



# **Quantum limits of deep space optical communication**

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**European  
Funds**  
Smart Growth



**Republic  
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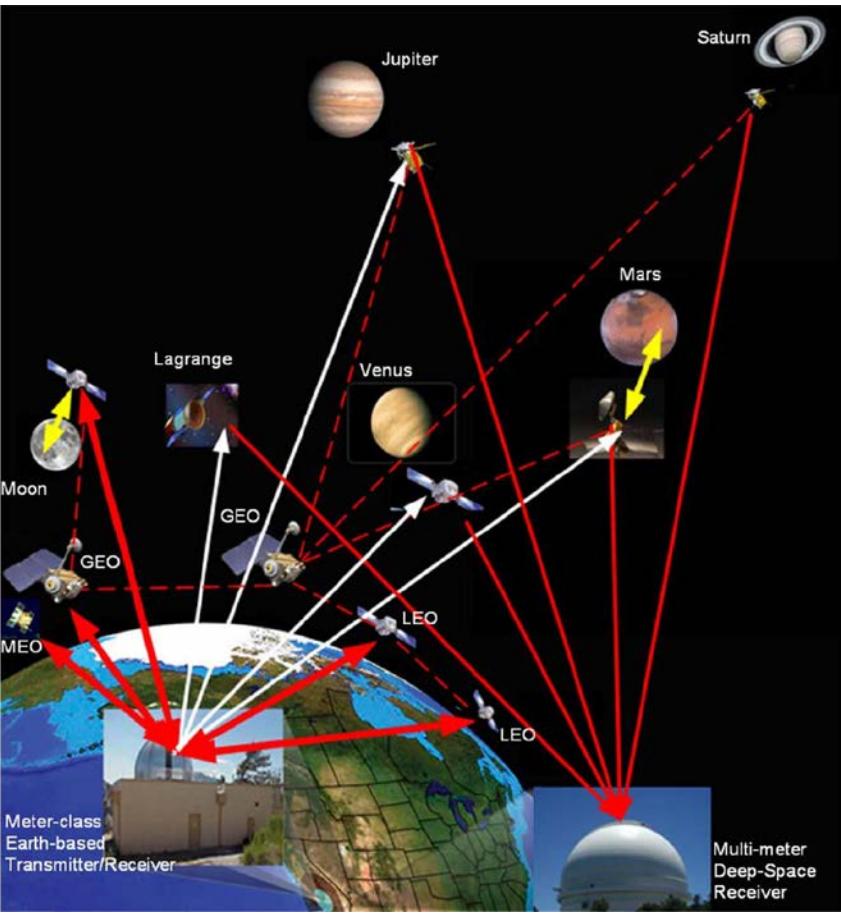


**Foundation for  
Polish Science**

**European Union  
European Regional  
Development Fund**



# Satellite optical communication



H. Hemmati, A. Biswas, and I. Djordjevic,  
*Deep-Space Optical Communications: Future  
Perspectives and Applications*,  
Proc. IEEE 99, 2020 (2011)

## Optical vs radio frequency communication

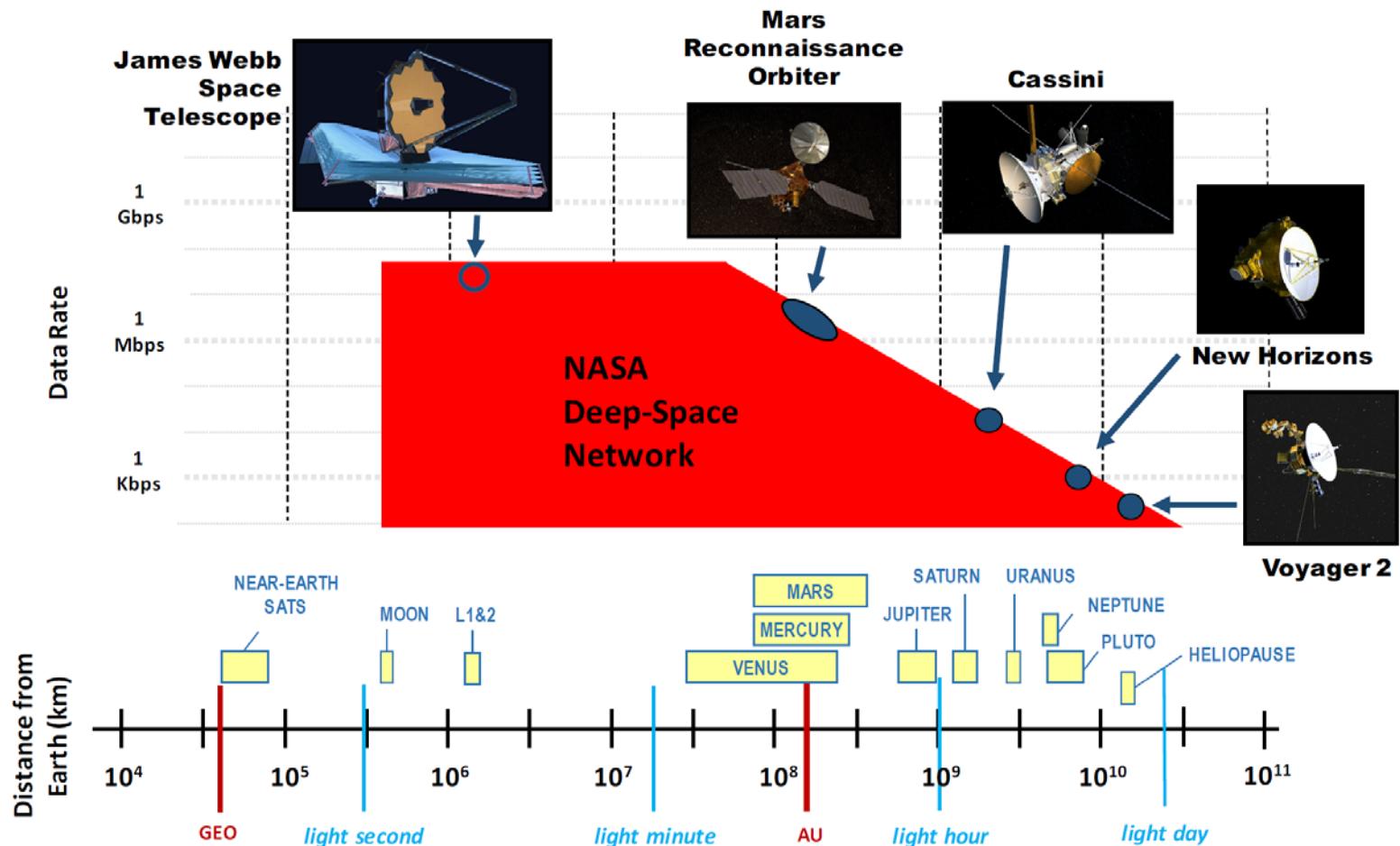
### Benefits:

- Access to higher bandwidths
- Lower diffraction losses
- Reduced regulatory requirements

### Challenges:

- Robustness against atmospheric conditions
- Wall-plug efficiency of onboard transceivers
- Pointing and tracking
- Antenna surface quality

# Deep-space rf communication links



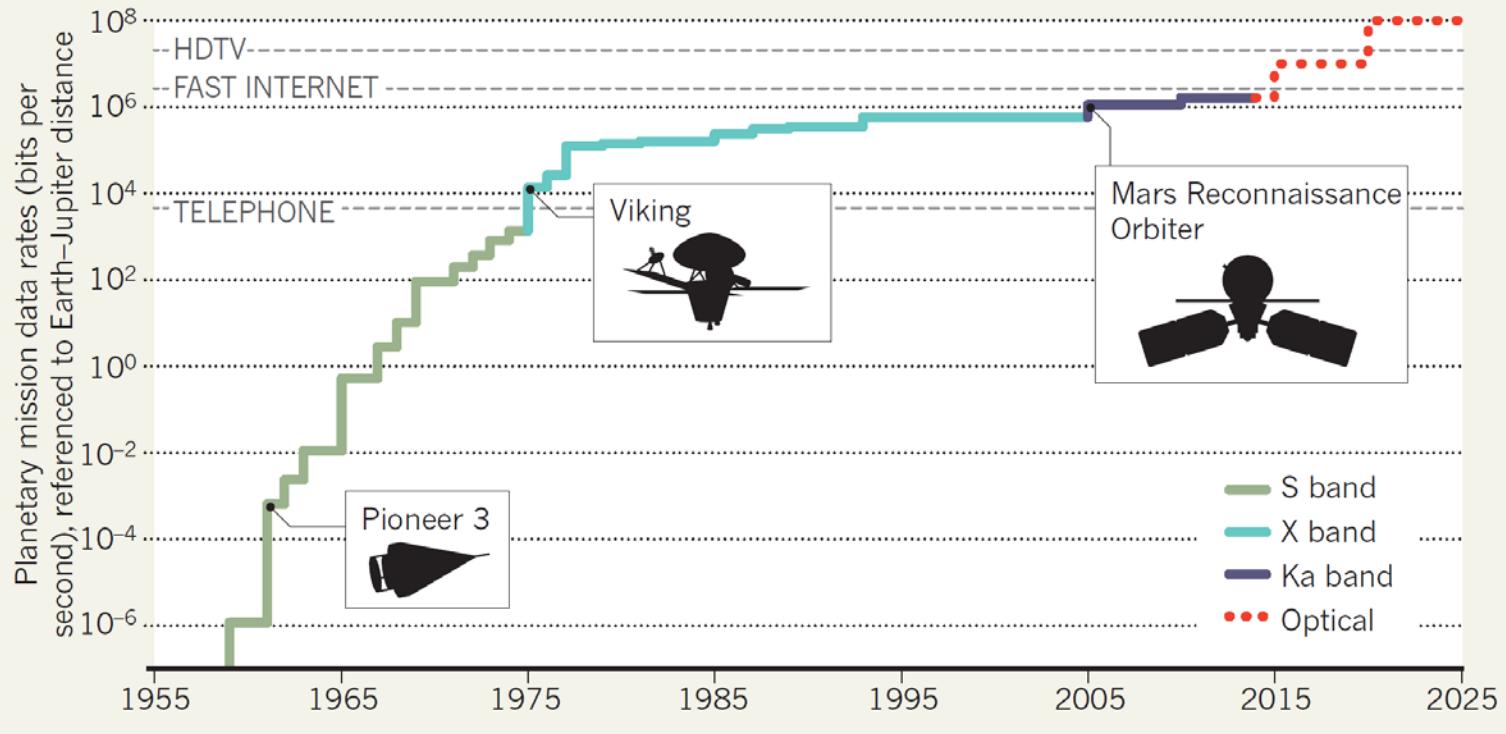
D. Boroson, *On achieving high performance optical communications from very deep space*, Proc. SPIE 10524, 105240B (2018)

