Protograph-Based LDPC Code Design for Shaped Bit-Metric Decoding

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Coded Modulation

Introduction

- Combination of **high-order constellations** and **coding** for **bandwidth limited** communication.
- Need for **binary labeling** of constellation points to be able to use binary codes.
- **Bit-Interleaved coded modulation** (BICM) is widelyb used in standards (DVB-S2, W-LAN, Wimax).
  - It employs a bit-metric decoder (BMD) at the receiver.
  - Resulting information theoretic system model consists of a set of parallel channels.

System Model

- Single-Input, Single Output AWGN channel with ASK input

$$\mathsf{Y} = \mathsf{X} + \mathsf{Z} = \Delta x_{\mathbf{B}} + \mathsf{Z}, \qquad \mathsf{Z} \sim \mathcal{N}(0,1)$$

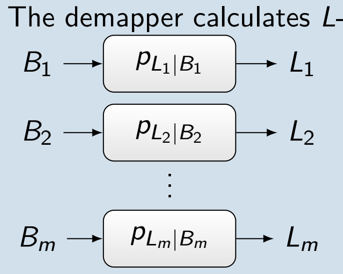
- 2^m ASK input constellation, labeled by a reflected Gray code

$$x_{\mathbf{B}} = \{x \in 2^m\text{-ASK} : \text{label}(x) = \mathbf{B} = (B_1, B_2, \dots, B_m)\}$$

- $\Delta \in \mathbb{R}^+$: constellation scaling, parameterizes the SNR

Bit-Metric Decoding

- The demapper calculates L-values



$$L_i = \underbrace{\log \frac{p_{Y|B_i}(Y|0)}{p_{Y|B_i}(Y|1)}}_{\text{channel}} + \underbrace{\log \frac{P_{B_i}(0)}{P_{B_i}(1)}}_{\text{a priori}}$$

for sign B_1 and amplitude label $B_2 B_3 \cdots B_m$.

- Achievable rate [1]:

$$\mathsf{R}_{\text{BMD}} = \sum_{i=1}^{m} I(\mathsf{B}_i;\mathsf{Y}) - \left(\sum_{i=1}^{m} \mathbb{H}(\mathsf{B}_i) - \mathbb{H}(\mathbf{B})\right)$$

Design Challenge

- The m binary input channels $p_{L_i|B_i}$ are of **different reliability**.The employed channel code has to take this into account.
  1. Need for structured ensembles to inherit the different bit-channels: multi-edge type (MET) or Protograph-based codes.
  2. Usual toolchain assumes binary-input symmetric-output (BISO) channels. Does not hold for $p_{L_i|B_i}$.

References

[1] G. Böcherer, "Probabilistic signal shaping for bit-metric decoding," in *Proc. IEEE Int. Symp. Inf. Theory (ISIT)*, pp. 431–435.

[2] G. Böcherer, P. Schulte, and F. Steiner, "Bandwidth efficient and rate-matched low-density parity-check coded modulation." [Online]. Available: http://arxiv.org/abs/1502.02733

[3] P. Schulte and G. Böcherer, "Constant composition distribution matching." [Online]. Available: http://arxiv.org/abs/1503.05133

[4] F. Steiner, G. Böcherer, and G. Liva, "Protograph-based LDPC code design for shaped bit-metric decoding." [Online]. Available: http://arxiv.org/abs/1504.03628

Probabilistic Amplitude Shaping – PAS [2]

Optimal Signaling

- Capacity $\frac{1}{2}\log_2(1+\text{SNR})$ is achieved by Gaussian input.
- Optimal constellation constrained capacity can be found as the solution of

