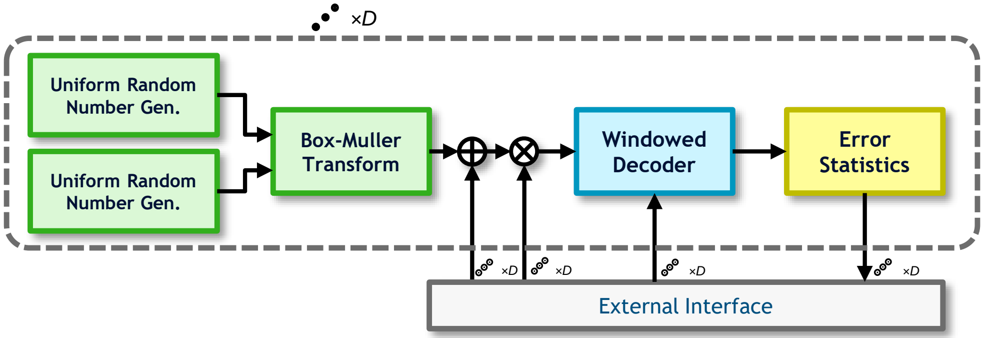


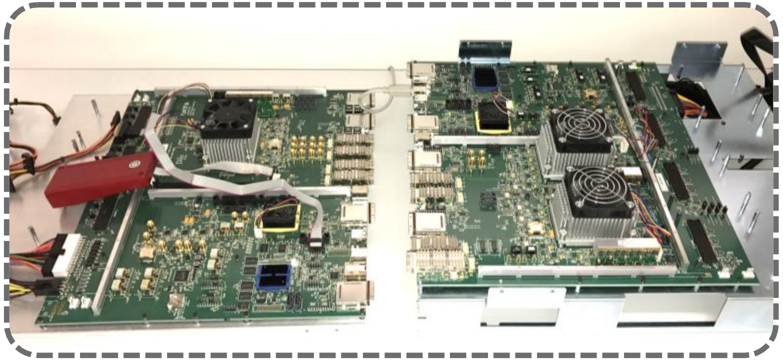
[SFH+12] B. Smith, A. Farhood, A. Hunt, F. Kschischang, J. Lodge, "Staircase Codes: FEC for 100 Gb/s OTN," *IEEE/OSA J. Lightw. Technol.*, 2012
[JPN+13] Y.-Y. Jian, H. Pfister, K. Narayanan, R. Rao, R. Mazahreh, "Iterative Hard-Decision Decoding of Braided BCH Codes for High-Speed Optical Communication," *Proc. GLOBECOM*, 2013

FPGA-Based Code Evaluation Platform

- **High throughputs & large coding gains** necessary in optical core networks & submarine cables
- **Required BER: around 0.0000000000001% (10^{-15})**
- Maximum 10 bit errors per day at line rate of 100 Gbit/s
- Requirements might become more strict in the future

Virtex-7 based, configurable FPGA emulator platform with windowed decoding





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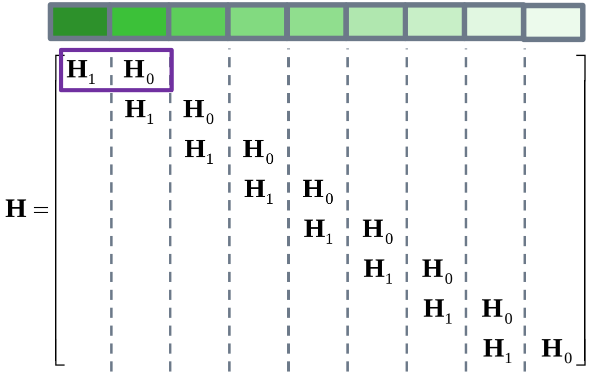
Results of FPGA-Based Code Evaluation ($R = 0.8$)

- Comparison of two different codes
  - Code A: optimized degree dist.
  - Code B: optimized for low floor

Single engine decoder, $I = 1$ iteration of layered decoder [H04]



[SAC+15] L. Schmalen, V. Aref, J. Cho, D. Suikat, D. Rösener, A. Leven “Spatially coupled soft-decision error correction for future lightwave systems,” *IEEE/OSA J. Lightw. Technol.*, 2015

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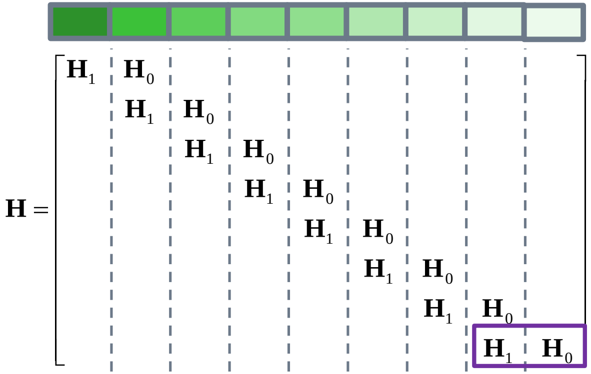

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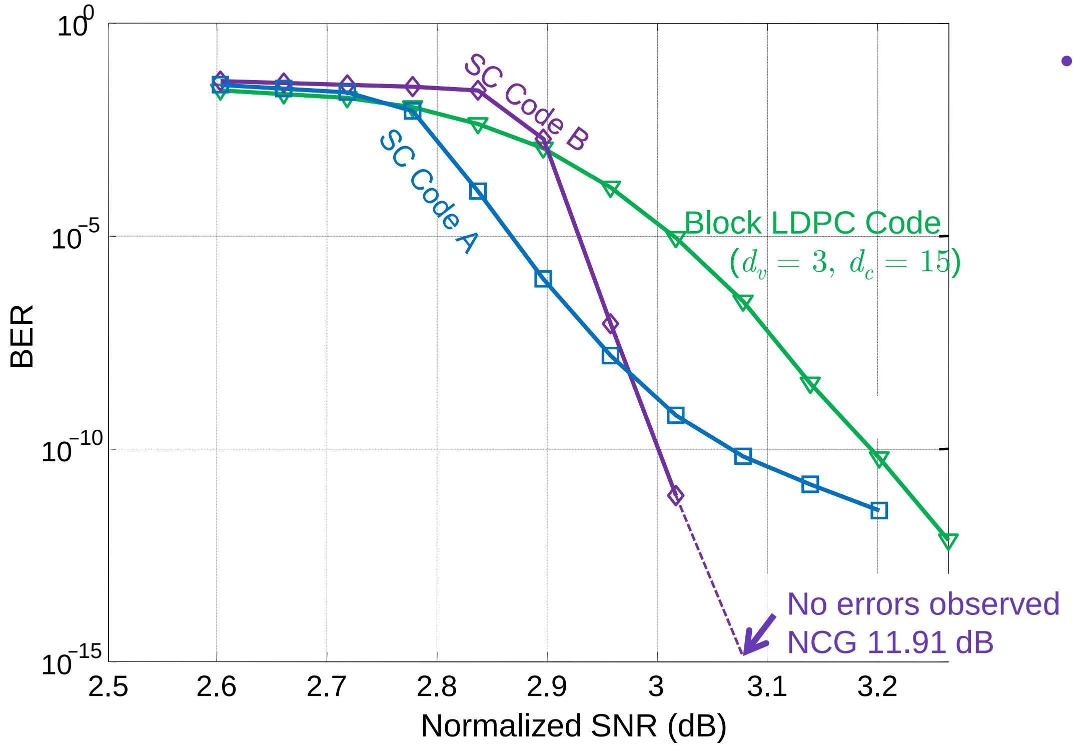

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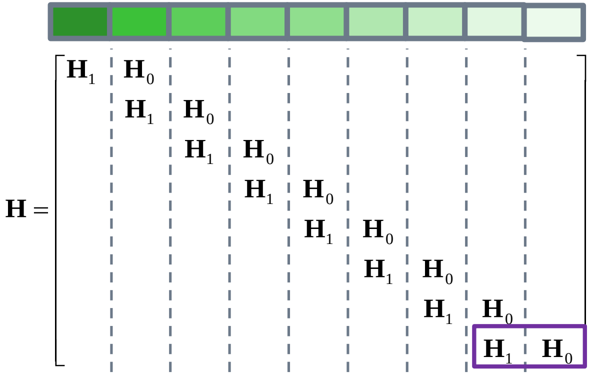

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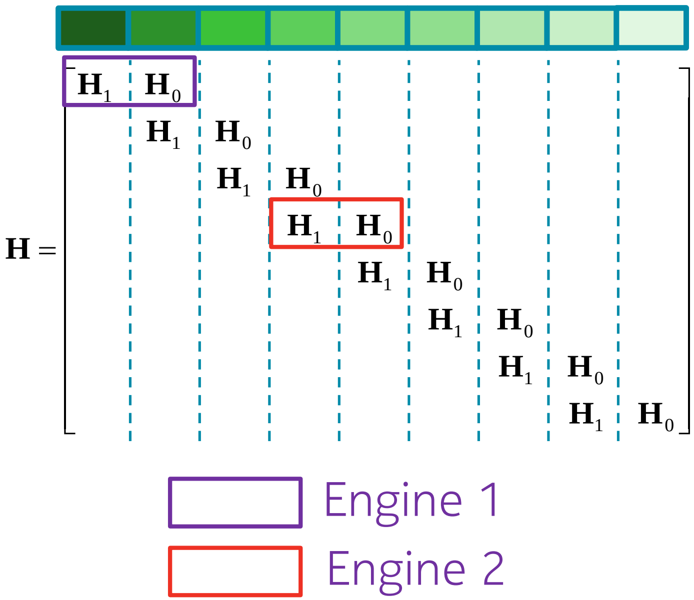
Hybrid Decoder with two engines



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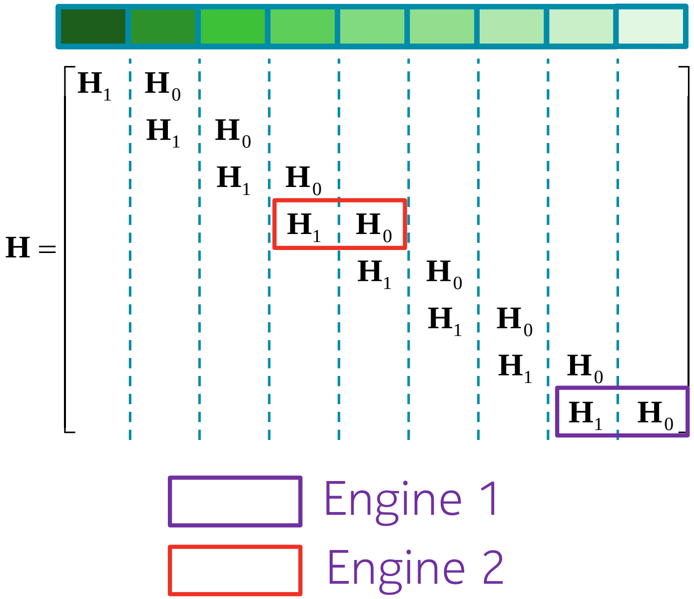

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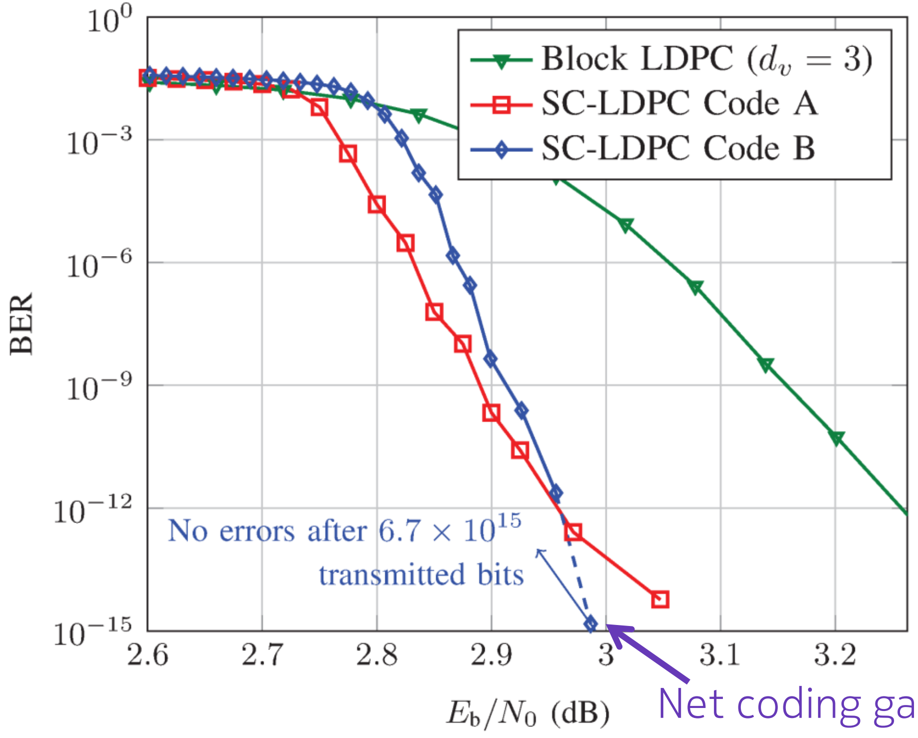
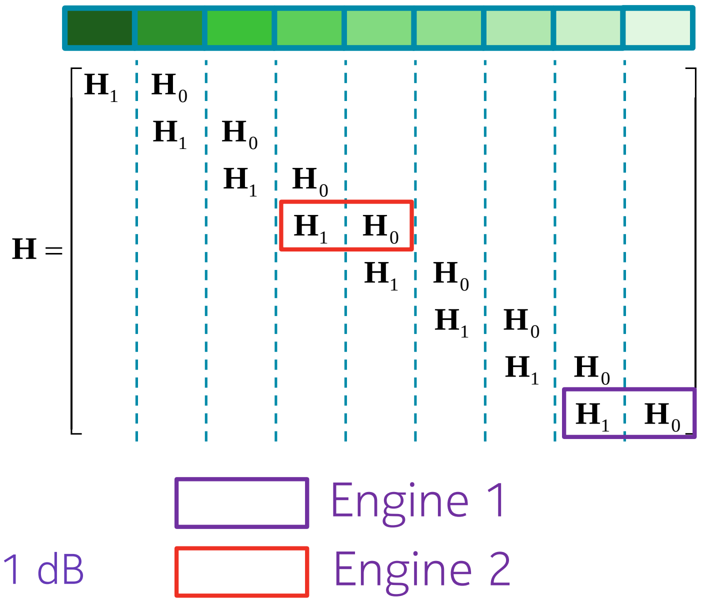


Net coding gain 12.01 dB

Hybrid Decoder with two engines



[SSR+15] L. Schmalen, D. Suikat, D. Rösener, V. Aref, A. Leven, S. ten Brink "Spatially coupled codes and optical fiber communications: An ideal match?," *Proc. Workshop on Signal Processing Advances in Wireless Communications (SPAWC)*, 2015

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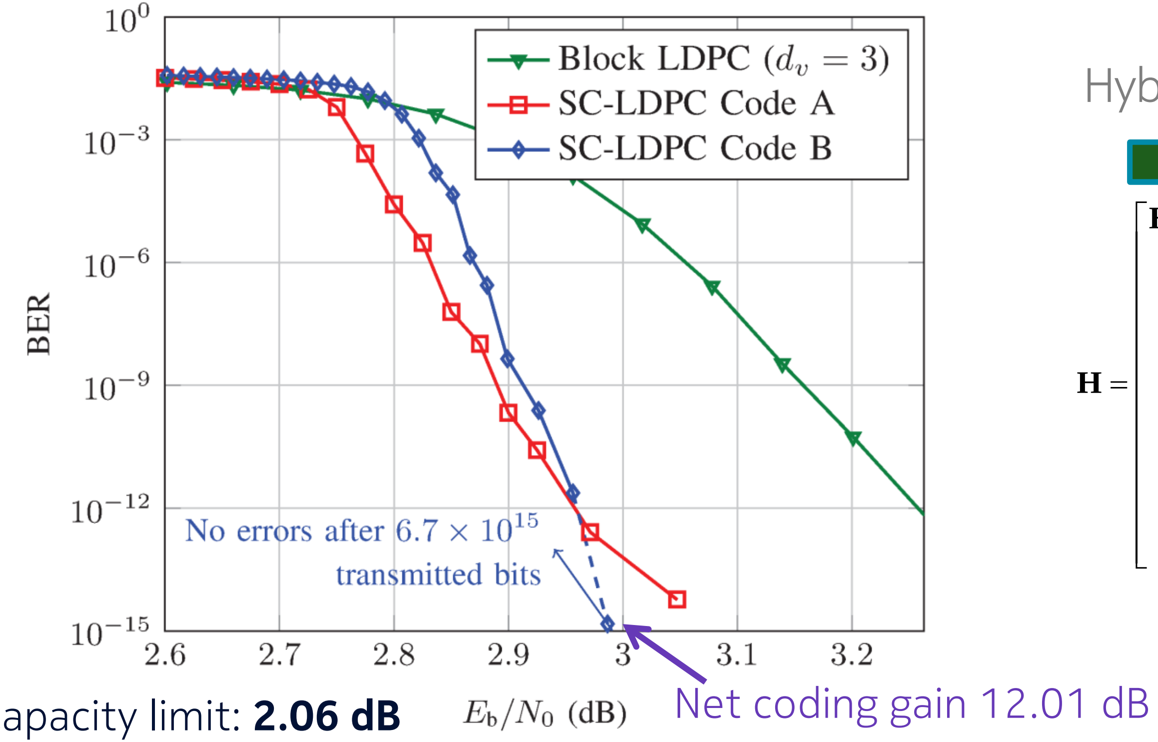
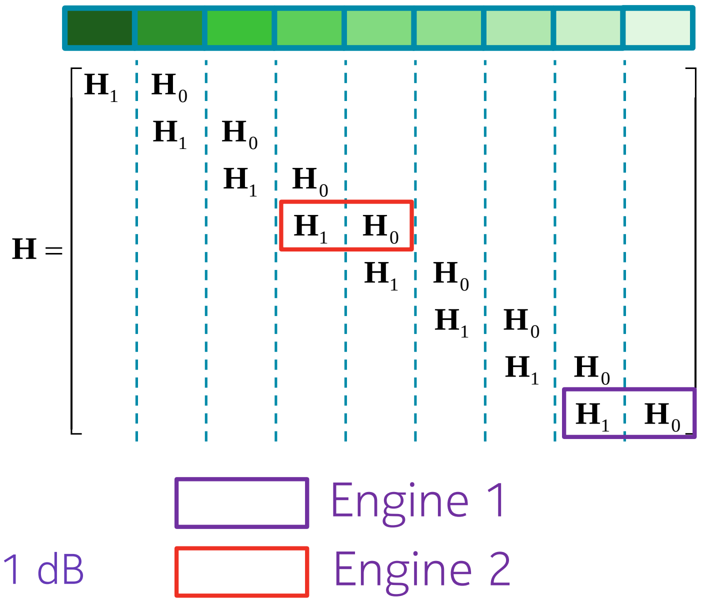
Results of FPGA-Based Code Evaluation ($R = 0.8$)



