Performance-Complexity Tradeoffs in Concatenated LDPC-Staircase Codes

Masoud Barakatain and Frank R. Kschischang
Department of Electrical & Computer Engineering
University of Toronto
{frank}@ece.utoronto.ca

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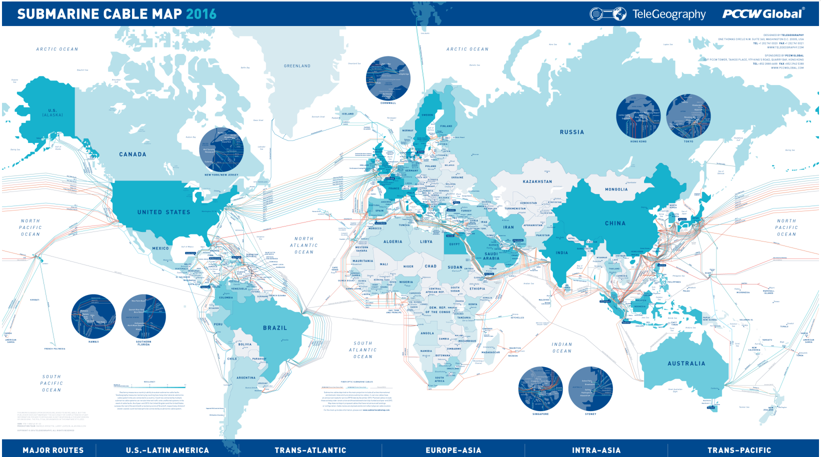
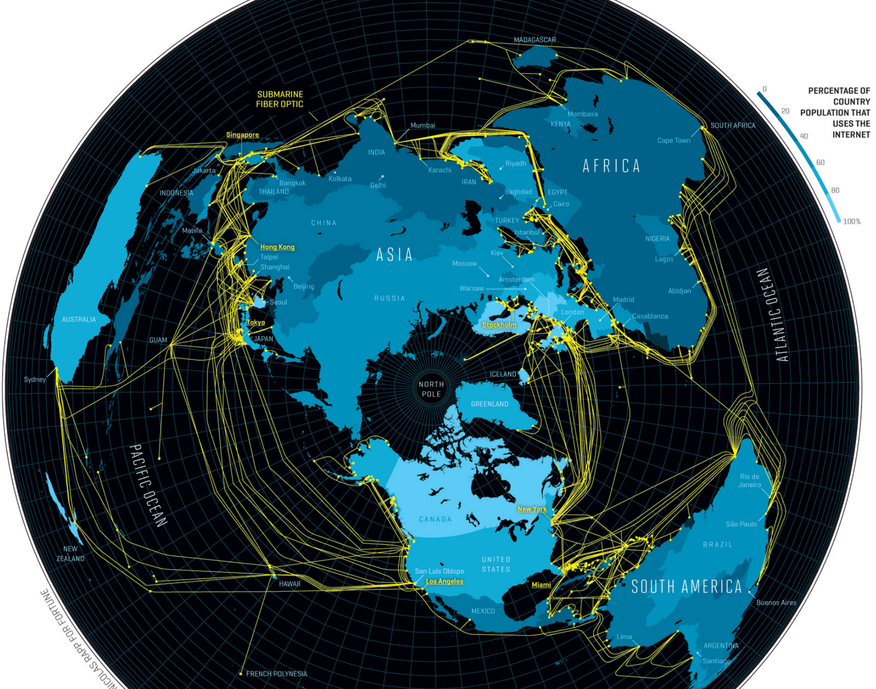
Motivation

Optical Transport Networks (OTNs)

- Long-haul fiber-optic communications
- Carrying trans- and inter-continental IP traffic and telecommunications





Source: Nicolas Rapp, *Fortune Magazine*, July 23, 2012

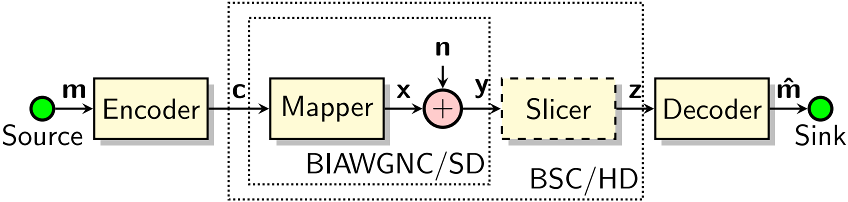
Forward Error Correction (FEC) for OTN

Decoder Requirements

- High throughput (100–400 Gbit/s, or more, per wavelength)
- Low complexity
- High reliability, $P_e < 10^{-15}$.

