Optical Communications Using the Nonlinear Fourier Transform

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Abstract

The Nonlinear Fourier Transform (NFT) uses the integrability of the Nonlinear Schroedinger Equation (NLSE) to represent signals in a nonlinear frequency domain, where the evolution equations are simple. The NFT degrees of freedom can be used for data communication. Different schemes of modulating the NFT in terms of eigenvalues and corresponding discrete spectral functions are investigated, focusing on spectral efficiency (SE).

Introduction

The pulse propagation on an optical fiber can be described by the **Nonlinear Schroedinger Equation (NLSE)** taking into account chromatic dispersion and Kerr-nonlinearity.

$$\frac{\partial A(T,\xi)}{\partial \xi} + j\frac{\partial^2 A(T,\xi)}{\partial T^2} + 2j|A(T,\xi)|^2 A(T,\xi) = 0$$

Governed by the NLSE, the evolution of signals on the fiber can become complicated in time- and frequency domain. However, a transformation into what is referred to as the nonlinear frequency domain by applying a **Nonlinear Fourier Transform (NFT)** can be done, where the evolution equations become simple.

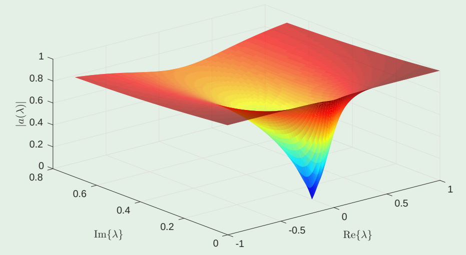
$$a(\lambda,\xi) = a(\lambda,\xi=0) = \lim_{T\to\infty} \vartheta_1^2 \exp(j\lambda T)$$

$$\vartheta_T = \begin{pmatrix} -j\lambda & A \\ -A^* & j\lambda \end{pmatrix} \vartheta$$

$$b(\lambda,\xi) = b(\lambda,\xi=0)\exp\left(-4j\lambda^2\xi\right) = \lim_{T\to\infty} \vartheta_2^2 \exp(-j\lambda T)$$

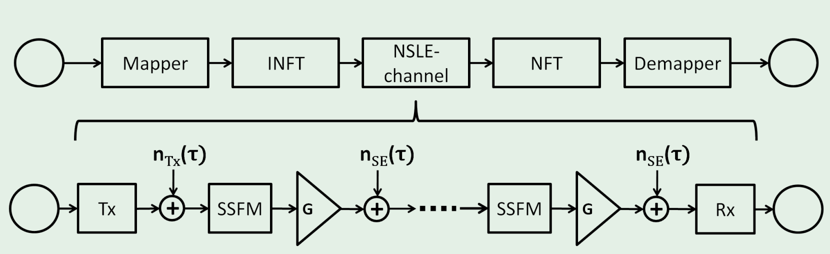
The NFT of a signal consists of:

- A **discrete spectrum** formed by the eigenvalues λ_i of the system for the respective impulse which corresponds to the solitonic part of the nonlinear spectrum, given by $a(\lambda_i) = 0$



- the **discrete spectral function** of each eigenvalue, $\tilde{U}(\lambda_i) = \frac{b(\lambda_i)}{a_\lambda(\lambda_i)}$
- the **continuous spectrum** which is the non-solitonic, dispersive part of the nonlinear spectrum (the ordinary Fourier Transform is the limit for small signal amplitudes where the solitonic part is absent)

For communications, bits are mapped onto these independent degrees of freedom at the transmitter and sent over the channel by applying the Inverse NFT (INFT).

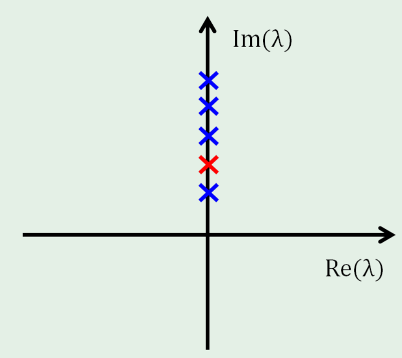


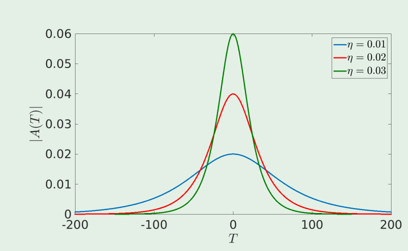
The simplified channel model, where the fiber is simulated by the **Split-Step-Fourier-Method (SSFM)** is shown above. INFT and NFT are numerically calculated by the **Darboux Transform** and the **Layer Peeling Method**, respectively.

