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Helmut Schmidt Universität
Universität der Bundeswehr Hamburg

2018 Munich Workshop on Information Theory of Optical Fiber

Machine Learning in Heterodyne Quantum Receivers

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FKZ: 16KIS0490

Motivation: Why Physical Layer Security?

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Public key method

- ► Logical layer
- ☺ Simple to implement
- ☹ Computational secure
- ☹ Vulnerable to quantum computers
- ☹ Threat of "store now, break later"

Quantum key distribution (QKD)

- ► Physical layer
- ☺ Unconditional security
- ☺ Attacker has to break the system when it is used
- ☹ Complex and costly
- ☹ State of the art key rate $G \approx 10^{-3}$ bit/symbol @ 90 km[1]

[1]D. Huang et al., Nature Scientific Reports, 2016, doi:10.1038/srep19201

Outline

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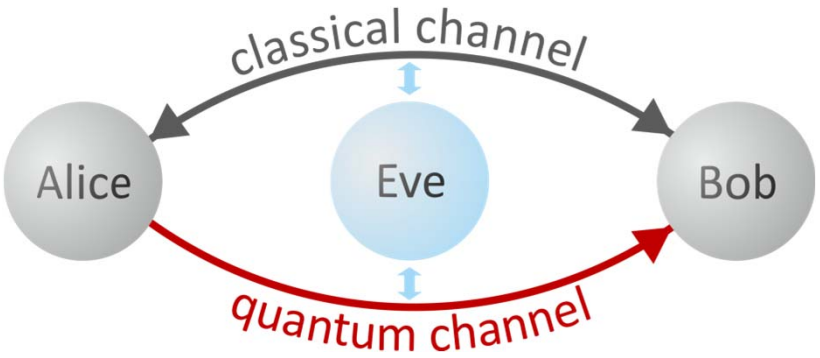
1. The QKD principle
2. Promises and challenges of coherent detection for QKD
3. Coherent quantum PSK
   - Mutual information
   - Key rate optimization
   - Excess noise
   - Experimental setup
4. DSP design for coherent quantum communications
5. Bayesian Inference & laser phase noise
6. Conclusion & Outlook

The QKD Principle



Quantum state transmission

Reconciliation (classical)
Error correction, privacy amplification

Secret key

Encrypt public channel

- Information advantage based on quantum properties
  - Non-orthogonality of coherent states (Heisenberg uncertainty) (CV)
  - Single photon or entanglement (DV)
- A key is not transmitted but **generated after the quantum state transmission** by **interactive reconciliation** via the classical channel

Secret Key Rate

- Key rate equals information advantage

$$G = \rho \cdot I_{\mathrm{AB}} - I_{\mathrm{Eve,max}}$$

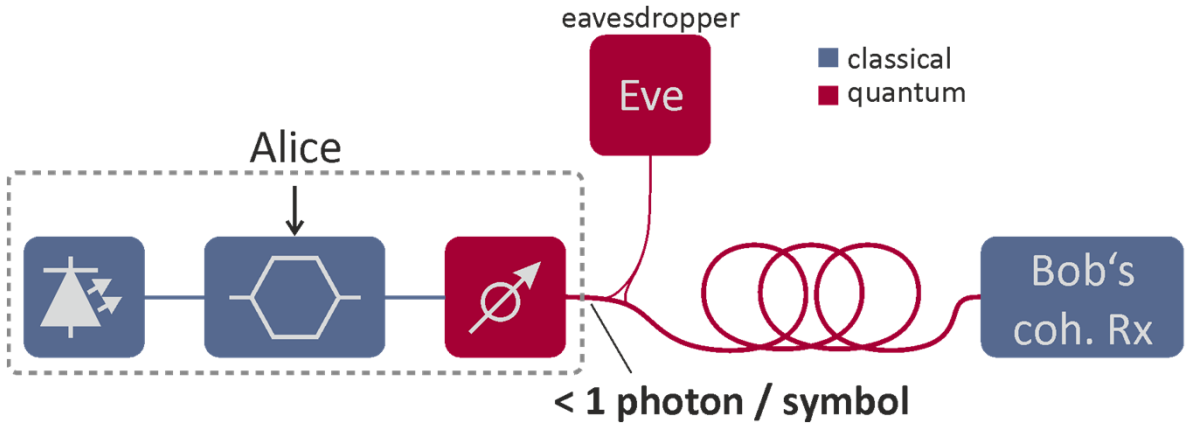
Reconciliation efficiency
- $0 \leq \rho \leq 1$

Mutual information of Alice and Bob
- Depends on signal power and receiver
- $0 \leq I_{\mathrm{AB}} \leq \log_2(M)$ [bit/symbol]

Eve's maximum information
- Depends on signal power, channel attenuation and excess noise ξ'
- $\xi' \geq 0$ [shot noise units]

- ξ': Unexplained noise power in the received signal
  - Assumed to be introduced by Eve
- For maximum key rate, the optimum signal power should be found
- **Usually $N_{\mathrm{Bob}} \ll 1$ photon per symbol**

The Coherent Quantum Channel I
Heterodyne Detection



$$\alpha_k = \sqrt{N_\mathrm{A}}\exp(j2\pi k/M)$$

$$\beta_k = \eta\alpha_k$$

η: channel transmittance

N_A: Sent photons/symbol

- Attenuation increases Heisenberg uncertainty
- Here: coherent M-PSK

$$k \xrightarrow{\text{modulate}} |\alpha_k\rangle \xrightarrow{\eta} |\beta_k\rangle \xrightarrow{\text{measure}} \beta \xrightarrow{\text{decode}} l$$

- After quantum state transmission: Estimation of $I_{\mathrm{Eve}}(\xi)$ necessary

The Optical Coherent Quantum Channel II
Heterodyne Detection

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Promises

- High quantum efficiency
- Spectral efficiency
- Standard telecom components
- Great selectivity, WDM tolerance due to LO

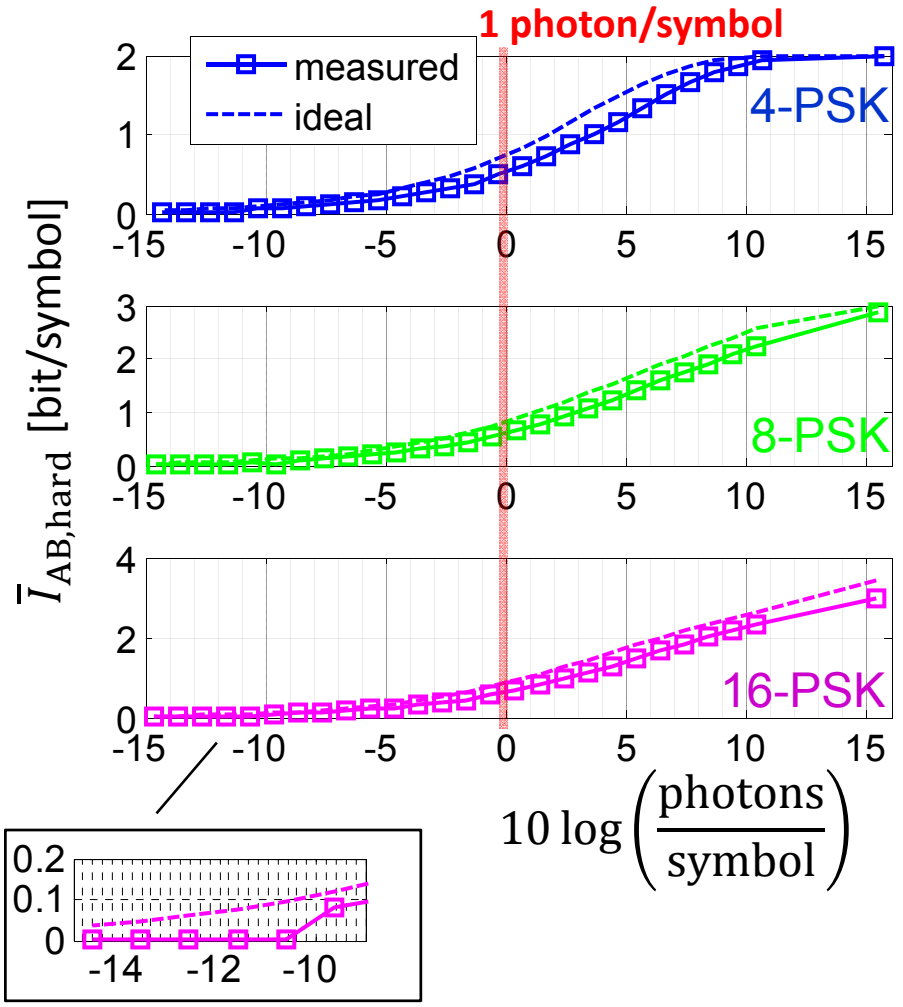
Challenges

- Local oscillator required
- Phase noise compensation
- Frequency estimation
- Synchronization
- Complex reconciliation procedure