Channel Models for OTN

For the purposes of code design, although the true channel is nonlinear, we adopt a simplistic (post-nonlinearity compensation) channel model:



Justifications

- Standard models for prior work on OTN FEC design
- Gray-labelled DP-QPSK with impairment compensation

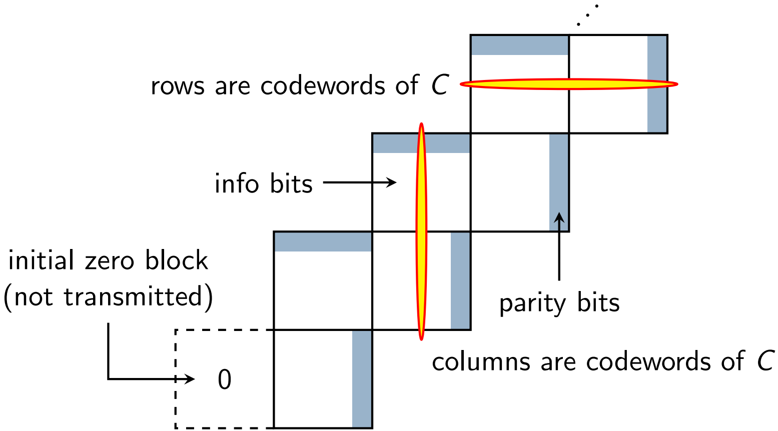
Staircase Codes

Staircase Codes

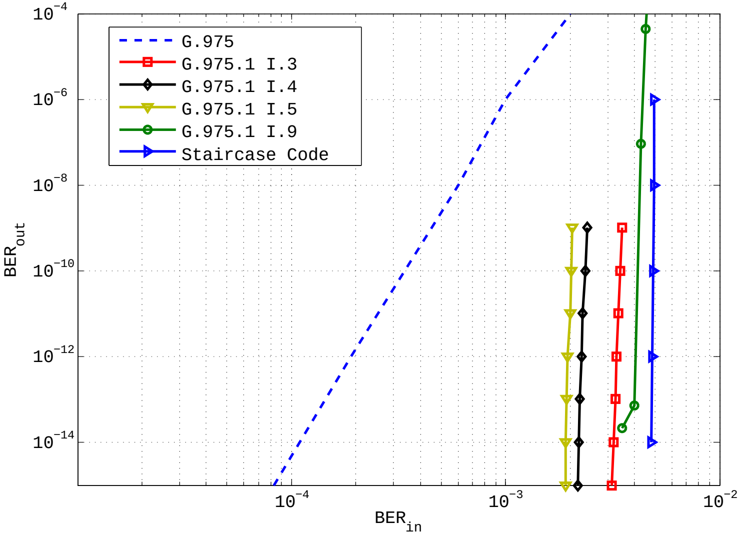
- Introduced in B. P. Smith, A. Farhood, A. Hunt, FRK, and J. Lodge, "Staircase Codes: FEC for 100 Gb/s OTN," *J. Lightwave Technology*, vol. 30, Jan. 2012, pp. 110–117.
- Spatially-coupled product codes
- Iterative algebraic decoding in a sliding window
- High-rate (low-complexity) BCH component decoders
- Low error floors with analytic bounds
- Adopted in standards:
  - OIF 400ZR (400 Gb/s, coherent), 2017 (inner Hamming, outer staircase)
  - ITU-T G.709.2 (OTU4 long-reach interface), 2018

Staircase Codes

• Each square "staircase block" is of size $m \times m$ bits. • C is a binary systematic code of length $2m$ (e.g., a t-error-correcting BCH code). • Iterative decoding occurs within a sliding window: "oldest" block shifted out when "newest block" is filled.



Staircase Code Performance (BSC)



Performance of G.709-compatible rate 239/255 staircase code, using a $t = 3$ shortened BCH component code.