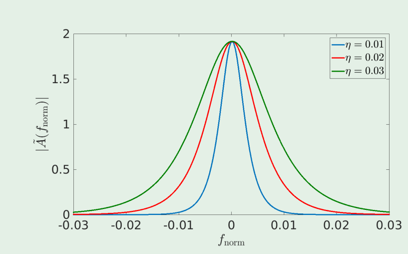
First Order Soliton - Eigenvalue Modulation

First order solitons only have **one eigenvalue** and no continuous spectrum. Two possible eigenvalue constellations are shown below.

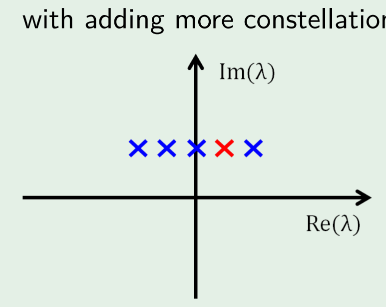
A **1 out of M** - constellation of the eigenvalues on the imaginary axis is similar to conventional AM. However, amplitude and duration of first order solitons are correlated. Necessary eigenvalue spacing is given by noise power. The time-bandwidth-product (TBP) increases linearly with the number of constellation points M, while $ld(M)$ bits per symbol can be transmitted.

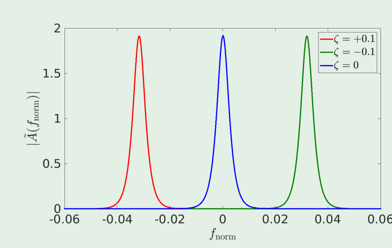






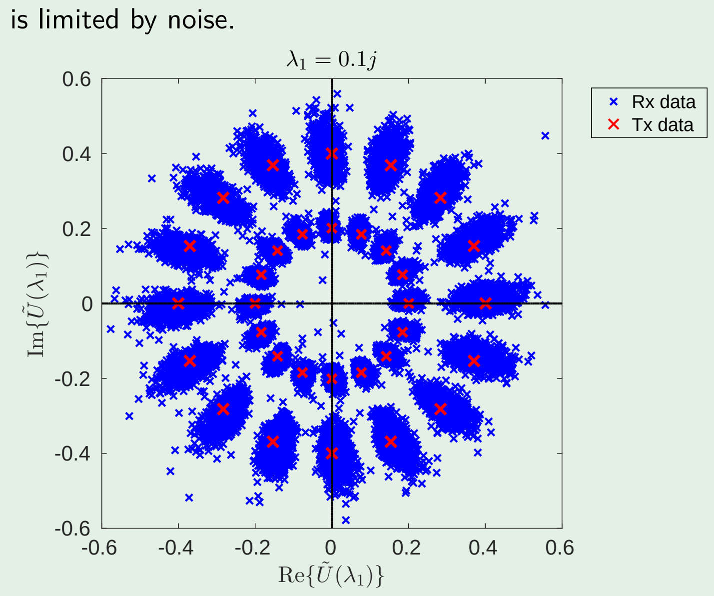
Constellation points can also be distinguished by their real part. The corresponding pulse shape will always be the same, but the different symbols are travelling with different velocities. This corresponds to a frequency shift. Consequently, the constellation bandwidth increases with adding more constellation points.



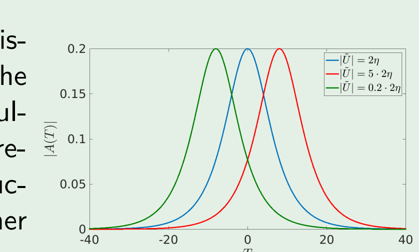


First Order Soliton - Spectral Function Modulation

Another possibility is modulating the discrete spectral function of the soliton eigenvalue. The figure below is the simulation result of a circular 32-QAM. The number of different phases and the amplitude spacing is limited by noise.

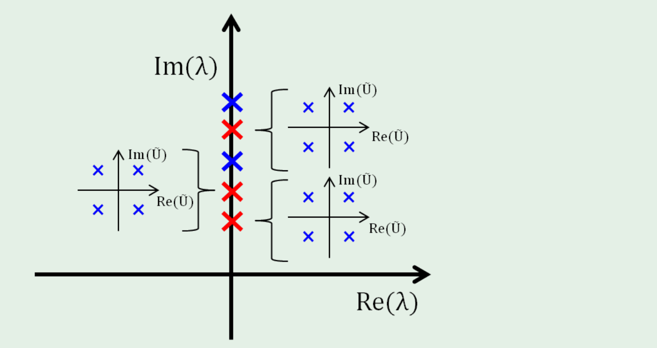


Scaling the amplitude of the discrete spectral function shifts the time center of the impulse. Multiple amplitude levels therefore require increased time slots, reducing the improvement of a higher order constellation.