# Deep-space optical communication

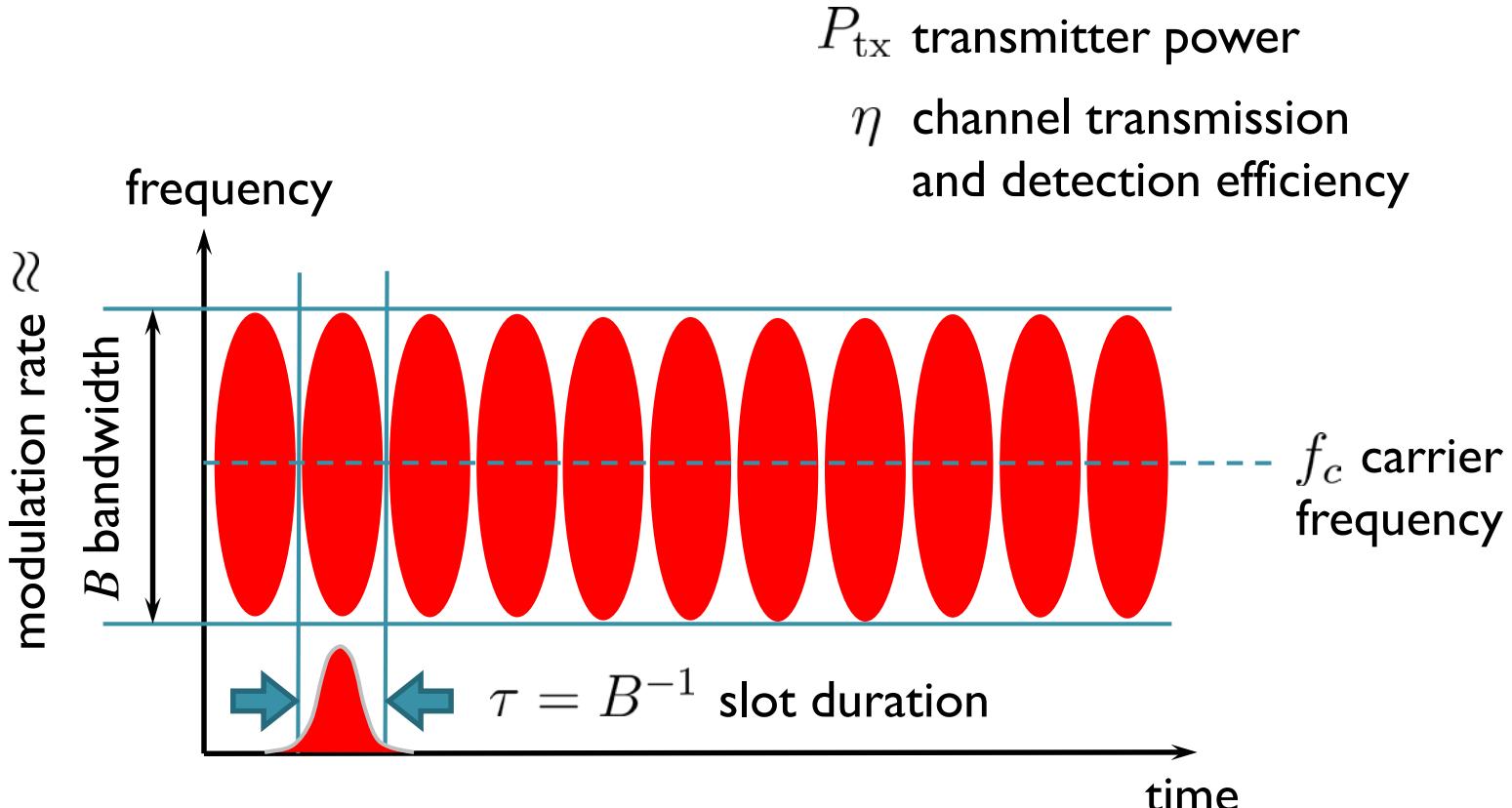
SOURCE: NASA/JPL-CALTECH

## TUNED IN

Interplanetary data transmission rates have shot up 10 orders of magnitude in the past 50 years, thanks in part to higher frequency bands of radio waves. Optical transmissions with lasers promise to extend that pace, to the point at which high-definition television broadcasts from Jupiter might be possible.



# Signal strength



Average detected number of photons per slot:

$$n_a = \frac{\eta P_{\text{tx}} \tau}{h f_c} = \frac{1}{h f_c} \cdot \frac{\eta P_{\text{tx}}}{B}$$

Planck's constant  $h = 6.626 \times 10^{-34} \text{ J} \cdot \text{s}$

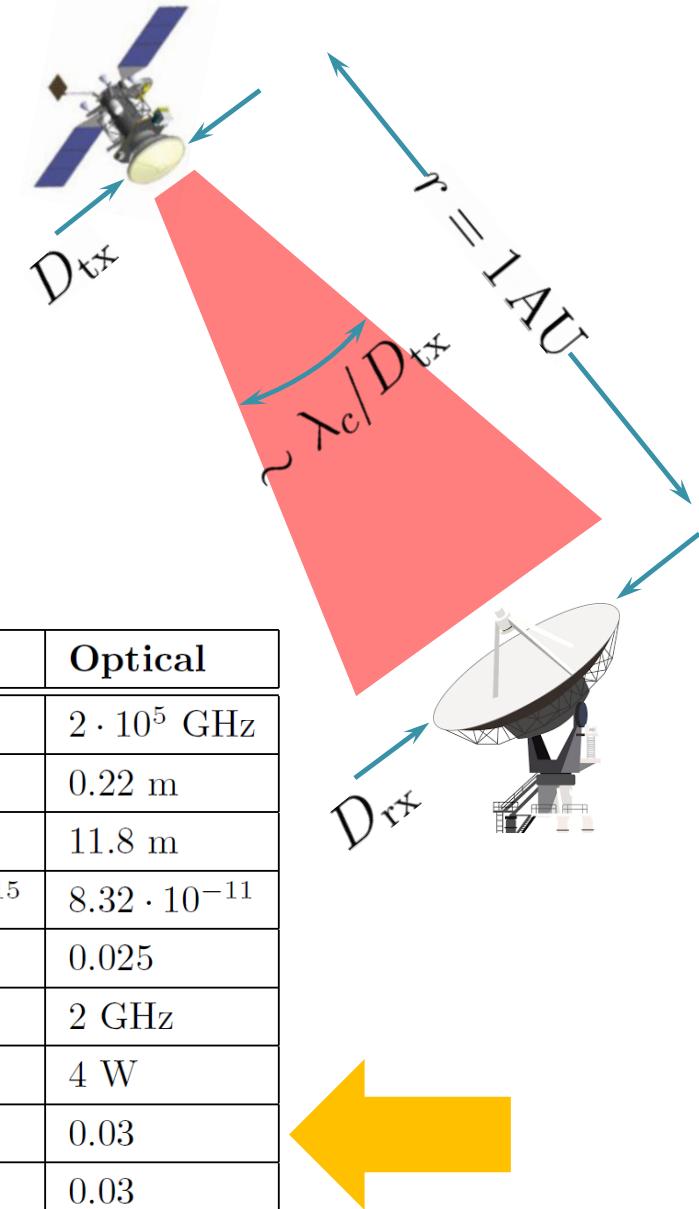
# System characteristics

Channel transmission:

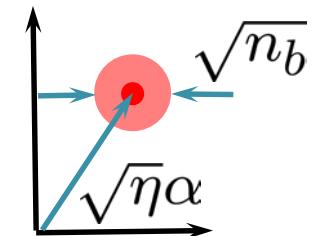
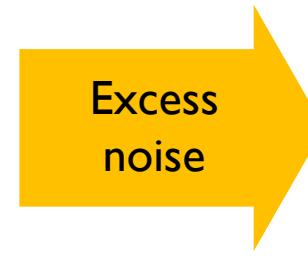
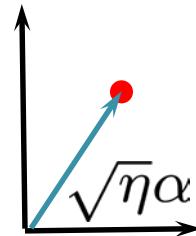
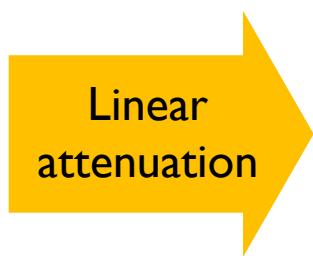
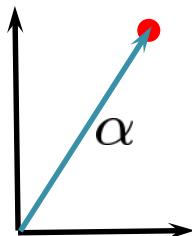
$$\eta_{\text{ch}} = \frac{1}{r^2} \cdot \left( \frac{\pi D_{\text{tx}} D_{\text{rx}}}{4 \lambda_c} \right)^2$$

Signal central wavelength  $\lambda_c = c/f_c$

Operating regime	RF	Optical
Carrier frequency $f_c$	32 GHz	$2 \cdot 10^5$ GHz
Transmit antenna diameter $D_t$	3 m	0.22 m
Receiver antenna diameter $D_r$	34 m	11.8 m
Channel transmission $\eta_{\text{ch}}$	$3.29 \cdot 10^{-15}$	$8.32 \cdot 10^{-11}$
Detector efficiency $\eta_{\text{det}}$	0.1	0.025
Bandwidth $B$	0.5 GHz	2 GHz
Transmit power $P$	35 W	4 W
Average output photon number $n_a$	1.08	0.03
Average noise photon number $n_b$	66.68	0.03