Selected Related Work

- Benjamin P. Smith, FRK, "A Pragmatic Coded Modulation Scheme for High-Spectral-Efficiency Fiber-Optic Communications", *JLT*, 2012.
- Jewong Yeon, Hanho Lee, "High-performance iterative BCH decoder architecture for 100 Gb/s optical communications", *ISCAS*, 2013.
- Yung-Yih Jian, Henry D. Pfister, Krishna R. Narayanan, Raghu Rao, Raied Mazahreh, "Iterative hard-decision decoding of braided BCH codes for high-speed optical communication", *GLOBECOM*, 2013.
- Christian Häger, Alexandre Graell i Amat, Fredrik Brännström, Alex Alvarado, Erik Agrell, "Terminated and Tailbiting Spatially Coupled Codes With Optimized Bit Mappings for Spectrally Efficient Fiber-Optical Systems", *JLT*, 2015.
- Carlo Condo, Francois Leduc-Primeau, Gabi Sarkis, Pascal Giard, Warren J. Gross, "Stall pattern avoidance in polynomial product codes", *GlobalSIP*, 2016.
- Pratana Kukieattikool, Norbert Goertz, "Variable-rate staircase codes with RS component codes for optical wireless transmission", *Transactions on Emerging Telecommunications Technologies*, 2016.
- Jakob Dahl Andersen, Knud J. Larsen, Christian Bering Bøgh, Søren Forchhammer, Francesco Da Ros, Kjeld Dalgaard, Shajeel Iqbal, "A configurable FPGA FEC unit for Tb/s optical communication", *IEEE* ICC, 2017.
- Christian Häger, Henry D. Pfister, Alexandre Graell i Amat, Fredrik Brännström, "Density Evolution for Deterministic Generalized Product Codes on the Binary Erasure Channel at High Rates", *IEEE Trans. Info. Theory*, 2017.
- Yung-Yih Jian, Henry D. Pfister, Krishna R. Narayanan, "Approaching Capacity at High Rates With Iterative Hard-Decision Decoding", *IEEE Trans. Info. Theory*, 2017.
- Lukas Holzbaur, Hannes Bartz, Antonia Wachter-Zeh, "Improved decoding and error floor analysis of staircase codes", *Designs, Codes and Cryptography*, 2018.
- Alireza Sheikh, Alexandre Graell i Amat, Gianluigi Liva, Fabian Steiner, "Probabilistic Amplitude Shaping With Hard Decision Decoding and Staircase Codes", *JLT*, 2018.
- Christian Häger, Henry D. Pfister, "Approaching Miscorrection-Free Performance of Product Codes With Anchor Decoding", *IEEE Trans. Commun.*, 2018.
- Christoffer Fougstedt, Per Larsson-Edefors, "Energy-Efficient High-Throughput VLSI Architectures for Product-Like Codes", *JLT*, 2019.

Staircase Code Library

R_{sc}	OH	M	p_{sc}	NCG Gap (dB)
15/16	6.67	704	5.0214×10^{-3}	0.462
10/11	10.00	440	7.5402×10^{-3}	0.587
8/9	12.50	360	9.2886×10^{-3}	0.684
6/7	16.67	252	1.2477×10^{-2}	0.784
5/6	20.00	216	1.4404×10^{-2}	0.920

- Staircase codes with $t = 4$ shortened BCH component codes.
- p_{sc} denotes the BSC crossover probability corresponding to an output BER of 10^{-15}.
- Overhead $\text{OH} = (1/R - 1) \times 100\%$
- $\text{NCG Gap (dB)} = 20 \log_{10} \text{erfc}^{-1}(2p_{sc}) - 20 \log_{10} \text{erfc}^{-1}(2H^{-1}(1 - R_{sc}))$
- Excellent HD codes, but they do not exploit soft information

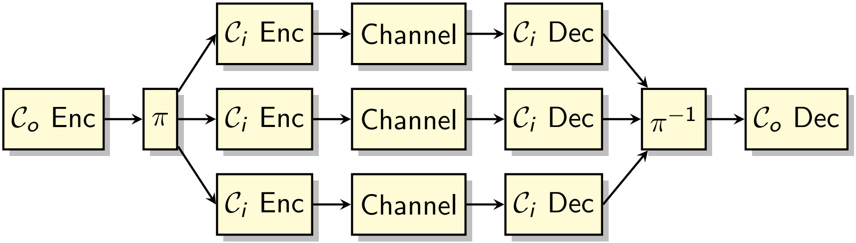
[L. M. Zhang and FRK, "Staircase Codes With 6% to 33% Overhead," *J. Lightwave Technology*, vol. 32, no. 10, pp. 1999-2002, May 2014.]