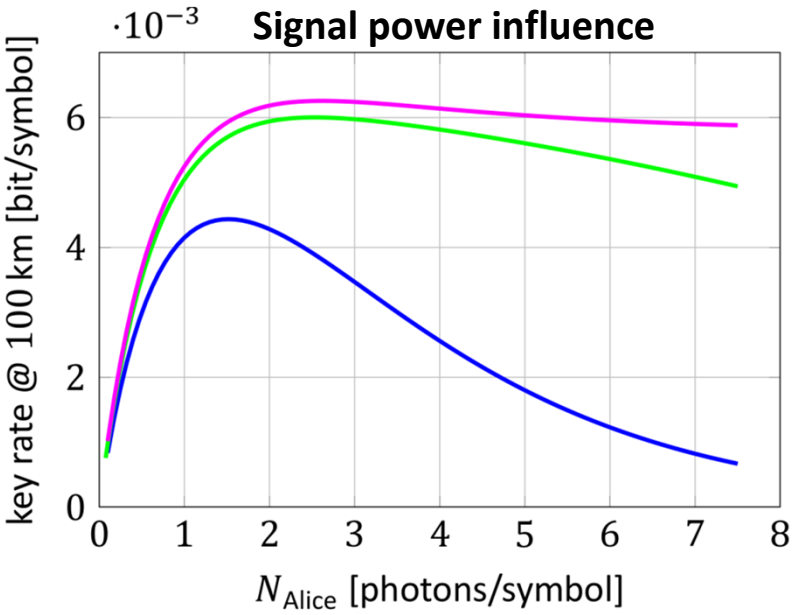
- Challenges not solved yet
- To date, only prototype systems for coherent QKD do exist

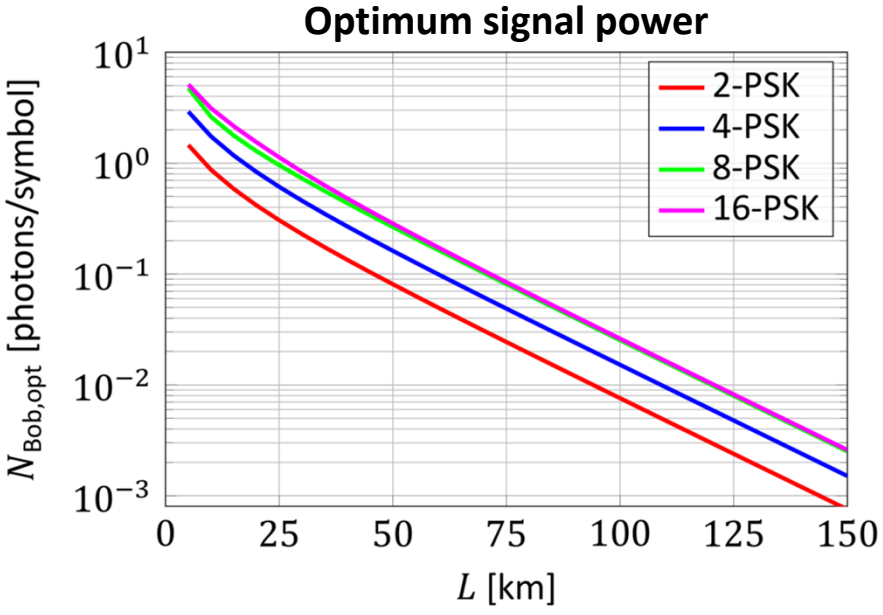
Typical Experimental Results on Mutual Information (Back to Back)

- Lowest power dependent on pilot signal power ratio (18 dB)
- Experimental evaluation of $\bar{I}_{\mathrm{AB,hard}}$
  - Penalty: ~ 2 dB
  - 1 dB due to Rx quantum efficiency
  - 0,5 dB due to electronic noise
- Experimental raw key rate
  - Optimize Alice's power level considering receiver characteristics
  - Found MI penalty serves as worst case estimate for key rate penalty



Properties of Quantum PSK





- Optimization of optical power
  - Beam splitter attack
  - Hard decision
  - Ideal reconciliation
- $\text{SNR}_{\text{ideal}} = N_{\text{Bob}}$

Very weak signal
at long distances

Excess Noise Estimation

- Key rate: $G = \rho \cdot I_{\mathrm{AB}} - I_{\mathrm{Eve,max}}(\xi')$
- Excess noise determines Eve's max. Information
- Alice reveals part of her symbols $\alpha(k)$
- Power components of the received signal

$\alpha(k)$	Alice's symbols
$\beta(k)$	Bob's noisy symbols
η	detector quantum efficiency
t	channel transmittance

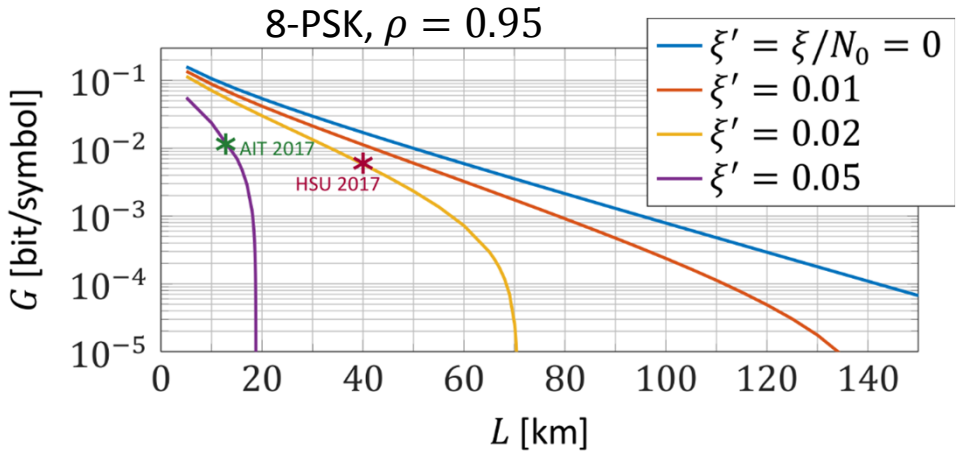
$$P_{\mathrm{Bob}} = \eta t P_{\mathrm{Alice}} + N_0 + N_{\mathrm{el}} + \eta t \xi$$

total power estimation
$\hat{P}_{Bob} = \overline{|\beta(k)|^2}$

signal power estimation
$\hat{P}_{\mathrm{Q}} = |\mathrm{Cov}(\alpha, \beta^*)|^2$

shot noise, electronic noise
calibrated before transmission

Excess noise
Residual power

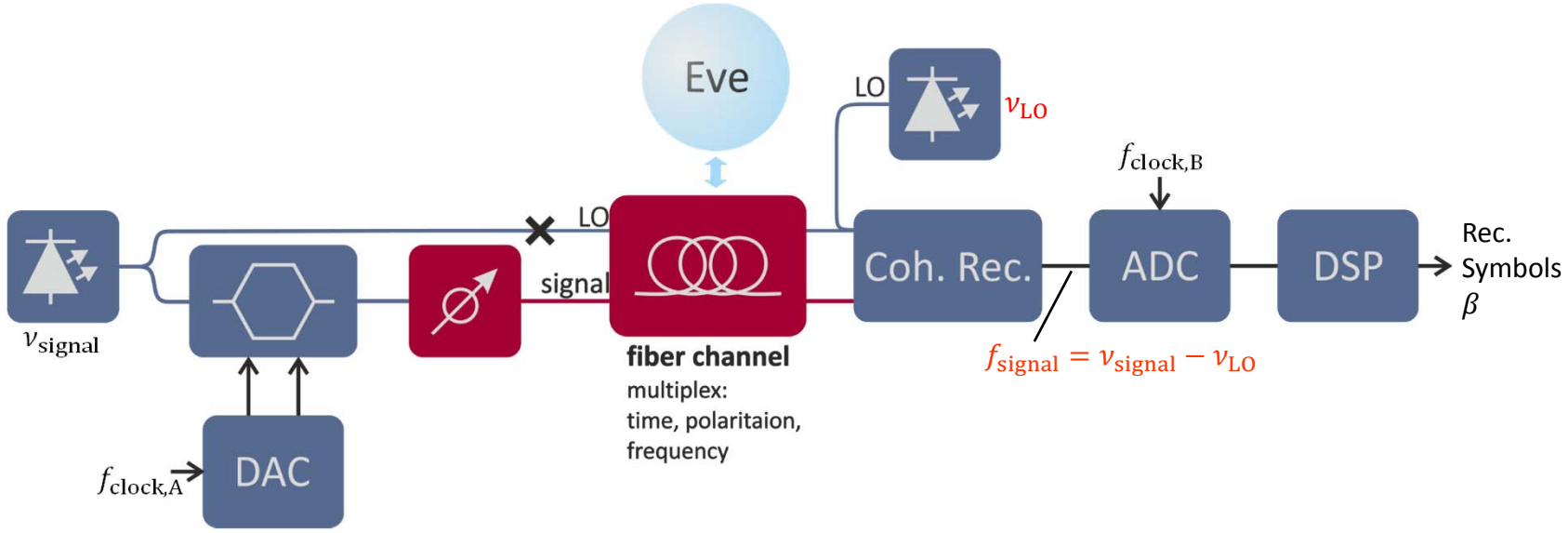


Key rate and achievable distance very sensitive to ξ'!