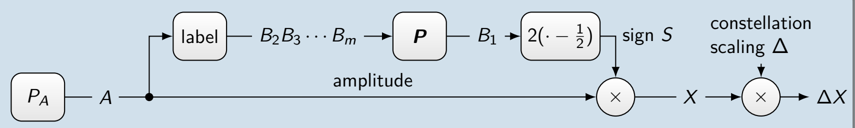
$$\max_{\Delta, P_{\mathbf{B}}:\ \mathrm{E}[|\Delta x_{\mathbf{B}}|^2] \le \text{SNR}} I(\Delta x_{\mathbf{B}}; Y).$$

- Optimal distribution $P_{\mathbf{B}} = P_X$ is **symmetric** and therefore exhibits an **Amplitude-Sign Factorization**:

$$P_X(x) = P_A(|x|) P_S(\text{sign}(x)).$$

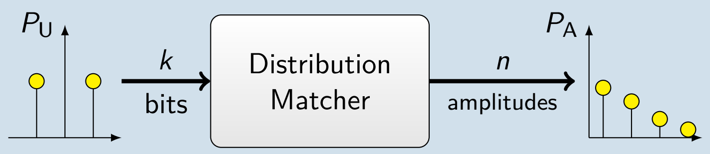
- Sign is uniformly distributed, i.e., $P_S(-1) = P_S(1) = \frac{1}{2}$.

Combining Linear Codes with Probabilistic Shaping: PAS



- ASK amplitude $A \sim P_A$.
- Systematic binary rate $(m-1)/m$ generator matrix $\boldsymbol{G} = [\boldsymbol{I}|\boldsymbol{P}]$.
- Encode binary amplitude labels by parity matrix $\boldsymbol{P}$ to get uniformly distributed check bits B_1.
- Use the check bit B_1 as sign S, i.e., $X = A \cdot S$.

Practical Shaping

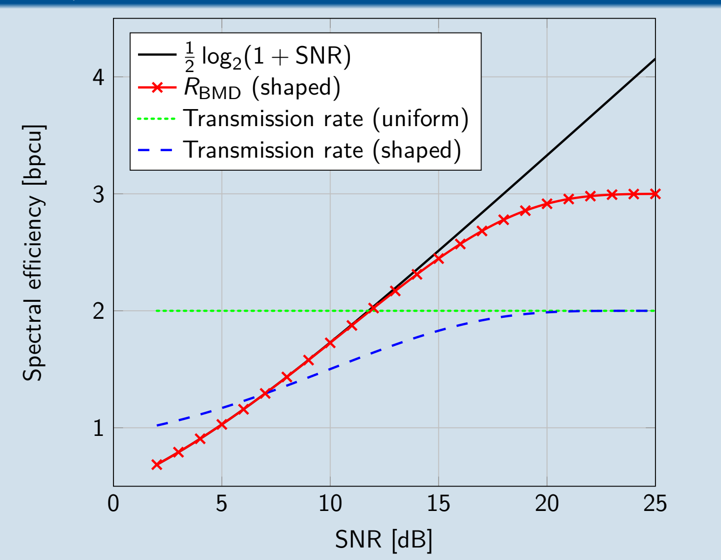


- Employ a block-to-block matcher [3].

PAS Principles

- Control rate $\mathbb{H}(A)$ via P_A
- Control power/channel uncertainty via Δ
- Choose P_A, Δ jointly $\Rightarrow$ SNR–dependent rate $\Rightarrow$ **Rate-adaptive**.

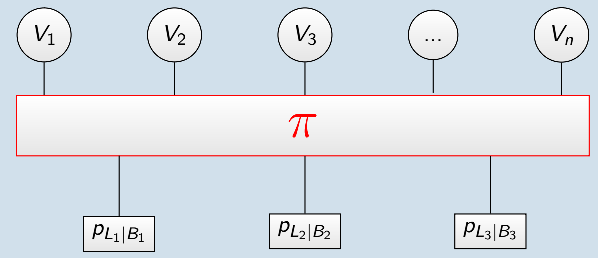
Rate curves, 8-ASK



Protograph-Based LDPC Code Design [4]

Existing approaches

- Use existing LDPC code and optimize the interleaver, i.e., assign the different coded bits belonging to the m channels to the different variable node degrees (e.g., Li, Ryan, 2005. Häger et al 2014.)