# Phase-insensitive Gaussian channel



$$\bar{n} = \langle |\alpha|^2 \rangle$$

$$n_a = \eta \bar{n}$$

Total:  $n_a + n_b$

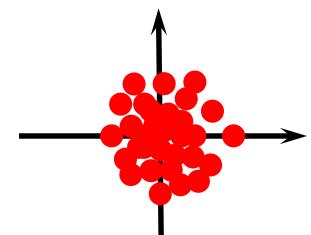
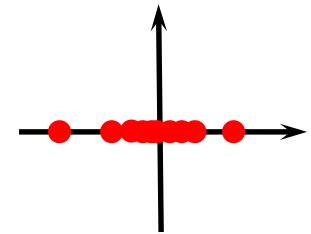
Shannon-Hartley theorem

Homodyne capacity per slot

$$C_{\text{hom}} = \frac{1}{2} \log_2 \left( 1 + \frac{4n_a}{1 + 2n_b} \right)$$

Heterodyne capacity per slot

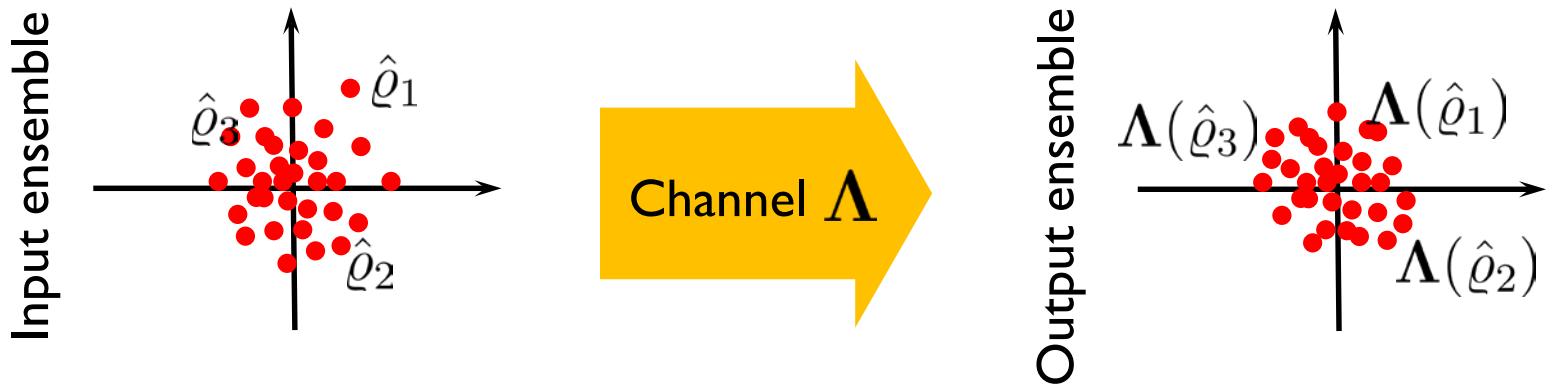
$$C_{\text{het}} = \log_2 \left( 1 + \frac{n_a}{1 + n_b} \right)$$



shot-noise limited detection

channel excess noise

# Quantum Shannon theory



**Holevo quantity  $\chi$ :** for any measurement on the output ensemble

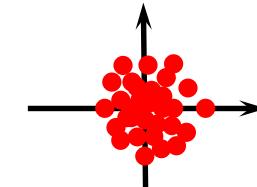
$$I \leq \chi = S\left(\sum_i p_i \Lambda(\hat{\varrho}_i)\right) - \sum_i p_i S(\Lambda(\hat{\varrho}_i))$$

where  $S(\hat{\varrho}) = -\text{Tr}(\hat{\varrho} \log_2 \hat{\varrho})$  is the von Neumann entropy.

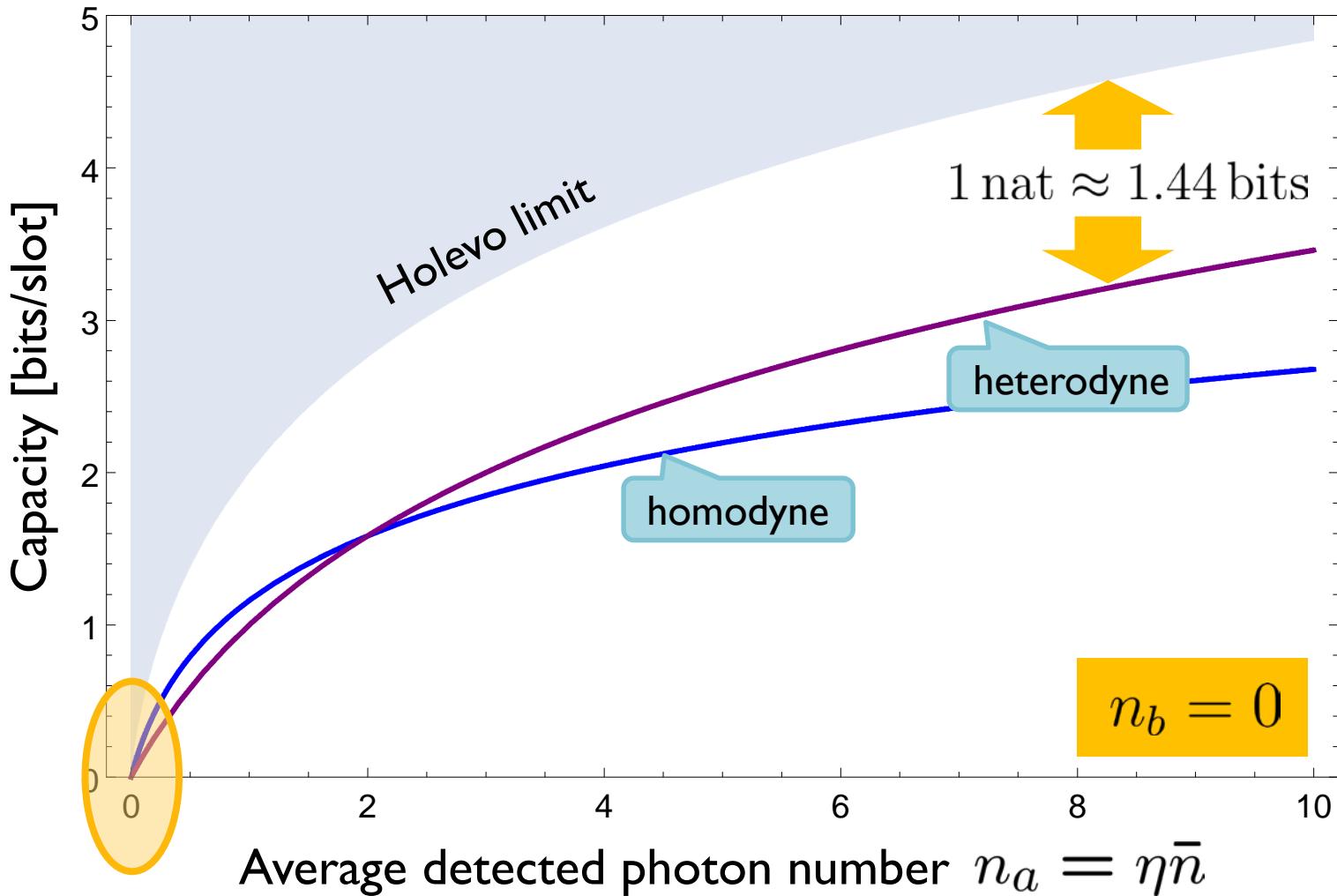
For a phase-insensitive Gaussian channel under average power constraint:

