28th February 2019

Oberpfaffenhofen Workshop on High Throughput Coding

Probabilistic Shaping with On-Off Keying

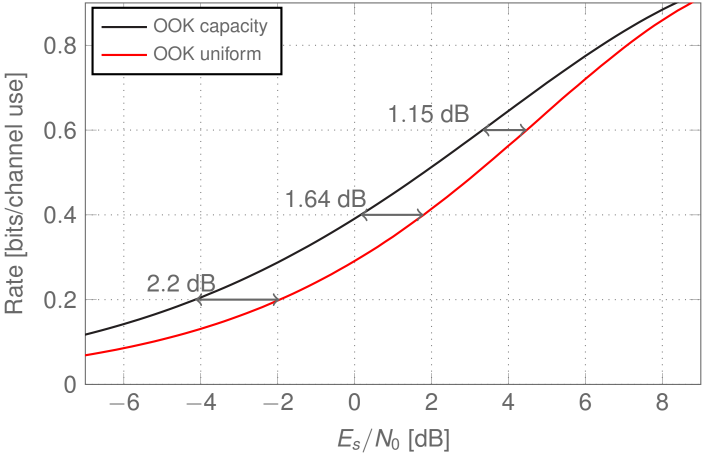
Balázs Matuz[1], Fabian Steiner[2], Alexandru Git[3]

[1] German Aerospace Center (DLR)
[2] Technical University of Munich (TUM)
[3] Mynaric AG

Motivation

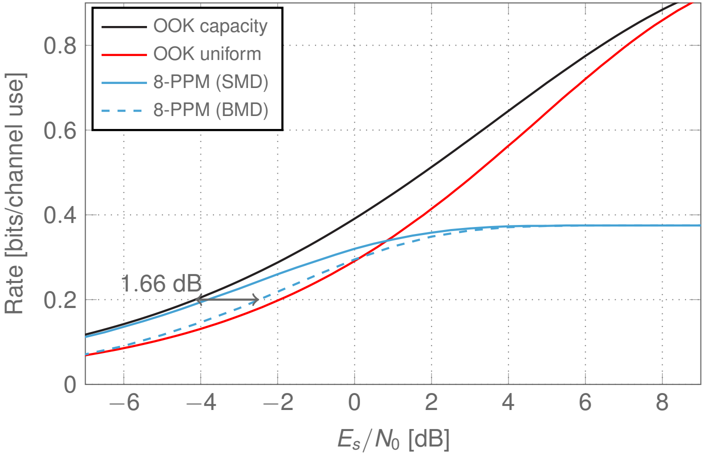
- Free-space optical (FSO) communications with increasing attention in the last years due to
  - large bandwidth/ high data rate
  - license free spectrum.
- Intensity modulation (IM) schemes, particularly for space links popular due to simple receivers.
- Commonly used IM schemes are
  - on-off keying (OOK)
  - pulse-position modulation (PPM).
- Additive white Gaussian noise (AWGN) channel model for direct detection.

OOK Rates



OOK capacity
OOK uniform
1.15 dB
1.64 dB
2.2 dB
0.8
0.6
0.4
0.2
0
−6
−4
−2
0
2
4
6
8
Rate [bits/channel use]
E_s/N_0 [dB]

PPM Rates


OOK capacity
OOK uniform
8-PPM (SMD)
8-PPM (BMD)
1.66 dB
Rate [bits/channel use]
E_s/N_0 [dB]
0
0.2
0.4
0.6
0.8
−6
−4
−2
0
2
4
6
8

How to reduce the gap to OOK capacity using binary codes?

$\rightarrow$ We try probabilistic shaping (PS) for OOK.

Outline

1. System Model
2. Probabilistic Shaping for OOK
3. Achievable Rates
4. Code Design
5. Numerical Results
6. Summary

Outline

Setup

- Consider transmission of OOK symbols X over an average power constrained AWGN channel with

$$Y = X + N$$

where $N \sim \mathcal{N}(0, \sigma^2)$, $X \in \mathcal{X} = \{0, A\}$, A being the amplitude of the OOK symbol.