Capacity limit: **2.06 dB**

Hybrid Decoder with two engines



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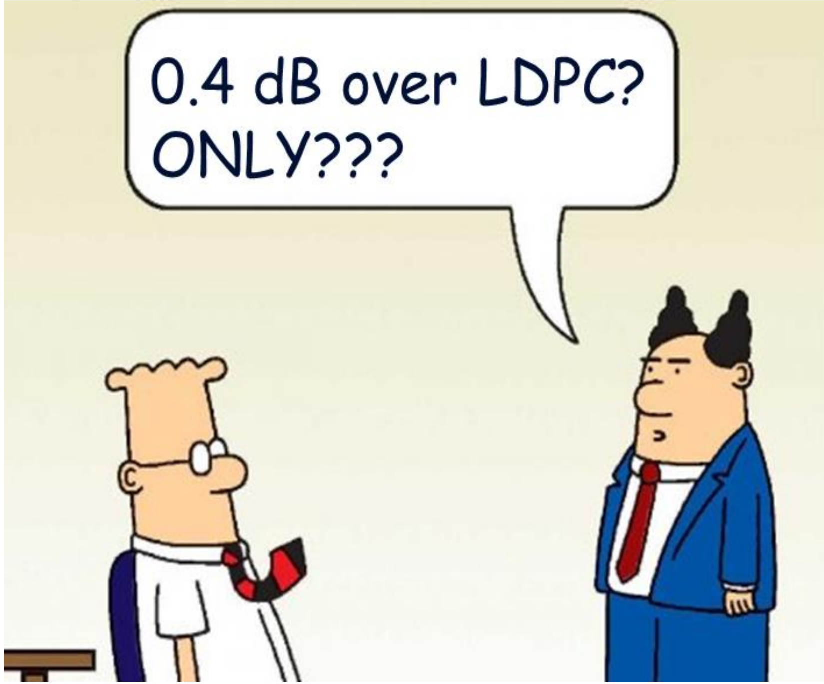
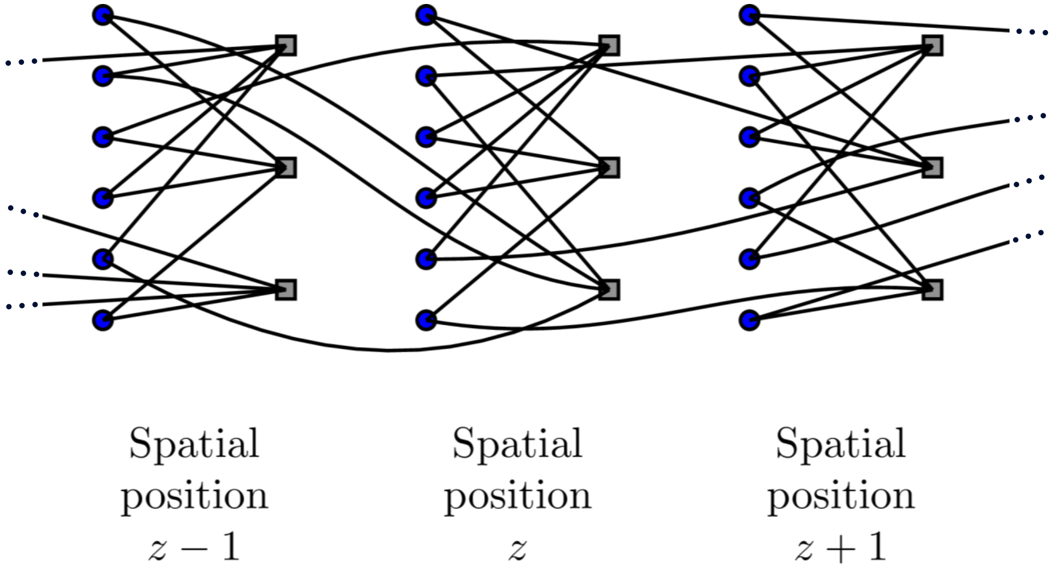


- **0.4dB** correspond to **900km reach increase** in trans-pacific cables
- Optical fiber communication systems **age** (material, lasers, photodiodes) and the **SNR will decay over time**
- In this case, additional gains increase lifetime/reduce margins of a system
- **More gains** are possible **with higher decoding complexity**
- However, **we want even more gains**!

[SSR+15] L. Schmalen, D. Suikat, D. Rösener, V. Aref, A. Leven, S. ten Brink "Spatially coupled codes and optical fiber communications: An ideal match?," *Proc. Workshop on Signal Processing Advances in Wireless Communications (SPAWC)*, 2015

New: Non-Uniformly Coupled LDPC Code Ensemble

Spatially Coupled LDPC Code with $w = 2$



Definition

Connect each edge from variable node at SP z to

- check node at position z **with probability α** and to
- Check node at position $z+1$ with **probability 1 − α**

[SAJ17] L. Schmalen, V. Aref, F. Jardel, "Non-Uniformly Coupled LDPC Codes: Better Thresholds, Smaller Rate-loss, and Less Complexity," *Proc. ISIT*, 2017

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Non-Uniformly Coupled LDPC Code Ensemble

Literature Review

- Optimized protographs with implicit non-uniform coupling **[MLC15]**
- Non-uniform protographs for coded modulation with spatially coupled codes **[StB13]**
- Non-uniform protographs for improved thresholds and unequal error prot. **[JB14]**
- Exponential, non-uniform coupling for anytime reliability **[NNL15]**
- Non-uniform coupling in spatially coupled compressed sensing **[KMS+12]**
- Rate loss mitigation by extra structure at the boundaries **[TKS12], [SP16]**
- …

[MLC15] D. Mitchell, M. Lentmaier, D. Costello, "Spatially coupled LDPC codes constructed from protographs," *IEEE Trans. Inf. Theory*, 2015

[StB13] L. Schmalen, S. ten Brink, "Combining spatially coupled LDPC codes with modulation and detection," *Proc. ITG SCC*, 2013

[JB14] F. Jardel, J. Boutros, "Non-uniform spatial coupling," *Proc. ITW*, 2014

[NNL15] M. Noor-A-Rahim, K. Nguyen, G. Lechner, "Anytime reliability of spatially coupled codes," *IEEE Trans. Commun.*, 2015

[KMS+12] F. Krzakala, M. Mézard, F. Sausset, Y. Sun, L. Zdeborová, "Statistical-physics-based reconstruction in compressed sensing," *Physical Review X*, 2012

[TKS12] K. Tazoe, K. Kasai, K. Sakinawa, "Efficient termination of spatially coupled codes," *Proc. ITW*, 2012

[SP16] M. Sanatkar, H. Pfister, "Increasing the rate of spatially-coupled codes via optimized irregular termination," *Proc. ISTC*, 2016

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Non-Uniformly Coupled LDPC Code Ensemble
BEC Density Evolution and Rate Loss

- BEC Density evolution for the generalized non-uniformly coupled ensemble

$$x_z^{(t+1)} = \varepsilon \left(1 - \sum_{i=0}^{w-1} \nu_i \left(1 - \sum_{j=0}^{w-1} \nu_j x_{z+i-j}^{(t)} \right)^{d_c-1} \right)^{d_v-1}$$

- In particular, for $w = 2$, we have $\nu = (\alpha, 1 - \alpha)$
- Rate of the generalized ensemble

$$R = \left(1 - \frac{d_v}{d_c}\right) - \frac{d_v}{d_c} \left(w - 1 - \sum_{k=0}^{w-2} \left[\left(\sum_{i=0}^{k} \nu_i \right)^{d_c} + \left(\sum_{i=k+1}^{w-1} \nu_i \right)^{d_c} \right] \right)$$

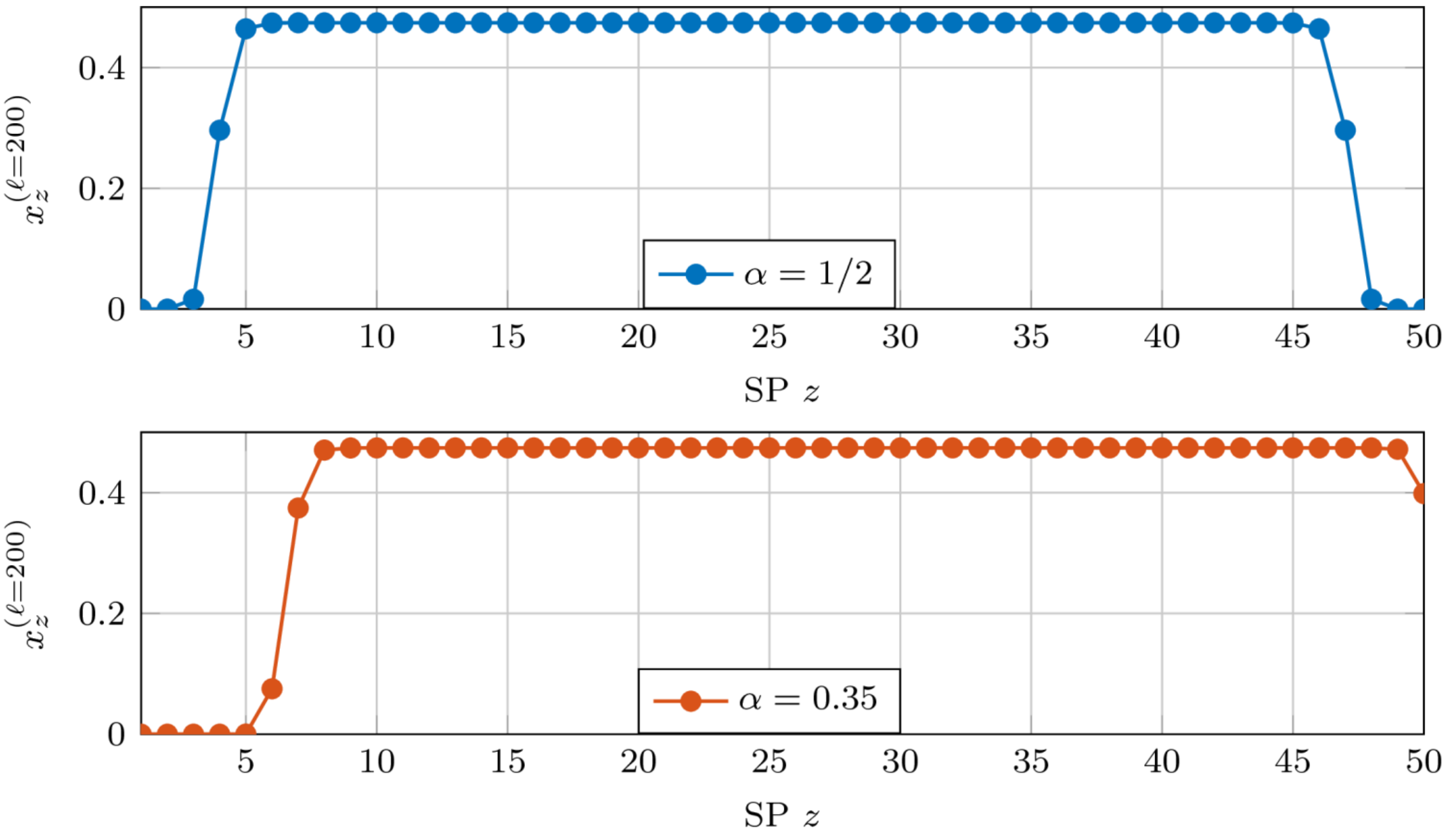
- For $w = 2$, rate is minimal for $\alpha = 1/2$, i.e., non-uniform coupling **reduces rate loss**

Density Evolution for ε = 0.48 , L = 50

Conventional Spatially Coupled LDPC Code
$d_v = 5,\ d_c = 10$

$I = 200$ iter.

New: Non-uniformly coupled code with
$d_v = 5,\ d_c = 10$
Single-side convergence



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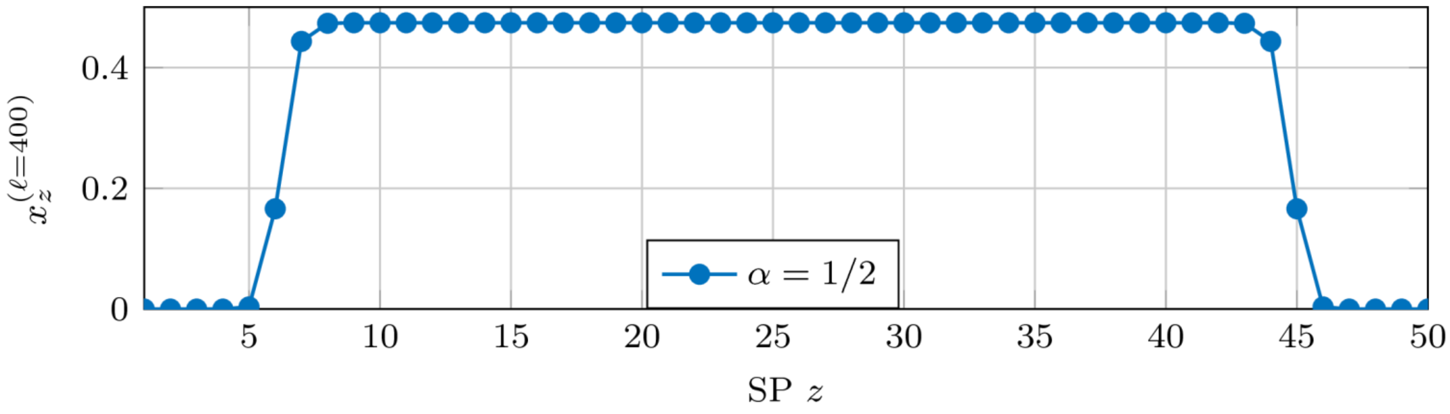
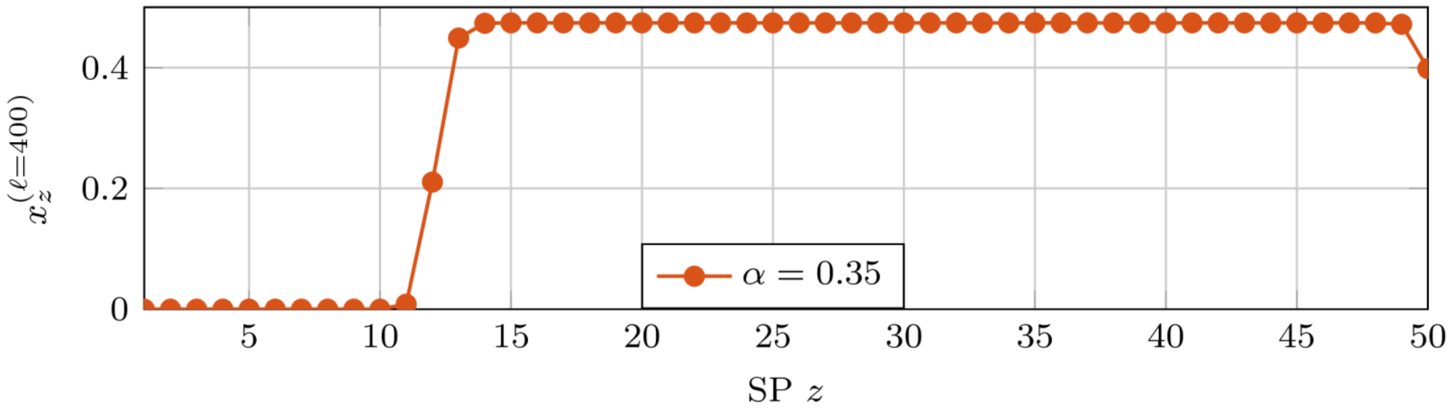
Density Evolution for ε = 0.48 , L = 50

Conventional Spatially Coupled LDPC Code
$d_v = 5,\ d_c = 10$

$I = 400$ iter.



New: Non-uniformly coupled code with
$d_v = 5,\ d_c = 10$
Single-side convergence

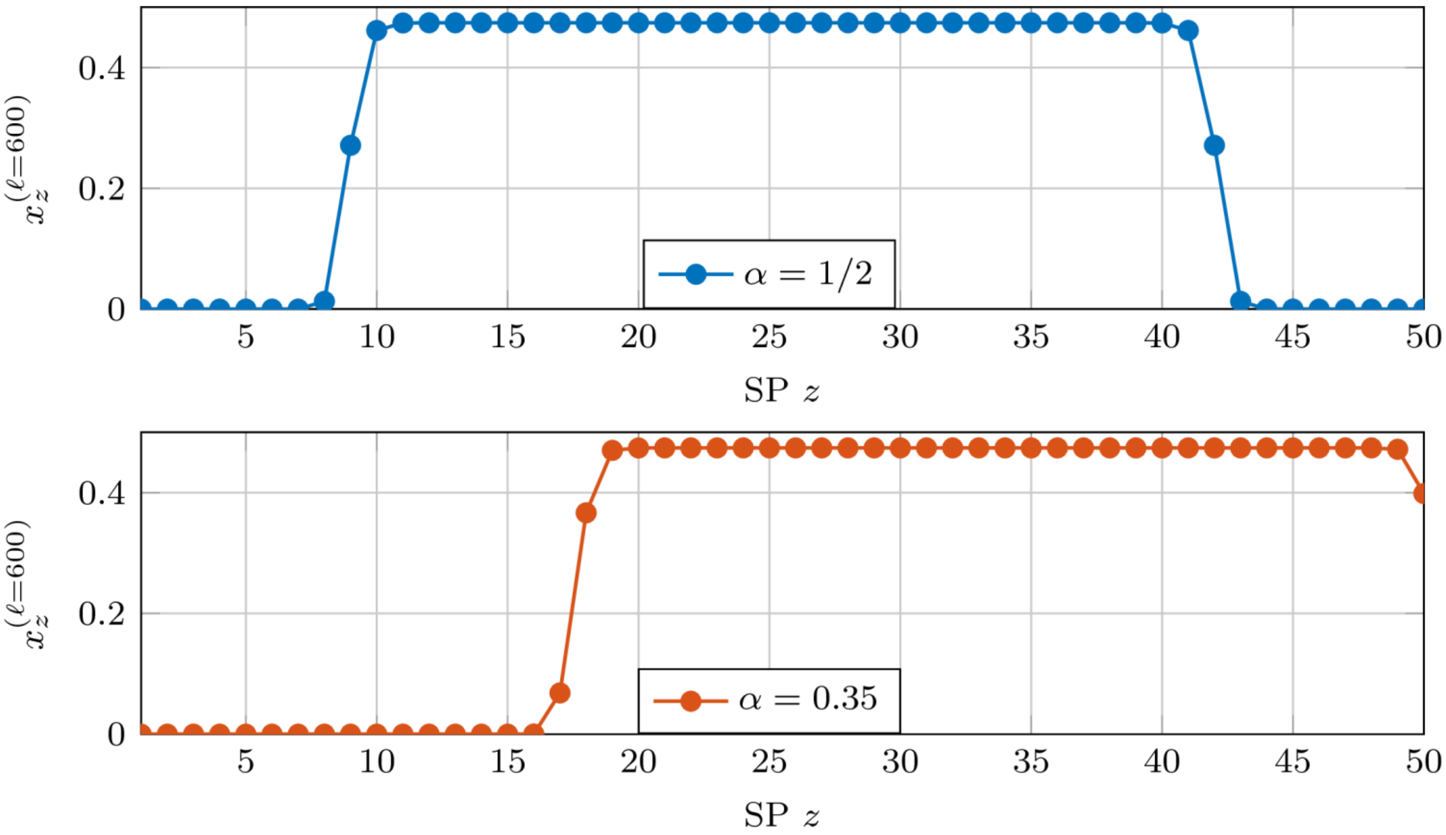


Density Evolution for ε = 0.48 , L = 50

Conventional Spatially Coupled LDPC Code
$d_v = 5,\ d_c = 10$

$I = 600$ iter.

