Massive MIMO:

Research Highlights and Opportunities

Erik G. Larsson

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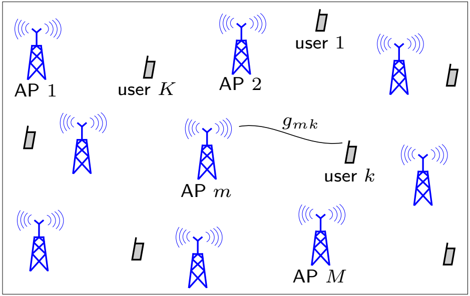
Div. of Communication Systems
Dept. of Electrical Engineering (ISY)
Linköping University
Linköping, Sweden

www.commsys.isy.liu.se

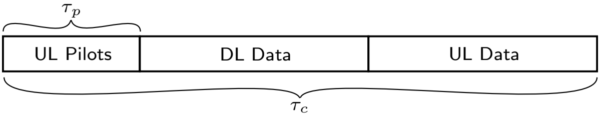
Massive MIMO without Cells

From http://arxiv.org/abs/1505.02617

Cell-Free Massive MIMO



- Channel model: $g_{mk} = \underbrace{\sqrt{\beta_{mk}}}_{\text{path loss and shadow fading}} \times \underbrace{h_{mk}}_{\text{small-scale fading}, \sim\mathcal{CN}(0,1)}$
- Hardening $\Rightarrow$ only APs need CSI



- Co-located Massive MIMO is a special case when $\beta_{mk} = \beta_k$

Uplink Pilots

- kth user sends $\tau_p \times 1$ pilot $\boldsymbol{\varphi}_k$

$$\boldsymbol{y}_m = \sqrt{\tau_p \rho_\mathrm{p}} \sum_{k=1}^{K} g_{mk} \boldsymbol{\varphi}_k + \underbrace{\boldsymbol{w}_m}_{\text{i.i.d. } \mathcal{CN}(0,1)}, \quad \|\boldsymbol{\varphi}_k\|^2 = 1$$

- MMSE estimate of g_{mk}:

$$\hat{g}_{mk} \propto \sqrt{\tau_p \rho_\mathrm{p}} g_{mk} + \underbrace{\sqrt{\tau_p \rho_\mathrm{p}} \sum_{k' \neq k}^{K} g_{mk'} \boldsymbol{\varphi}_k^H \boldsymbol{\varphi}_{k'}}_{\text{pilot contamination}} + \boldsymbol{\varphi}_k^H \boldsymbol{w}_m$$

Downlink Data Transmission

- maximum-ratio processing:

$$x_m = \sqrt{\rho_{\mathrm{d}}} \sum_{k=1}^{K} \sqrt{\eta_{mk}} \hat{g}_{mk}^{*} q_k$$

- Power constraint per AP:

$$\mathbb{E}\left\{|x_m|^2\right\} = \rho_{\mathrm{d}} \sum_{k=1}^{K} \eta_{mk}\gamma_{mk} \leq \rho_{\mathrm{d}}, \quad \gamma_{mk} \triangleq \mathbb{E}\left\{|\hat{g}_{mk}|^2\right\}$$

- Power control coefficients $\{\eta_{mk}\}$ satisfy: $\sum_{k=1}^{K} \eta_{mk}\gamma_{mk} \leq 1$

- RX signal at the kth user:

$$r_k = \sum_{m=1}^{M} g_{mk} x_m + \underbrace{n_k}_{\mathcal{CN}(0,1)} = \sqrt{\rho_{\mathrm{d}}} \sum_{m=1}^{M} \sum_{k'=1}^{K} \sqrt{\eta_{mk'}} g_{mk} \hat{g}_{mk'}^{*} q_{k'} + n_k$$

$$= \left(\sqrt{\rho_{\mathrm{d}}} \sum_{m=1}^{M} \sqrt{\eta_{mk}} g_{mk} \hat{g}_{mk}^{*} \right) q_k + \cdots$$

Ergodic Capacity Lower Bound (DL)