ECOC 20117: HSU P2.SC6.26
Influence of the SNR of Pilot Tones on the Carrier Phase Estimation in Coherent Quantum Receivers, *Sebastian Kleis;*
AITR P2.SC6.10
High-Rate Continuous-Variables Quantum Key Distribution with Piloted-Disciplined Local Oscillator,*Bernhard Schrenk*

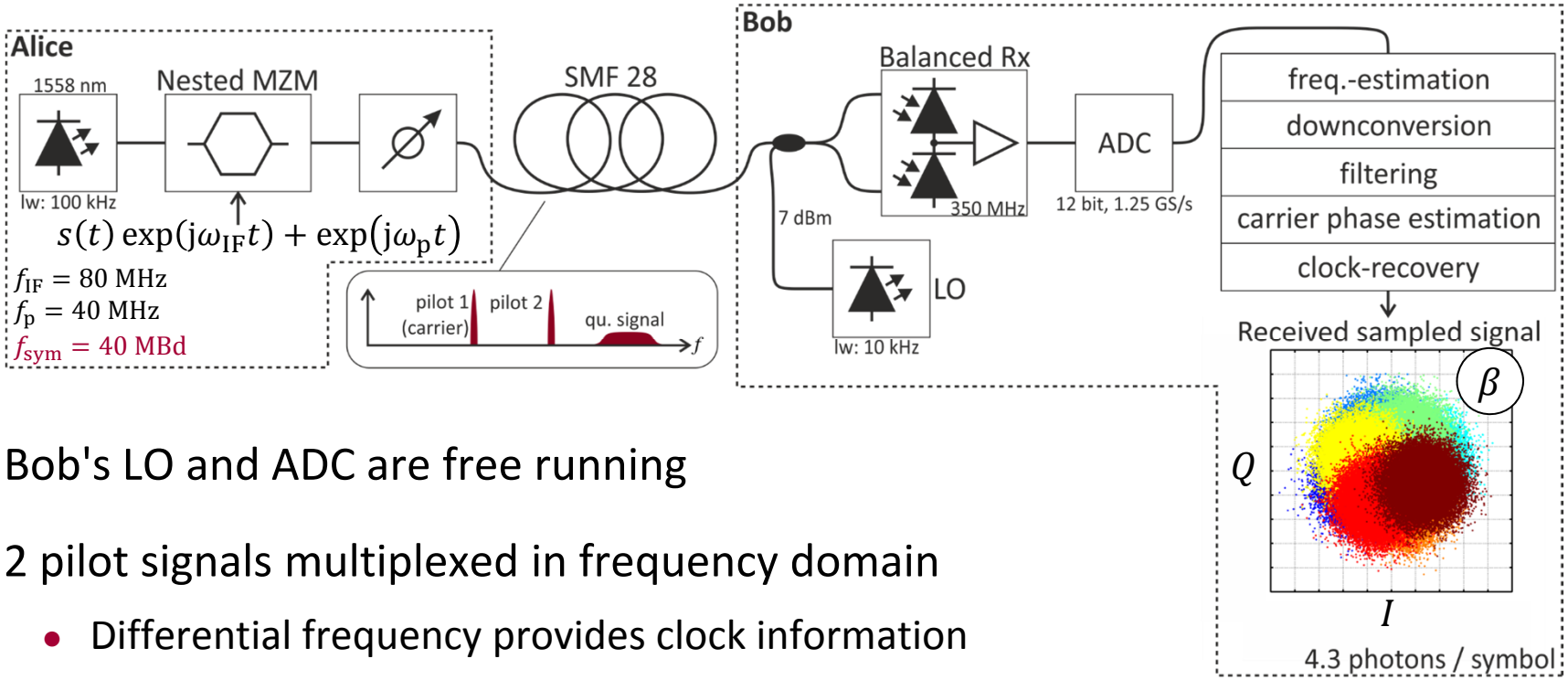
General Coherent Quantum System

- Major challenges: Laser phase noise and clock synchronization



- Remote LO is a common approach but problematic
  - Eve has access to the LO
  - Limited reach due to attenuated LO
- Our approach: Heterodyne with real LO
  - The DSP has to compensate laser freqency noise and perform clock recovery!

Experimental Heterodyne Quantum PSK System[2]

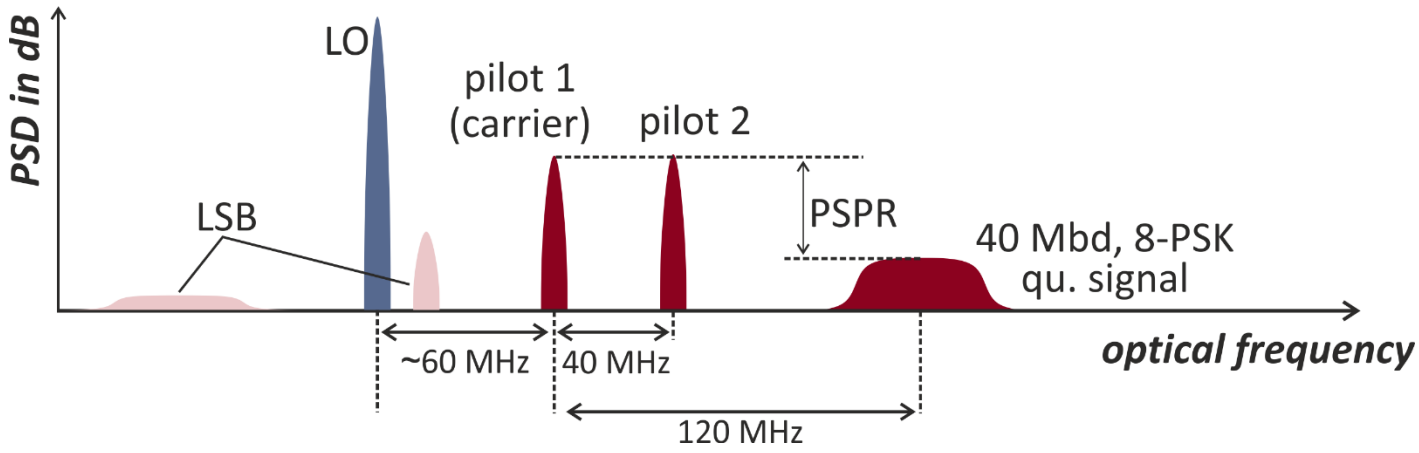


- Bob's LO and ADC are free running
- 2 pilot signals multiplexed in frequency domain
  - Differential frequency provides clock information
- Power ratio between pilots and signal limited by dynamic range of the components (DAC, modulator, balanced Rx, ADC)
  - Pilots exhibit low SNR, too

[2]S. Kleis and C. G. Schaeffer, Optics Letters, 2017, doi:10.1364/OL.42.001588

Details of The Received Signal

- Received optical signal before balanced detection



- Pilots are equal in power
- The **pilot to signal power ratio (PSPR)** is the power ratio between one pilot and the quantum signal
- Pilot 2 provides clock information

Design of DSP for Ultra Low SNR

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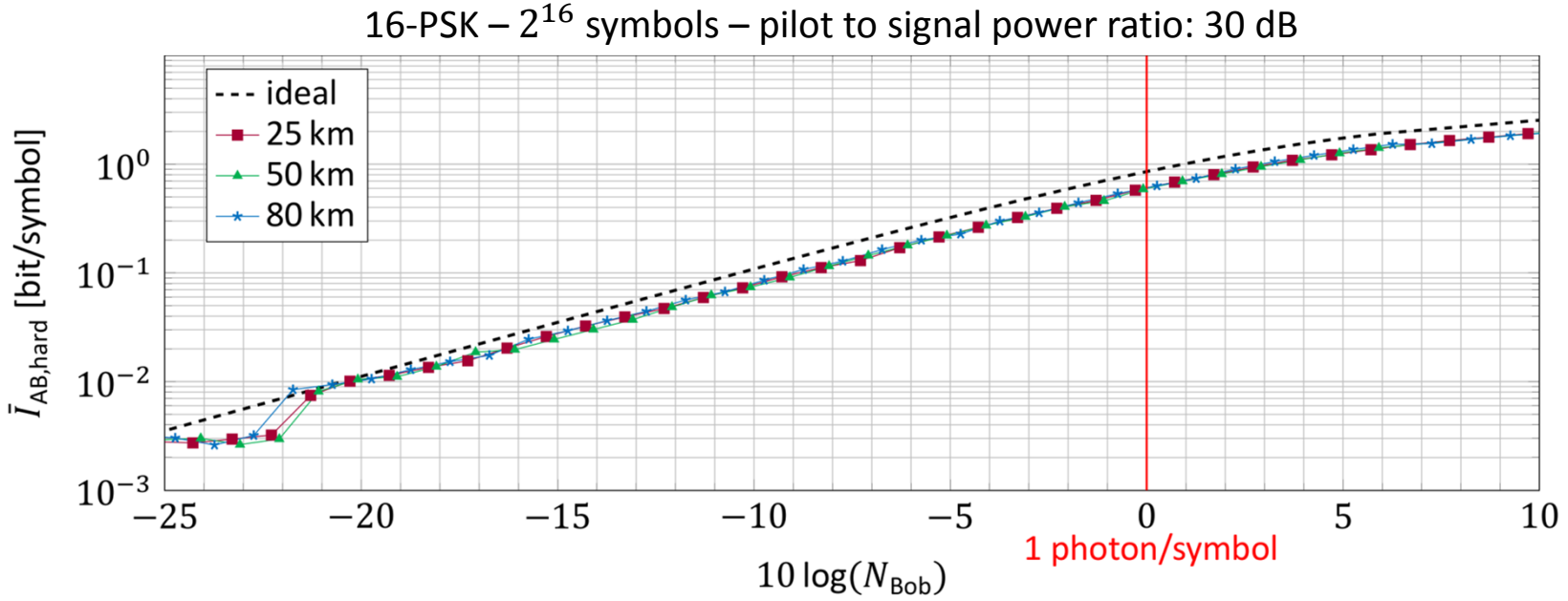
- No known algorithms can deal with such low SNR → **pilot signals necessary!**
- Frequency estimation
  - *Classical system:*
    Coarse estimation only, residual offset is corrected by carrier phase estimation
  - *Quantum system:*
    Critical problem, residual offsets directly translate into phase errors
- Carrier phase estimation
  - *Classical system*: Based on modulated signal, e. g. "Viterbi & Viterbi"
  - *Quantum system*:
    Based on pilot signals, accuracy very important for the key rate
- Clock/timing recovery
  - *Classical system*: Based on modulated signal, e. g. "filter and square"
  - *Quantum system:*
    Pilot signals must contain clock information, precision critical for the key rate