New: Non-uniformly coupled code with
$d_v = 5,\ d_c = 10$
Single-side convergence



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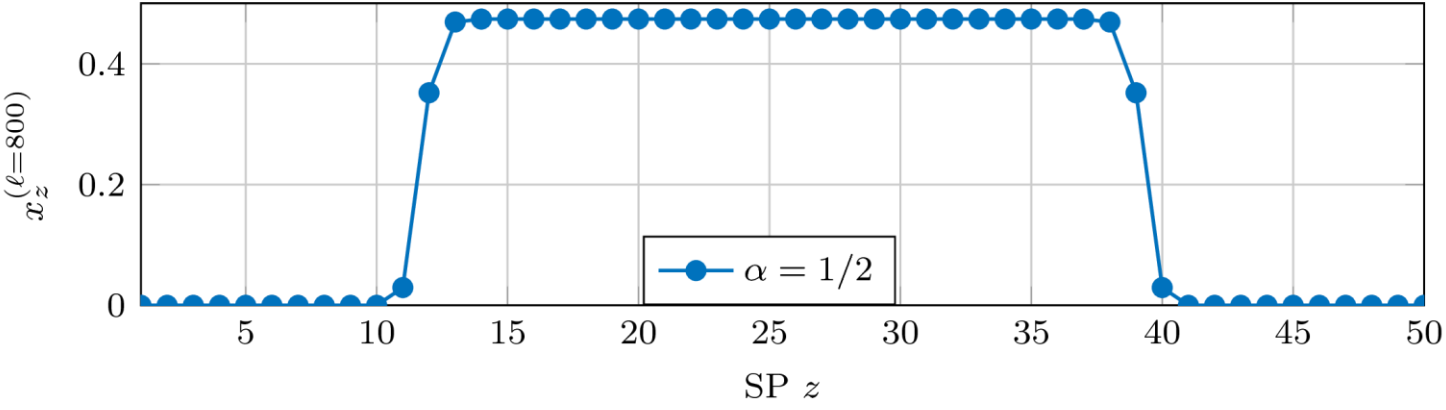
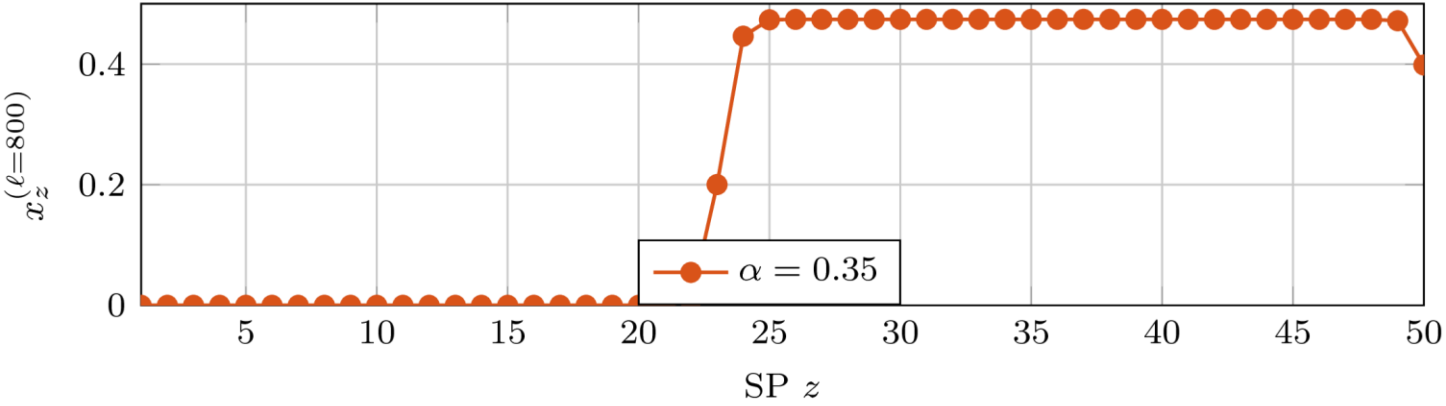
Density Evolution for ε = 0.48 , L = 50

Conventional Spatially Coupled LDPC Code
$d_v = 5,\ d_c = 10$

$I = 800$ iter.



New: Non-uniformly coupled code with
$d_v = 5,\ d_c = 10$
Single-side convergence



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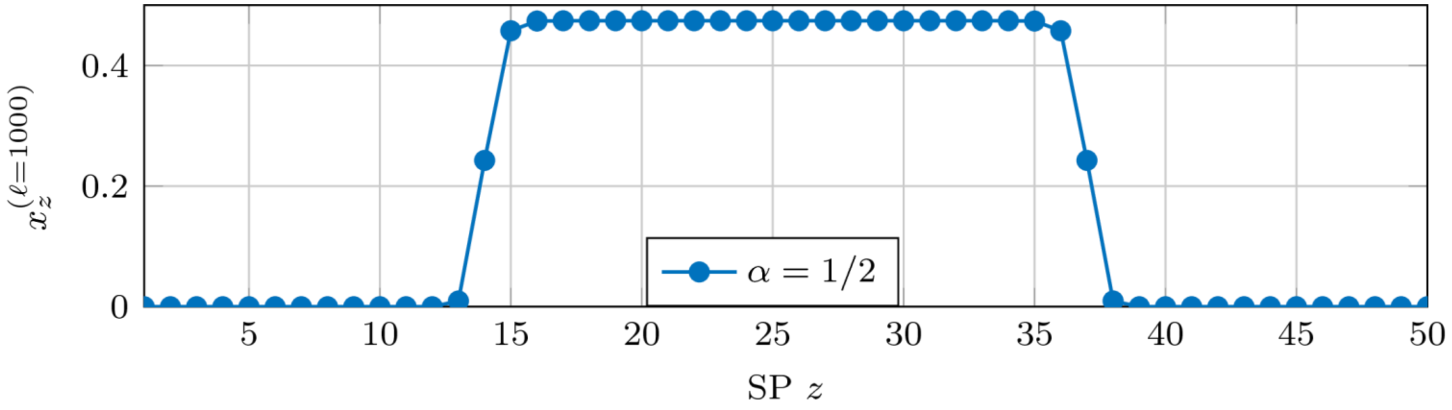
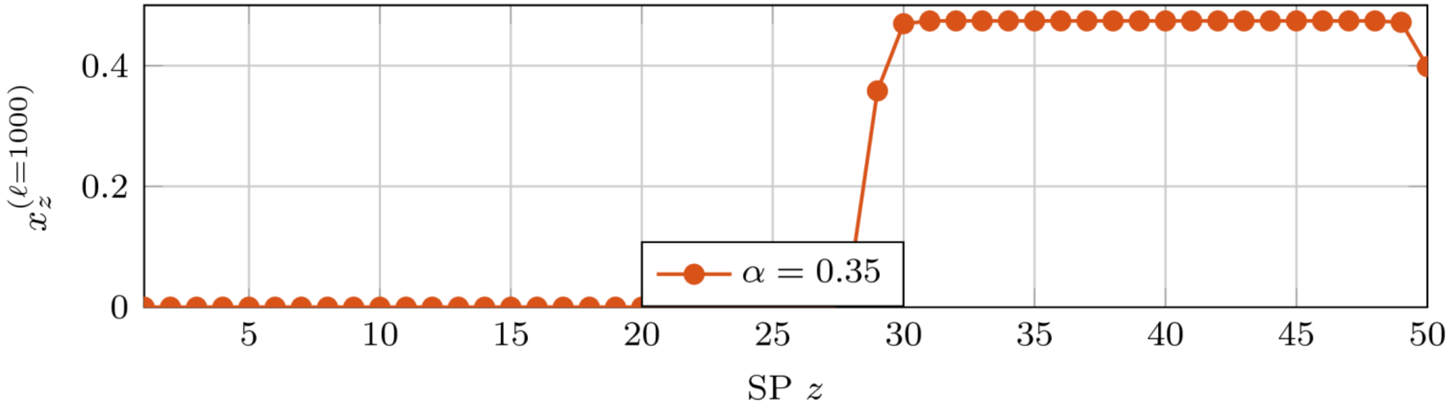
Density Evolution for ε = 0.48 , L = 50

Conventional Spatially Coupled LDPC Code
$d_v = 5,\ d_c = 10$

$I = 1000$ iter.



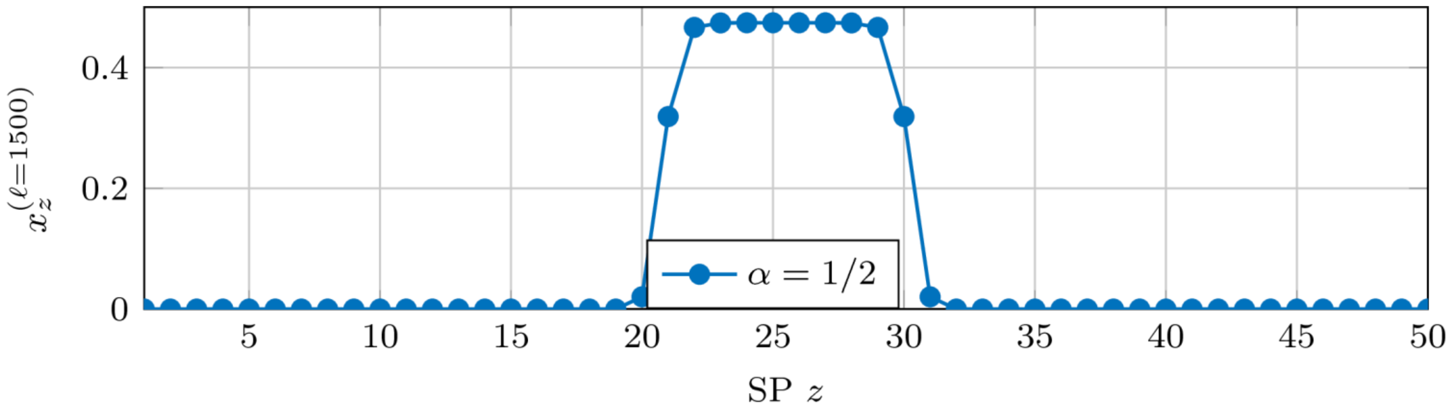
New: Non-uniformly coupled code with
$d_v = 5,\ d_c = 10$
Single-side convergence



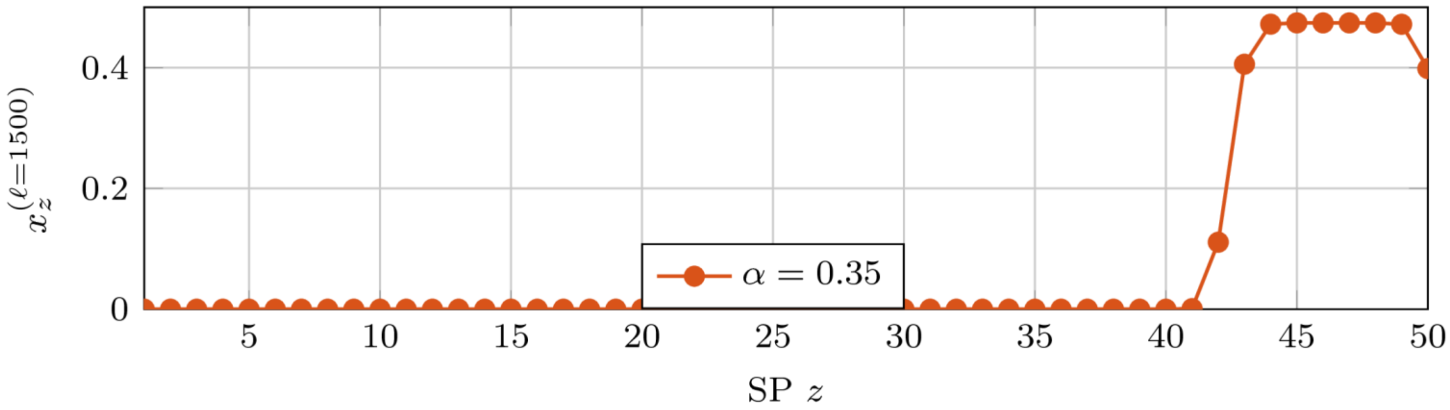
Density Evolution for ε = 0.48 , L = 50

Conventional Spatially Coupled LDPC Code
$d_v = 5,\ d_c = 10$

$I = 1500$ iter.



New: Non-uniformly coupled code with
$d_v = 5,\ d_c = 10$
Single-side convergence



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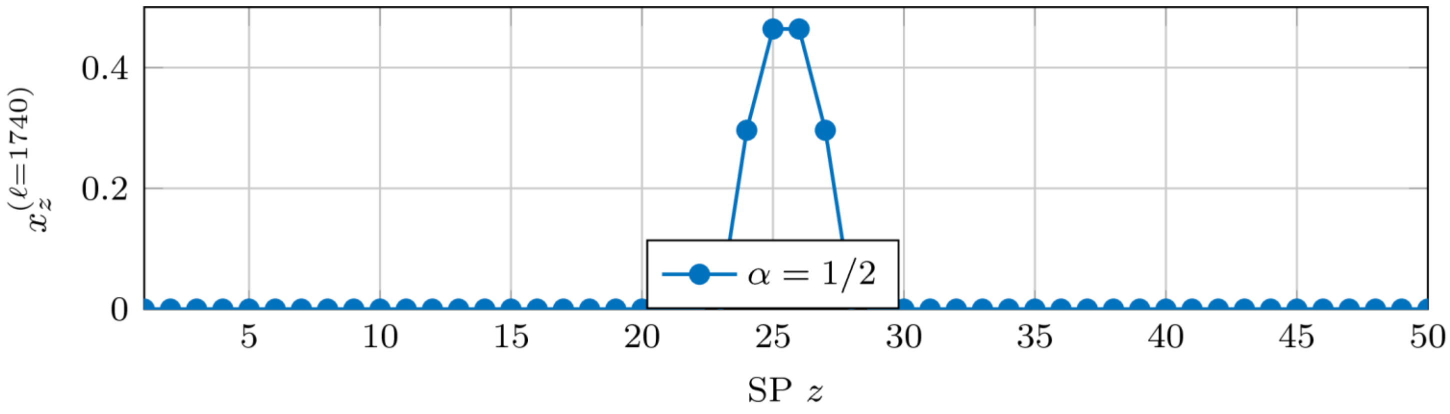
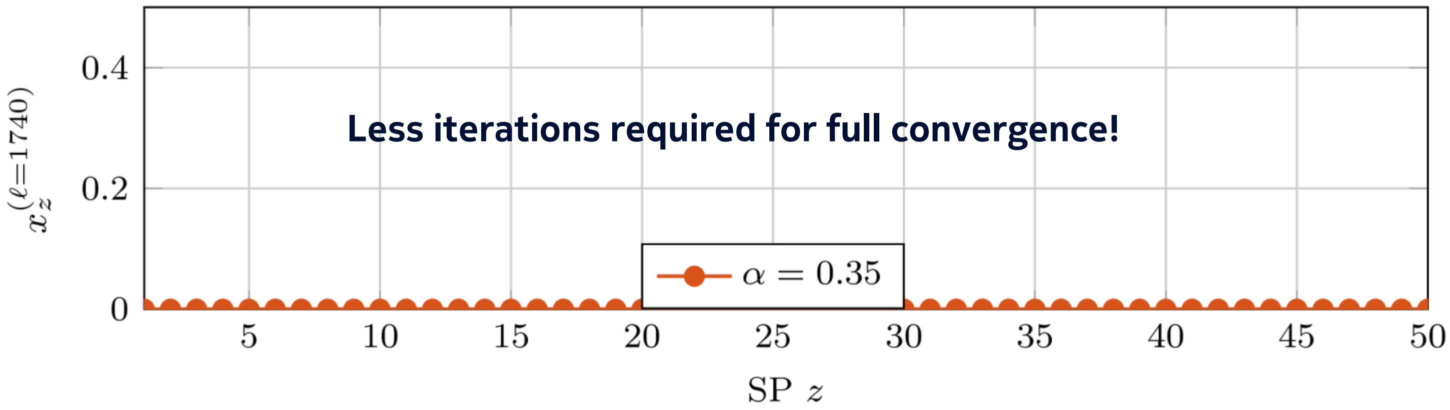
Density Evolution for ε = 0.48 , L = 50

Conventional Spatially Coupled LDPC Code
$d_v = 5,\ d_c = 10$

$I = 1740$ iter.