Motivation

Key Question

Can staircase codes be leveraged in an efficient, low-complexity, error-correction scheme for the SD channel in OTN systems?

Concatenation



Elements

- $\mathcal{C}_o$ – Staircase outer code
- π – Interleaver to reduce correlation between bit errors
- $\mathcal{C}_i$ – Low-complexity soft-decision decodable inner code

$\mathcal{C}_i$: Error-reducing, couples the channel to the staircase code

Concatenated LDPC-Staircase Code

Key Ideas

- LDPC code with SD decoding for error **reduction**:

$$p_{\mathrm{ch}} \sim 10^{-2} \rightarrow p_{\mathrm{sc}} \sim 10^{-3}$$

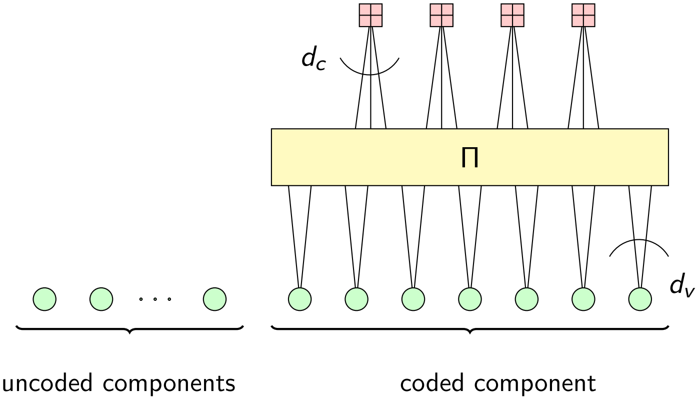
- Staircase code with HD decoding for error **correction**:

$$p_{\mathrm{sc}} \sim 10^{-3} \rightarrow 10^{-15}$$

Advantages

- Low-complexity design of LDPC ensemble
- Staircase code has low and analytical error-floor

The LDPC Inner-Code



Tanner graph of the inner-code ensemble. Degree distributions to be designed – degree-zero and degree-one nodes are allowed!

Design Parameters

Outer Code

- Staircase code parameters: R_{sc}, P_{sc}

Inner code

- Portion of uncoded bits: L_0
- Variable node degree distribution: $\lambda(x) = \sum_i \lambda_i x^{i-1}$
- Concentrated check node degree distribution: $\bar{d}_c$
- Degree-one variable nodes per check node: ν

Concatenated Code Design

Given an overall rate R_{cat} and a staircase code library:

Design Objective

Determine the Pareto frontier between SNR (or NCG) and complexity (η) for concatenated coding scheme.

Complexity Score

Given sparse-graph code with:

- Number of edges: E
- Maximum iterations: I
- Number of information bits: K
- Post-processing complexity, per information bit, in outer code decoding: P (very small in comparison)

Decoding Complexity

$$\eta = \frac{EI}{K} + P = \frac{1 - R_{in}}{R_{sc} R_{in}} (\bar{d}_c - \nu) I + P$$

Complexity-Optimized Design – Minimum Iterations

Estimating Number of Iterations

Smith *et al.* derived an approximation for the number inner decoding iterations, based on the code EXIT function $f_{\boldsymbol{\lambda}}(p)$:

$$I \approx \int_{p_t}^{p_0} \frac{dp}{p \log\left(\frac{p}{f_{\boldsymbol{\lambda}}(p)}\right)}$$

[B. P. Smith, M. Ardakani, W. Yu and FRK, "Design of Irregular LDPC Codes with Optimized Performance-Complexity Tradeoff," *IEEE Trans. Commun.*, vol. 58, no. 2, pp. 489-499, Feb 2010.]