Experimental Results at Different Fiber Lengths



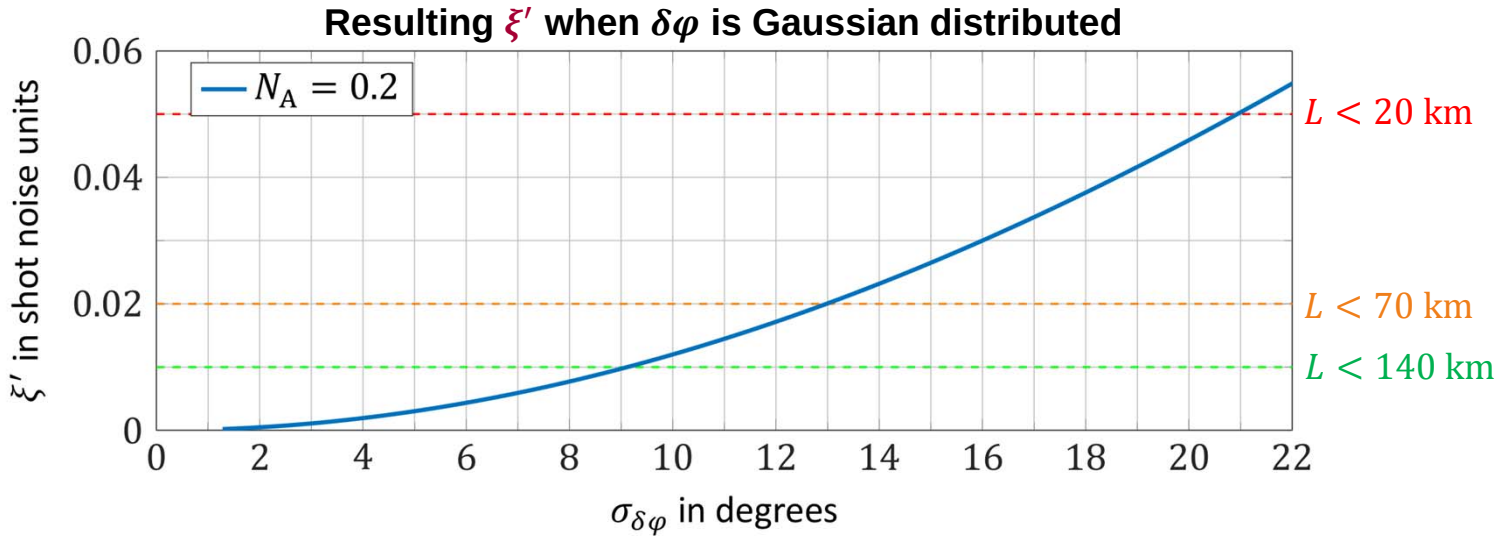
- Here, no influence of fiber length → CD compensation not necessary
- Penalty of < 2 dB (thermal noise, quantum efficiency)
- Less than 10^{-2} photons per symbol detectable!
- Setup shows great stability, repeatability of results

Impact of a Phase Error $\delta\varphi$ in the Received Symbols

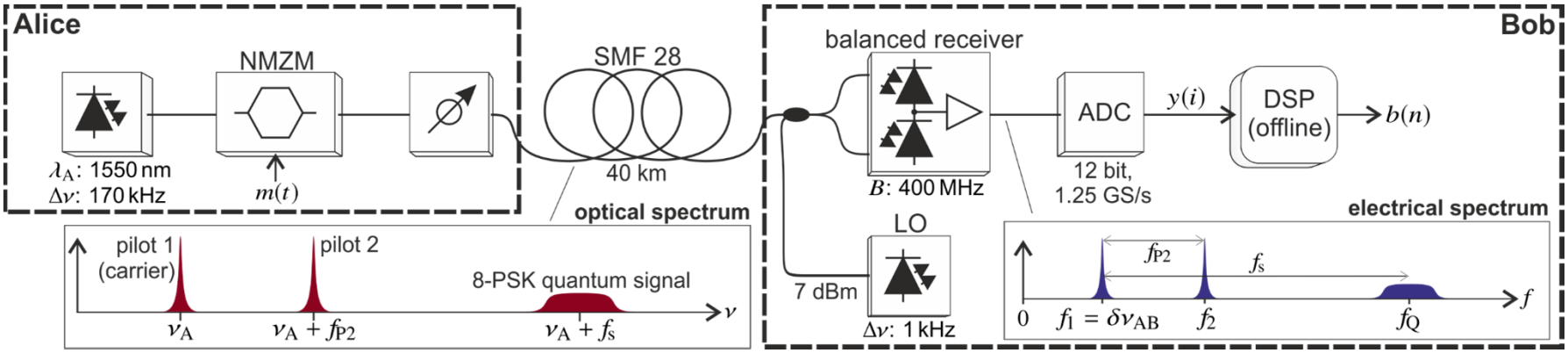
- With phase/frequency distortion: $\langle \hat{P}_{\mathrm{Q}} \rangle < P_{\mathrm{Q}}$
- Underestimation of P_{Q} ⇒ Overestimation of ξ'

$$\xi' = 2N_{\mathrm{A}}\left[1 - \frac{\hat{P}_{\mathrm{Q}}}{P_{\mathrm{Q}}}\right]$$
$$= 2N_{\mathrm{A}}\Delta\hat{P}_{\mathrm{Q}}$$
$$= 2N_{\mathrm{A}}[1 - \langle\cos(\delta\varphi)\rangle^2]$$

$\hat{P}_{\mathrm{Q}}$	Estimated quantum Signal power
$\Delta\hat{P}_{\mathrm{Q}}$	Signal power underestimation factor
N_{A}	Alice's power in photons/symbol
$\delta\varphi$	Phase error
ξ'	Excess noise



Experimental heterodyne quantum communication system[1]



Experimental parameters:

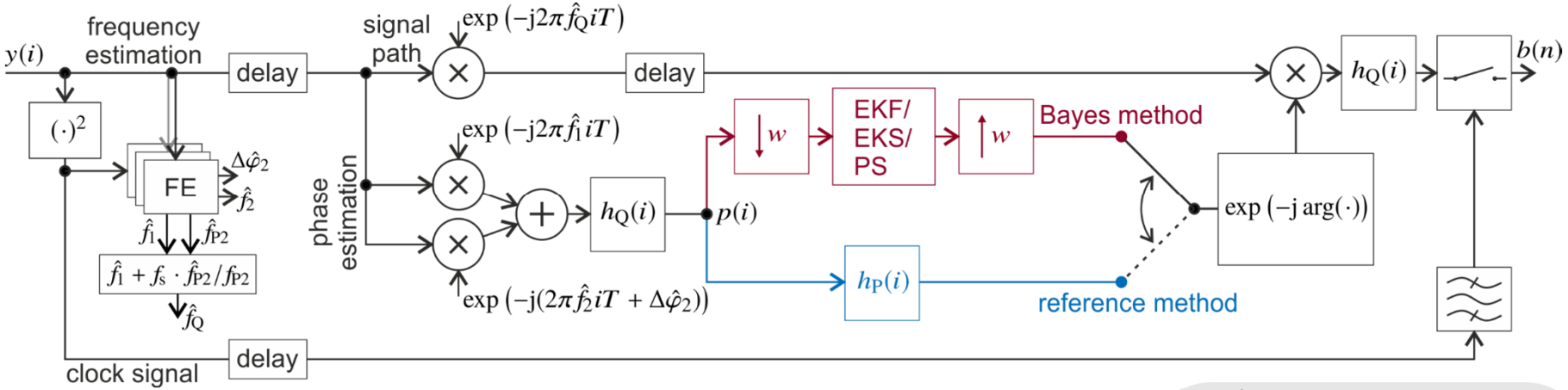
Format	8-PSK
Symbol rate	40 MBd
f_{P2}	40 MHz
f_s	120 MHz

- Bob's LO and ADC are free running
- Two pilots provide frequency, phase and clock information
  - Fixed pilot to signal power ratio (PSPR)
  - Limited by linearity and dynamic range of components
  - The pilots should be weak, too!

[1] S. Kleis, C. G. Schaeffer, *"Continuous variable quantum key distribution with a real local oscillator using simultaneous pilot signals"*, Optics Letters 42(8), 2017.