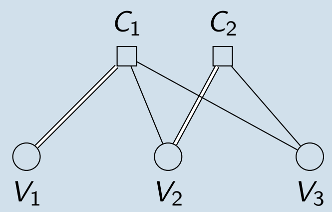
- "Wholistic approach": tackle problem of interleaver and code design at the same time. (Zhang, Kschischang, 2013. Uses multidimensional extension of EXIT analysis and channel adapters.)

Our contribution

- Wholistic approach based on protographs.
- Threshold analysis via P-EXIT and surrogate channels.
- New matching criteria based on rate backoff.

Getting structure: Protographs

- Introduced for building more "structured" ensembles via the basematrix $\boldsymbol{A} \in \{0,\dots,S\}^{M \times N}$.
- Based on small tanner graphs which are then "copy-and-permuted" by a factor Q.



$$\boldsymbol{A} = \begin{pmatrix} 2 & 1 & 1 \\ 0 & 2 & 1 \end{pmatrix}$$

$M = 2, N = 3, S = 2$

P-EXIT Analysis

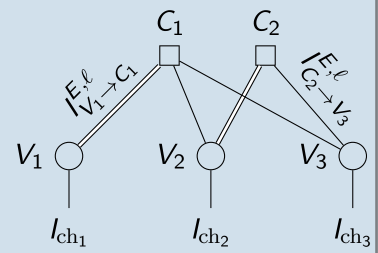
- We employ an extension of EXIT-charts to protographs: P-EXIT. For given $\boldsymbol{A} \in \{0,\dots,S\}^{M \times N}$ and $l \in [1:M], k \in [1:N]$ define the recursion:

$$I^{E,\ell}_{V_k \to C_l} = f^{V}_{k,l}(\boldsymbol{I}^{E,\ell-1}_{C \to V_k}, \boldsymbol{I}_{\text{ch}})$$

$$\boldsymbol{I}^{E,\ell}_{C \to V_k} = \left(I^{E,\ell}_{C_1 \to V_k}, \dots, I^{E,\ell}_{C_M \to V_k}\right)$$

$$I^{E,\ell}_{C_l \to V_k} = f^{C}_{k,l}(\boldsymbol{I}^{E,\ell-1}_{V \to C_l})$$

$$\boldsymbol{I}^{E,\ell}_{V \to C_l} = \left(I^{E,\ell}_{V_1 \to C_l}, \dots, I^{E,\ell}_{V_N \to C_l}\right)$$



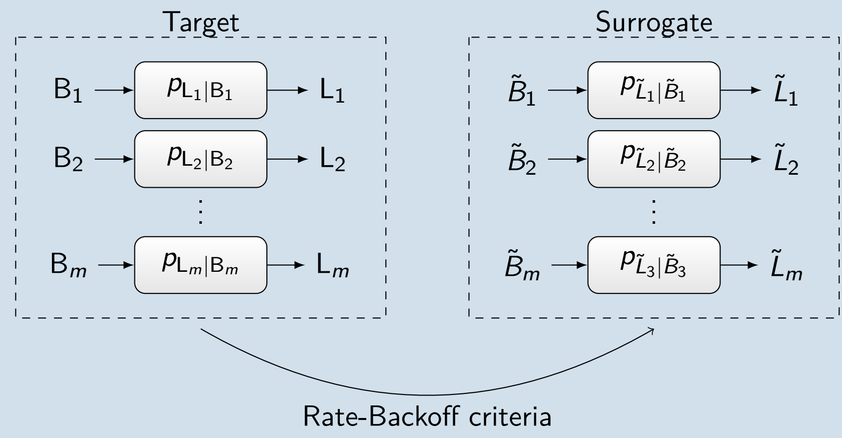
- $I^{\text{APP},l}_k$: mutual information between aposteriori loglikelihood ratio and coded bits of k-th channel at iteration l
- Convergence region

$$\mathcal{C}_I = \left\{ \boldsymbol{I}^{\text{ch}} \in [0,1]^N : I^{\text{APP},l}_k \to 1, \forall k, l \to \infty \right\}.$$

- Alternatively, identify convergence region by general channel parameter $\mathcal{C}_\xi$.