- User does not know g_{mk}, but relies on hardening:

$$r_k = \sqrt{\rho_{\mathrm{d}}}\,\mathbb{E}\left\{\sum_{m=1}^{M}\sqrt{\eta_{mk}}g_{mk}\hat{g}_{mk}^*\right\}q_k + \underbrace{\sqrt{\rho_{\mathrm{d}}}\sum_{m=1}^{M}\sum_{k'\neq k}^{K}\sqrt{\eta_{mk'}}g_{mk}\hat{g}_{mk'}^*q_{k'}}_{(1)}$$

$$+\underbrace{\sqrt{\rho_{\mathrm{d}}}\left(\sum_{m=1}^{M}\sqrt{\eta_{mk}}g_{mk}\hat{g}_{mk}^* - \mathbb{E}\left\{\sum_{m=1}^{M}\sqrt{\eta_{mk}}g_{mk}\hat{g}_{mk}^*\right\}\right)q_k}_{(2)} + n_k$$

- Capacity lower bound: $R_k = \log_2\left(1 + \underbrace{\frac{\rho_{\mathrm{d}}\left|\mathbb{E}\left\{\sum_{m=1}^{M}\sqrt{\eta_{mk}}g_{mk}\hat{g}_{mk}^*\right\}\right|^2}{\mathsf{Var}((1)) + \mathsf{Var}((2)) + 1}}_{\triangleq \mathsf{SINR}_k}\right)$

$$\mathsf{SINR}_k = \frac{\rho_{\mathrm{d}}\left(\sum_{m=1}^{M}\sqrt{\eta_{mk}}\gamma_{mk}\right)^2}{\underbrace{\rho_{\mathrm{d}}\sum_{k'=1}^{K}\sum_{m=1}^{M}\eta_{mk'}\gamma_{mk'}\beta_{mk}}_{\text{inter-user interference + gain uncertainty}} + \underbrace{\rho_{\mathrm{d}}\sum_{k'\neq k}^{K}\left(\sum_{m=1}^{M}\sqrt{\eta_{mk'}}\gamma_{mk'}\frac{\beta_{mk}}{\beta_{mk'}}\right)^2\left|\boldsymbol{\varphi}_{k'}^H\boldsymbol{\varphi}_k\right|^2}_{\text{pilot contamination}} + 1}$$

Capacity of DL and UL Cell-Free Massive MIMO

$$\mathsf{SINR}_k^{\mathsf{DL}} = \frac{\rho_{\mathrm{d}} \left(\sum_{m=1}^{M} \sqrt{\eta_{mk}} \gamma_{mk} \right)^2}{\rho_{\mathrm{d}} \sum_{k'=1}^{K} \sum_{m=1}^{M} \eta_{mk'} \gamma_{mk'} \beta_{mk} + \rho_{\mathrm{d}} \sum_{k' \neq k}^{K} \left(\sum_{m=1}^{M} \sqrt{\eta_{mk'}} \gamma_{mk'} \frac{\beta_{mk}}{\beta_{mk'}} \right)^2 |\boldsymbol{\varphi}_{k'}^H \boldsymbol{\varphi}_k|^2 + 1}$$

$$\mathsf{SINR}_k^{\mathsf{UL}} = \frac{\rho \eta_k \left(\sum_{m=1}^{M} \gamma_{mk} \right)^2}{\rho \sum_{k' \neq k}^{K} \eta_{k'} \left(\sum_{m=1}^{M} \gamma_{mk} \frac{\beta_{mk'}}{\beta_{mk}} \right)^2 |\boldsymbol{\varphi}_k^H \boldsymbol{\varphi}_{k'}|^2 + \rho \sum_{k'=1}^{K} \eta_{k'} \sum_{m=1}^{M} \gamma_{mk} \beta_{mk'} + \sum_{m=1}^{M} \gamma_{mk}}$$

- Co-located Massive MIMO as special case:

$$\mathsf{SINR}_k^{\mathsf{DL}} = \frac{M \rho_{\mathrm{d}} \gamma_k \eta_k}{\rho_{\mathrm{d}} \beta_k \sum_{k'=1}^{K} \eta_{k'} + 1}, \quad \mathsf{SINR}_k^{\mathsf{UL}} = \frac{M \rho \gamma_k \eta_k}{\rho \sum_{k'=1}^{K} \beta_{k'} \eta_{k'} + 1}$$

$\eta_{mk} = \eta_k / (M \gamma_{mk})$ for DL

Pilot Assignment

- **Random assignment** from pre-determined set $\mathcal{S}$

- **Greedy assignment**:
  - Start with random assignment
  - User k^* with smallest SINR selects new pilot:

$$\arg \min_{\boldsymbol{\varphi}_{k^*} \in \mathcal{S}} \sum_{m=1}^{M} \sum_{k' \neq k^*}^{K} \beta_{mk'} \left| \boldsymbol{\varphi}_{k^*}^H \boldsymbol{\varphi}_{k'} \right|^2$$