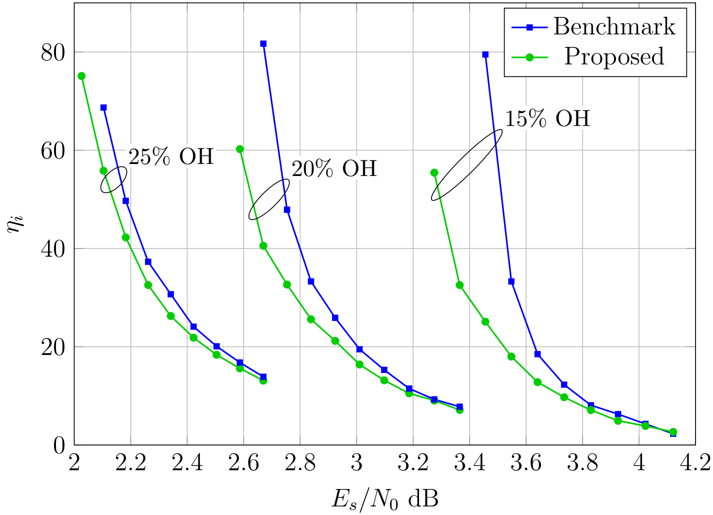
Optimization Problem Formulation

Given R_{cat}, iterate over R_{sc},d_c, ν, and SNR, minimize

$$\eta = \frac{1 - R_{\rm in}}{R_{\text{sc}} R_{\rm in}} (d_c - \nu) \int_{p_{t,\text{SNR}}}^{p_{0,\text{SNR}}} \frac{\mathrm{d}p}{p \log\left(\frac{p}{f_{\boldsymbol{\lambda},\text{SNR}}(p)}\right)}$$

s.t. $\sum_{i=1}^{D_v} \frac{\lambda_i}{i} \geq \frac{1 - L_0}{d_c(1 - R_{\rm in})}$	achieve rate R_{in}
$0 \leq \lambda_i \quad \forall i \in \{1, \ldots, D_v\}$	no negative degrees
$\sum_{i=1}^{D_v} \lambda_i = 1$	valid degree distribution
$\lambda_1 d_c = \nu$	desired degree-one nodes
$0 \leq L_0 \leq L_0^{\max}$	degree-zero nodes in range
$f_{\boldsymbol{\lambda},\text{SNR}}(p) < p \quad \forall p \in [p_{t,\text{SNR}}, p_{0,\text{SNR}}]$	EXIT chart open
$L_0 p_{0,\text{SNR}} + (R_{\rm in} - L_0) P_{t,\text{SNR}} \leq R_{\text{in}} p_{\rm sc}$	staircase BER achieved.

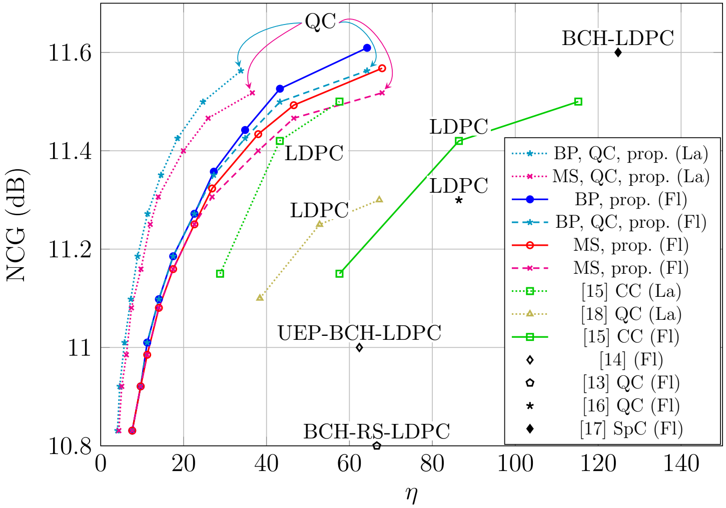
Results – Pareto Frontiers



Pareto frontiers, for OH $\in \{0.15, 0.2, 0.25\}$.

All designs chose the highest rate outer staircase codes!

Comparison



NCG and complexity of the best known FEC schemes at 20% OH.