New: Non-uniformly coupled code with
$d_v = 5,\ d_c = 10$
Single-side convergence



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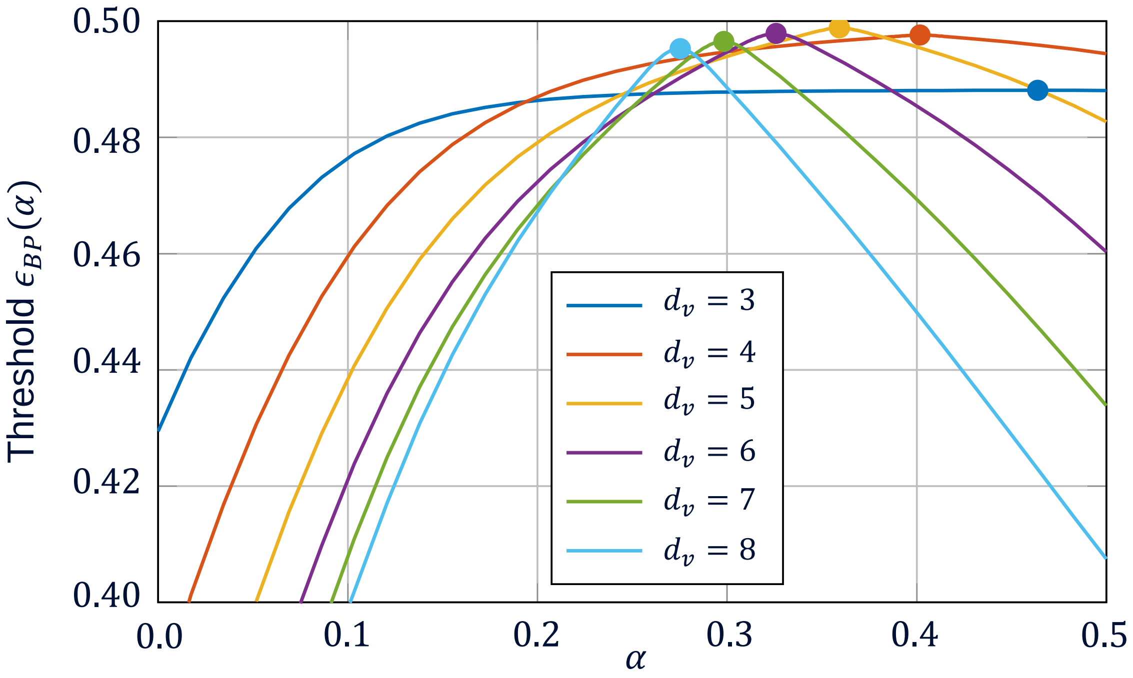
Non-Uniform Coupling vs. Conventional Uniform Coupling (1)

BP decoding thresholds: SC-LDPC $(d_v, 2d_v, w = 2, \alpha, L = 100)$ over the BEC



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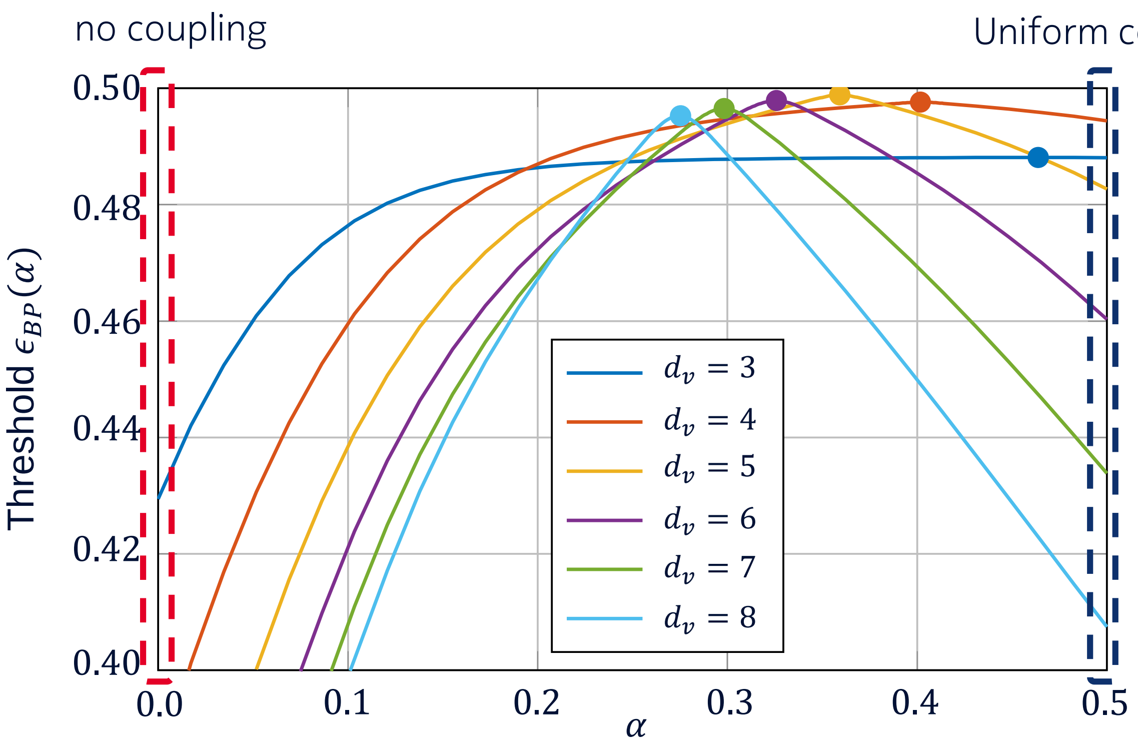

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Non-Uniform Coupling vs. Conventional Uniform Coupling (1)

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d_v	α^*	ϵ_{BP} uncoupled	ϵ_{MAP}	$\epsilon_{BP}(\alpha = 0.5)$	$\epsilon_{BP}(\alpha^*)$
3	0.4517	0.4294	0.48815	0.488(8)	0.4881(0)
4	0.4017	0.3834	0.49774	0.4944	0.4976
5	0.359	0.3415	0.49949	0.4827	0.4989
6	0.3252	0.3075	0.49988	0.4603	0.4979
7	0.2978	0.2798	0.49997	0.4338	0.4965
8	0.2745	0.257	0.49999	0.4074	0.4953
9	0.2544	0.2378	0.49999	0.3829	0.4943
10	0.2368	0.2215	0.49999	0.3606	0.4936

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increasing

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increasing decreasing

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Non-Uniform Coupling vs. Conventional Uniform Coupling (1)

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ϵ_{MAP}: increasing

$\epsilon_{BP}(\alpha = 0.5)$: decreasing

$\epsilon_{BP}(\alpha^*)$: increasing (d_v = 3 to 5); Almost unchanged (d_v = 6 to 10)

Non-Uniform Coupling vs. Conventional Uniform Coupling (1)

BP decoding thresholds: SC-LDPC $(d_v,\ 2d_v,\ w=2,\ \alpha,\ L=100)$ over the BEC

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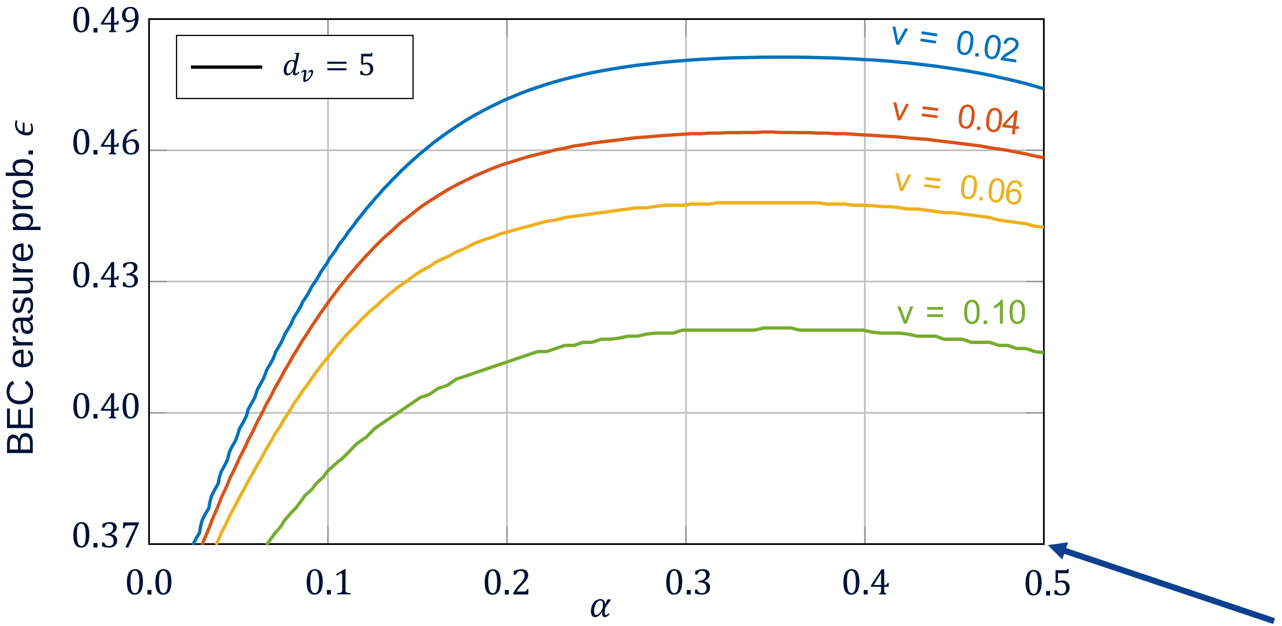
Non-Uniform Coupling vs. Conventional Uniform Coupling (2)

Complexity of Decoding (i.e., Number of Iterations)



Decoding speed contour plots for the random SC-LDPC$(d_v,\ 2d_v,\ w = 2,\ \alpha,\ L = 100)$ ensemble.

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Non-Uniform Coupling vs. Conventional Uniform Coupling (2)

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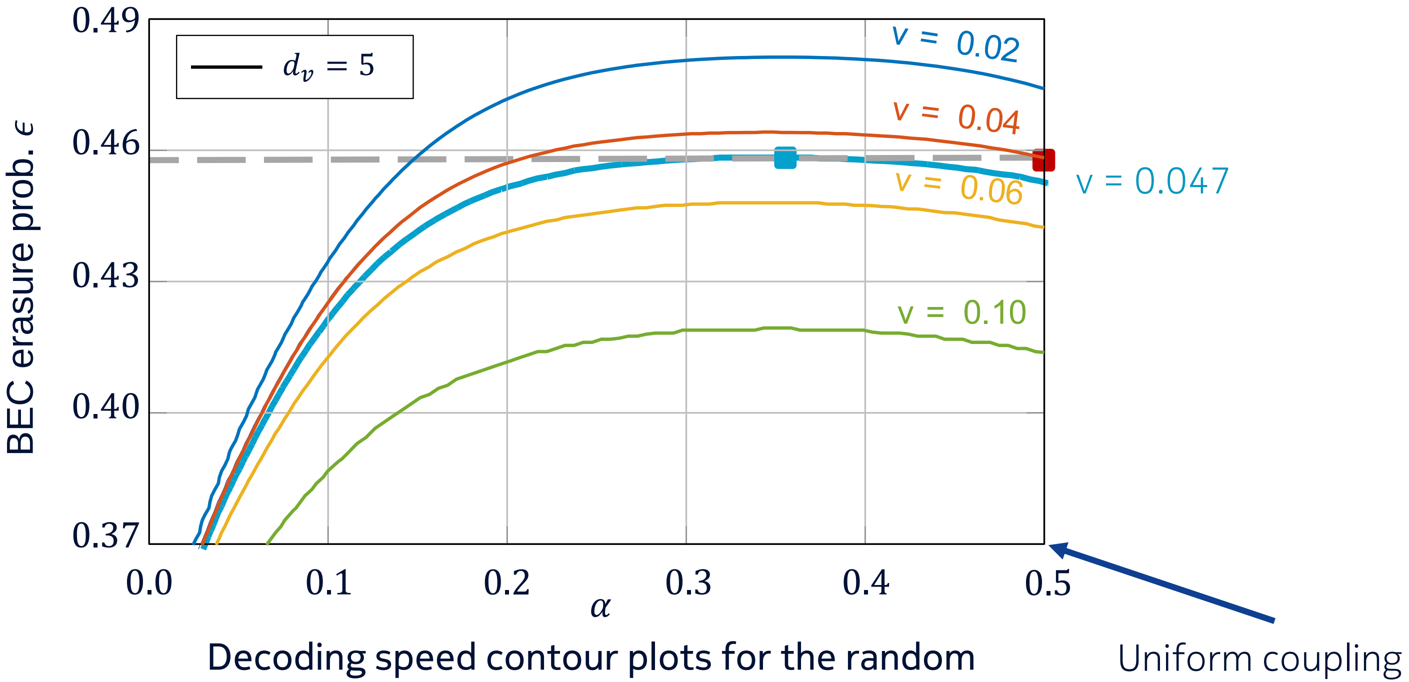

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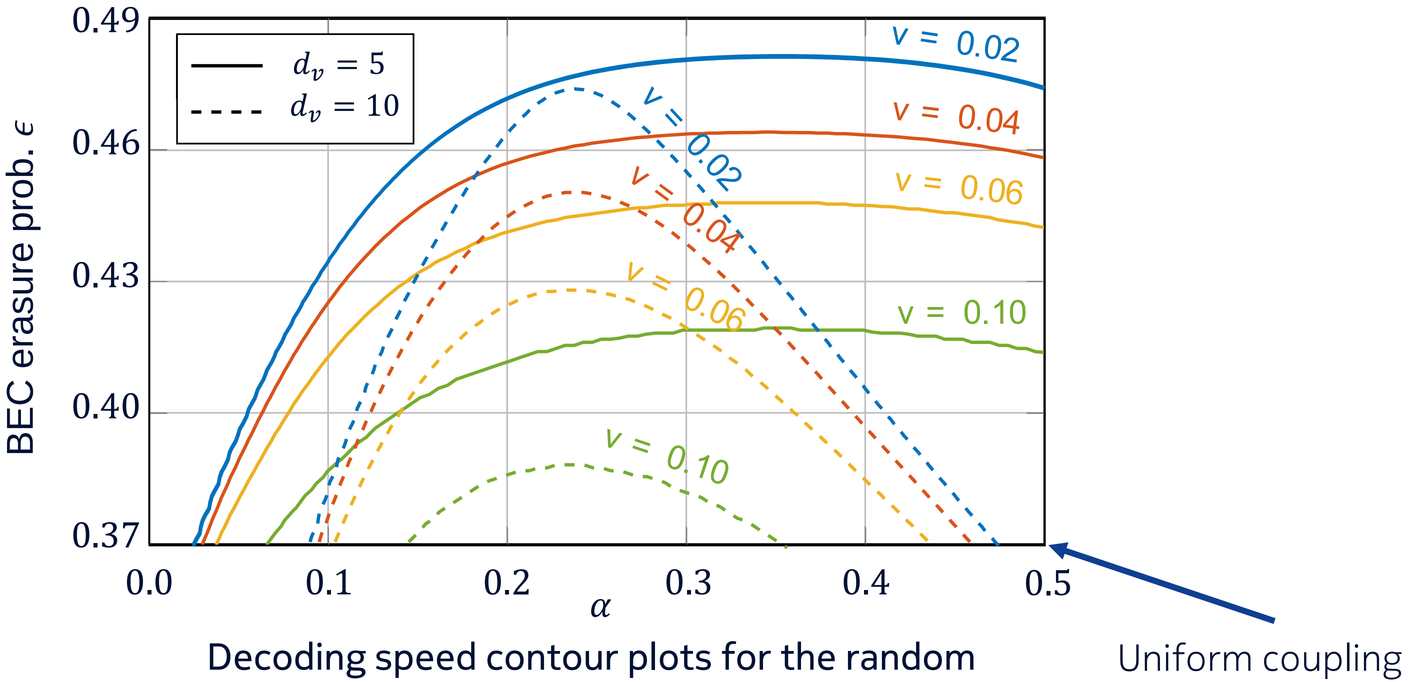

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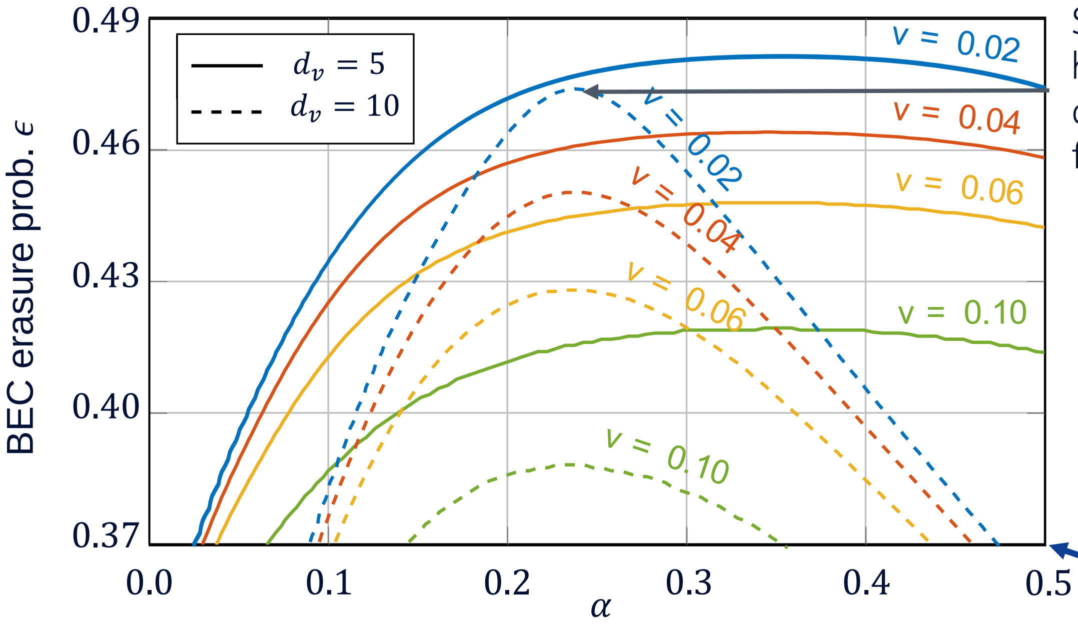
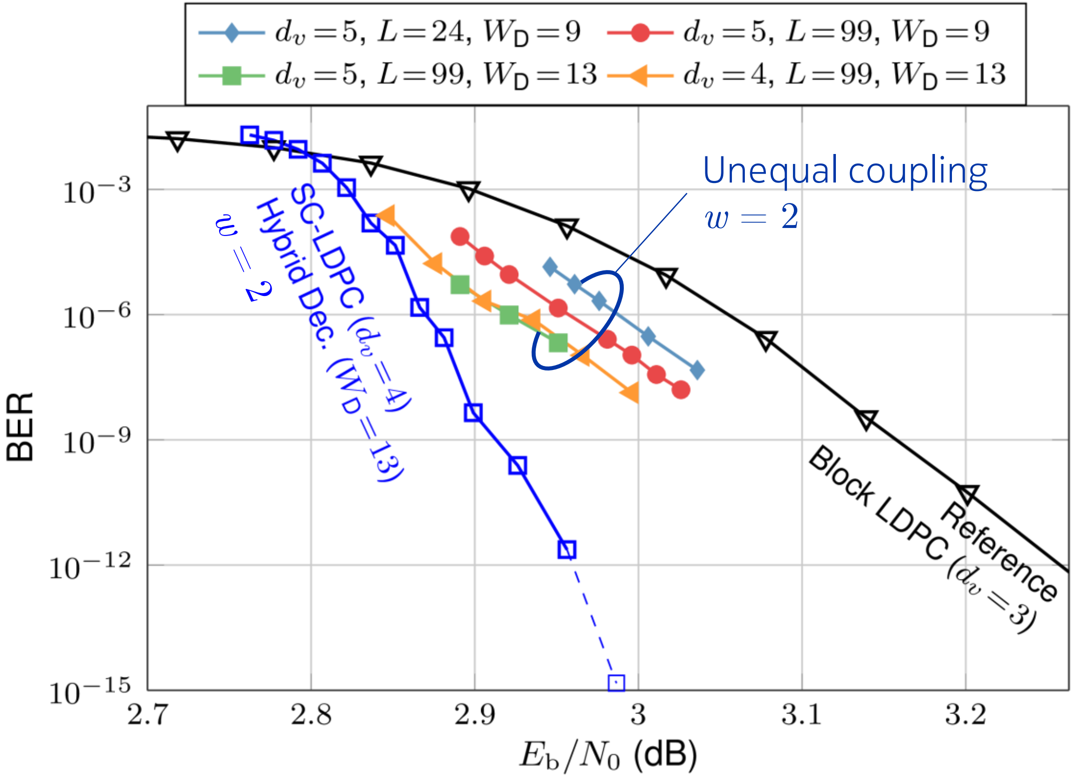


Same velocity with higher variable node degree → lower error floors

Decoding speed contour plots for the random SC-LDPC$(d_v, 2d_v, w = 2, \alpha, L = 100)$ ensemble.