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Numerical Results

- $M = 100$ APs, $K = 40$ users uniformly distributed in 1000×1000 m^2
- $\tau_c = 200$ symbols coherence
- Large-scale fading:
  - $\beta_{mk} = \mathsf{PL}_{mk} \cdot 10^{\frac{z_{mk}}{10}}$
  - $z_{mk} \sim \mathcal{N}(0, \sigma_{\mathsf{sh}}^2)$, $\sigma_{\mathsf{sh}} = 8$ dB
  - PL_{mk}: three-slope + Hata-COST231 propagation model

$$\mathsf{PL}_{mk} = \begin{cases} -L - 35\log_{10}(d_{mk}) & \text{if } d_{mk} > 50 \text{ m} \\ -L - 15\log_{10}(d_1) - 20\log_{10}(d_{mk}) & \text{if } 10 \text{ m} < d_{mk} \leq 50 \text{ m} \\ -L - 15\log_{10}(d_1) - 20\log_{10}(d_0) & \text{if } d_{mk} \leq 10 \text{ m} \end{cases}$$

- "No power control" so far:

$$\begin{cases} \eta_{mk} = \dfrac{1}{\sum_{k'=1}^{K} \gamma_{mk'}}, & \text{for DL} \\ \eta_k = 1, & \text{for UL} \end{cases}$$

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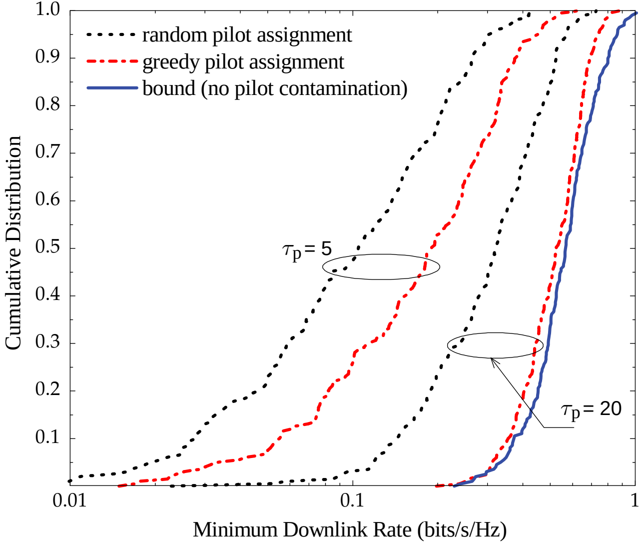
Random Pilot vs. Greedy Pilot Assignment (DL)



Max-Min Power Control for DL

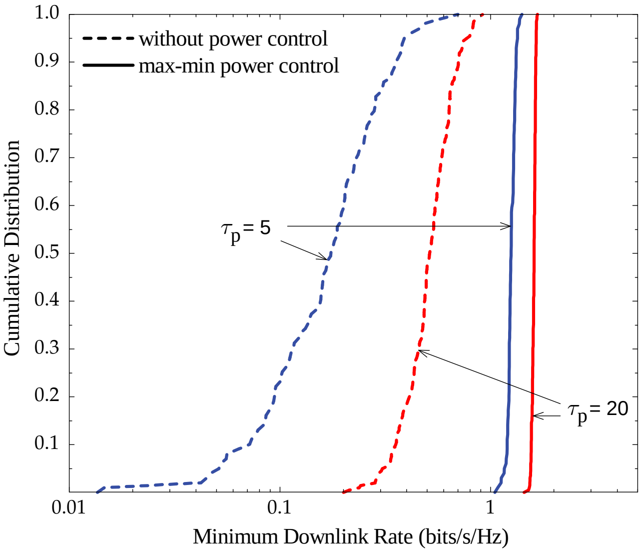
$$\max_{\{\eta_{mk}\}} \quad \min_k R_k$$
$$\text{subject to} \quad \sum_{k=1}^{K} \eta_{mk}\gamma_{mk} \leq 1$$
$$\eta_{mk} \geq 0$$