Multi-SNR Code Optimization

Flexibility towards channel quality

- A *design set* $S = \{\mathrm{SNR}_1, \mathrm{SNR}_2, \ldots, \mathrm{SNR}_n\}$
- Obtain the complexity-optimized concatenated code for SNR_j, and for all $j \in \{1, 2, \ldots, n\}$
- Given η_j^*'s, design a code with minimized maximum complexity deviation from η_j^*'s

Optimization Problem Formulation – Multi-SNR

Given R_{cat}, S, iterate over R_{sc},d_c, ν, minimize

$$\gamma = \max\left(\frac{\eta_1}{\eta_1^*}, \frac{\eta_2}{\eta_2^*}, \ldots, \frac{\eta_n}{\eta_n^*}\right),$$

subject to:

$\sum_{i=1}^{D_v} \frac{\lambda_i}{i} \geq \frac{1 - L_0}{d_c(1 - R_{\text{in}})}$	achieve rate R_{in}
$0 \leq \lambda_i \quad \forall i \in \{1, \ldots, D_v\}$	no negative degrees
$\sum_{i=1}^{D_v} \lambda_i = 1$	valid degree distribution
$\lambda_1 d_c = \nu$	desired degree-one nodes
$0 \leq L_0 \leq L_0^{\max}$	degree-zero nodes in range

Optimization Problem Formulation – Multi-SNR

Given R_{cat}, S, iterate over R_{sc}, d_c, ν, minimize

$$\gamma = \max\left(\frac{\eta_1}{\eta_1^*}, \frac{\eta_2}{\eta_2^*}, \ldots, \frac{\eta_n}{\eta_n^*}\right),$$

and also subject to:

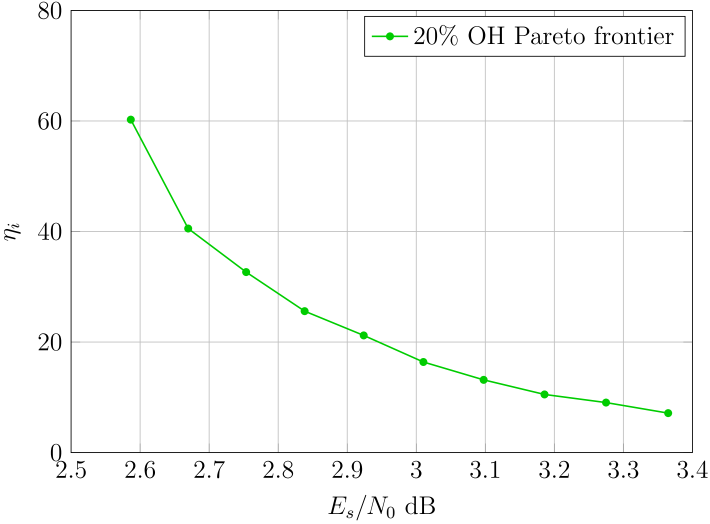
$$\begin{aligned}
f_{\lambda,\text{SNR}_1}(p) &< p \quad \forall p \in [p_{t,\text{SNR}_1}, p_{0,\text{SNR}_1}],\\
f_{\lambda,\text{SNR}_2}(p) &< p \quad \forall p \in [p_{t,\text{SNR}_2}, p_{0,\text{SNR}_2}],\\
&\vdots\\
f_{\lambda,\text{SNR}_n}(p) &< p \quad \forall p \in [p_{t,\text{SNR}_n}, p_{0,\text{SNR}_n}],
\end{aligned}$$

EXIT chart open

$$\begin{aligned}
L_0 p_{0,\text{SNR}_1} + (R_{\text{in}} - L_0) P_{t,\text{SNR}_1} &\le R_{\text{in}} p_{\text{sc}},\\
L_0 p_{0,\text{SNR}_2} + (R_{\text{in}} - L_0) P_{t,\text{SNR}_2} &\le R_{\text{in}} p_{\text{sc}},\\
&\vdots\\
L_0 p_{0,\text{SNR}_n} + (R_{\text{in}} - L_0) P_{t,\text{SNR}_n} &\le R_{\text{in}} p_{\text{sc}}.
\end{aligned}$$

staircase BER achieved.