- Average power constraint

$$\mathrm{E}\left[X^2\right] = A^2 P_X(A) = 1.$$

- Signal-to-noise ratio (SNR)

$$E_s/N_0 = 1/\sigma^2.$$

OOK Capacity

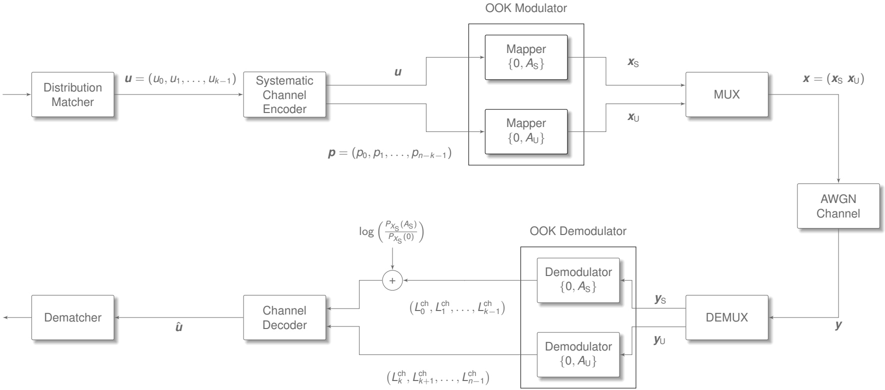
- Channel capacity under OOK modulation constraint

$$\mathsf{C} = \max_{P_X} \mathsf{I}(X;Y) \quad \text{subject to} \quad A^2 P_X(A) = 1.$$

- The optimization over $P_X = (P_X(0)\ P_X(A))$ is an optimization over both the probability weights and the support.
- In fact, this implies an optimization only over $P_X(A)$, since
  - $P_X(0) = 1 - P_X(A)$ and
  - $A = \sqrt{1/P_X(A)}$.
- For $P_X = (\frac{1}{2}\ \frac{1}{2})$ we obtain rates for uniform OOK.

Outline

Transceiver Overview



DLR

Shaping and Channel Coding

- Distribution (constant composition) matcher encodes k' uniformly distributed bits into a length k shaped information bit sequence $\boldsymbol{u}$

$$\mathrm{R}_{\mathrm{DM}} = \frac{k'}{k}.$$

- Systematic channel encoder generates n code bits $\boldsymbol{c}$ out of k information bits $\boldsymbol{u}$ with

$$\underbrace{\boldsymbol{c}}_{(\boldsymbol{u}|\boldsymbol{p})} = \boldsymbol{u}\underbrace{\boldsymbol{G}}_{(\boldsymbol{I}|\boldsymbol{P})}.$$

DLR

Modulation

- We distinguish between OOK modulated information symbols X_{S} and modulated parity symbols X_{U} with
  - $X_{\mathrm{S}} \in \{0, A_{\mathrm{S}}\}$
  - $X_{\mathrm{U}} \in \{0, A_{\mathrm{U}}\}$
  - $P_{X_{\mathrm{S}}} = P_U$
  - $P_{X_{\mathrm{U}}} \approx (\frac{1}{2}\ \frac{1}{2})$.
- We consider two cases:
  - i $A = A_{\mathrm{S}} = A_{\mathrm{U}}$
  - ii A_{S} possibly different from A_{U}.
- Reverse operations at receiver (exploiting knowledge P_U).

Outline

Time Sharing Rate

- Transmission rate:

$$\mathrm{R_{TX}} = \mathrm{R_{DM}} \cdot \mathrm{R_C} \approx \mathrm{H}\left(X_{\mathrm{S}}\right) \cdot \mathrm{R_C}.$$

- $\mathrm{R_{TX}}$ is directly related to $P_{X_{\mathrm{S}}}(A_{\mathrm{S}}) = \mathrm{p}_1$ via

$$\mathrm{p}_1 = \mathrm{H}^{-1}\left(\frac{\mathrm{R_{TX}}}{\mathrm{R_C}}\right). \tag{1}$$

- An achievable rate of the time sharing (TS) scheme is given by

$$\mathrm{R_{TS}} = \mathrm{R_C}\mathrm{I}(X_{\mathrm{S}}; Y_{\mathrm{S}}) + (1 - \mathrm{R_C})\mathrm{I}(X_{\mathrm{U}}; Y_{\mathrm{U}}).$$