Uniform coupling

What Happens with Optimized SC-LDPC Codes ($R = 0.8$)

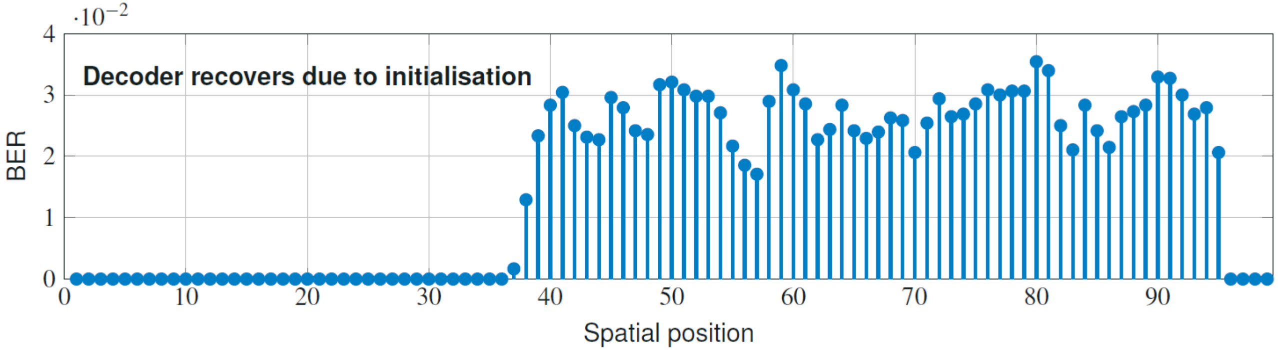


- FPGA simulation results of QC versions of these codes
- Degraded performance of optimized, unequally coupled codes under windowed decoding **[SSA+16]**
- Performance does not correspond to predicted threshold
- What is happening?

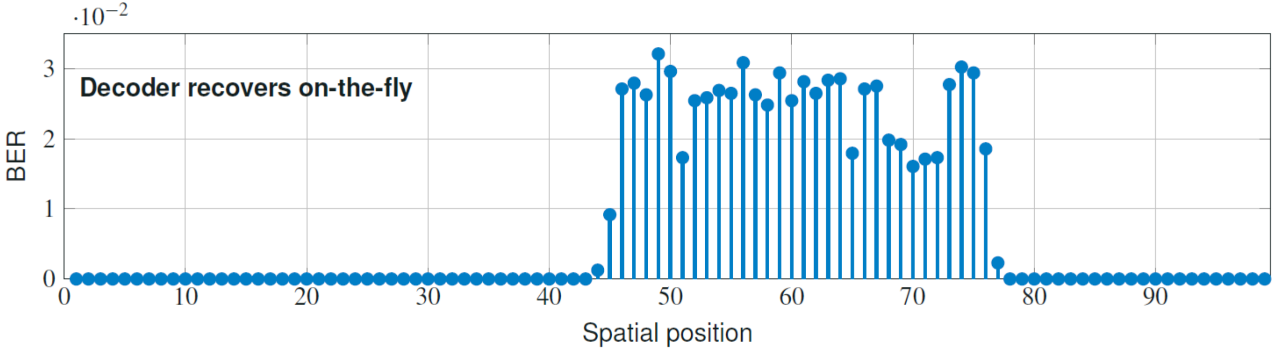
[SSA+16] L. Schmalen, D. Suikat, V. Aref, D. Rösener, "On the design of capacity approaching unit-memory spatially coupled LDPC codes for optical communications," *Proc. ECOC*, 2016

Windowed Decoder Stall

Exemplary Error Patterns AFTER Decoding



- In rare cases, decoder gets stuck
- Subsequent spatial positions are also stuck



- *Burst-like error pattern*

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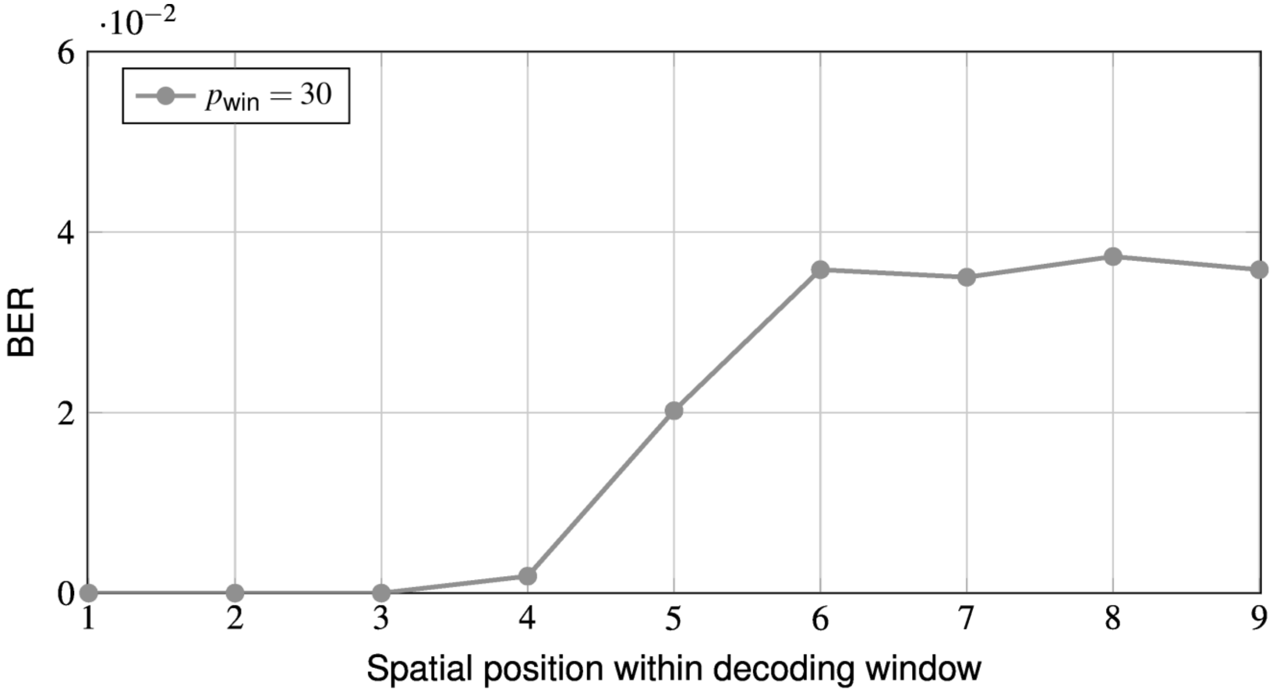
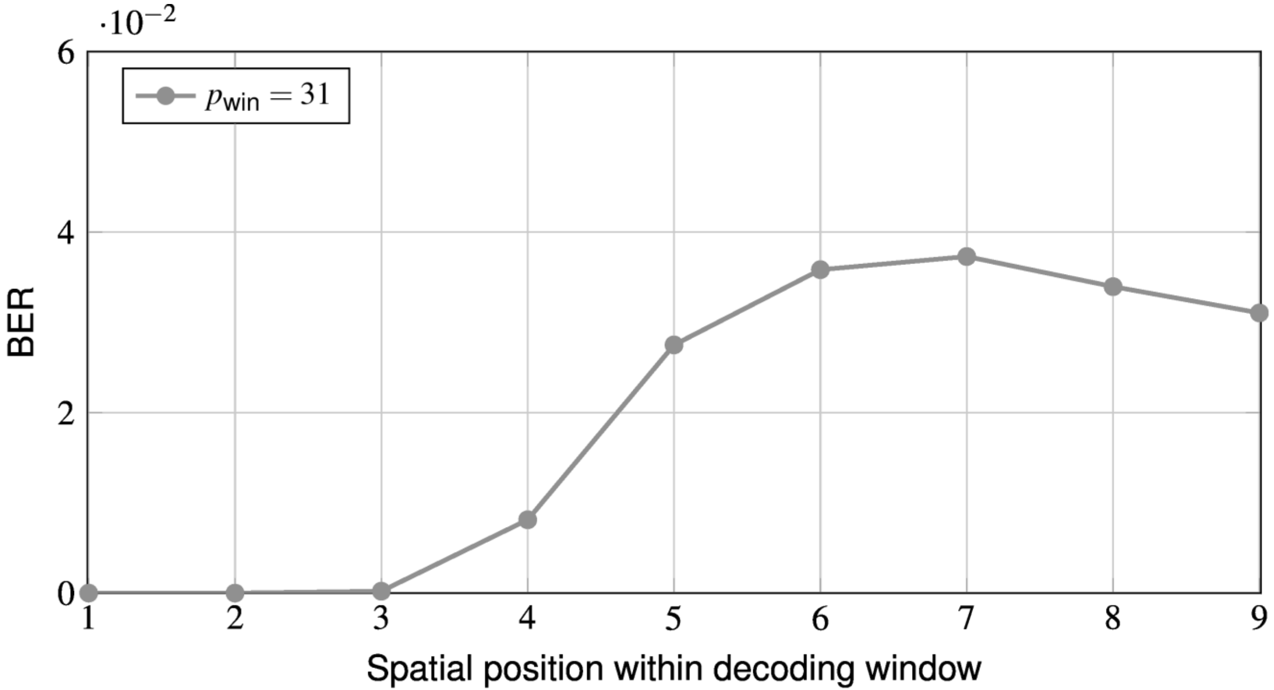
Observations Inside the Decoding Window



- $p_{\text{win}} \in (1, N_W)$ denotes the window position
- Decoder gets stuck around $p_{\text{win}} = 37$
- Leftmost position(s) needs to be error-free before decoding

[KCS+18] K. Klaiber, S. Cammerer, L. Schmalen, S. ten Brink, "Avoiding Burst-like Error Patterns in Windowed Decoding of Spatially Coupled LDPC Codes," *Proc. ISTC*, 2018

Observations Inside the Decoding Window



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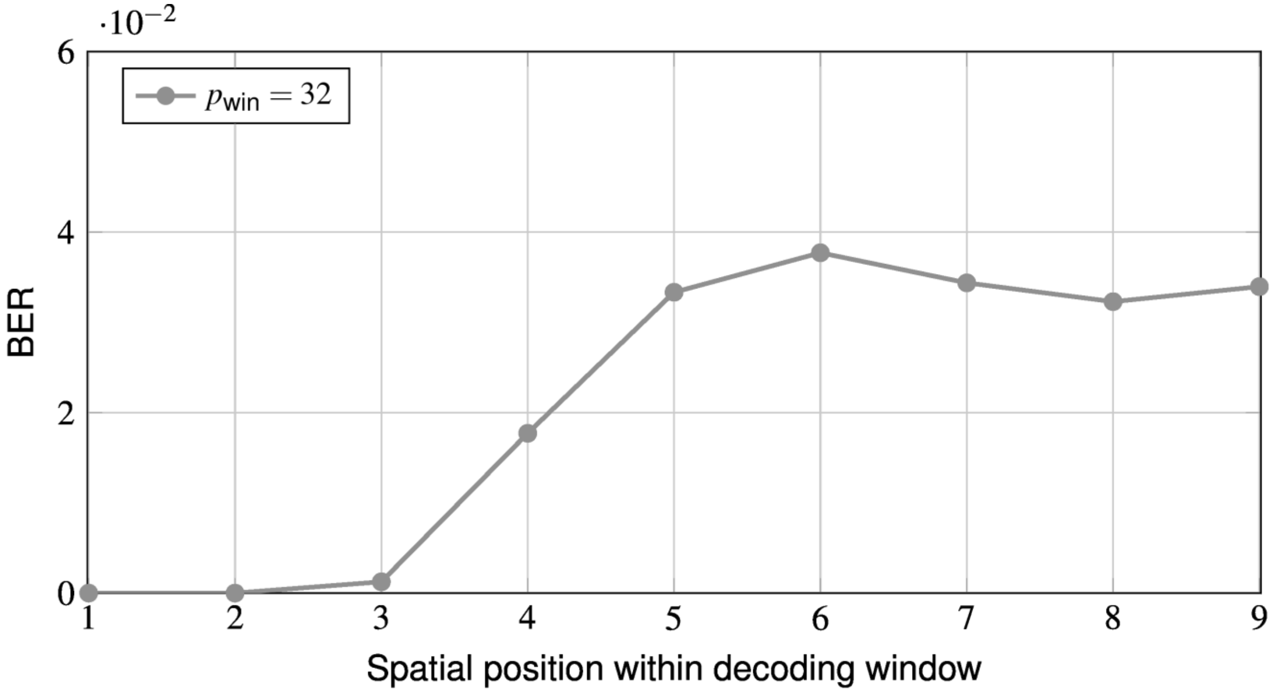

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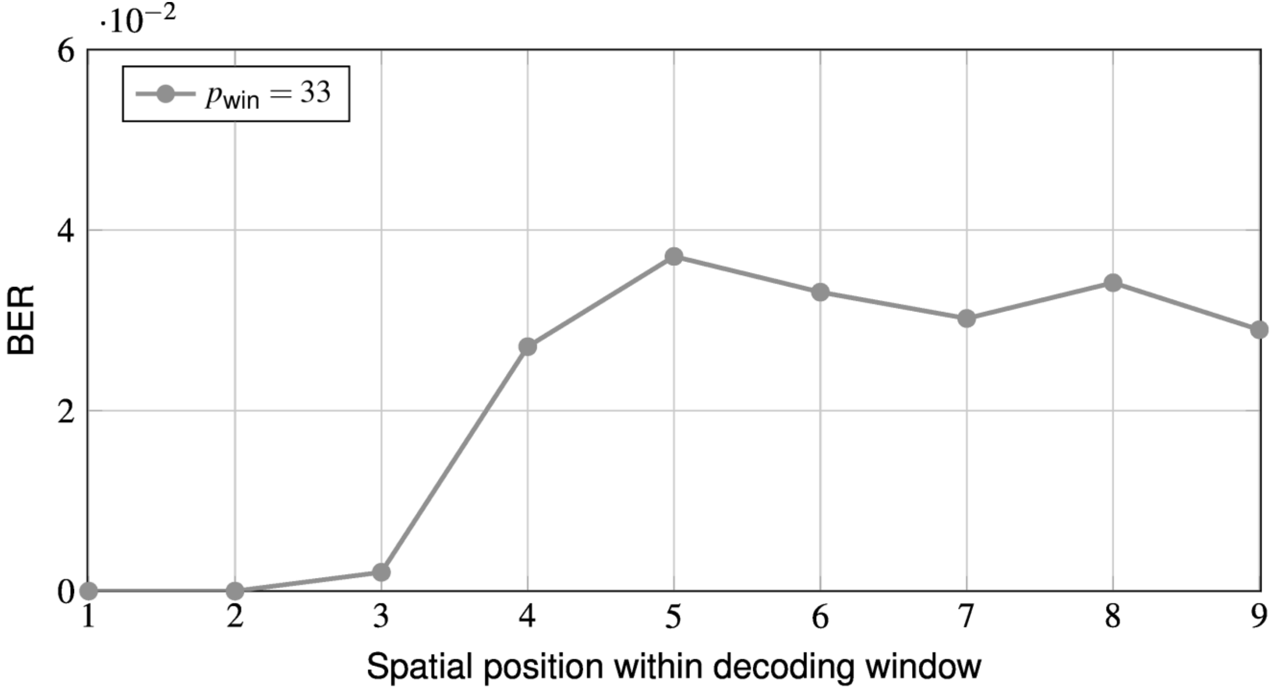

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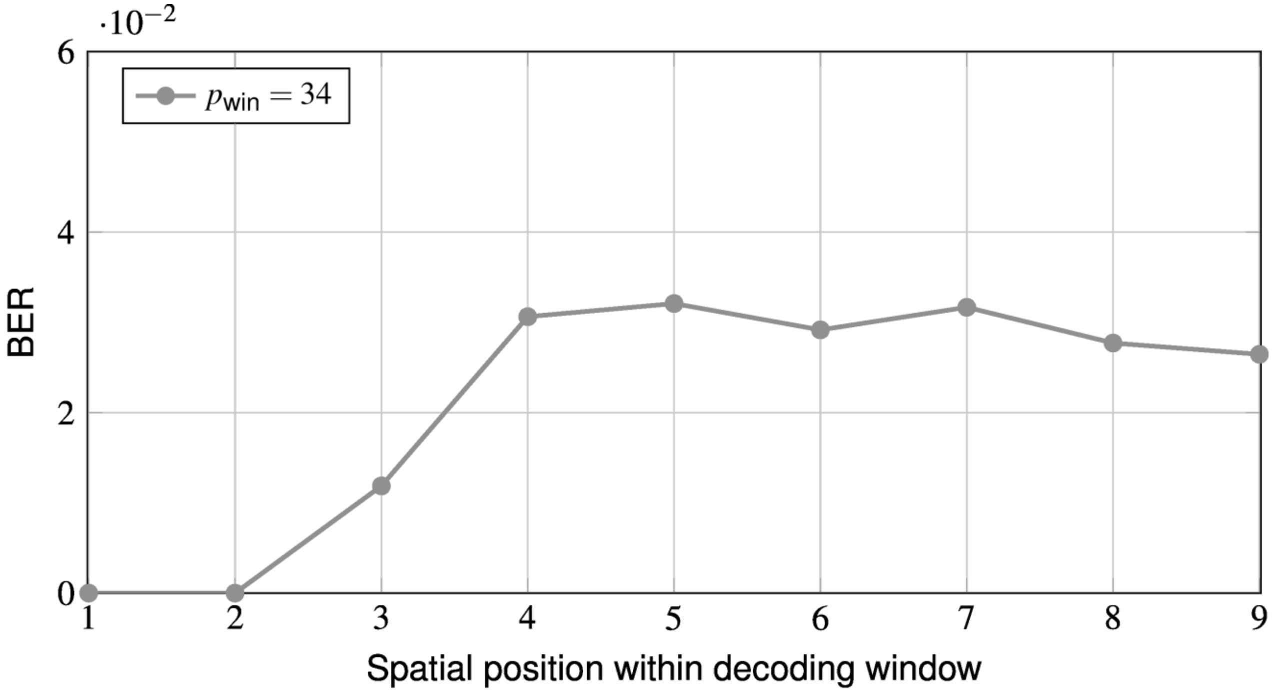