$$C_{\text{Hol}} = g(n_a + n_b) - g(n_b)$$

where  $g(x) = (x + 1) \log_2(x + 1) - x \log_2 x$



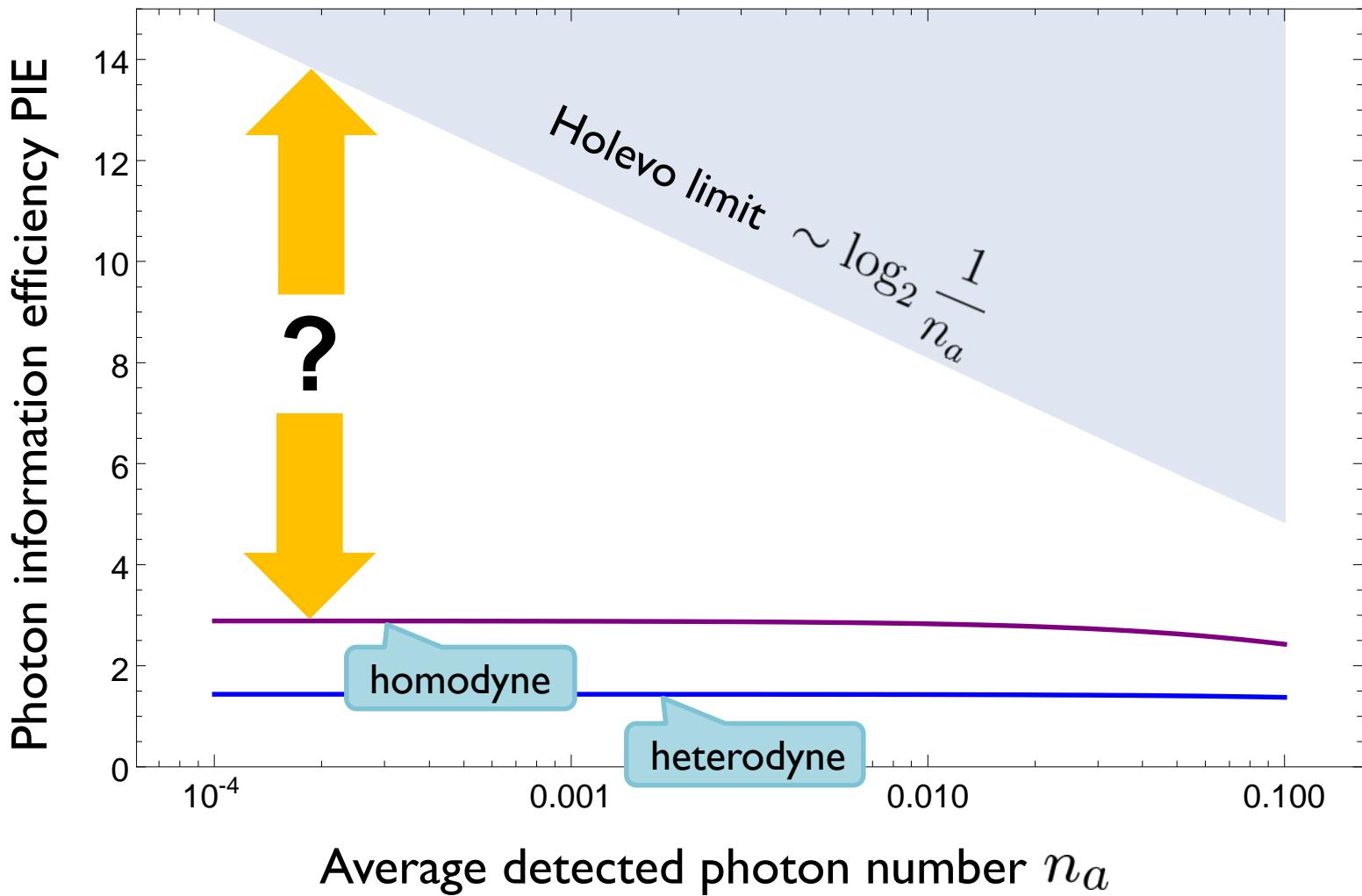
# Pure loss channel



$$C_{\text{Hol}} - C_{\text{het}} = n_a \log_2 \left( 1 + \frac{1}{n_a} \right) \xrightarrow{n_a \gg 0} \log_2 e$$

# Photon information efficiency

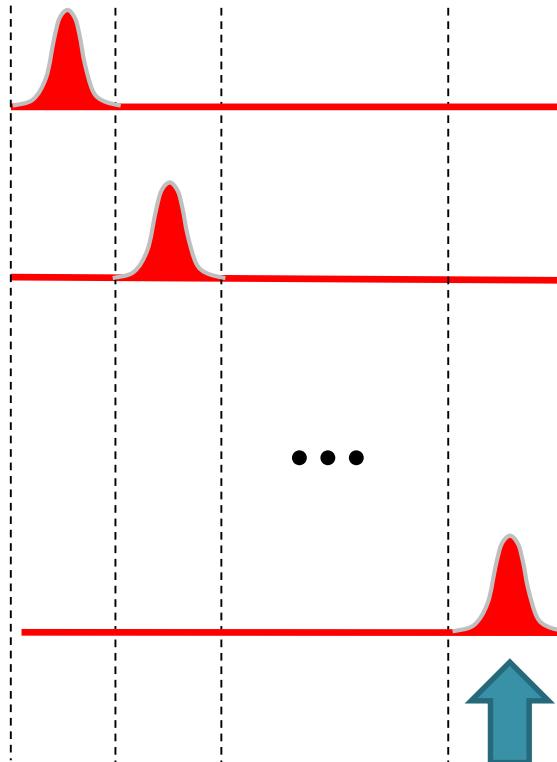
Information rate [bits/s]:  $R = B \cdot C = B \cdot n_a \cdot \text{PIE}$



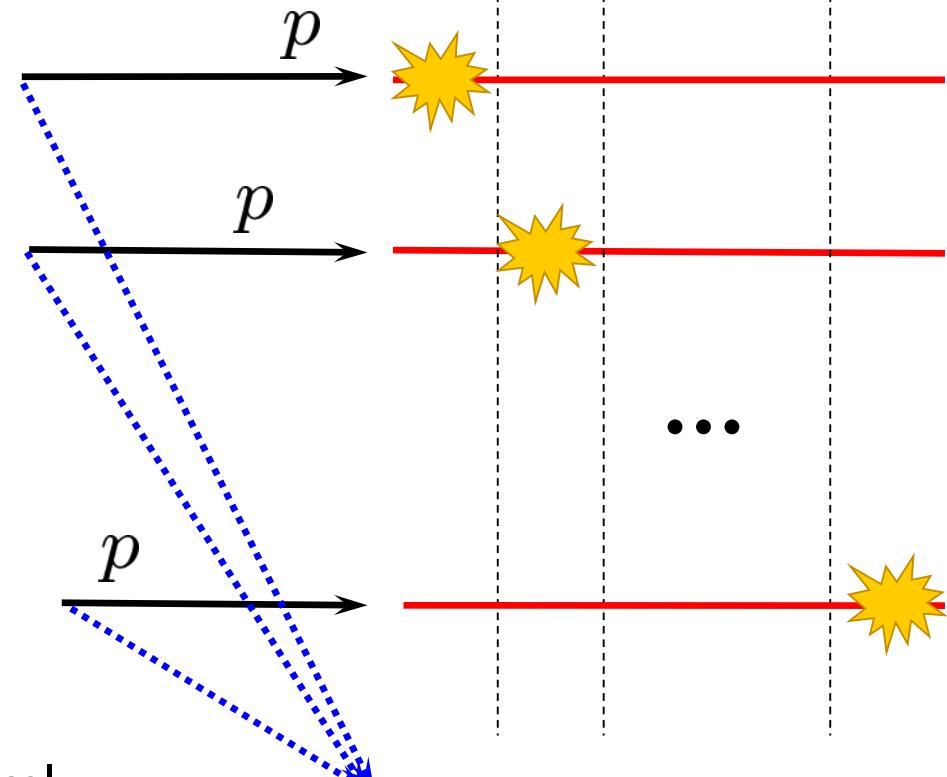
# PPM – Pulse Position Modulation

Symbols:

1    2    ...     $M$



Geiger-type  
direct detection



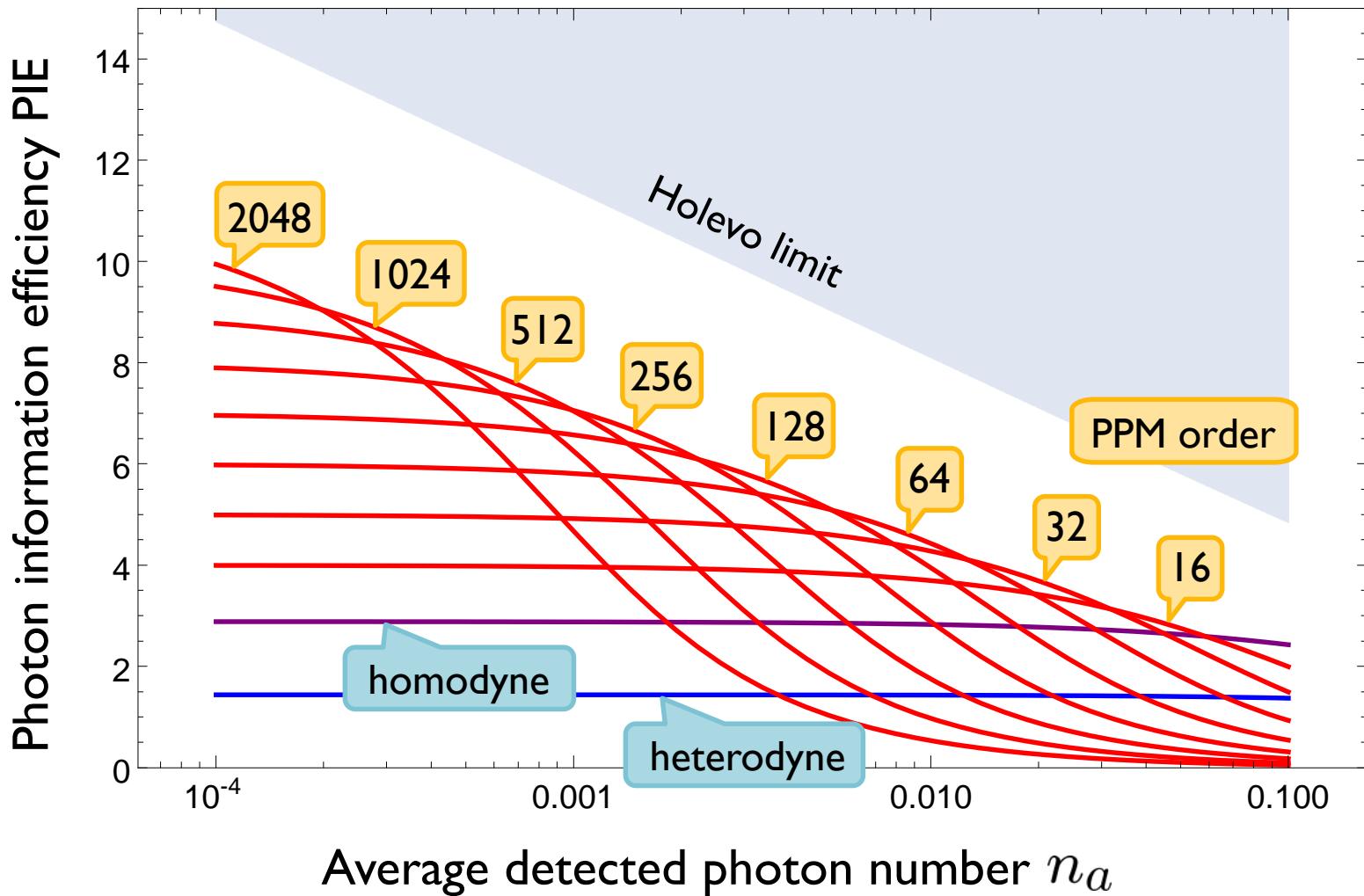
pulse optical  
energy  $Mn_a$

erasure

Photocount probability (symbol recovery):  $p = 1 - e^{-Mn_a}$

# PPM photon information efficiency

$$\text{PIE} = \frac{1 - e^{-Mn_a}}{Mn_a} \log_2 M \xrightarrow{n_a \rightarrow 0} \log_2 M$$