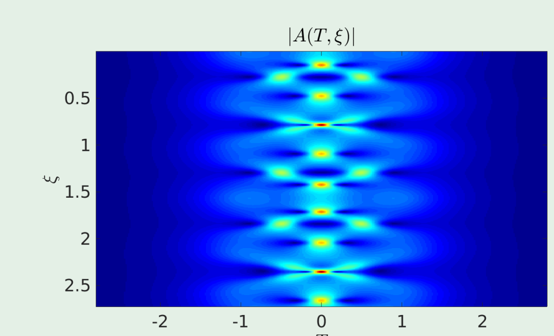


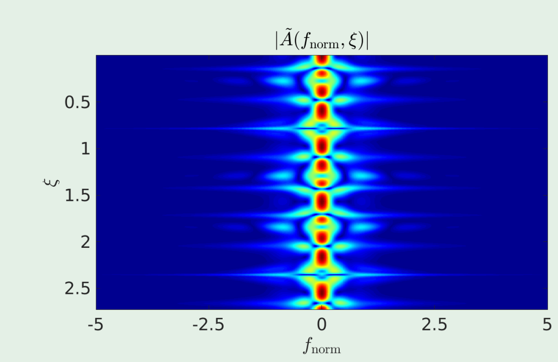
Higher Order Soliton Modulation

Transmitting **multiple eigenvalues** in parallel generates **higher order solitons**. One can generally consider a **k out of M** - constellation on the imaginary axis. Different combinations can be sent to transmit information. Each eigenvalue can have different discrete spectral amplitudes. It is also possible to fix k eigenvalues and only modulate their discrete spectral functions. Mapping bits to eigenvalues directly has no significant advantage, as 1 out of M is a subset of this scheme why the same limitations hold.

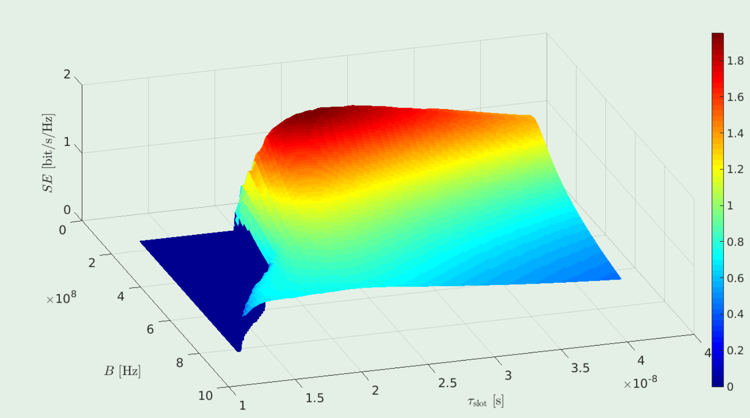


The eigenvalues and the corresponding discrete spectral functions determine the duration and bandwidth of the soliton, as well as the temporal and spatial periodicity of the impulse. However these relations are not obvious, especially not for higher soliton orders, as analytical solutions are generally not available. Therefore, comprehensive simulation is necessary to find spectrally efficient eigenvalue combinations. The example shows a third order soliton.





A brute force approach simulates all possibilities for a **second order soliton** with a 15-point imaginary constellation $\eta = [0.01, 0.02]$ and a **16-PSK** for the discrete spectral function. As some solitons have bad TBP properties, the overall SE is poor. Sorting out all solitons with time- and bandwidth above a certain cutoff threshold can improve the performance. Below, the SE as a function of the temporal and spectral **cutoff parameters** is shown. This approach is a way to screen suitable higher order eigenvalue combinations.



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Research Questions for Further Investigation

- How are temporal and spectral width of higher order solitons related to their eigenvalues?
- Can the SE be improved by increasing the soliton order?
- Can the shown modulation schemes be combined and optimized?
- How do temporal and spectral neighboring solitons interfere?
- What about modulating the continuous spectrum?
- What improved numerical methods should be used?
- More precise noise model?
- Discrete vs. distributed amplification?

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