Protograph-LDPC Code Design

Circumventing BISO necessity: Surrogate channels



Design Paradigm: Rate-Backoff as universality criteria

- Transmission rate R.
- Asymptotically achievable rate R^* of considered code ensemble on original channel.
- Same rate-backoff $R^* - R$ on different channels, results in similar code performance.

- For BMD: $R^* - R = (1-c)\cdot m - \sum_{i=1}^{m} \mathbb{H}(\mathsf{B}_i|\mathsf{L}_i)$ $\implies$ Match in $\mathcal{U} = \mathcal{U}(\text{SNR}) = \{\mathbb{H}(\mathsf{B}_i|\mathsf{L}_i)\}_{i=1}^{m}$.

Use biAWGN as surrogate channel

$$\tilde{\mathsf{L}}_i = x_{\tilde{\mathsf{B}}_i} + \tilde{Z}_i, \quad \text{where } x_{\tilde{\mathsf{B}}_i} \in \{\pm\sigma^2_{\text{ch}_i}/2\}, \tilde{Z} \sim \mathcal{N}(0, \sigma^2_{\text{ch}_i}).$$

Solve $\mathbb{H}\left(\tilde{\mathsf{L}}_i|\tilde{\mathsf{B}}_i\right) \stackrel{!}{=} \mathbb{H}(\mathsf{B}_i|\mathsf{L}_i)$ to obtain $\sigma^2_{\text{ch}_i}$.

- To allow for D different variable node degrees per bit-channel, it must hold $N = m \cdot D$. Define $T(k) = \lceil k/D \rceil$, $k \in [1:N]$.
- Define $\boldsymbol{\sigma}^{\mathcal{U}(\text{SNR})}_{\text{ch}}$ as

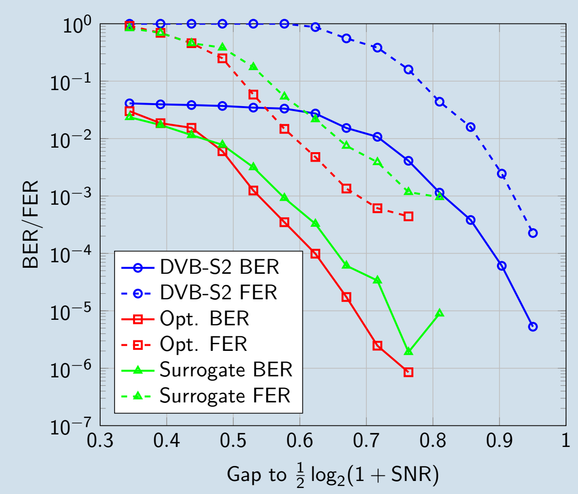
$$\boldsymbol{\sigma}^{\mathcal{U}(\text{SNR})}_{\text{ch}} = \left(\sigma^2_{\text{ch}_{T(1)}}, \sigma^2_{\text{ch}_{T(2)}}, \dots, \sigma^2_{\text{ch}_{T(N)}}\right).$$

- Decoding threshold of a given ensemble then given as

$$\min \text{SNR} \quad \text{s.t.} \quad \boldsymbol{\sigma}^{\mathcal{U}(\text{SNR})}_{\text{ch}} \in \mathcal{C}_{\sigma_{\text{ch}}}.$$

- Find ensemble with lowest decoding threshold by means of differential evolution.

Simulation Results, 64-ASK Shaped, Blocklength 64800, 100 Iter.



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