Modeling the Fiber Optical MIMO Channel

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Introduction

Extremely high data rate telecommunications demand capable enough backhaul networks and force the realization of fiber optical MIMO. In this work, we give, for the first time, the model of a fiber optical MIMO channel under the assumptions of crosstalking between the subchannels, some *Mode Dependent Loss* (MDL)and zero backscattering. The mixing between the propagating modes is random over different frequency bands.

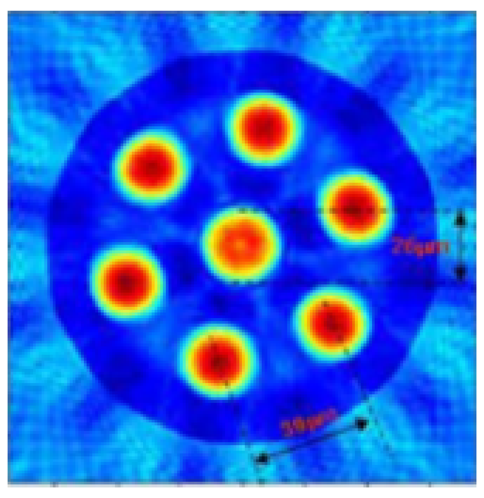

Figure 1: Multicore fiber cross section.[1]

The Optical Channel

We assume a non-correlated optical N-channel.Total input power into the fiber equals the output power $\mathbf{v}_{out} = \mathbf{S}\mathbf{v}_{in}$ where $\mathbf{S}$ is a $2N \times 2N$ unitary scattering matrix. For $\mathbf{S}$ we also assume zero backscattering.
The analytic expression of the $2N \times 2N$ scattering matrix $\mathbf{S}$ reads [2] :

$$\mathbf{S} = \mathbf{I}_{2N} - 2\pi i \mathbf{Q}^{\dagger}(\mathcal{H} + i\pi\mathbf{Q}\mathbf{Q}^{\dagger})^{-1}\mathbf{Q}$$

where $\mathcal{H}$ is the $2N \times 2N$ channel hamiltonian which eigenvalues are the eigenfrequencies of the fiber and $\mathbf{Q}$ $2N \times 2N$ matrix is formed by the coupling constants of the fiber to the "world". For perfect leads we can safely assume that $\mathbf{Q} \propto \mathbf{I}_{2N}$.Operationally, the uniform absorption can equivalently be taken into account by a purely imaginary shift of the scattering energy $\mathcal{H} + i\mathbf{\Gamma}$ [3].
To incorporate the idea of a fading and crosstalking channel we assume that $\mathcal{H}$ consists of a deterministic $\mathbf{H}$ plus random part $\mathbf{W}$.

- $\mathbf{H}$.Two contributors :1) linear part $\mathbf{H}_{lin}$ due to "line-of-sight" component. 2) non-linear part $\mathbf{H}_{nl}$ due to the average of the non-linear phenomena.
- $\mathbf{W}$. Two contributors : 1) linear part $\mathbf{W}_{lin}$ which is the cross-talking between the various subchannels (due to energy diffusion and mechanical reasons i.e bending of the fiber).2)non-linear part $\mathbf{W}_{nl}$ due to the variations of the non linearities inside the fiber. $\mathbf{W}$ is GUE.

The mutual information of an optical MIMO channel is the well known

$$\mathcal{I}(y;x|\mathbf{U}) = \mathrm{E}\left[\log\det\left(\mathbf{I} + \rho_0 \mathbf{U}^{\dagger}\mathbf{U}\right)\right]$$

Where $\mathbf{U}^{\dagger}\mathbf{U}$:

$$\mathbf{U}^{\dagger}\mathbf{U} = 4\pi^2 a^4 \frac{1}{(\mathbf{H}+\mathbf{W})^2 + \mathbf{\Gamma}^2}$$

and $\mathbf{U}$ is the upper-left corner sub-matrix of $\mathbf{S}$ which connects the incoming power to the outgoing one.