Digital Signal Processing Routine

- Block wise procedure for signals of arbitrary length



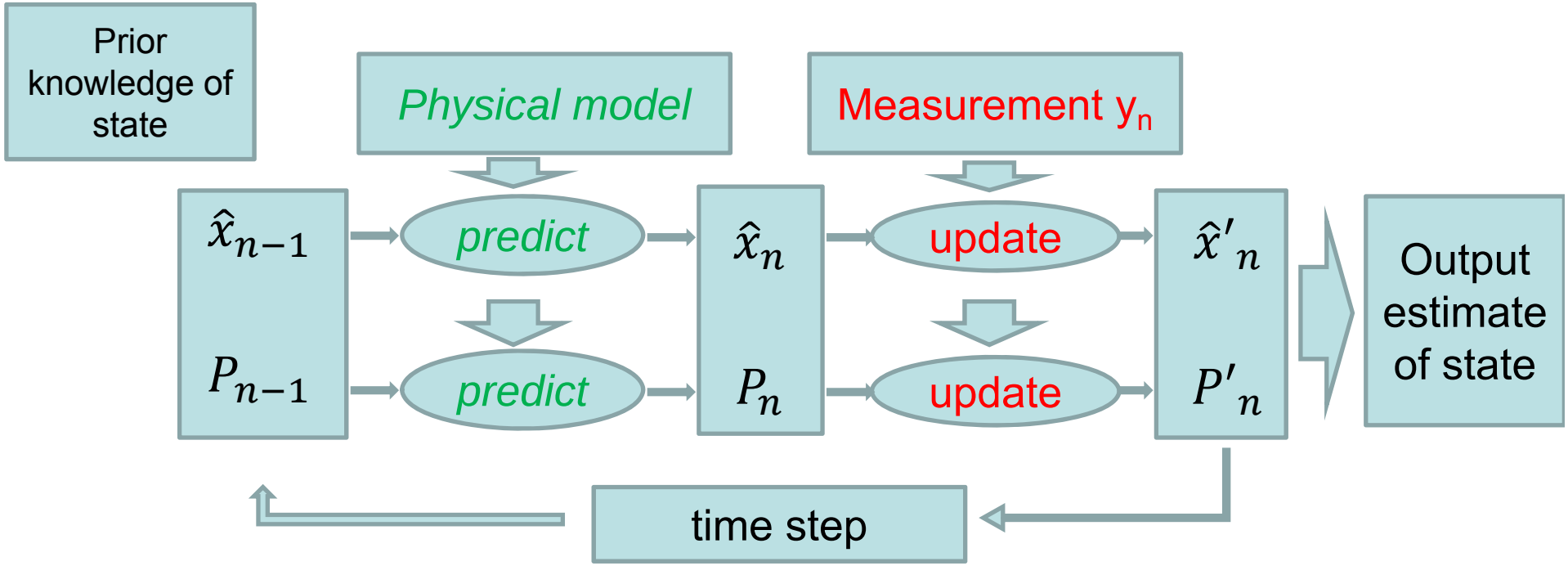
- Pilot SNR improvement by coherent superposition
- Approach for phase estimation
  - **Previously**: Nyquist filter with optimized bandwidth
  - **Novel**: Bayesian inference
  - Methods can be switched for comparison

$\hat{f}$	Estimated frequencies
$\Delta\hat{\varphi}_2$	Estimated initial pilot phase
$h(i)$	Zero roll-off Nyquist filter
$p(i)$	Combined pilot
EKF/ EKS/ PS	Extended Kalman Filter/ Extended Kalman Smoother/ Particle Smoother
w	Resampling factor

Bayesian Inference Methods [2]

- Purpose is to compute the state of a phenomenon when only the measurements are observed
- Recursive methods: $\hat{x}_n = f(\hat{x}_{n-1}, y_n)$
- The information of a new measurement is used to update the old information

$\hat{x}_n$	n-th estimate
y_n	n-th measurement



[2] S. Särkkä, Bayesian filtering and smoothing. Cambridge University Press, 2013, vol. 3.

Bayesian Inference Methods [2]

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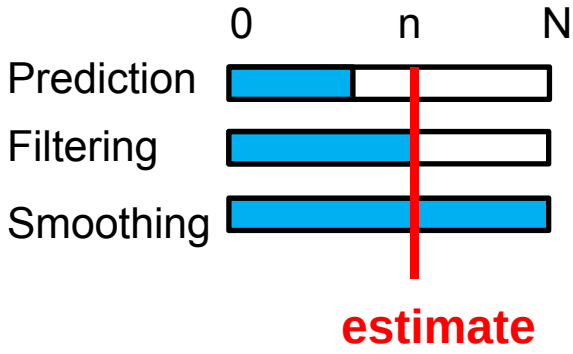
$\hat{x}_n$	n-th estimate
y_n	n-th measurement

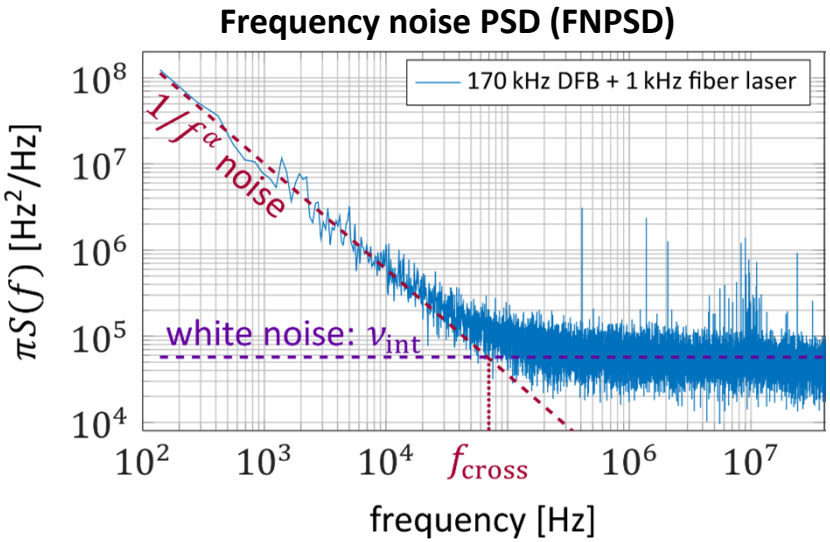
- Recursive methods: $\hat{x}_n = f(\hat{x}_{n-1}, y_n)$
- Provide optimal estimates if the state space model is exact
- **Filters:** To estimate x_n, measurements $y_{1:n}$ are taken into account
- **Smoothers:** For x_n, all measurements $y_{1:N}$ are taken into account
- **Extended Kalman filter/smoother**
  - Analytical method
  - Extension of Kalman filter to non-linear models
  - Approximation by linearization

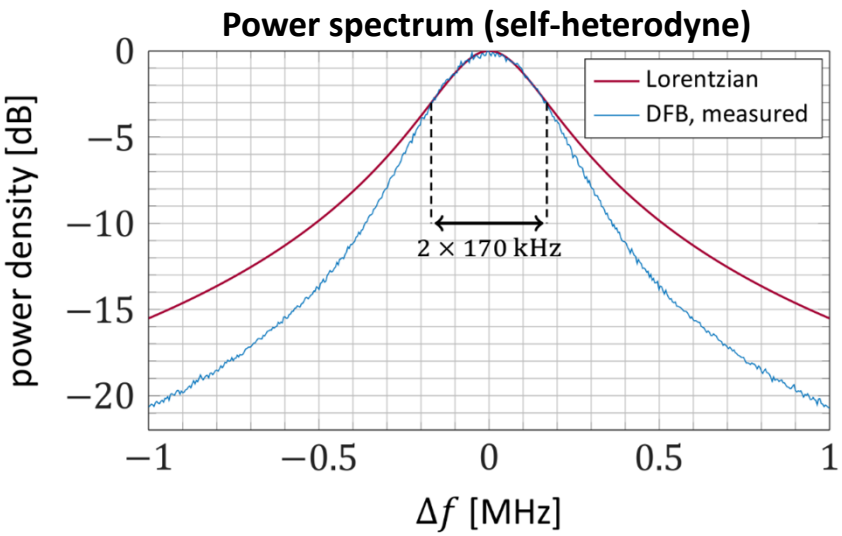


- **Particle filter/smoother**
  - Statistical method (Particles are randomly chosen)
  - No approximation involved
  - Converges to optimum for large number of particles (computationally heavy)

[2] S. Särkkä, Bayesian filtering and smoothing. Cambridge University Press, 2013, vol. 3.

Measured Laser Phase Noise Characteristics





- White frequency noise ⟺ Lorentzian line, **not here!**
- Strong $1/f^{\alpha}$ frequency noise component
- Three parameters describe the frequency noise: ν_{int}, f_{cross}, α
- These parameters should be included in the state space model for Bayesian inference

State space model including $1/f^2$ noise

- Process

$$\mathbf{x}_n = f(\mathbf{x}_{n-1}) + \mathbf{q}_{n-1}$$

$$\begin{bmatrix}\Omega_n \\ \varphi_n\end{bmatrix} = \begin{bmatrix}1 & 0 \\ 1 & 1\end{bmatrix}\begin{bmatrix}\Omega_{n-1} \\ \varphi_{n-1}\end{bmatrix} + \begin{bmatrix}q_{\Omega,n-1} \\ q_{\varphi,n-1}\end{bmatrix}$$

Ω_n:	Frequency to include $1/f^2$ - noise
φ_n:	Signal phase
$\mathbf{q}_n$, $\mathbf{r}_n$:	WGN processes