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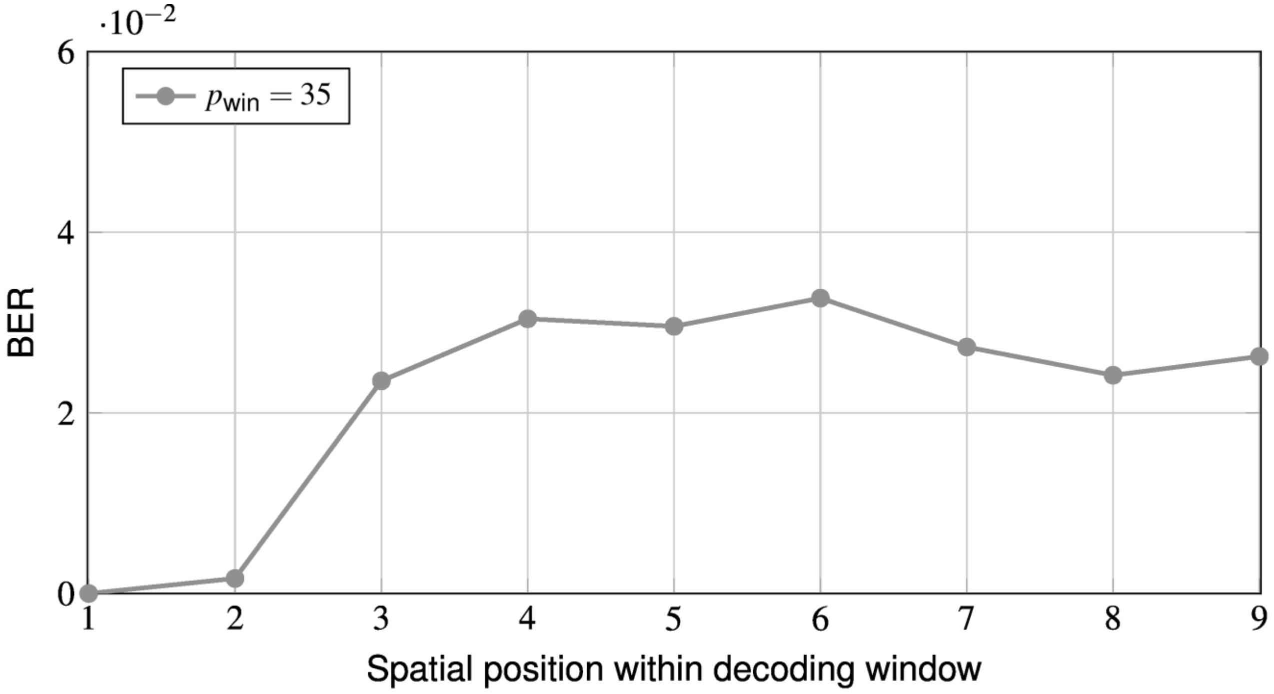
Observations Inside the Decoding Window



- $p_{\text{win}} \in (1, N_W)$ denotes the window position
- Decoder gets stuck around $p_{\text{win}} = 37$
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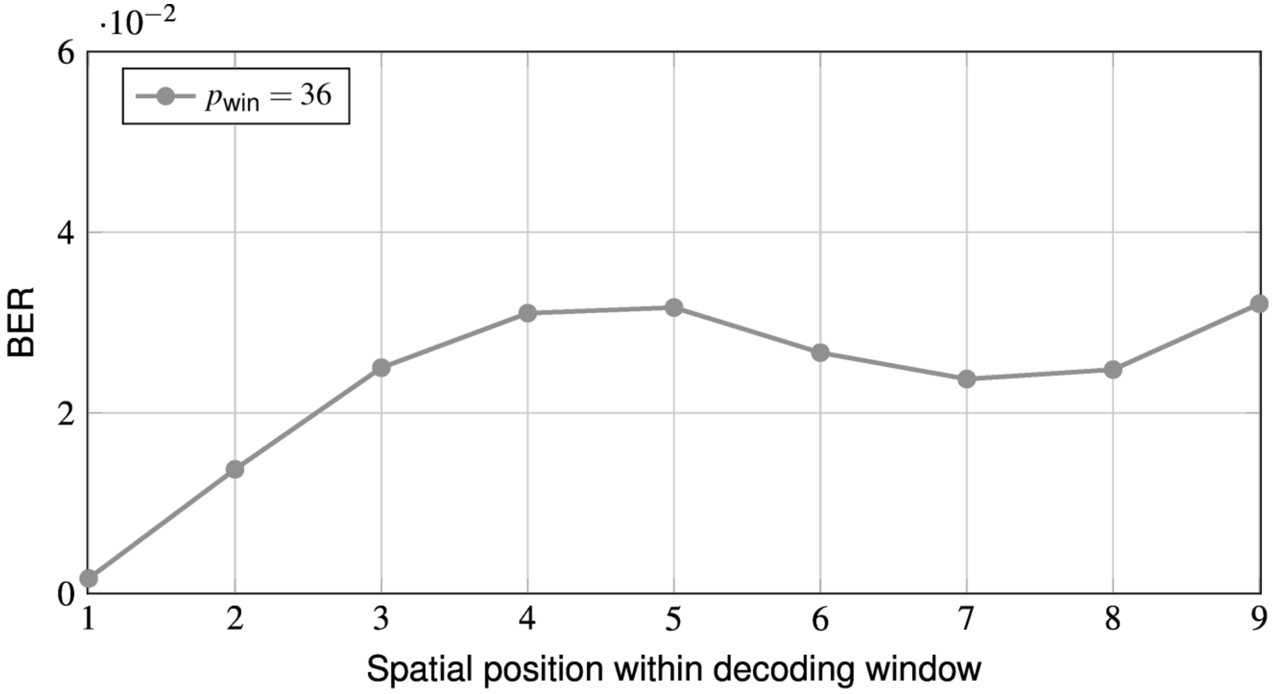

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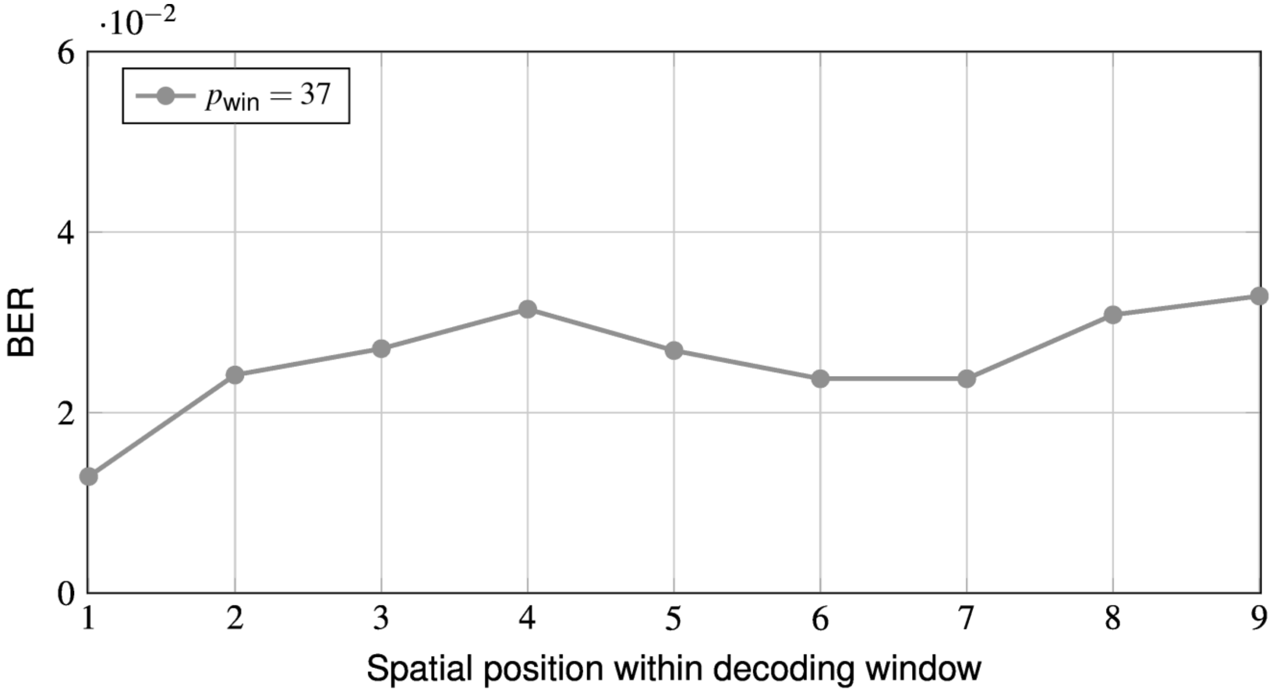

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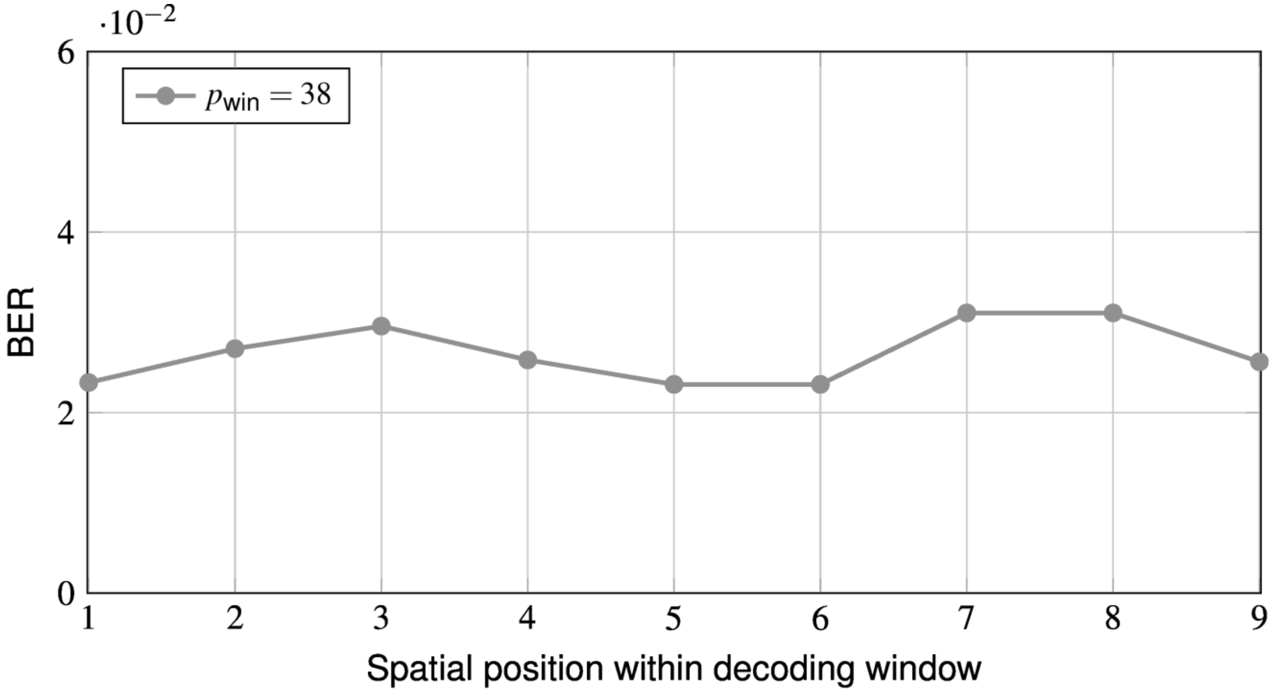

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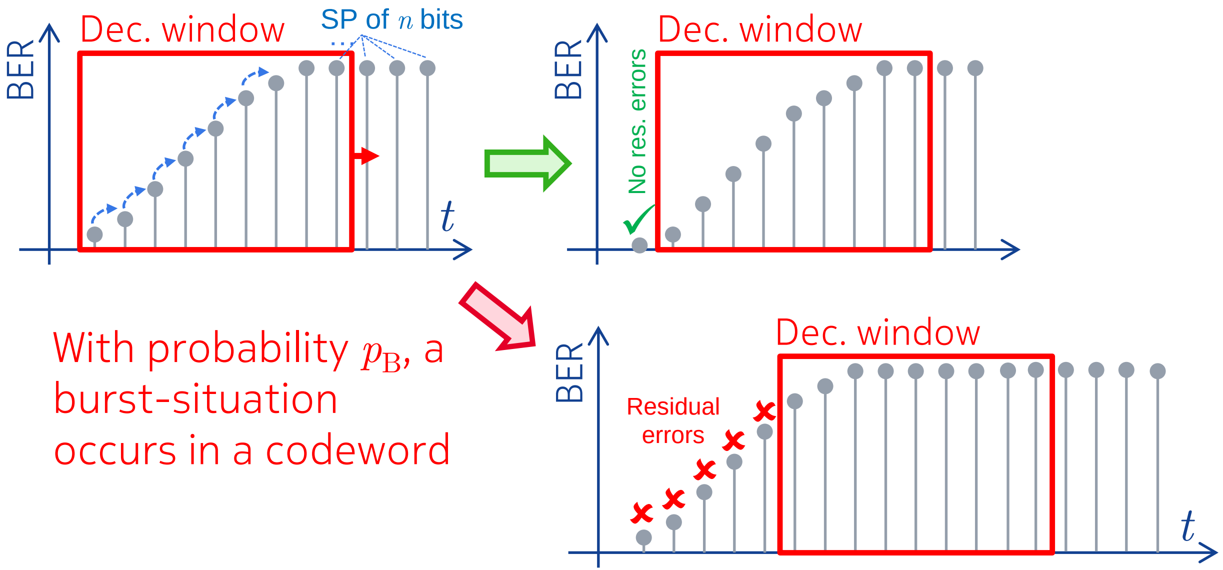
Decoding Window Loses Track of Decoding Wave



With probability p_{B}, a burst-situation occurs in a codeword

[SSA+16] L. Schmalen, D. Suikat, V. Aref, D. Rösener, "On the design of capacity approaching unit-memory spatially coupled LDPC codes for optical communications," *Proc. ECOC,* 2016

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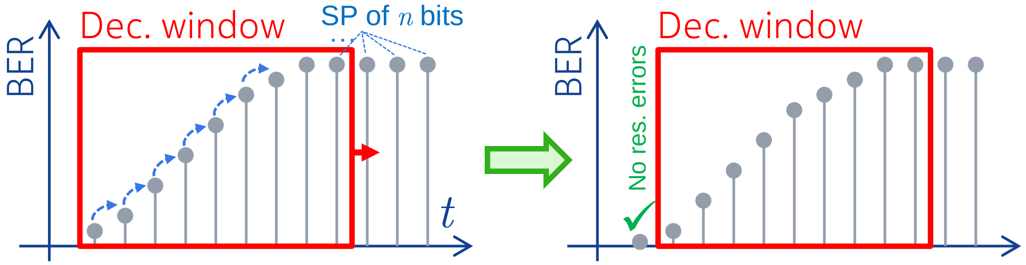
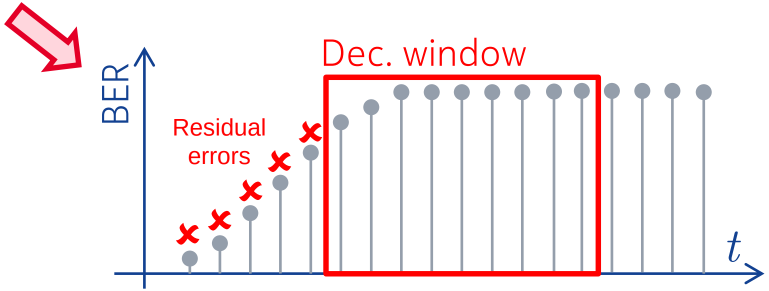


- Estimated p_B for the codes used in previous simulation:

With probability p_B, a burst-situation occurs in a codeword



E_b/N_0 (dB)	p_B for $L = 24$	p_B for $L = 99$
2.84	$1.0{\cdot}10^{-3}$	$5.6{\cdot}10^{-2}$
2.87	$1.6{\cdot}10^{-4}$	$6.3{\cdot}10^{-4}$
2.90	$2.0{\cdot}10^{-5}$	$8.9{\cdot}10^{-5}$
2.93	$3.2{\cdot}10^{-6}$	$1.3{\cdot}10^{-5}$

[SSA+16] L. Schmalen, D. Suikat, V. Aref, D. Rösener, "On the design of capacity approaching unit-memory spatially coupled LDPC codes for optical communications," *Proc. ECOC*, 2016

[SLO16] M. Stinner, L. Barletta, P. Olmos, "Finite-length scaling based on belief propagation for spatially coupled LDPC codes," *Proc. ISIT*, 2016

Stall Prediction

- **Solution 1**: Increase number of decoding operations
  - Will increase complexity, hence not recommended in high-throughput cases

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Stall Prediction

- **Solution 1**: Increase number of decoding operations
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- **Idea 2**: *Foresightful* Stall Prediction
    - Only increase number of iterations when needed
    - Prediction on channel output of current SP not easily possible
        - Current SP may lead to a stall few W_{D} positions away
        - See **[KCS+18]** for details and examples of cases where this doesn't work

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- **Idea 3**: *Stall Detection*
    - React when stall is about to happen

[KCS+18] K. Klaiber, S. Cammerer, L. Schmalen, S. ten Brink, "Avoiding Burst-like Error Patterns in Windowed Decoding of Spatially Coupled LDPC Codes," *Proc. ISTC*, 2018

Decoder Stall Detection

Decoder stall detection

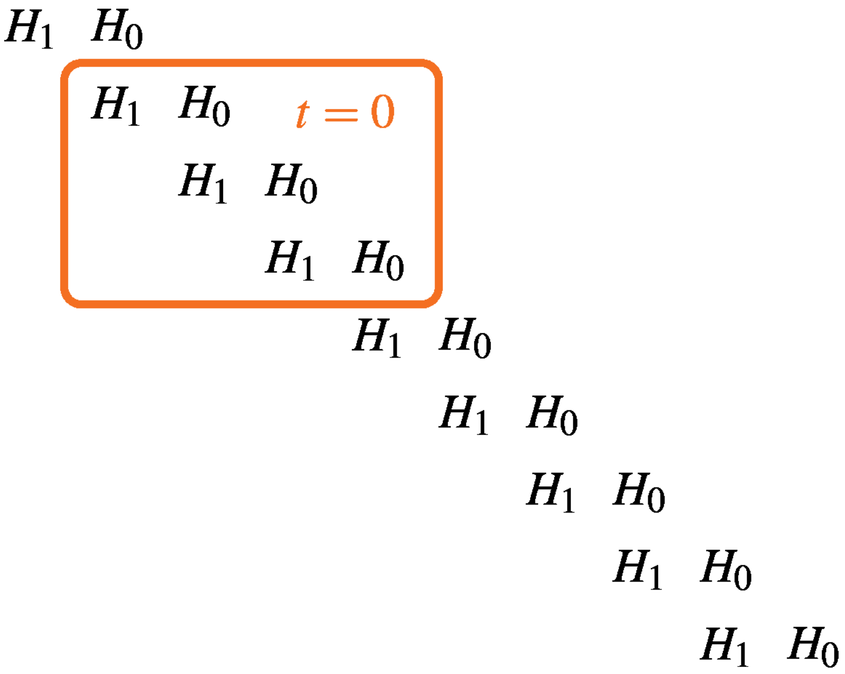
- Variant A: Stall detection based on fulfilled parity checks (HD)
- **Variant B**: Stall detection based on estimated BER (SD)
  - Estimate BER within SP inside windowed decoder as **[HIS00]**

$$\mathrm{BER}_i = \frac{1}{M}\sum_{k=1}^{M}\frac{1}{1+\exp(|L_{i,k}|)}$$

  - Use BER_i thresholds to estimate position of wave inside decoder
  - React by carrying out more iterations or shifting window (Strat. A, B, C)