# PPM PIE asymptotics

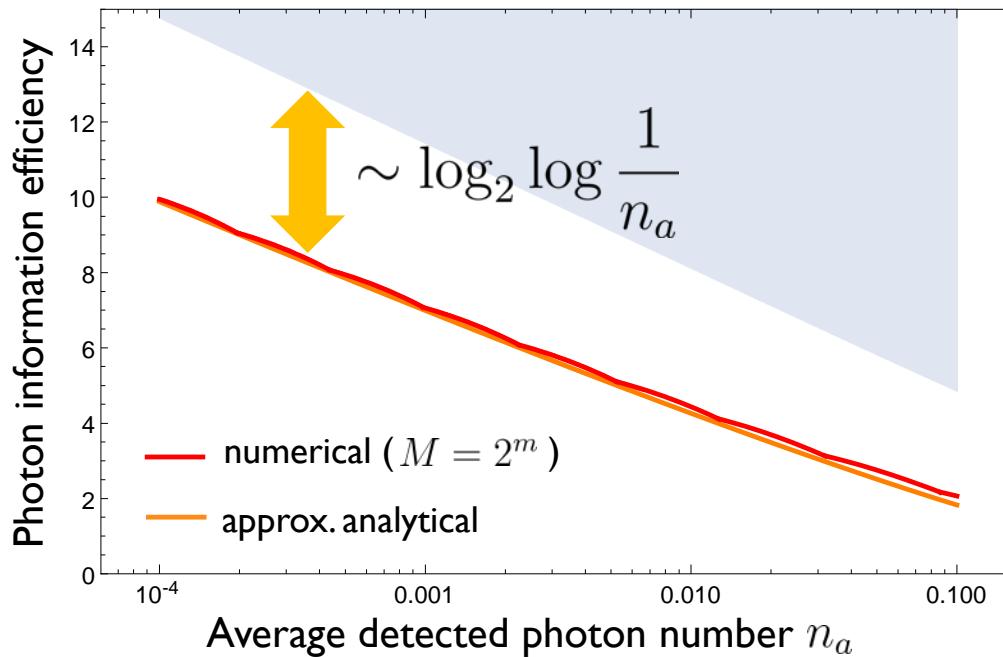
M. Jarzyna, P. Kuszaj, K. Banaszek, Opt. Express **23**, 3170 (2015)

Photocount probability:

$$p = 1 - \exp(-Mn_a) \approx Mn_a - \frac{1}{2}(Mn_a)^2$$

Approximate analytical expression:

$$\text{PIE}_{\text{PPM}} \approx \left( W\left(\frac{2e}{n_a}\right) - 2 + \left[ W\left(\frac{2e}{n_a}\right) \right]^{-1} \right) \log_2 e$$



Lambert function  $W(x)$   
 $\approx \log x - \log \log x$   
for  $x \gg 1$

Optimal PPM order

$$M \approx \frac{2}{n_a} \left[ W\left(\frac{2e}{n_a}\right) \right]^{-1}$$

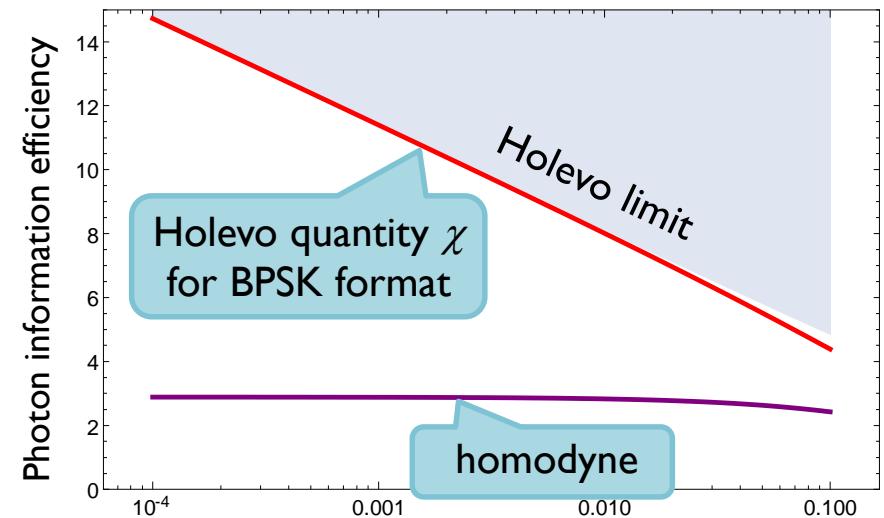


# **Superadditivity of accessible information**

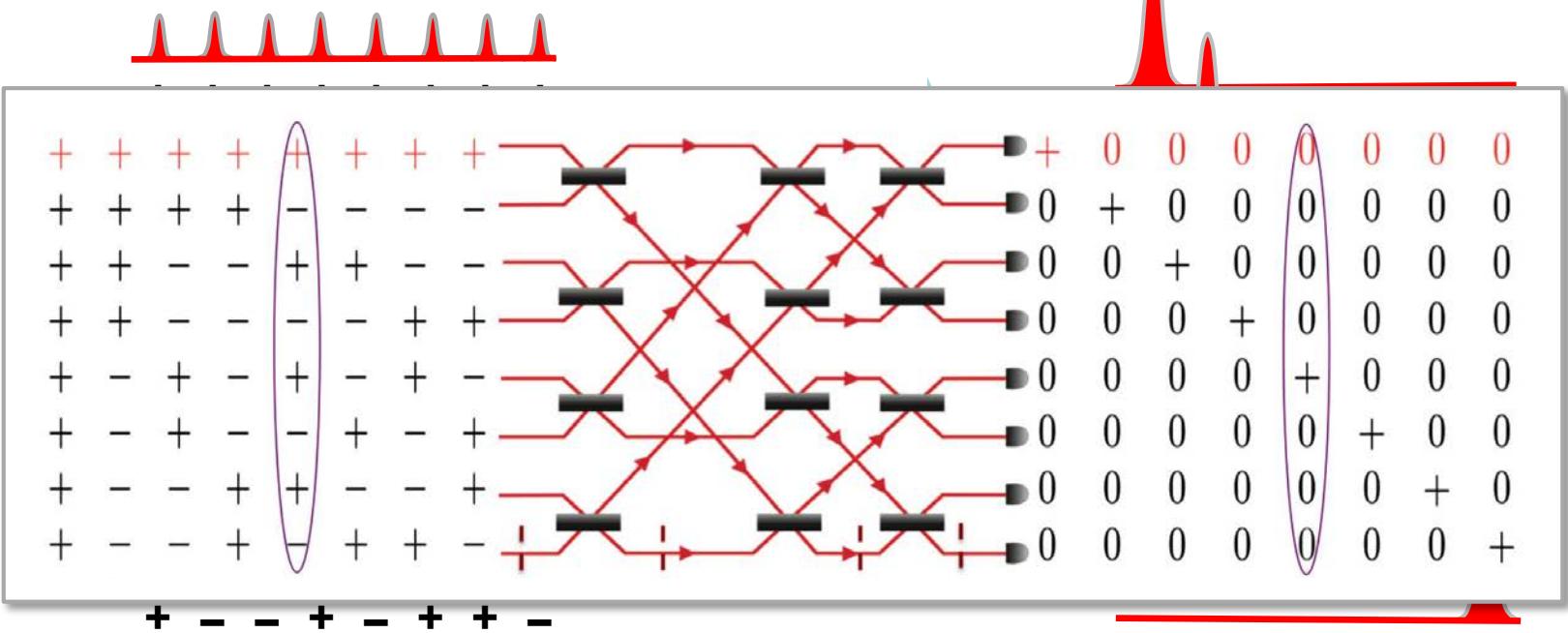
BPSK:

## Holevo quantity assumes:

- preparation of codewords
  - collective detection of multiple symbols

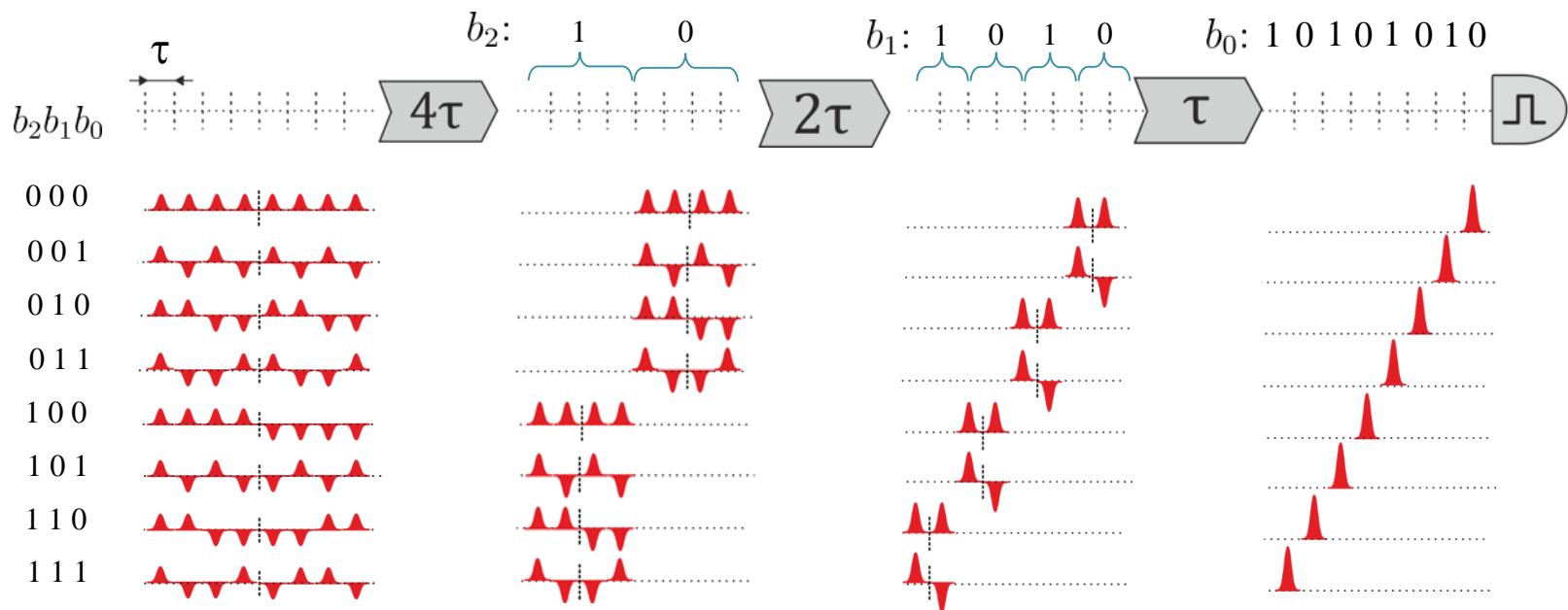
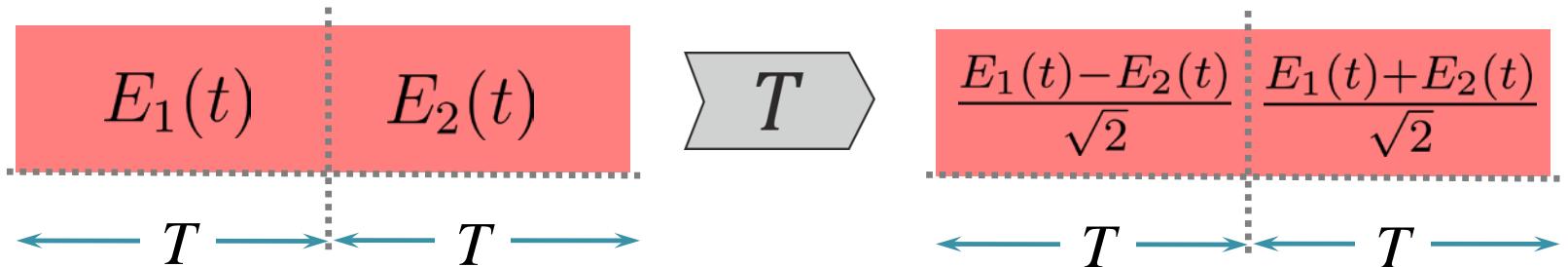