Results – Operating Codes at Non-Optimal SNRs



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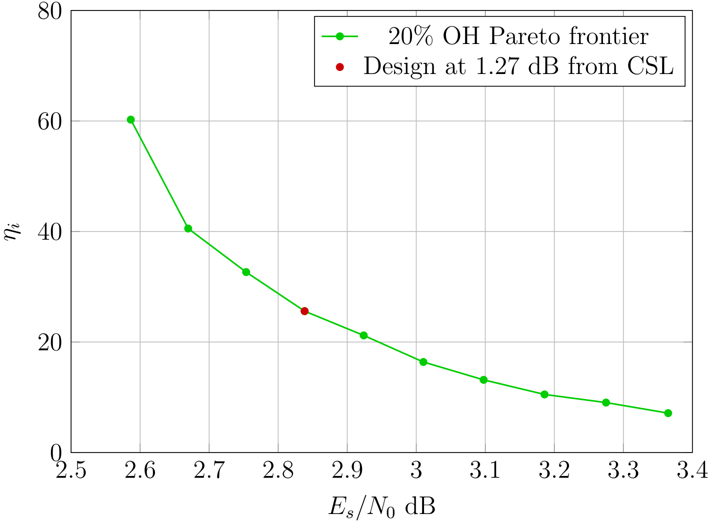

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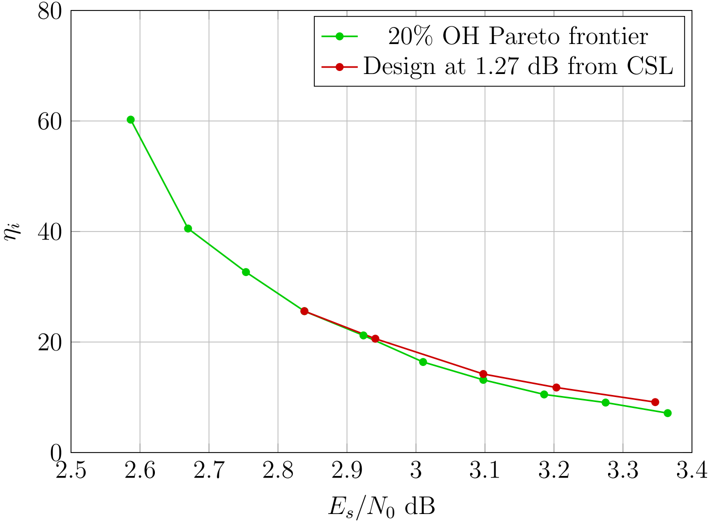
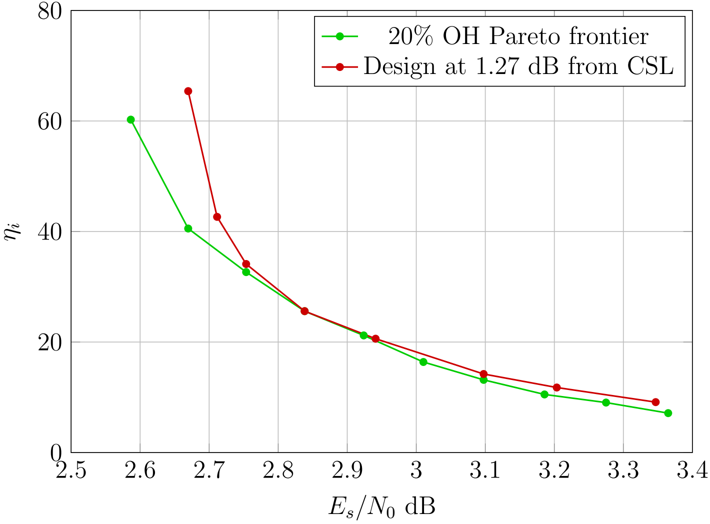
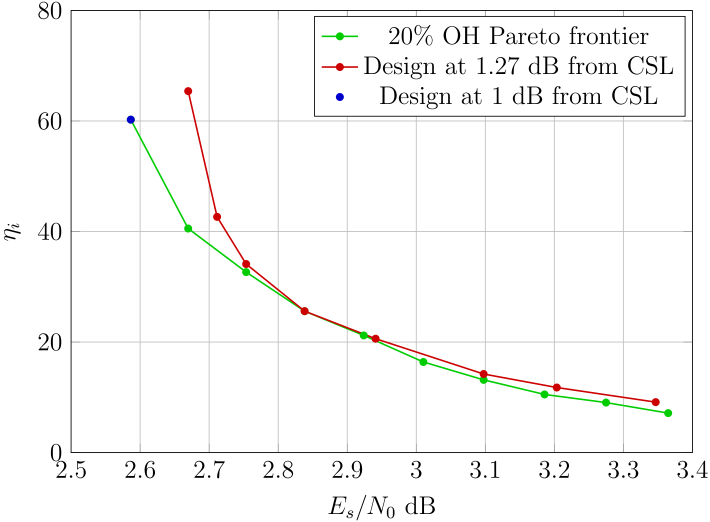