Mutual Information through Replicas

Following the saddle point analysis,the moment generating function is $g(\nu_1,\nu_2)_{\mathcal{I}} = \int d\mu(\mathcal{R},\mathcal{T},\mathcal{P},\mathcal{Q})_{1,2}e^{-S}$. Therefore, setting as γ the "randomness" factor we have, with $\rho = 2\rho_0\pi a^2 + 1 > 0$ [4]

$$\begin{aligned}\mathcal{I}(y;x|\mathbf{U}) &= \mathrm{E}\left[\log\det\left[(\mathbf{H}+\gamma\mathbf{W})^2 + \mathbf{\Gamma}^2 + \rho\mathbf{I}\right]\right] \\ &\quad -\mathrm{E}\left[\log\det\left[(\mathbf{H}+\gamma\mathbf{W})^2 + \mathbf{\Gamma}^2\right]\right] = \mathcal{I}_1 - \mathcal{I}_2.\end{aligned}$$

That way, the mean is $\mu(\mathcal{I}) = \mu(\mathcal{I}_1) - \mu(\mathcal{I}_2)$ and the variance is $\mathrm{var}(\mathcal{I}) = \mathrm{var}(\mathcal{I}_1) + \mathrm{var}(\mathcal{I}_2) - 2\mathrm{covar}(\mathcal{I}_{12})$.

Mutual Information through Replicas(...cont.)

For the simple case of $\mathcal{I}_1$ it is

$$\begin{aligned} S &= \log\det\left[(\rho\mathbf{I} + \mathbf{\Gamma} + \gamma\mathcal{T})(\mathbf{I} + \gamma\mathcal{R})\right. \\ &\quad \left. + (\mathbf{H} + \gamma(\mathcal{P} - \mathcal{Q}))(\mathbf{H} + \gamma(\mathcal{P} + \mathcal{Q}))\right] \\ &\quad - N\mathrm{Tr}(\mathcal{T}\mathcal{R} - \mathcal{Q}^2 - \mathcal{P}^2)\end{aligned}$$

and for the second order term

$$S_2 = \frac{1}{2N}\mathrm{Tr}\left\{\begin{pmatrix}\delta\mathcal{T}\\ \delta\mathcal{R}\\ \delta\mathcal{P}\\ \delta\mathcal{Q}\end{pmatrix}^T \mathbf{V} \begin{pmatrix}\delta\mathcal{T}\\ \delta\mathcal{R}\\ \delta\mathcal{P}\\ \delta\mathcal{Q}\end{pmatrix}\right\}$$

where $\mathbf{V}$ is given by

$$\mathbf{V} = \begin{bmatrix} -\mathrm{tr}\left(\frac{\gamma^2}{Z^2}(\mathbf{I}+\gamma r)^2\right) & -\mathrm{tr}\left(\frac{\gamma^2}{Z^2}(\mathbf{I}+\gamma r)(\Delta+\gamma t) + \frac{\gamma^2}{Z} - 1\right) & -\mathrm{tr}\left(\frac{2\gamma^2}{Z^2}(\mathbf{I}+\gamma r)(\mathbf{H}-\gamma p)\right) & 0 \\ \mathrm{tr}\left(\frac{\gamma^2}{Z^2}(\mathbf{I}+\gamma r)(\Delta+\gamma t) + \frac{\gamma^2}{Z} - 1\right) & -\mathrm{tr}\left(\frac{\gamma^2}{Z^2}(\Delta+\gamma t)^2\right) & -\mathrm{tr}\left(\frac{2\gamma^2}{Z^2}(\Delta+\gamma t)(\mathbf{H}-\gamma p)\right) & 0 \\ -\mathrm{tr}\left(\frac{2\gamma^2}{Z^2}(\mathbf{I}+\gamma R)(\mathbf{H}-\gamma p)\right) & -\mathrm{tr}\left(\frac{2\gamma^2}{Z^2}(\Delta+\gamma t)(\mathbf{H}-\gamma p)\right) & -\mathrm{tr}\left(\frac{4\gamma^2}{Z^2}(\mathbf{H}-\gamma p)^2 + \frac{2\gamma^2}{Z} - 2\right) & 0 \\ 0 & 0 & 0 & 2N - \mathrm{tr}\left(\frac{2\gamma^2}{Z}\right) \end{bmatrix}$$

where $\Delta = \gamma\mathbf{I} + \mathbf{\Gamma}$, $Z = (\Delta + \gamma t)(\mathbf{I} + \gamma r) + (\mathbf{H} - \gamma p)^2$ and r, t, p are the solutions of S at the saddle point.

