A Strictly Increasing Lower Bound on the Capacity of the Fiber Optical Channel

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Abstract

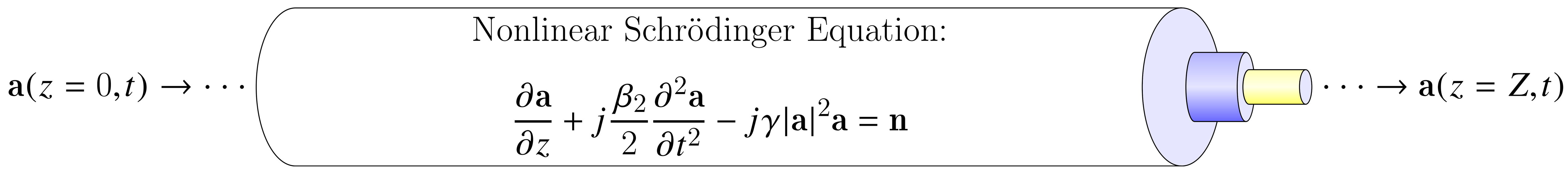
Abstract: An achievable rate is derived for the fiber-optical channel, described by the nonlinear Schrödinger equation and discretized in time and space. The model takes into account the effect of nonlinearity and dispersion. The obtained achievable rate goes to infinity with a pre-log factor of one half as the power grows large. Since any achievable rate is a lower bound on the capacity of the same channel, the result proves that the capacity of the discretized fiber-optical channel grows unboundedly.

Introduction and Motivation

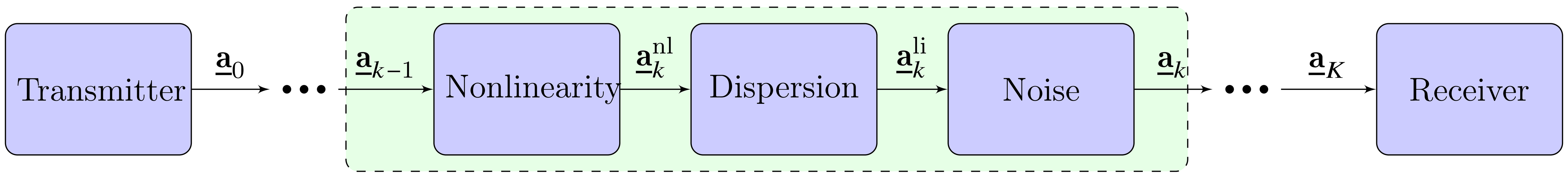
- A large gap exists between the known lower and upper bounds on the capacity of optical fiber channel.
- The only available upper bound [2], [3] is $\log(1+\rho)$, where ρ is the signal to noise ratio.
- All previous lower bounds either saturate or fall to zero in the high power regime.
- The proposed lower bound grows with a pre-log factor of one half.

Channel Model

- Continuous channel:



- Split step Fourier method (SSFM) channel (the same model has been used in [2]):



Results

Main Theorem:
The capacity of the SSFM channel in bit per channel use under the constraint $E\left[\|\underline{\mathbf{a}}_0\|^2\right] \leq E_0$, when the number of segments K goes to infinity, is lower-bounded by

$$\lim_{K\to\infty} \mathcal{C}_K \geq \frac{1}{2}\log\left(\frac{e}{2\pi}\left(1+\frac{\rho^2}{2\rho+1}\right)\right).$$

Here $\rho = E_0/E_n$ is the receiver signal-to-noise ratio. This result extends to lumped amplification and dual polarizations.

Proof: For an iid complex Gaussian input vector $\underline{\mathbf{a}}_0$,

$$h(\underline{\mathbf{a}}_K) \geq L\log\left(\frac{\pi e}{L}(E_n+E_0)\right)$$

The proof of this inequality employs entropy power inequality.

$$\lim_{K\to\infty} h(\underline{\mathbf{a}}_K \,|\, \underline{\mathbf{a}}_0) \leq \frac{L}{2}\log\left(\frac{2\pi^3 e}{L^2}\left(2E_nE_0+E_n^2\right)\right)$$

The proof of this inequality relies on the following two lemmas. For complete proofs, see [1].

Lemma 1: Let $(\underline{\mathbf{x}},\underline{\mathbf{y}})$ be a pair of L-dimensional complex random vectors, distributed according to an arbitrary joint probability density function.

$$h(\underline{\mathbf{y}}|\underline{\mathbf{x}}) \leq \frac{L}{2}\log\left(2\pi^3 e\frac{\kappa(\underline{\mathbf{y}}\,|\,\underline{\mathbf{x}})}{L}\right),$$

$$\kappa(\underline{\mathbf{y}}\,|\,\underline{\mathbf{x}}) = \sum_{i=0}^{L-1} E_{\mathbf{y}_i}\left[|\mathbf{y}_i|^4\right] - \sum_{i=0}^{L-1} E_{\underline{\mathbf{x}}}\left[E_{\mathbf{y}_i}\left[|\mathbf{y}_i|^2|\underline{\mathbf{x}}\right]^2\right].$$

The proof employs the maximum entropy result in the polar coordinate system.

Lemma 2: For the SSFM channel with iid complex Gaussian input the following holds

- $\kappa\left(\underline{\mathbf{a}}_k^{\mathrm{nl}}\,|\,\underline{\mathbf{a}}_0\right) = \kappa\left(\underline{\mathbf{a}}_{k-1}\,|\,\underline{\mathbf{a}}_0\right)$.
- $\kappa\left(\underline{\mathbf{a}}_k^{\mathrm{li}}\,|\,\underline{\mathbf{a}}_0\right) = \kappa\left(\underline{\mathbf{a}}_k^{\mathrm{nl}}\,|\,\underline{\mathbf{a}}_0\right) + \mathcal{O}(1/K^2)$.
- $\kappa\left(\underline{\mathbf{a}}_k\,|\,\underline{\mathbf{a}}_0\right) = \kappa\left(\underline{\mathbf{a}}_k^{\mathrm{li}}\,|\,\underline{\mathbf{a}}_0\right) + L\sigma_n^4 + 2E\left[\|\underline{\mathbf{a}}_k^{\mathrm{li}}\|^2\right]\sigma_n^2$.
- $E\left[\|\underline{\mathbf{a}}_k^{\mathrm{li}}\|^2\right] = E\left[\|\underline{\mathbf{a}}_0\|^2\right] + L(k-1)\sigma_n^2$.

[1] K. Keykhosravi, E. Agrell, and G. Durisi, "A monotonically increasing lower bound on the capacity of the fiber optical channel," *arxiv.org,* Dec. 2015.
[2] G. Kramer, M. I. Yousefi, and F. R. Kschischang, "Upper bound on the capacity of a cascade of nonlinear and noisy channels," *IEEE Inf. Theory Workshop,* Apr–May, 2015.
[3] M. I. Yousefi, G. Kramer, and F. R. Kschischang, "An upper bound on the capacity of the single-user nonlinear Schrödinger channel," in *Can. Workshop Inf. Theory,* July 2015, Online at: http://arxiv.org/abs/1502.06455.

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