- Quasi-concave problem reformulation:

$$\max_{\{\varsigma_{mk},\varrho_{k'k},\vartheta_m\}} \quad \min_k \frac{\left(\sum_{m=1}^{M} \gamma_{mk}\varsigma_{mk}\right)^2}{\sum\limits_{k'\neq k}^{K} |\boldsymbol{\varphi}_{k'}^H \boldsymbol{\varphi}_k|^2 \varrho_{k'k}^2 + \sum\limits_{m=1}^{M} \beta_{mk}\vartheta_m^2 + \frac{1}{\rho_{\mathrm{d}}}}$$
$$\text{subject to} \quad \sum_{k'=1}^{K} \gamma_{mk'}\varsigma_{mk'}^2 \leq \vartheta_m^2$$
$$\sum_{m=1}^{M} \gamma_{mk'} \frac{\beta_{mk}}{\beta_{mk'}} \varsigma_{mk'} \leq \varrho_{k'k}, \ \forall k' \neq k$$
$$0 \leq \vartheta_m \leq 1$$
$$\varsigma_{mk} \geq 0$$

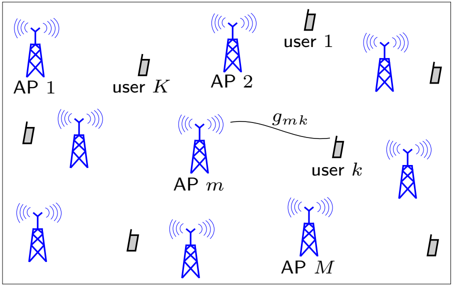
- Use bisection + SOCP solver

Power Control versus No Power Control, DL, Greedy Pilot Assignment



How Much Better is Cell-Free Massive MIMO than Small Cells?

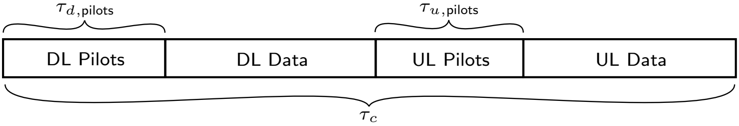
Small-Cell System



- User k is served by only one AP m_k:

$$m_k = \arg\max_{m \in \{\text{available APs}\}} \beta_{mk} \quad (\text{one round } k = 1, \ldots, K)$$

- No hardening, both APs and users need CSI



Small-Cell System

- Downlink pilots:
  - m_kth AP sends a pilot $\sqrt{\tau_{d,\text{pilots}}}\boldsymbol{\phi}_k$, $\|\boldsymbol{\phi}_k\|^2 = 1$
  - kth user estimates the channel $g_{m_k k} \Rightarrow \hat{g}_{m_k k}$
- Downlink data transmission:

$$y_k = \sqrt{\rho_\text{d}} \sum_{k'=1}^{K} g_{m_{k'} k} \sqrt{\eta_{k'}} q_{k'} + w_k$$

$$= \sqrt{\rho_\text{d}} \hat{g}_{m_k k} \sqrt{\eta_k} q_k + \underbrace{\sqrt{\rho_\text{d}} \tilde{g}_{m_k k} \sqrt{\eta_k} q_k}_{(1)\ \text{CSI error}} + \underbrace{\sqrt{\rho_\text{d}} \sum_{k' \neq k}^{K} g_{m_{k'} k} \sqrt{\eta_{k'}} q_{k'}}_{(2)\ \text{interference}} + w_k$$

- Achievable DL rate: $R_k = \mathbb{E}\left\{ \log_2 \left(1 + \frac{\rho_\text{d} \eta_k \left| \hat{g}_{m_k k} \right|^2}{1 + \text{Var}(1) + \text{Var}(2)} \right) \right\}$
- Net throughput:

$$S_{\text{cell-free},k} = \frac{B}{2}\left(1 - \frac{\tau_p}{\tau_c}\right) R_k, \quad S_{\text{small-cell},k} = \frac{B}{2}\left(1 - \frac{\tau_{u,\text{pilots}} + \tau_{d,\text{pilots}}}{\tau_c}\right) R_k$$

Small-Cell: Max-Min Power Control for DL

- Achievable downlink rate:

$$R_k = f\left(\frac{\rho_{\mathrm{d}}\eta_k\gamma_{m_k k}}{\rho_{\mathrm{d}}\eta_k(\beta_{m_k k} - \gamma_{m_k k}) + \rho_{\mathrm{d}}\sum_{k'\neq k}^{K}\eta_{k'}\beta_{m_{k'} k} + 1}\right)$$

- $f(x) \triangleq -(\log_2 e)e^{1/x}\mathsf{Ei}\left(-\frac{1}{x}\right)$ is a non-decreasing function