Results

In the context of this work we provided a model for the optical fiber MIMO channel and successfully, calculated analytically the corresponding capacity under some assumptions.

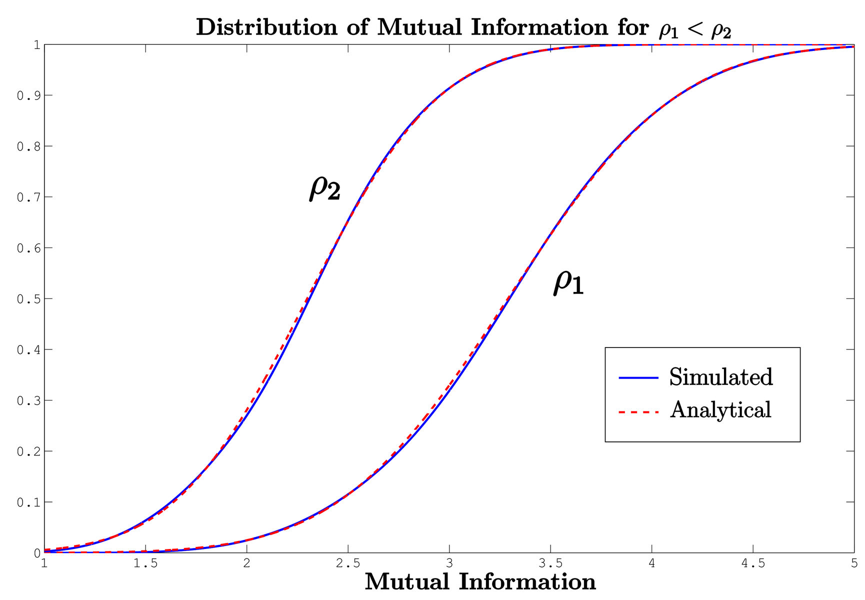

Figure 2: Cumulative distribution function (CDF) of mutual informaiton for $N = 6$ and $\rho_1 < \rho_2$. Runs $= 10^6$

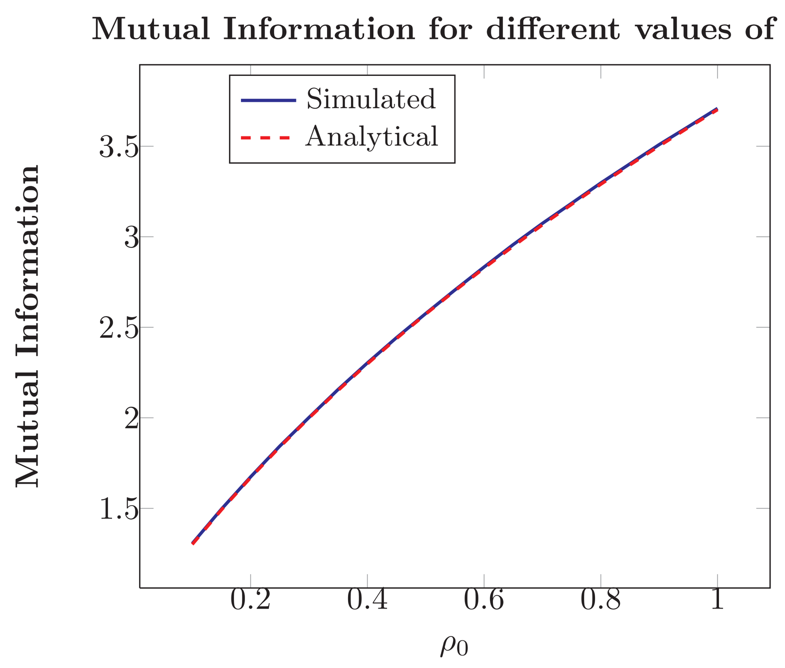

Figure 3: $N = 6$. Runs $= 10^6$

References

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