Achievable Rates for Fixed $\mathrm{R_C}$

- Case i: $A_\mathrm{S} = A_\mathrm{U} = A$:

$$\mathrm{R_{TS}}{}_1^* = \max_{\mathrm{p}_1} \mathrm{R_{TS}}$$

$$\text{subject to} \quad \left(\mathrm{R_C p_1} + (1-\mathrm{R_C})\frac{1}{2}\right) A^2 = 1$$

with

$$A = 1/\sqrt{\mathrm{R_C p_1} + (1-\mathrm{R_C})/2}.$$

- Case ii: A_S and A_U possibly different:

$$\mathrm{R_{TS}}{}_2^* = \max_{\mathrm{p}_1, A_\mathrm{S}} \mathrm{R_{TS}}$$

$$\text{subject to} \quad \mathrm{R_C p_1} A_\mathrm{S}^2 + (1-\mathrm{R_C})\frac{1}{2} A_\mathrm{U}^2 = 1$$

with

$$A_\mathrm{U} = \sqrt{(1-\mathrm{R_C p_1} A_\mathrm{S}^2)/((1-\mathrm{R_C})/2)}.$$

Pragmatic Approach

- Previous optimization provides for each $\mathrm{R_C}$ and E_s/N_0 an achievable rate $\mathrm{R_{TS}}^*$, such that $\mathrm{R_{TX}} \leq \mathrm{R_{TS}}^*$.
- In practice, for a fixed $\mathrm{R_{TX}}$ we are interested in the code rate $\mathrm{R_C}^*$ which requires the lowest E_s/N_0.
- Best code rate $\mathrm{R_C}^*$ among $\mathcal{R}_\mathrm{C} = \{0.25, 0.33, 0.5, 0.67, 0.75, 0.8, 0.9\}$.

$\mathrm{R_{TX}}$	$\mathrm{R_C}^*$ case i	$\mathrm{R_C}^*$ case ii
0.2	0.33	0.67
0.25	0.5	0.67
0.33	0.5	0.67
0.5	0.67	0.67
0.67	0.75	0.8
0.75	0.8	0.8
0.85	0.9	0.9

Outline

Protograph Low-Density Parity-Check (LDPC) Codes

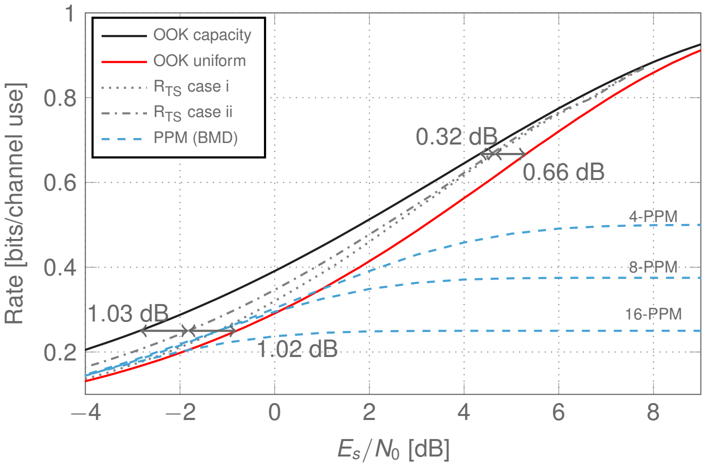
- Protographs can be seen as a compact representation of (an ensemble of) structured LDPC codes.
- Protograph structure yields performance and hardware-friendly implementations.
- Standard tools to design good protographs:
  - Optimization algorithm that evaluates a cost function and tries to minimize it.
  - Cost function associated to protograph: iterative decoding threshold.
- Extension of standard tools to consider prior (shaping).