- Max-min power control problem

$$\begin{aligned} \max_{\{\eta_k\}} \quad & \min_k R_k \\ \text{subject to} \quad & 0 \leq \eta_k \leq 1 \end{aligned}$$

is quasi-linear:

$$\begin{aligned} \max_{\{\eta_k\},t} \quad & t \\ \text{subject to} \quad & t \leq \frac{\rho_{\mathrm{d}}\eta_k\gamma_{m_k k}}{\rho_{\mathrm{d}}\eta_k(\beta_{m_k k}-\gamma_{m_k k})+\rho_{\mathrm{d}}\sum_{k'\neq k}^{K}\eta_{k'}\beta_{m_{k'} k}+1} \\ & 0 \leq \eta_k \leq 1 \end{aligned}$$

Shadowing Correlation Model

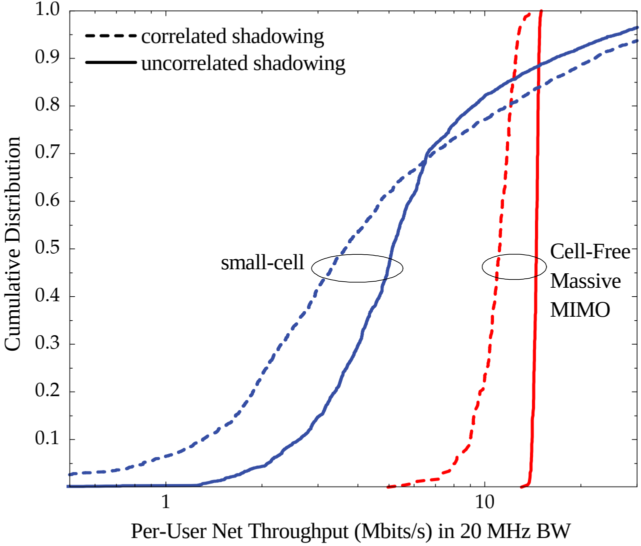
$$z_{mk} = \sqrt{\alpha}a_m + \sqrt{1-\alpha}b_k$$

- a_m and b_k are independent $\mathcal{N}(0,1)$
- α $(0 \leq \alpha \leq 1)$: cross-correlation at the AP side
- $1-\alpha$: cross-correlation at the user side
- spatial correlation:

$$\mathbb{E}\left\{a_m a_{m'}\right\} = 2^{-\frac{\text{distance(AP } m\text{, AP } m'\text{)}}{d_{\text{decorr}}}}$$

$$\mathbb{E}\left\{b_k b_{k'}\right\} = 2^{-\frac{\text{distance(user } k\text{, user } k'\text{)}}{d_{\text{decorr}}}}$$

Downlink, $\tau_p = \tau_{u,\text{pilots}} = \tau_{d,\text{pilots}} = K$, $\tau_c = 200$, $K = 40$, $M = 100$



Cell-Free Massive MIMO versus Small Cells

	Co-located	Distributed
architecture	compact arrays	distributed antennas, no cells
coverage	high	higher
backhaul	less	more
favorable propagation	yes	yes and better
channel hardening	yes	some
macro diversity	some	more
open problems	many	even more

Open research directions:

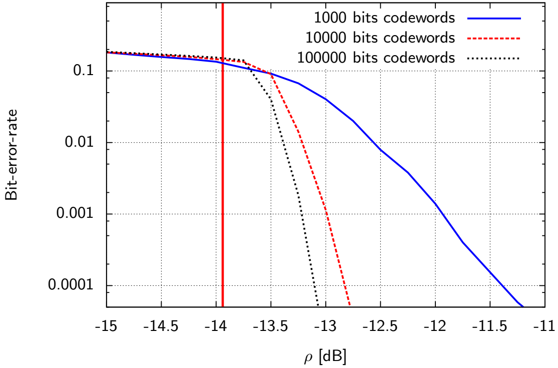
- scalability (computation, power control)
- ZF or other processing
- better pilot assignment & power control
- simpler system model with less backhaul connections

Are the Massive MIMO Closed-Form Capacity Results for Real?

From http://arxiv.org/abs/1503.06854

Co-Located UL, 100×30, MRC, QPSK + rate-1/2, 30 b/s/Hz

- 1 ms $\times$ 200 kHz channel coherence
- one uplink pilot symbol per terminal



Ngo-Larsson-Marzetta'13 formula (vertical line):

$$K \cdot \log_2 \left(1 + \frac{M}{K} \frac{\rho}{(\rho + \frac{1}{K})(1 + \frac{1}{\rho K})} \right) = 30 \text{ b/s/Hz} \;\Rightarrow\; \rho \approx -13.9 \text{ dB}$$

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