Results – Operating Codes at Non-Optimal SNRs



Results – Operating Codes at Non-Optimal SNRs



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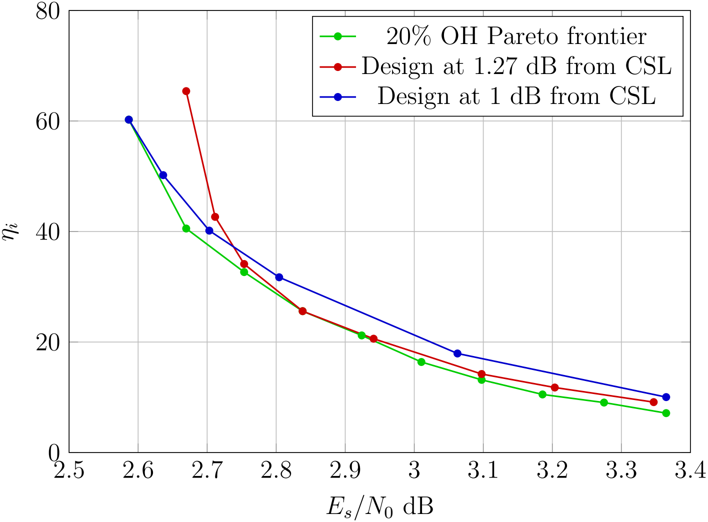
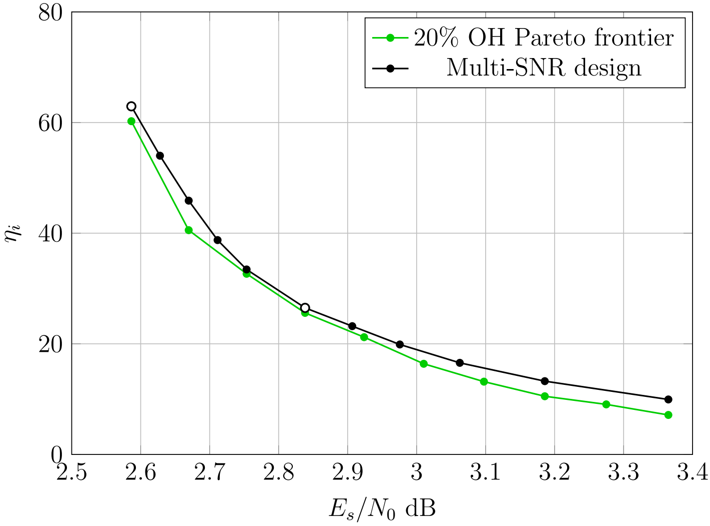


Results – Multi-SNR Code



Two-SNR optimized code, $\gamma = 1.046$

References

[1] M. Barakatain, F. R. Kschischang, **Low-Complexity Concatenated LDPC-Staircase Codes**, *Journal of Lightwave Technology*, 2018.

[2] B. P. Smith, M. Ardakani, W. Yu and F. R. Kschischang, **Design of Irregular LDPC Codes with Optimized Performance-Complexity Tradeoff**, *IEEE Trans. Commun.*, vol. 58, no. 2, pp. 489-499, Feb 2010.

[3] L. M. Zhang and F. R. Kschischang, **Staircase Codes With 6% to 33% Overhead**, *Journal of Lightwave Technology*, vol. 32, no. 10, pp. 1999-2002, May 2014.

Thank You!

Results

OH (%)	SNR set (dB)	$L(x)$	$R(x)$	I	γ
15	3.36 3.64	$0.211865 + 0.188406x + 0.051091x^3 + 0.548638x^4$	x^{35}	5 13	1.054
20	2.84 2.59	$0.139598 + 0.166667x + 0.038726x^3 + 0.613364x^4, 0.041646x^9$	x^{28}	8 19	1.046
25	2.34 2.10	$0.073696 + 0.264000x + 0.348856x^4 + 0.313448x^5$	x^{22}	8 16	1.067

Table: Multi-SNR inner-code designs for 15%, 20%, and 25% OHs