[HIS00] P. Hoeher, I. Land, U. Sorger, "Log-likelihood values and Monte Carlo simulation-some fundamental results," *Proc. ISTC*, 2000
[KCS+18] K. Klaiber, S. Cammerer, L. Schmalen, S. ten Brink, "Avoiding Burst-like Error Patterns in Windowed Decoding of Spatially Coupled LDPC Codes," *Proc. ISTC*, 2018

Strategy A - Adaptive Iterations Decoder



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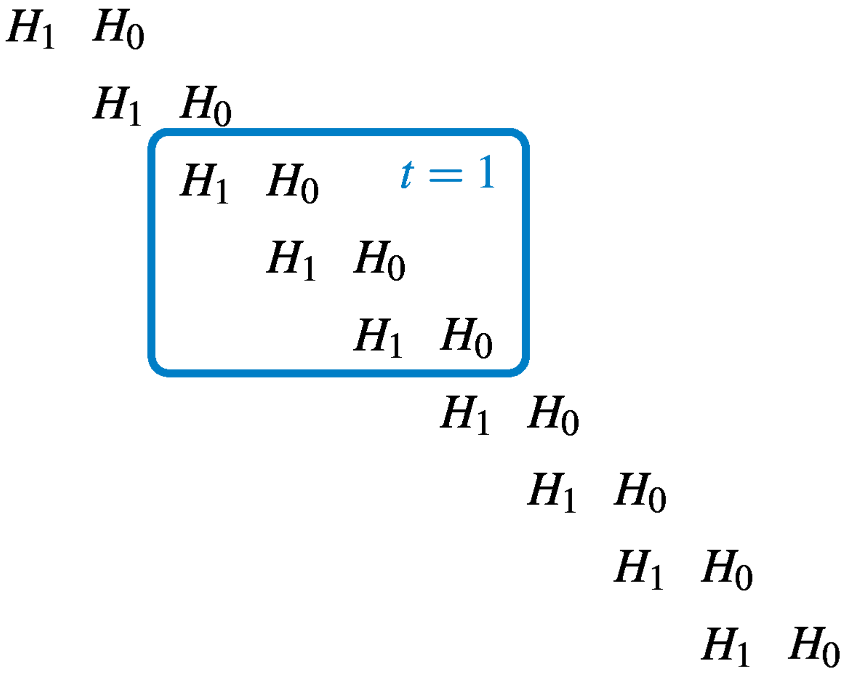

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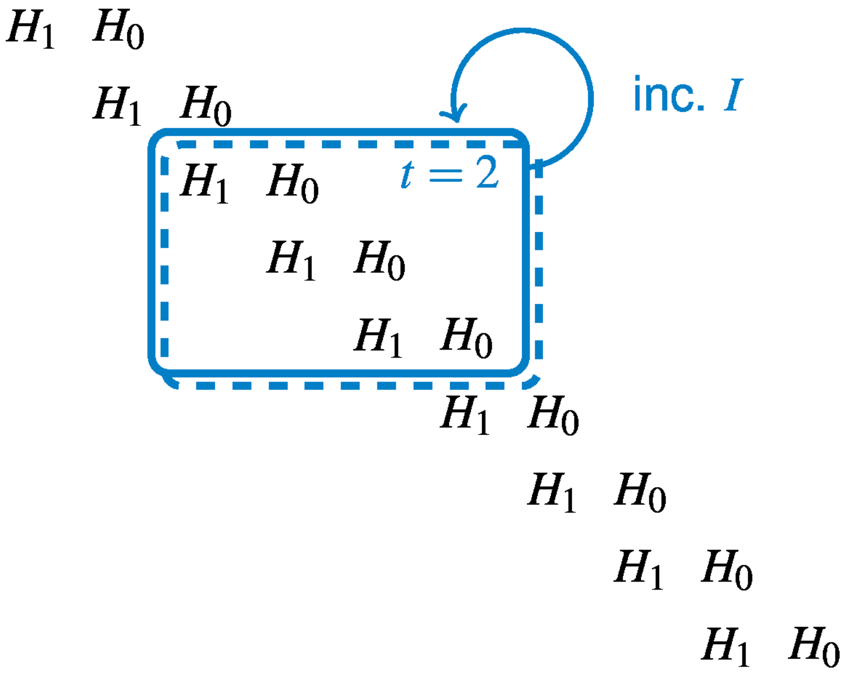

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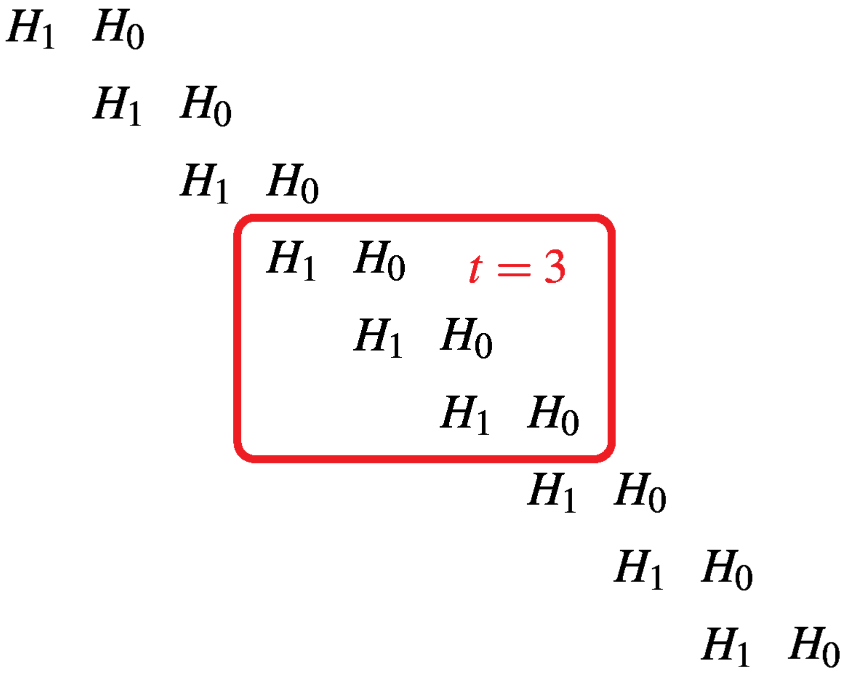

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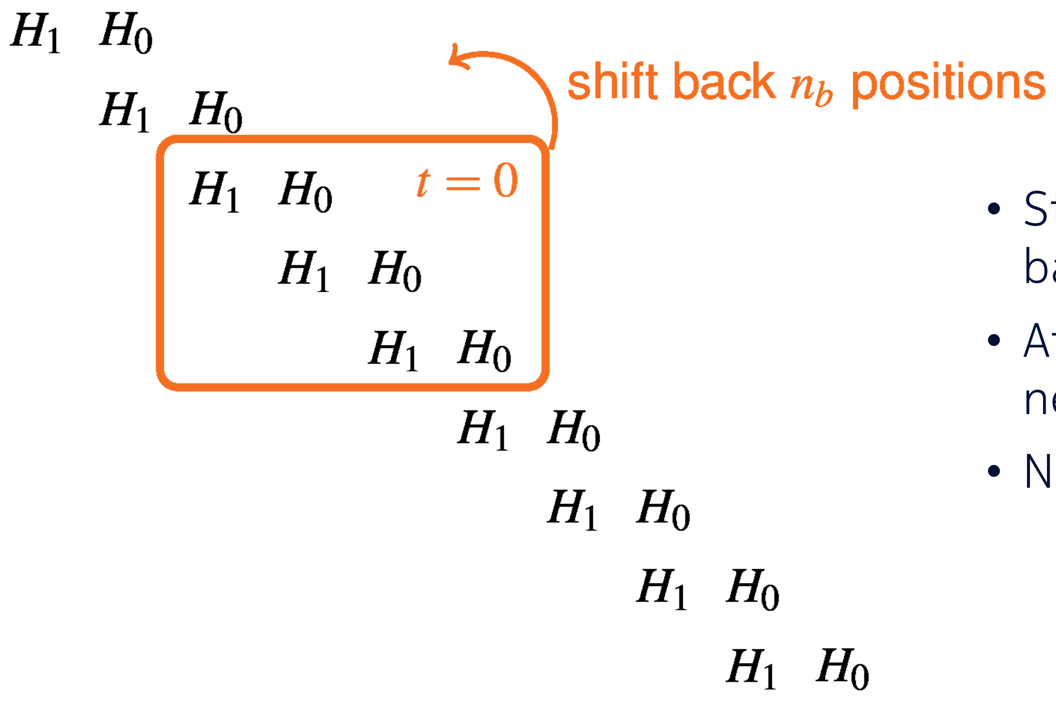

- Stall detected: Shift window backwards
- After I iterations, continue with next window
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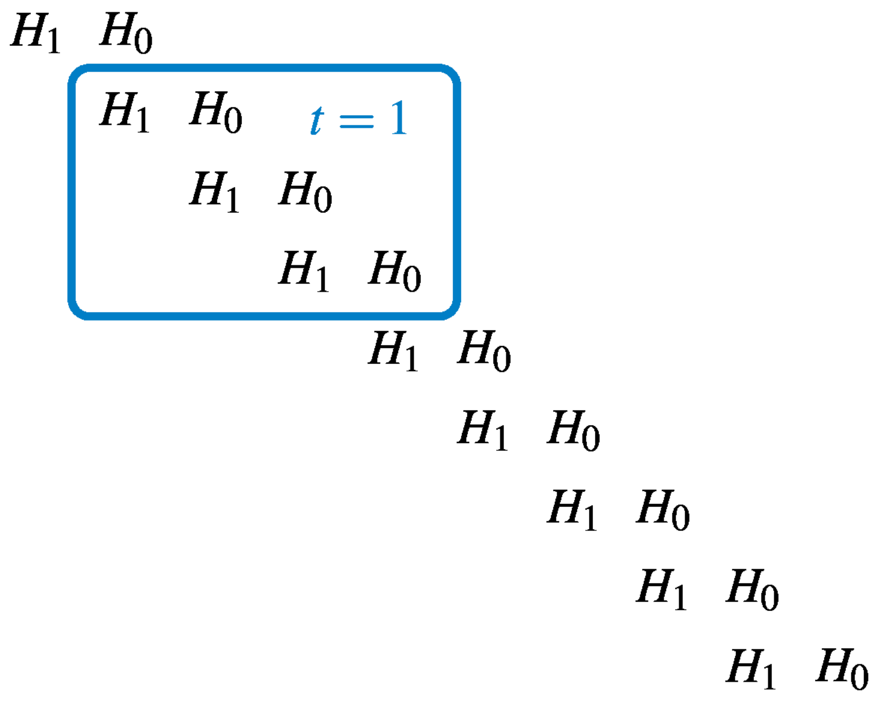

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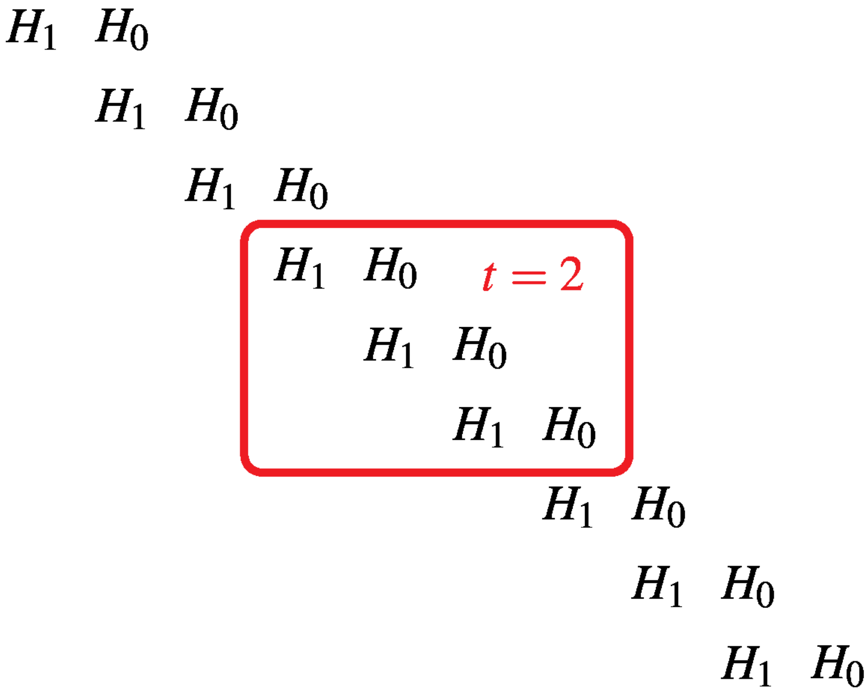

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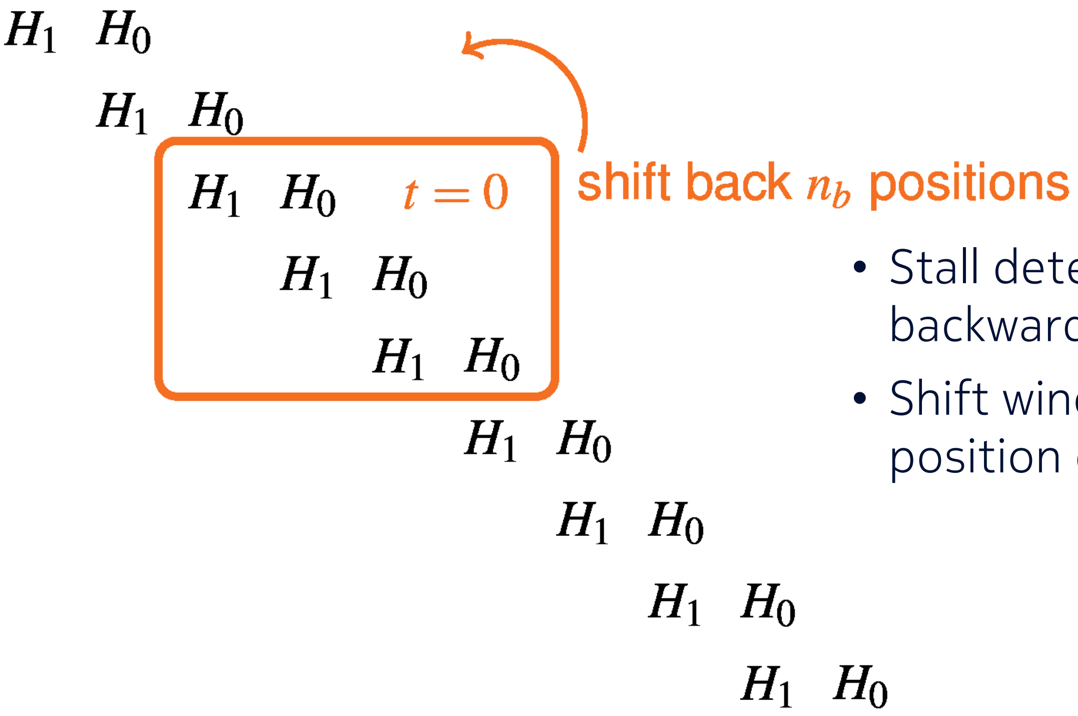

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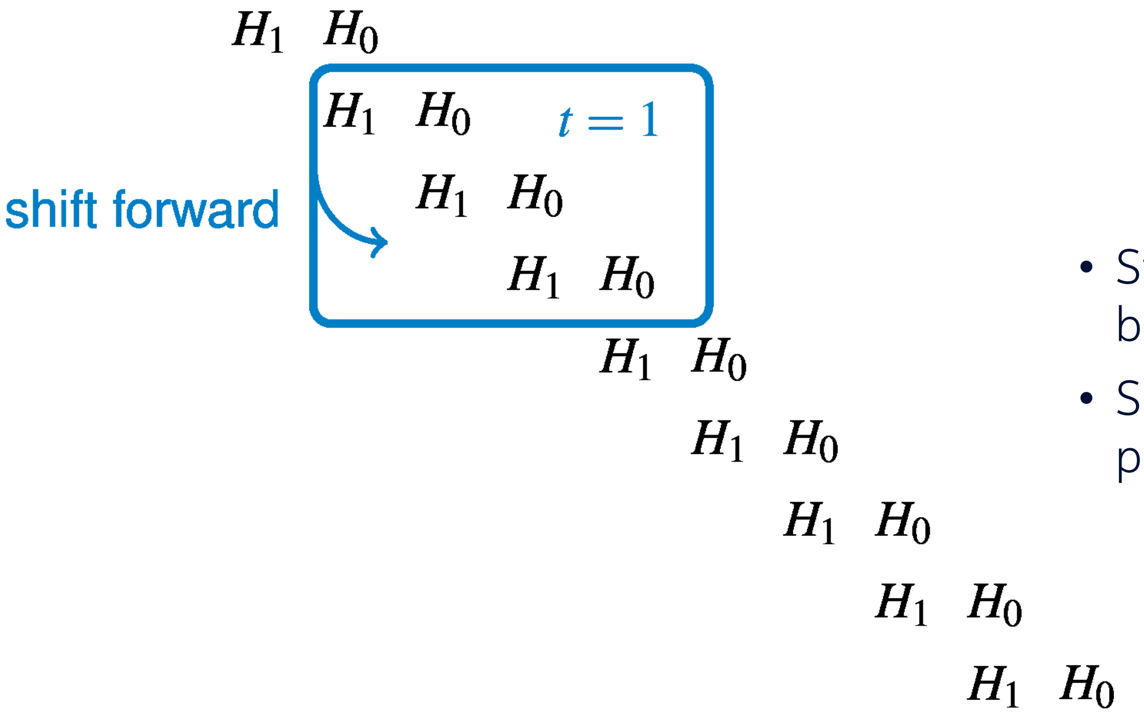

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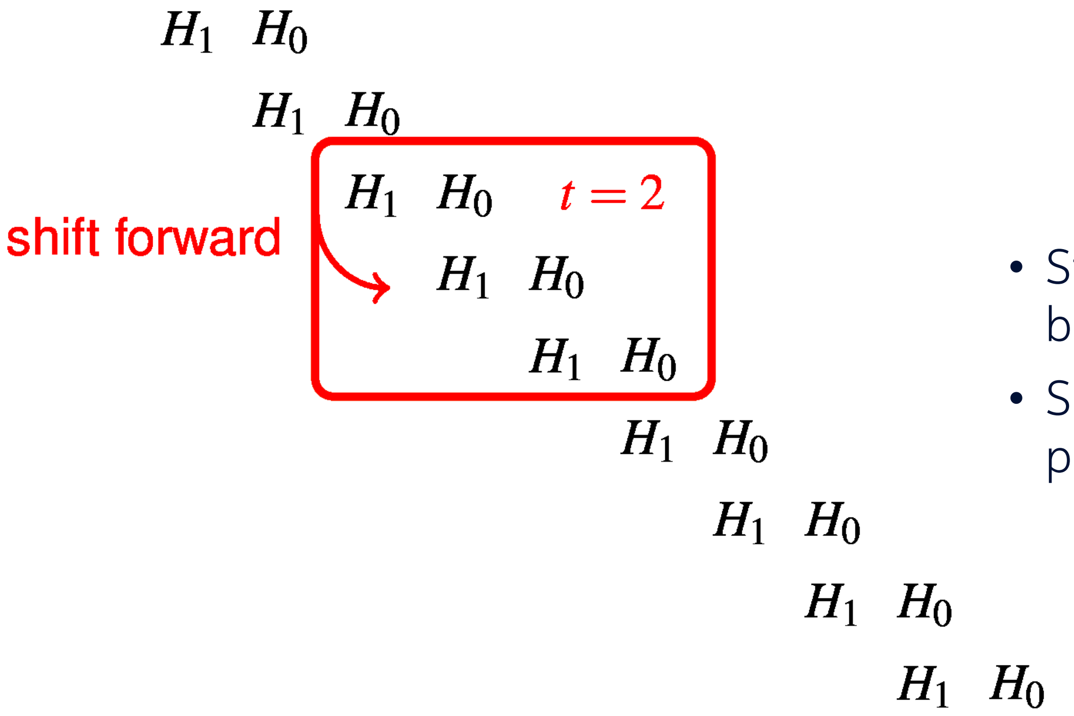