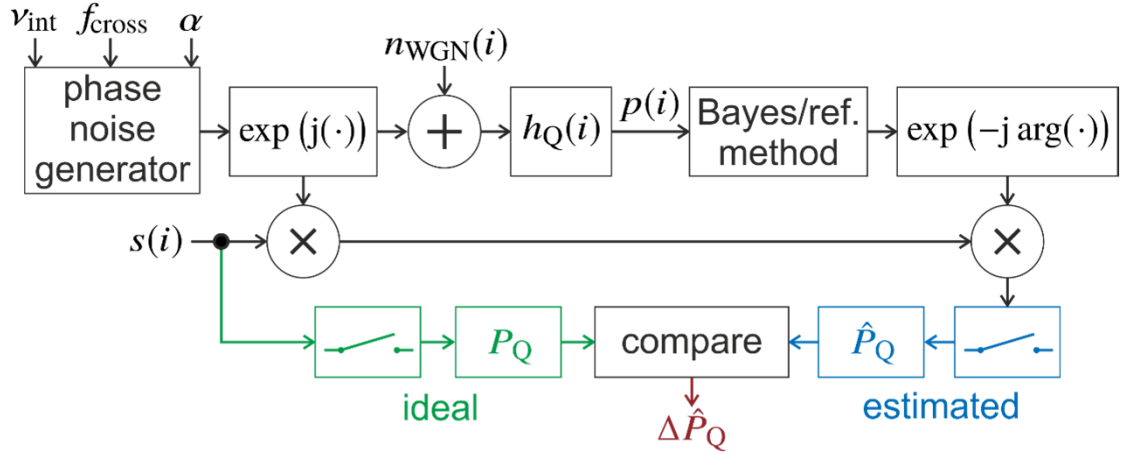
- Exact model for $\frac{1}{f^2}$ - noise ($\alpha = 2$)
- $\frac{1}{f^\alpha}$ -noise with $\alpha \neq 2$ not feasible due to an infinite number of poles [3]
  - Results in an infinite number of state space variables
  - **Measured:** $\alpha \approx 1.23$
  - To reduce the impact of that mismatch, $f_{\text{cross,model}} := \frac{f_{\text{cross}}}{2}$
- Measurement of the pilot quadratures with a coherent receiver

$$\mathbf{y}_n = \mathbf{x}_n + \mathbf{r}_n \qquad \begin{bmatrix}y_{\text{I},n} \\ y_{\text{Q},n}\end{bmatrix} = \hat{y}\begin{bmatrix}\cos(\varphi_n) \\ \sin(\varphi_n)\end{bmatrix} + \begin{bmatrix}r_{\text{I},n} \\ r_{\text{Q},n}\end{bmatrix}$$

[3] N. J. Kasdin, “Discrete simulation of colored noise and stochastic processes and $1/f^\alpha$ power law noise generation,” Proceedings of the IEEE, vol. 83, no. 5, pp. 802–827, 1995.

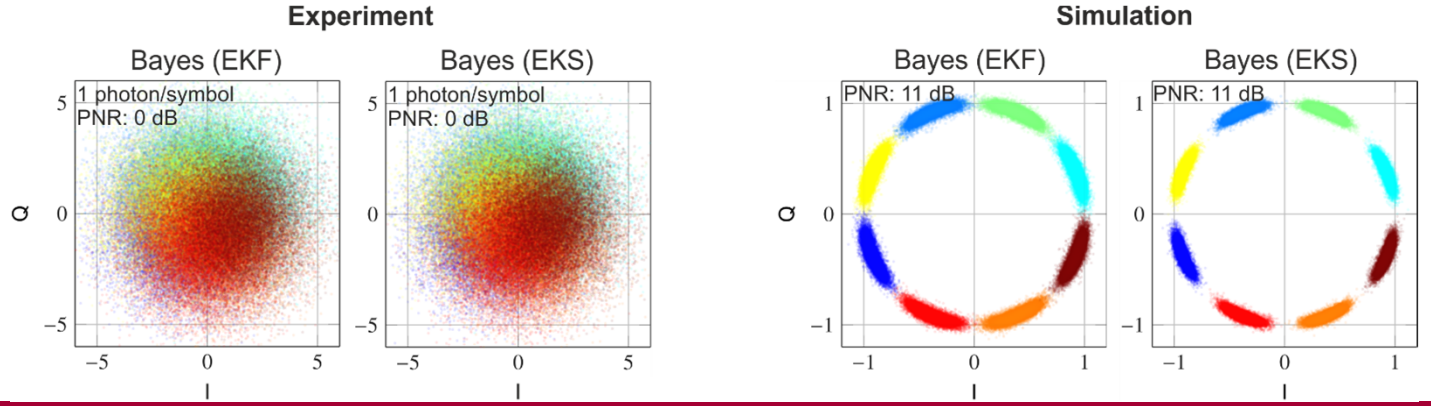
Simulation Model for $\Delta\hat{P}_{\mathrm{Q}}$

- Block diagram

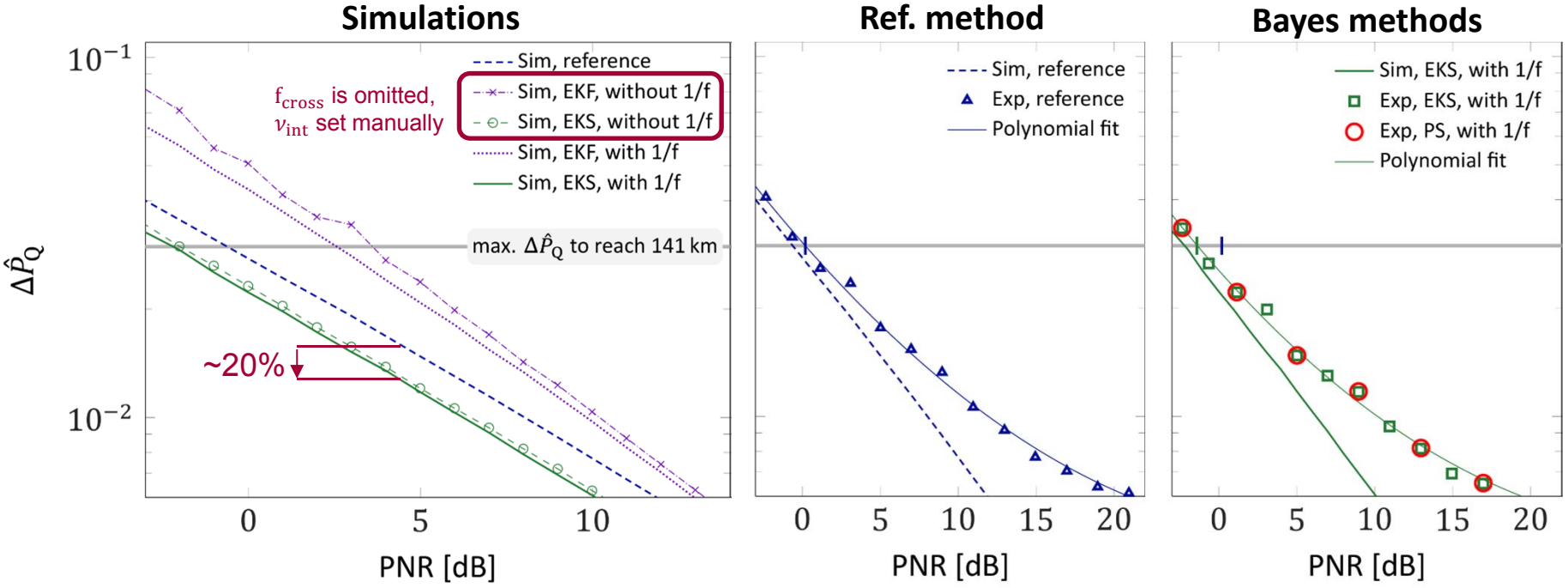


$s(i)$	Nyquist-shaped 8-PSK signal
$\hat{P}_{\mathrm{Q}}$	Power estimator for quantum signal
$\Delta\hat{P}_{\mathrm{Q}}$	Signal power underestimation factor
$n_{\mathrm{WGN}}(i)$	White Gaussian noise
PNR	Pilot to noise ratio (SNR of $p(i)$)

- Only additive Gaussian noise and phase noise included
- Signal constellations after phase correction



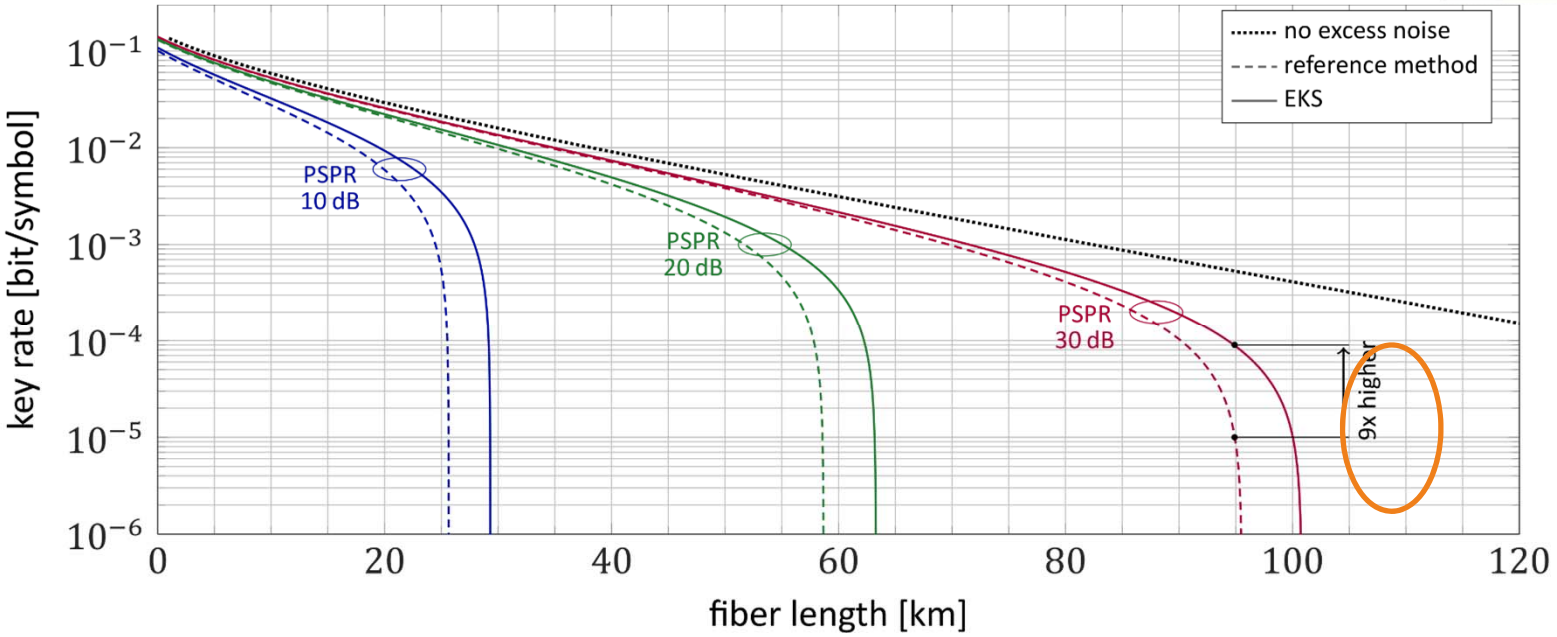
Experimental and Simulation Results



- It is beneficial to include f_{cross} in the model
  - Better performance / no manual adjustment necessary
- The EKS method outperforms the reference by **20%**
- PS and EKS show same performance
  → Indicates optimum for given state space model

S. Kleis, C. G. Schaeffer, *"Improving the Secret Key Rate of Coherent Quantum Key Distribution with Bayesian Inference"*, JLT October 2018.