S. Guha, Phys. Rev. Lett. 106, 240502 (2011)



# Scalable structured receiver

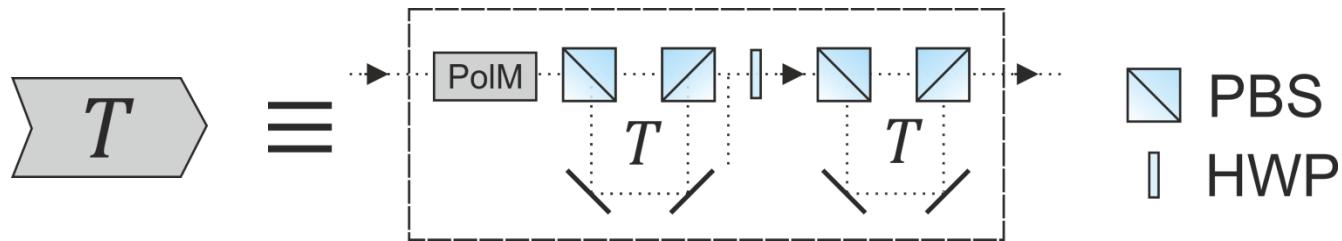
K. Banaszek and M. Jachura, Proc. IEEE ICSOS 2017, pp. 34-37



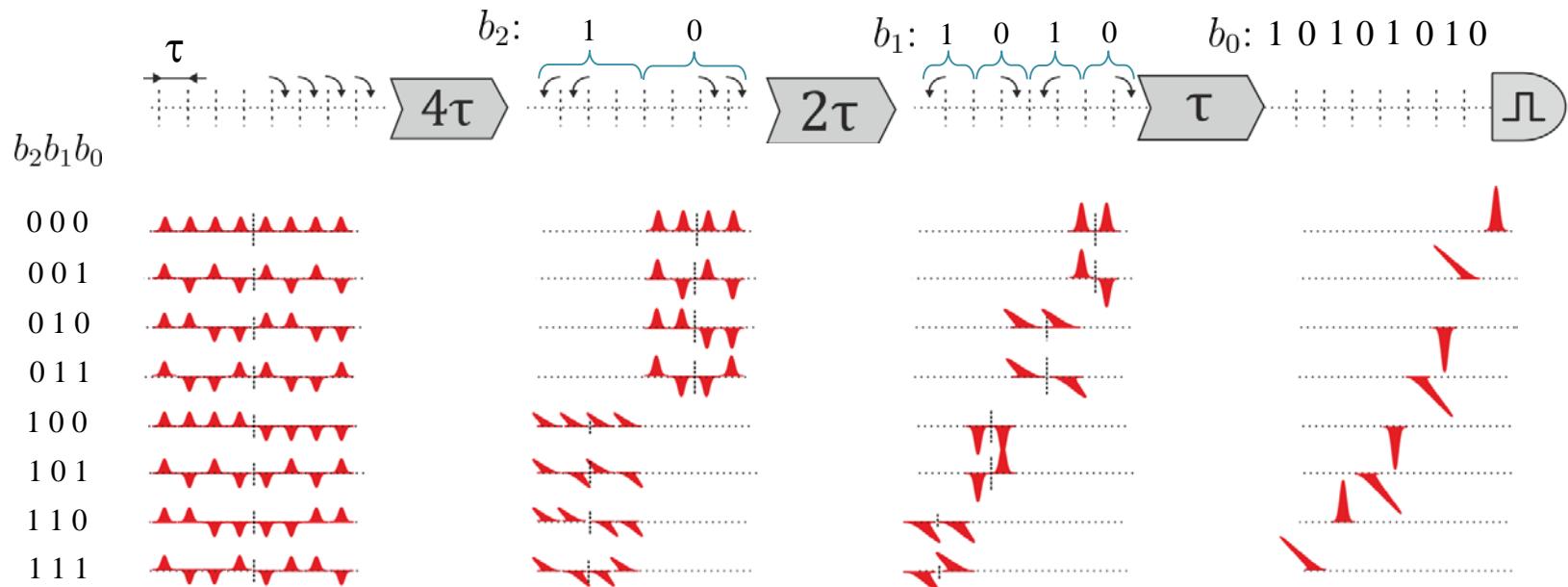
Pulse position:  $\sum_k b_k 2^k$

# Realization

K. Banaszek and M. Jachura, Proc. IEEE ICSOS 2017, pp. 34-37



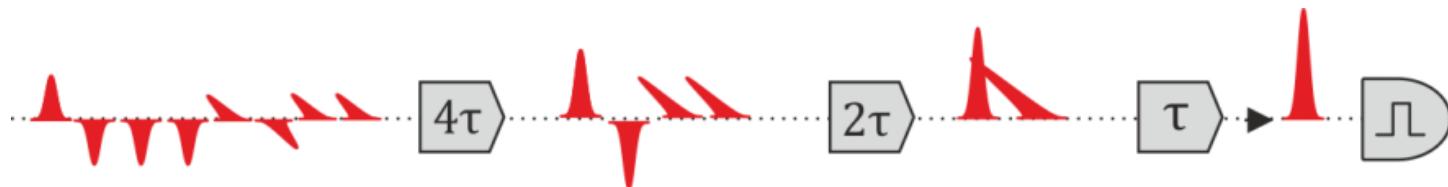
↷↷ polarization switching



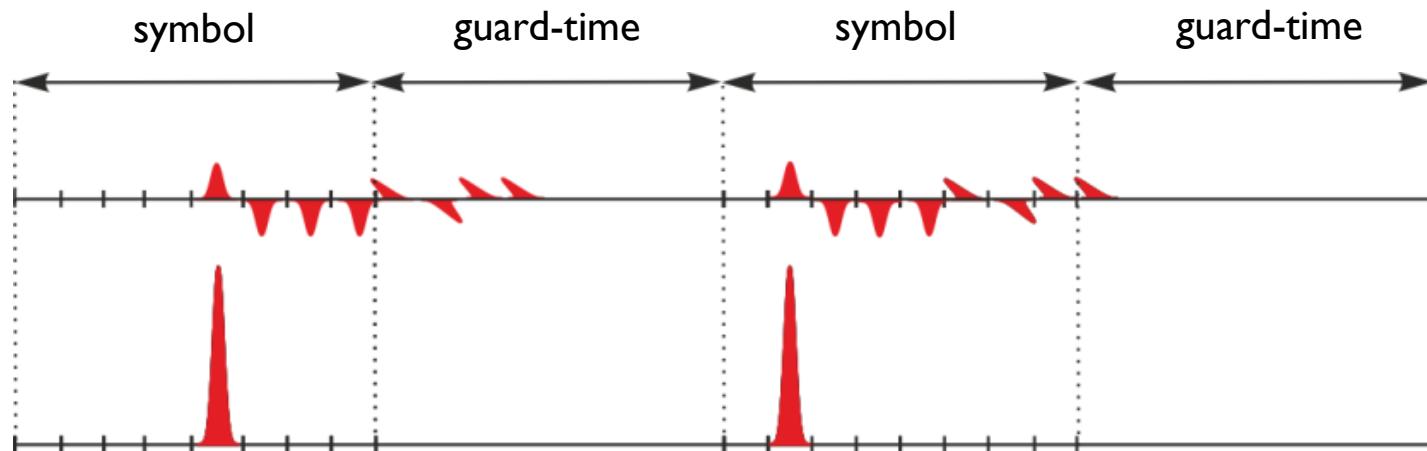
$$\text{Pulse position: } \sum_k b_k 2^k$$

# Phase-polarization patterns

K. Banaszek and M. Jachura, Proc. IEEE ICSOS 2017, pp. 34-37



Polarization-dependent  
delay + rotation by  $45^\circ$



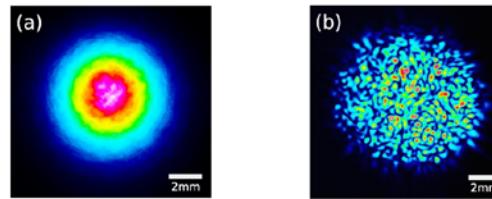
PPM encoding achieved by shifting the entire pattern in time