Definition (Iterative decoding threshold)

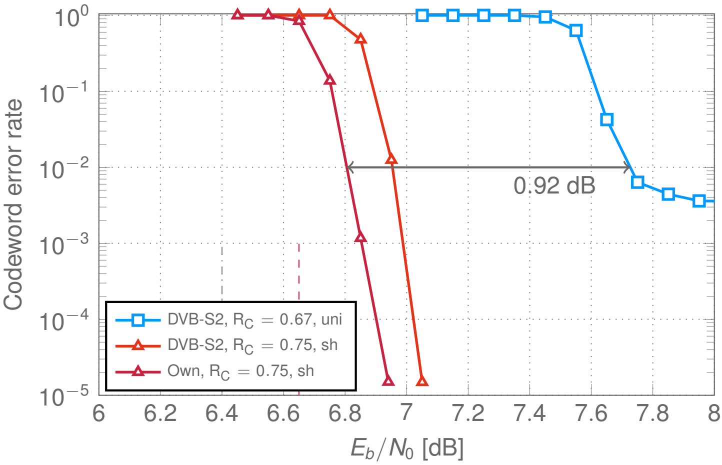
Lowest E_s/N_0, such that probability of symbol error after decoding vanishes as the block length and number of iterations go to infinity.

Outline

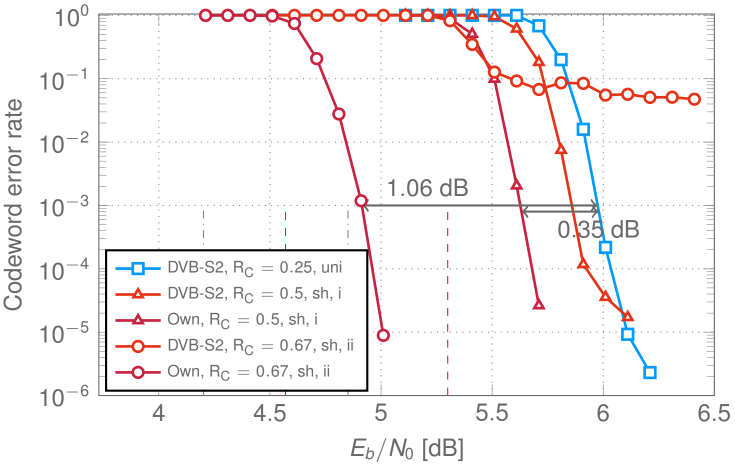
Achievable Rates



CER versus E_b/N_0 for $\mathrm{R_{TX}} = 0.67$ bpcu, Case i



CER versus E_b/N_0 for $\mathrm{R_{TX}} = 0.25$ bpcu, Cases i, ii



Outline

Summary

- We propose a transceiver architecture for shaped OOK and perform a dedicated code design.
- Use of binary LDPC codes with standard decoders.
- No need for iterations between demodulator and decoder.
- Only minor modifications at the receiver/transmitter required yielding visible gains:
  - 0.9 dB gain w.r.t. uniform OOK at $R_{TX} = 0.67$ bpcu for case i
  - 1.1 dB gain w.r.t. uniform OOK at $R_{TX} = 0.25$ bpcu for case ii.

Thank you for your attention!

Questions?

Contact:
Balázs Matuz
German Aerospace Center (DLR)
balazs.matuz@dlr.de

Protographs

In the following, we provide the optimized base matrices $\boldsymbol{B}_1$, $\boldsymbol{B}_2$ and $\boldsymbol{B}_3$ for code with $\mathrm{R_C} = 0.5, \mathrm{R_C} = 0.67$ and $\mathrm{R_C} = 0.75$, respectively. For $\boldsymbol{B}_1$ the first column is punctured.

$$\boldsymbol{B}_1 = \begin{pmatrix} 3 & 0 & 0 & 1 & 2 & 0 & 0 \\ 1 & 0 & 2 & 0 & 0 & 1 & 2 \\ 3 & 0 & 1 & 2 & 2 & 1 & 1 \\ 2 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \qquad \boldsymbol{B}_2 = \begin{pmatrix} 1 & 0 & 1 & 0 & 2 & 0 & 0 & 0 & 3 \\ 4 & 2 & 3 & 2 & 4 & 2 & 1 & 1 & 3 \\ 3 & 1 & 4 & 1 & 1 & 1 & 2 & 1 & 1 \end{pmatrix}$$

$$\boldsymbol{B}_3 = \begin{pmatrix} 4 & 0 & 1 & 4 & 0 & 3 & 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 1 & 2 & 3 & 1 & 1 & 1 & 2 & 2 & 1 & 3 & 2 \\ 3 & 2 & 1 & 4 & 1 & 2 & 2 & 1 & 2 & 1 & 1 & 1 \end{pmatrix}$$