Impact on the Secret Key Rate



- The realized PSPR has a very strong influence
- The EKS method improves the system
  - Significantly higher key rate
  - Extended reach
  - More efficient tuning (no optimization required)

A. Becir et al., "Continuous-Variable Quantum Key Distribution Protocols With Eight-State Discrete Modulation," International Journal of Quantum Information, vol. 10, no. 1, p. 1250004, 2012.

Combining the Quantum Channel with Infinera's Commercial WDM System

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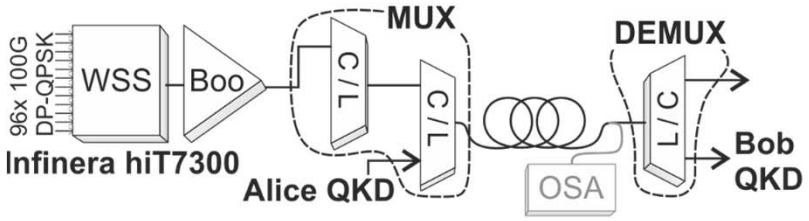
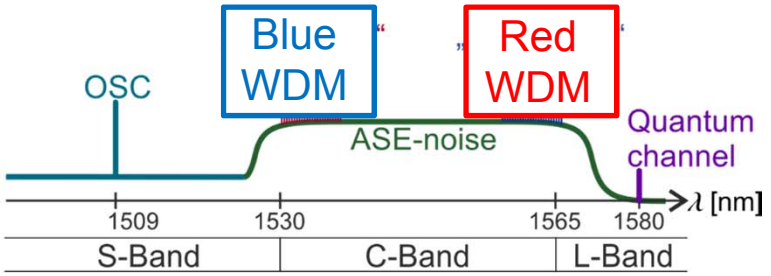
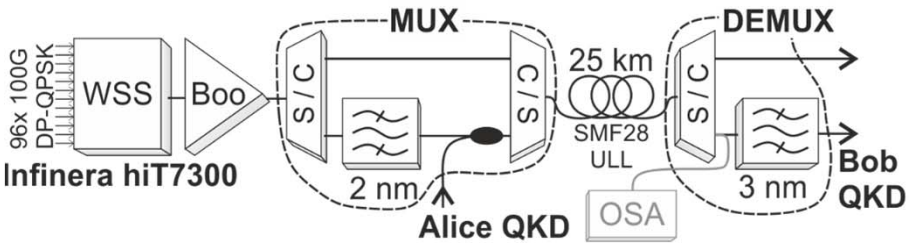
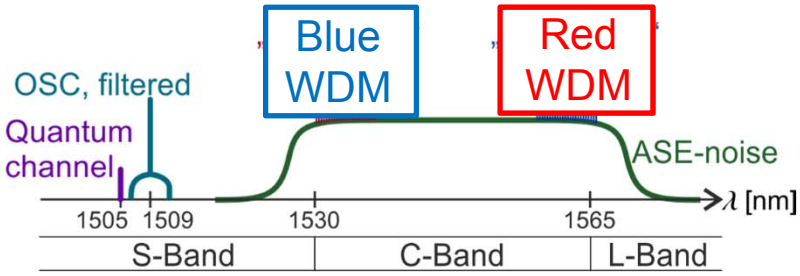
- L-Band configuration





- S-Band configuration

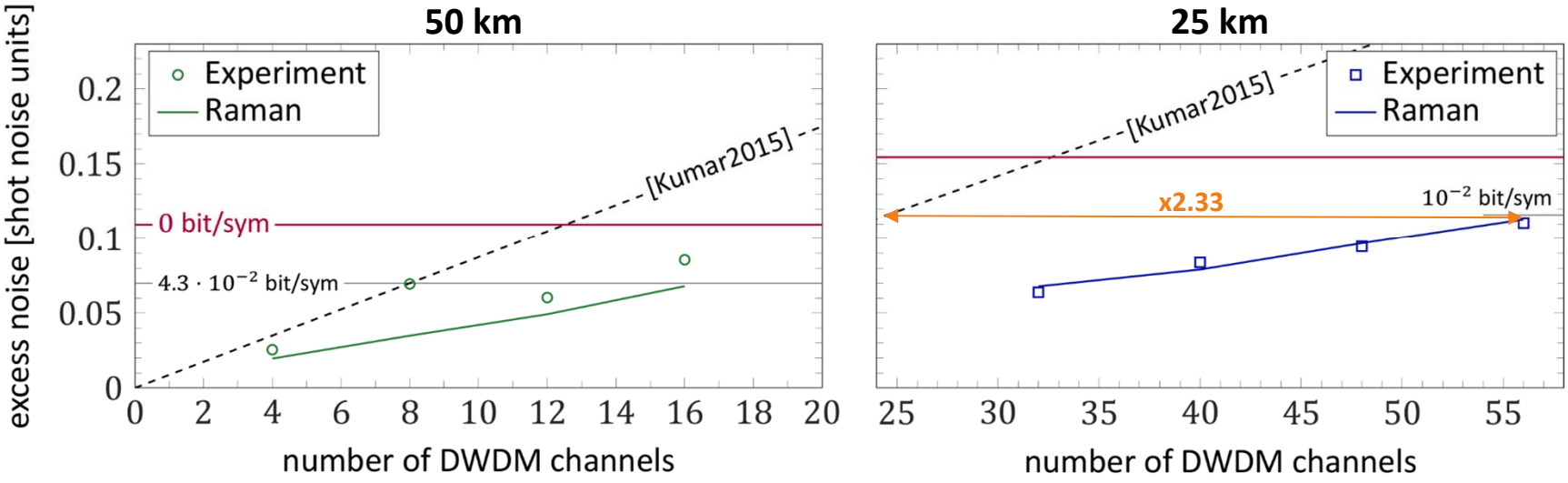




- Prior to experiments, noise contributions are measured with an OSA to predict the performance
- Quantum signal is Gaussian modulated!

infinera
coriant

Experimental Results for S-Band – „Red WDM“ – -3dBm/ch



- High tolerance also at large number of channels!
  - More than 56 of the 96 C-Band channels at 25km!
  - High flexibility for the C-Band DWDM system
  - Advantage compared to [Eriksson2018], where a narrow local minimum is used

[Eriksson2018]: 18ch, 13.7dBm, 10km
This work: 56ch, 14.5dBm, 25km

Parameters for 25 km (50 km)	
Launch power	2.42 ph/sym (1.99)
Reconciliation eff.	0.95
El. Noise power	0.18 SNU
Key rate w/o excess noise	5.3×10^{-2} bit/sym (1.4×10^{-2})
No. evaluated qu. symbols per data point	1.5×10^{8}

Conclusion

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- Quantum communications can be realized very similar to classical systems
- The DSP must perform the same tasks but under significantly different conditions
- Shown: Feasibility for signal powers lower than 10^{-2} photons per symbol with standard components only
- Laser phase noise is a limiting factor for the achievable reach in CV-QKD systems
- Bayesian smoothers can improve the system
  - Better performance in terms of key rate and reach
  - No try and error optimization required
  - EKS and PS show best performance
  - EKS is preferred due to lower computational complexity
- Coexistence of QKD and WDM investigated

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Thanks for your attention.

Some references:

Zibar, Darko ; Piels, Molly ; Jones, Rasmus Thomas ; Schaeffer, C. G.: „ Machine Learning Techniques in Optical Communication“, 41st European Conference on Optical Communication (ECOC), paper **Th.2.6.1**

S. Kleis, C. G. Schaeffer, "Improving the Secret Key Rate of Coherent Quantum Key Distribution with Bayesian Inference", IEEE JLT October 2018.

S. Kleis, C. G. Schaeffer, "Continuous variable quantum key distribution with a real local oscillator using simultaneous pilot signals", Optics Letters 42(8), 2017.