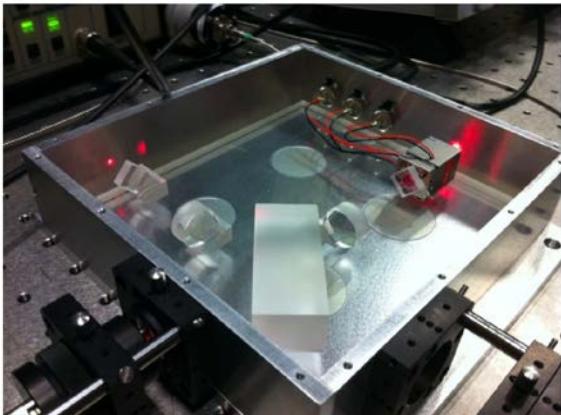
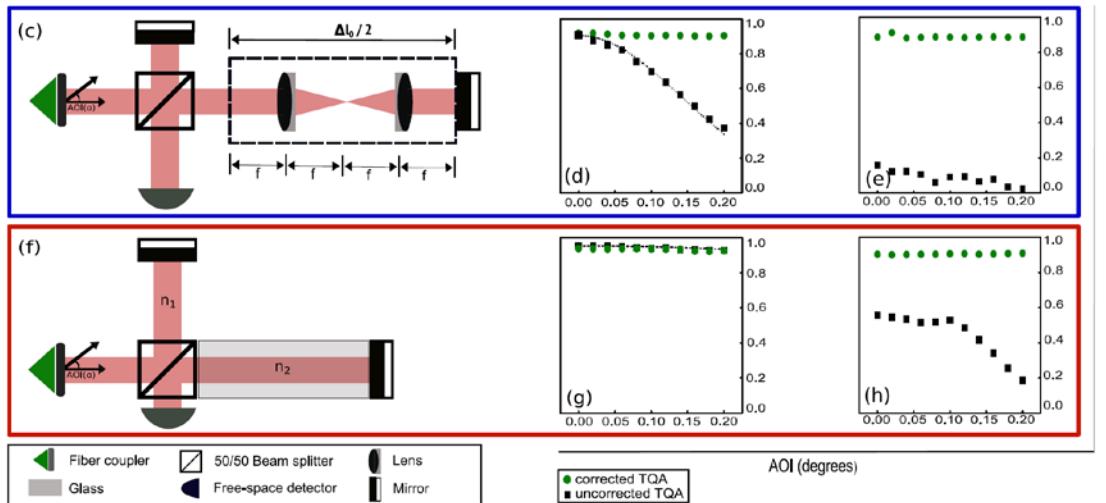
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OF WARSAW