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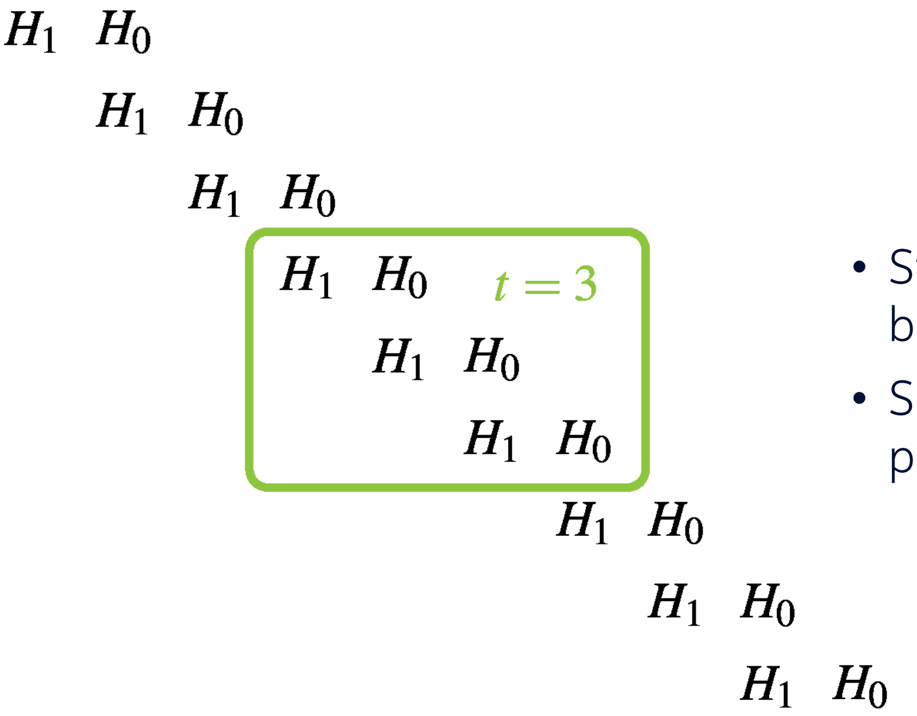

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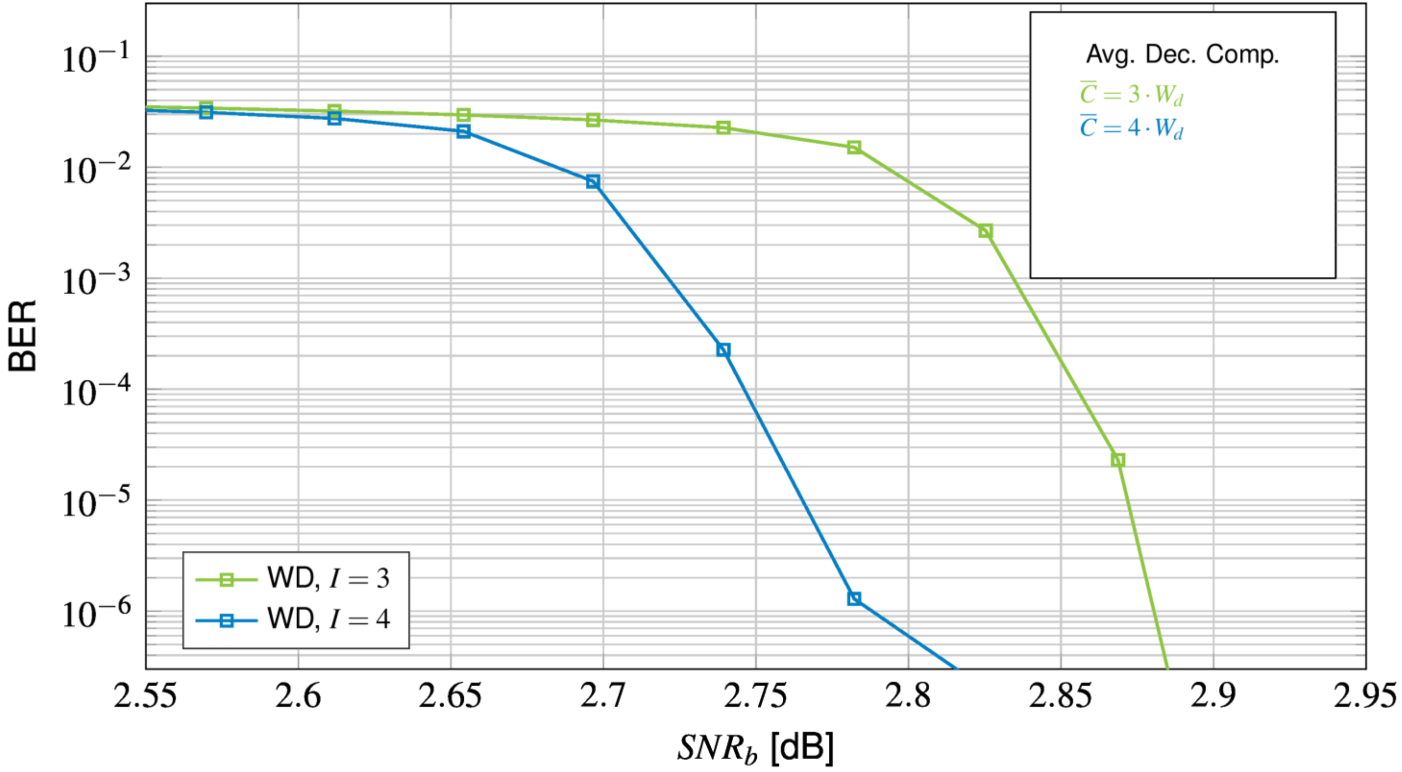
Simulation Results



- Average decoding complexity $\bar{C}$: average number of iterations per spatial position
- Code rate $R = 0.8$

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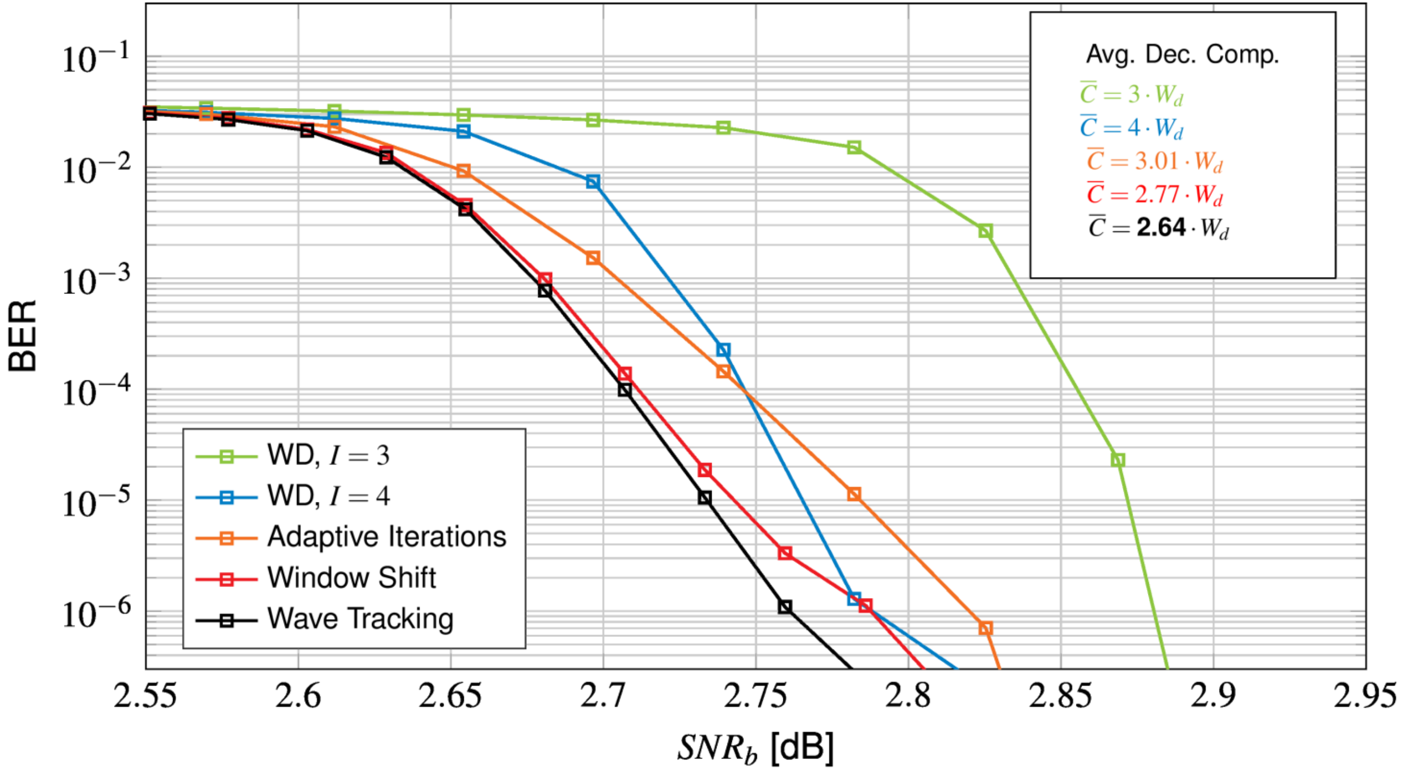

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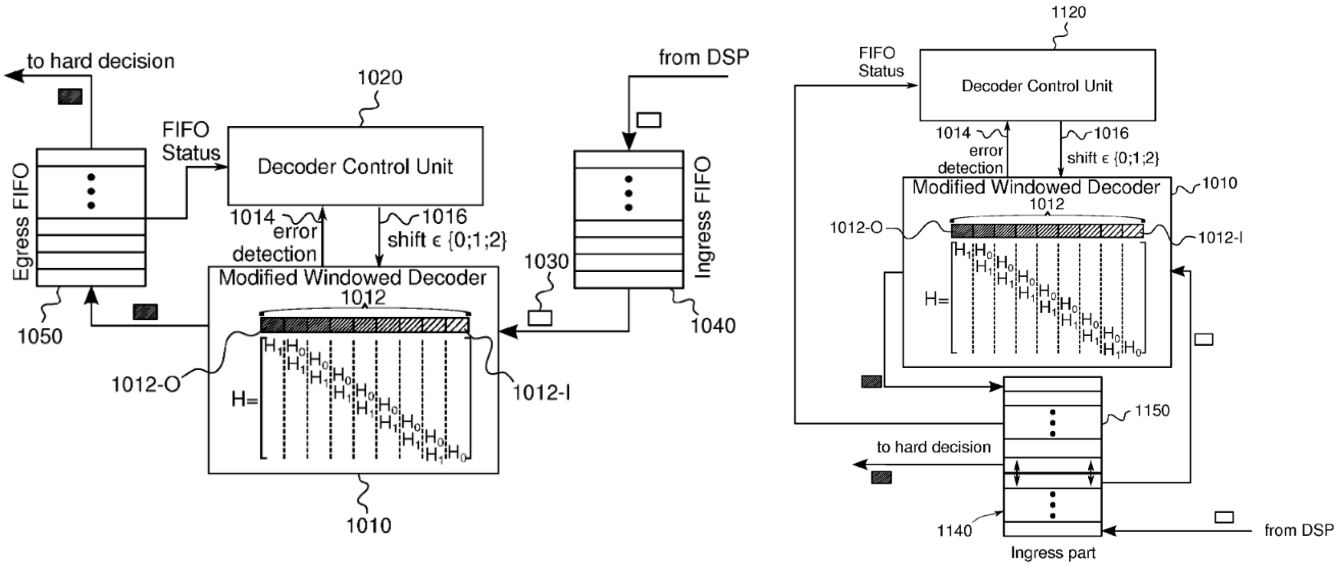

- Adaptive shifting can be implemented using some simple buffering and control **[SL14]**

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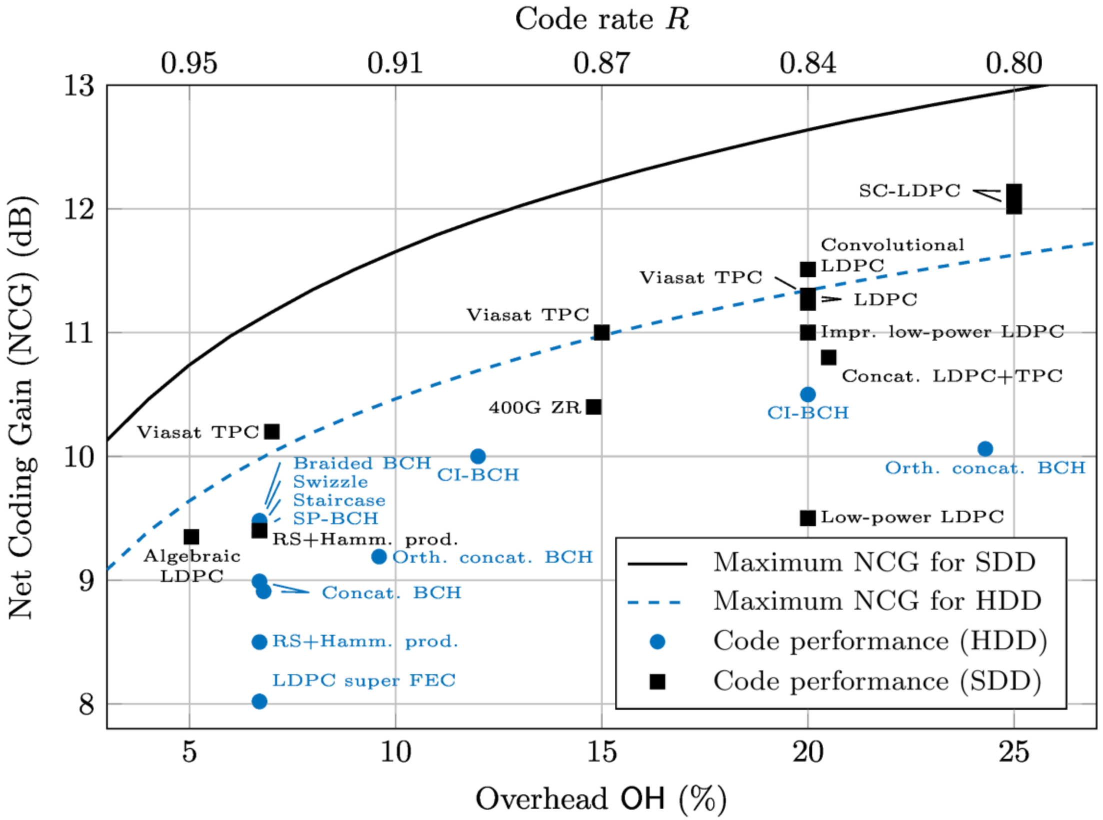
- Feasible coding scheme promising additional gains, but need HW architectures

Comparison of Coding Schemes in Optical Communications



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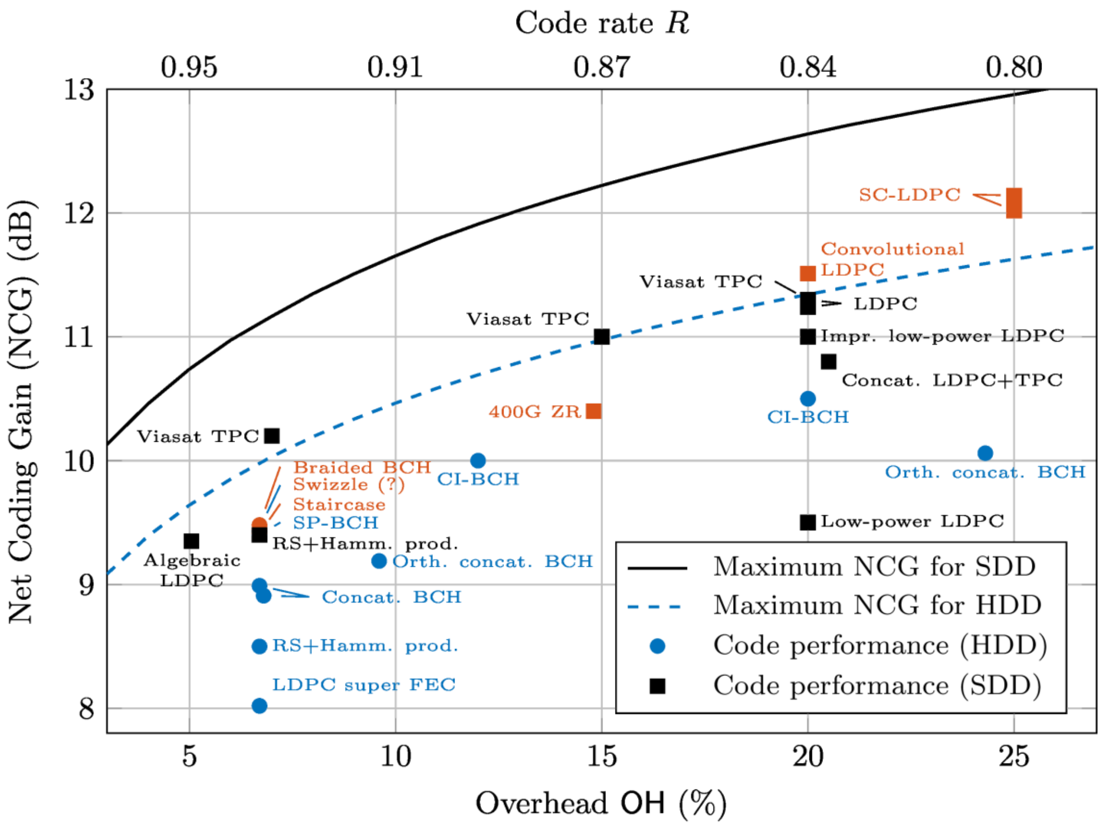

- State-of-the-art FEC schemes proposed for practical implementation
- Performance verified or reasonably estimated at 10^{-15} BER
- The best performing schemes are **spatially coupled codes**

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