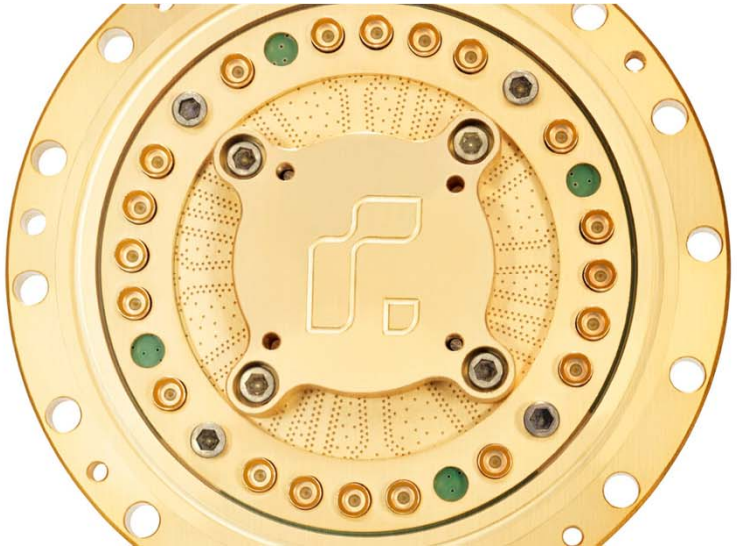
Quantum Computers

- They are coming:

Google unveiled Bristlecone, with 72 quantum qubits

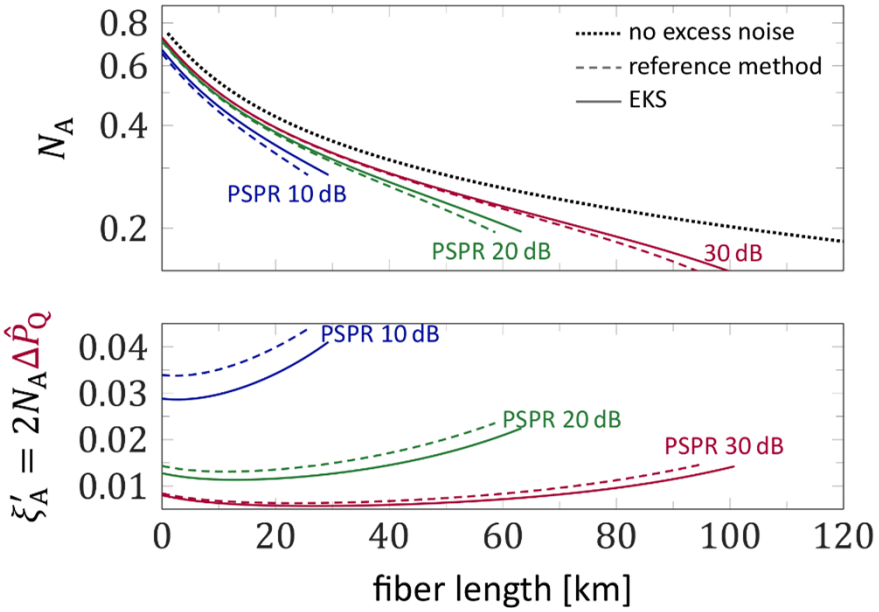
The next phase of quantum Computing: Rigetti 128

https://www.rigetti.com/



Key Rate Optimization Details

- Key rates are calculated according to 8-PSK security proof [3]
  - Optimization of N_A required
  - Key rate also depends on: $\Delta\hat{P}_\mathrm{Q}$, η, ρ, L, N_el
  - $\Delta\hat{P}_\mathrm{Q}$ depends on N_A and the PNR
    → Polynomial fits are included in the optimization



N_A	Alice's photons/symbol
$\Delta\hat{P}_\mathrm{Q}$	Signal power underestimation factor
η	Receiver efficiency
ρ	Reconciliation efficiency
L	Fiber length
N_el	Electronic noise power
PNR	Pilot to noise ratio

Scenario parameters

fiber loss	0.2 dB/km
ρ	0.95
N_el	0.2 N_0
η	0.51

[3] A. Becir et al., "Continuous-Variable Quantum Key Distribution Protocols With Eight-State Discrete Modulation," International Journal of Quantum Information, vol. 10, no. 1, p. 1250004, 2012.

The Extended Kalman Filter algorithm

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- Prediction

$$\hat{\varphi}_n^- = \hat{\varphi}_{n-1}$$

$$\mathbf{P}_n^- = \mathbf{P}_{n-1} + \mathbf{Q}_{n-1}$$

φ_n:	Measured phase
$\hat{\varphi}_n$:	Filtered phase
$\mathbf{Q}_n$, $\mathbf{R}_n$:	Covariance matrices of, $\mathbf{q}$, $\mathbf{r}$
$\mathbf{H}_\mathbf{x}(\varphi)$:	Jacobian matrix of h with respect to $\mathbf{x}$ around φ

- Update

$$v_n = \varphi_n - h(\hat{\varphi}_n^-)$$

$$S_n = \mathbf{H}_\mathbf{x}(\hat{\varphi}_n^-)\mathbf{P}_n^-\mathbf{H}_\mathbf{x}^\mathrm{T}(\hat{\varphi}_n^-) + \mathbf{H}_\mathbf{r}(\hat{\varphi}_n^-)\mathbf{R}_n^-\mathbf{H}_\mathbf{r}^\mathrm{T}(\hat{\varphi}_n^-)$$

$$K_n = \mathbf{P}_n^-\mathbf{H}_\mathbf{x}^\mathrm{T}(\hat{\varphi}_n^-)S_n^{-1}$$

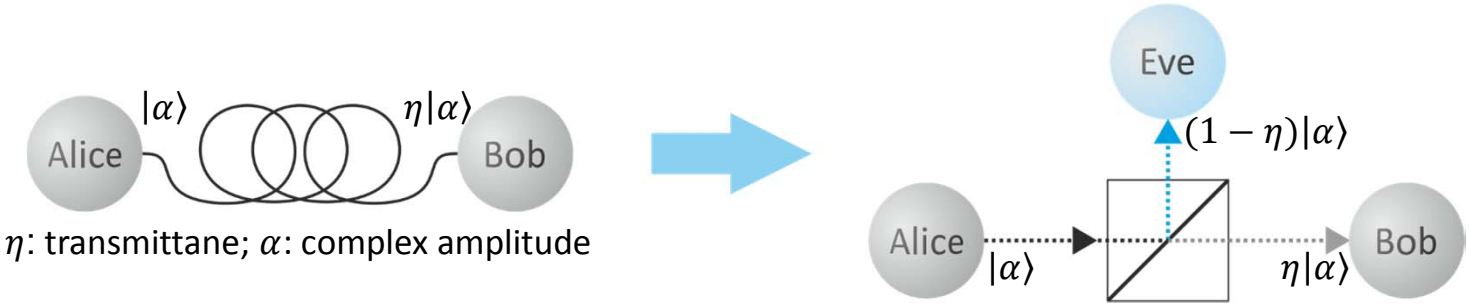
$$\hat{\varphi}_n = \hat{\varphi}_n^- + K_n v_n$$

$$\mathbf{P}_n = \mathbf{P}_n^- - K_n S_n K_n^{-1}$$

- Currently in progress: Integration into existing DSP procedure

Strongest Passive Attack[5]

- Beam splitter attack: Eve replaces channel by loss-less beam splitter



- Direct reconciliation:

$$G = \rho I_{\mathrm{AB}}(\eta, |\alpha|) - I_{\mathrm{AE}}(1-\eta, |\alpha|) \rightarrow \text{no key for } \eta < 0.5$$

- Reverse reconciliation:

$$G = \rho I_{\mathrm{AB}}(\eta, |\alpha|) - I_{\mathrm{BE}}(1-\eta, |\alpha|) \rightarrow \text{key for any } \eta$$

[5]G. van Assche, *"Quantum Cryptography and Secret Key Distillation"*, Cambridge University Press, 2006

Eve‘s Information

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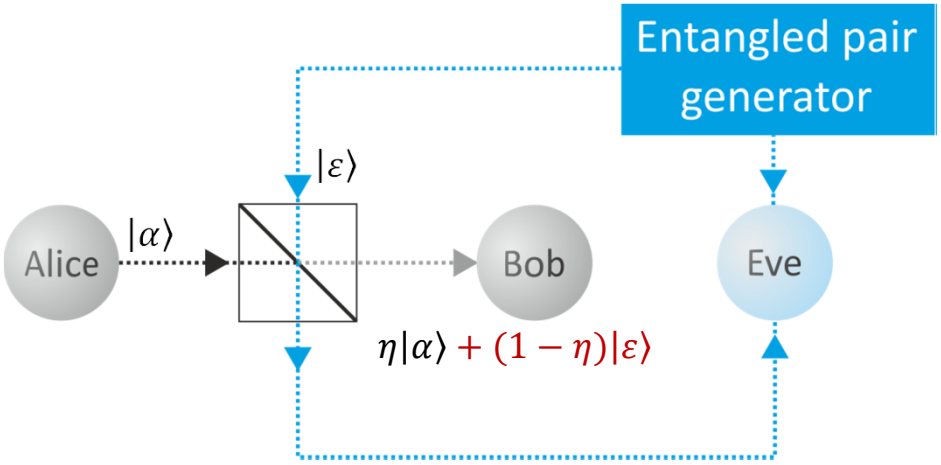
- Scenario:
  - Beam splitter attack
  - Eve uses a perfect coherent receiver (with measurement outcome ε)
  - Bob applies hard decision

$$I_{\mathrm{BE,hard}}(\varepsilon) = \log_2(M) + \sum_{l=0}^{M} p(l|\varepsilon)\log_2[p(l|\varepsilon)]$$

$$I_{\mathrm{BE,hard}} = \int p(\varepsilon) I_{\mathrm{BE,hard}}(\varepsilon)\,\mathrm{d}\varepsilon$$

General Active Attack Scenario (with Excess Noise)

- Entanglement cloner attack[2]
- Generalization of the beam splitter attack



- Eve increases I_{BE} by introducing one half of an entangled pair $|\varepsilon\rangle$, generated by her.
- This additional correlation between Eve and Bob induces excess noise ξ' in Bob‘s received signal

[2]G. van Assche, *“Quantum Cryptography and Secret Key Distillation“*, Cambridge University Press, 2006