# Atmospheric turbulence

J. Jin et al., Demonstration of analyzers for multimode photonic time-bin qubits, Phys. Rev. A **97**, 043847 (2018)

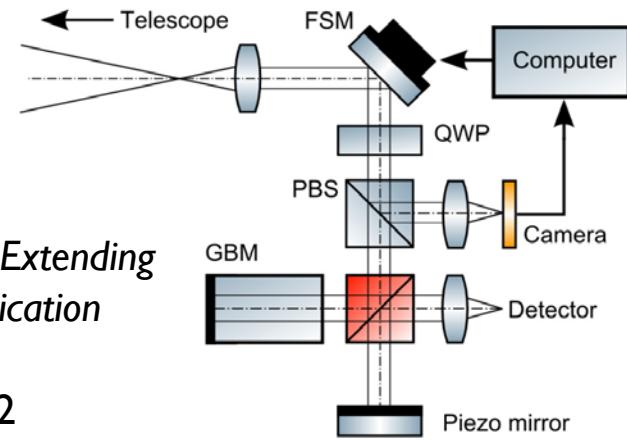


wavefront distorted signal

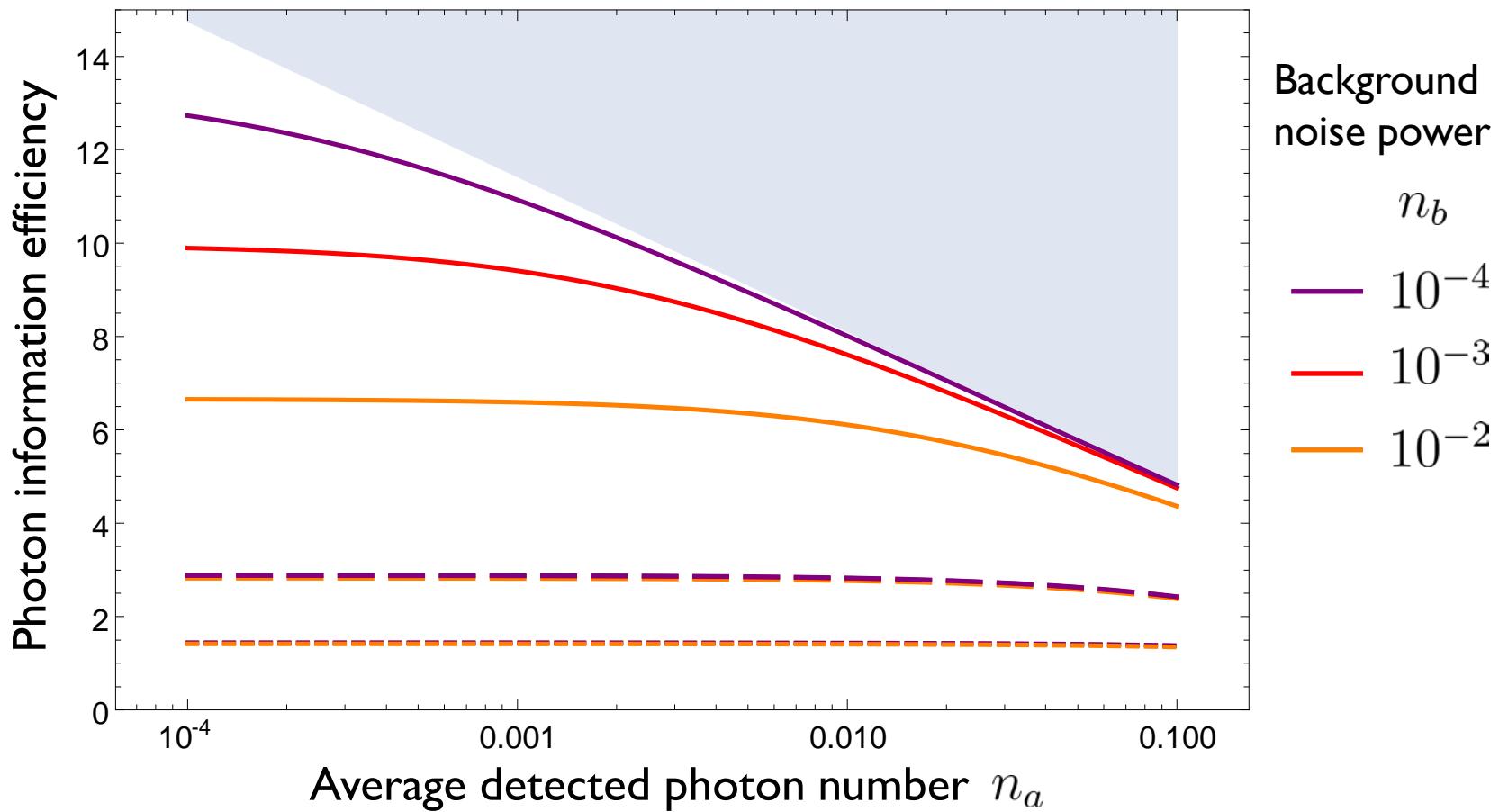


Multimode DPSK receiver

Z. Sodnik and M. Sans, Extending EDRS to Laser Communication from Space to Ground, Proc. ICSOS 2012, 13-2



# Noisy channel asymptotics



PIE  $\xrightarrow[n_a \rightarrow 0]{} \frac{\log_2 e}{1 + n_b}$

heterodyne

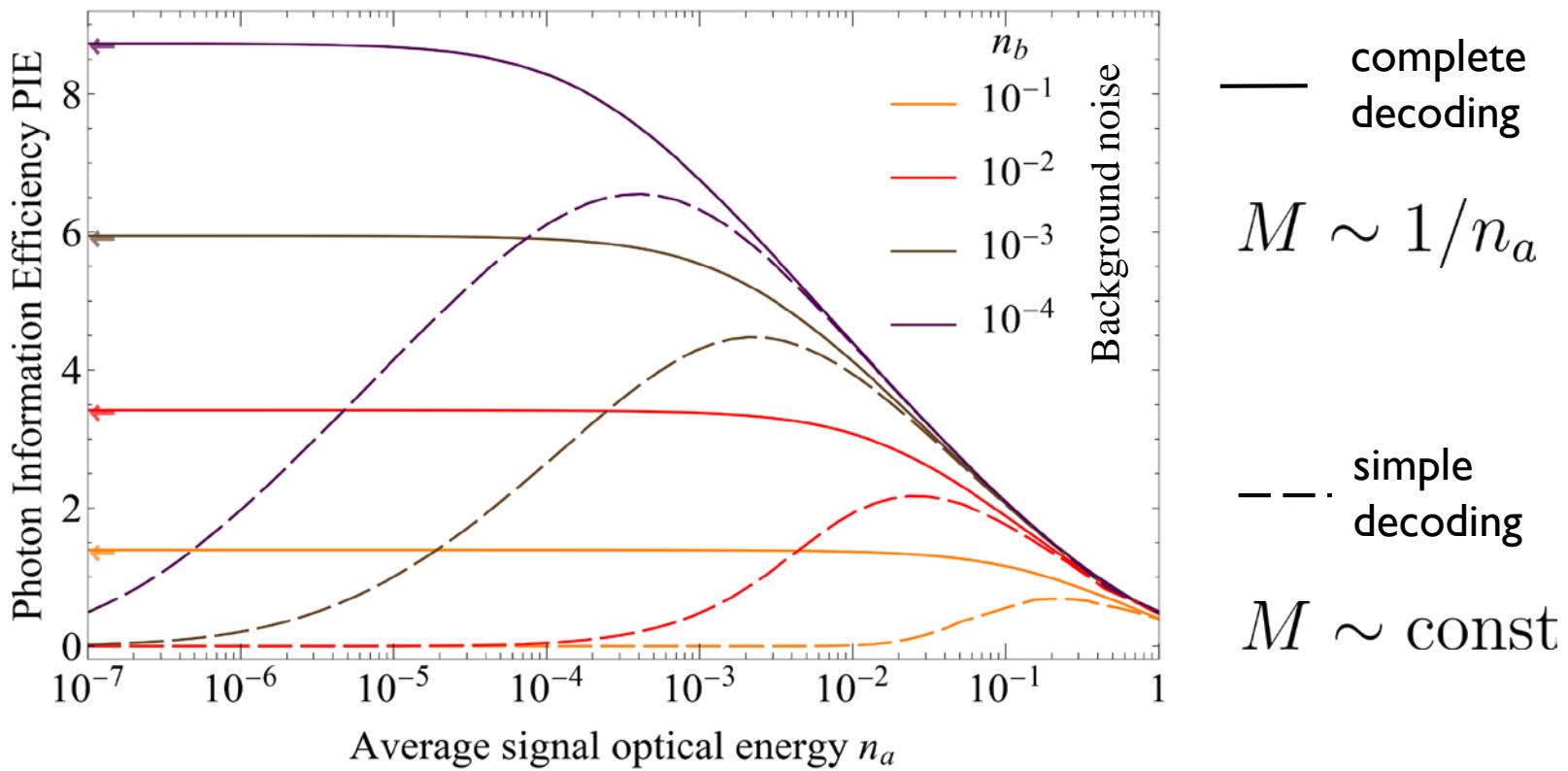
homodyne

Holevo

$$\frac{2 \log_2 e}{1 + n_b}$$
$$\log_2 \left( 1 + \frac{1}{n_b} \right)$$

# Optimized PPM with background noise

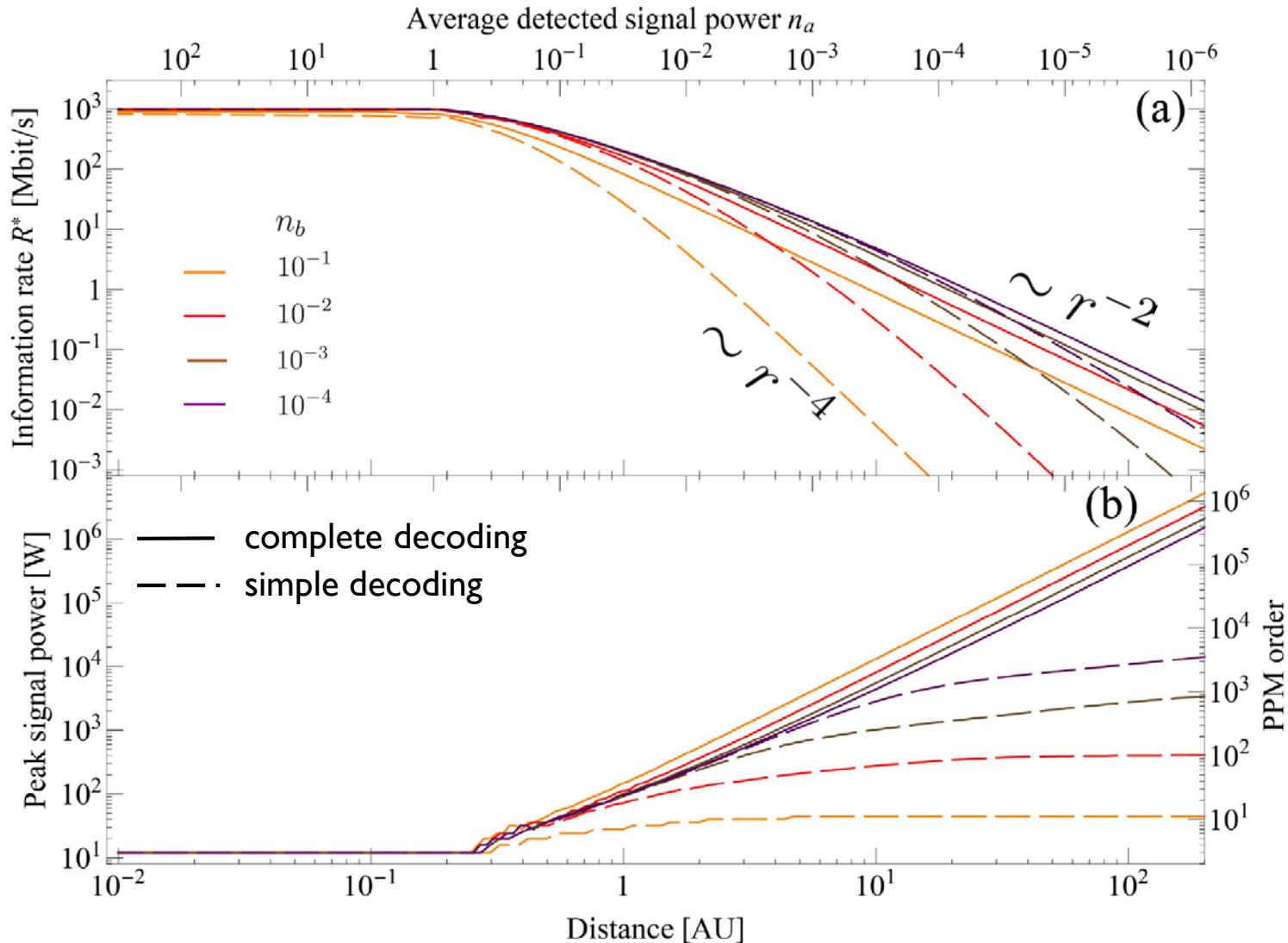
W. Zwoliński, M. Jarzyna, and K. Banaszek, Opt. Express **26**, 25827 (2018)



- multimode background noise yielding Poissonian count statistics
- Geiger-type direct detection
- unconstrained peak-to-average power ratio (PPM order)

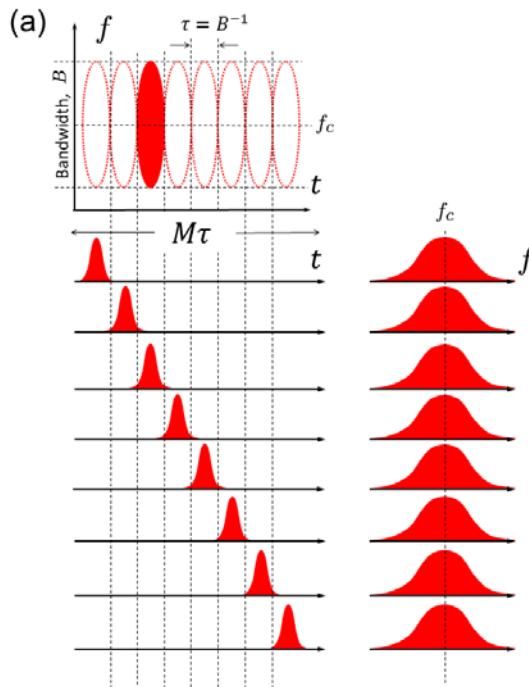
# Range dependence

W. Zwoliński, M. Jarzyna, and K. Banaszek, Opt. Express **26**, 25827 (2018)

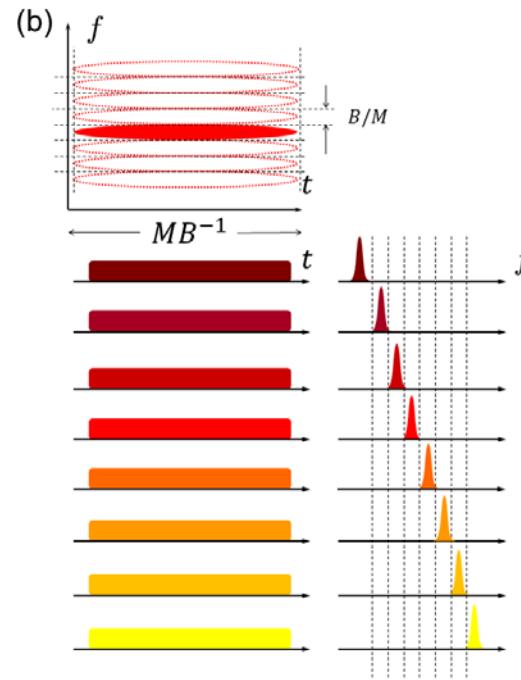


# High-order modulation formats

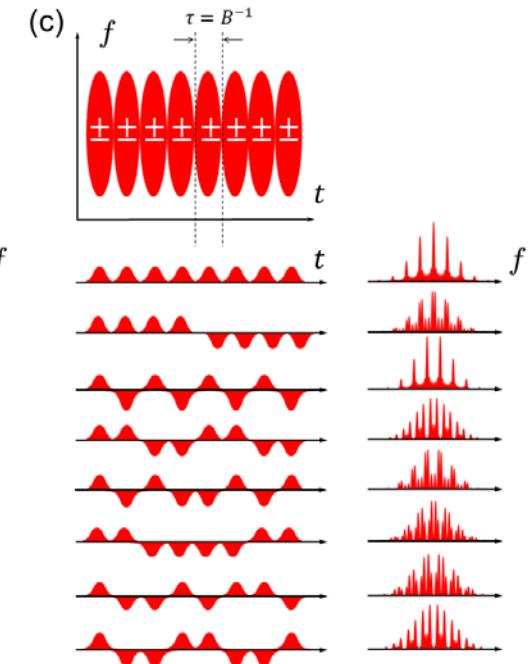
pulse  
position  
modulation



frequency  
shift keying



BPSK  
Hadamard  
codewords



K. Banaszek, M. Jachura, W. Wasilewski, *Utilizing time-bandwidth space for efficient deep-space communication*, Proc. International Conference on Space Optics 2018, paper P22



# **Thank you!**



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