Non-Binary LDPC Erasure Codes with Separated Low-Degree Variable Nodes

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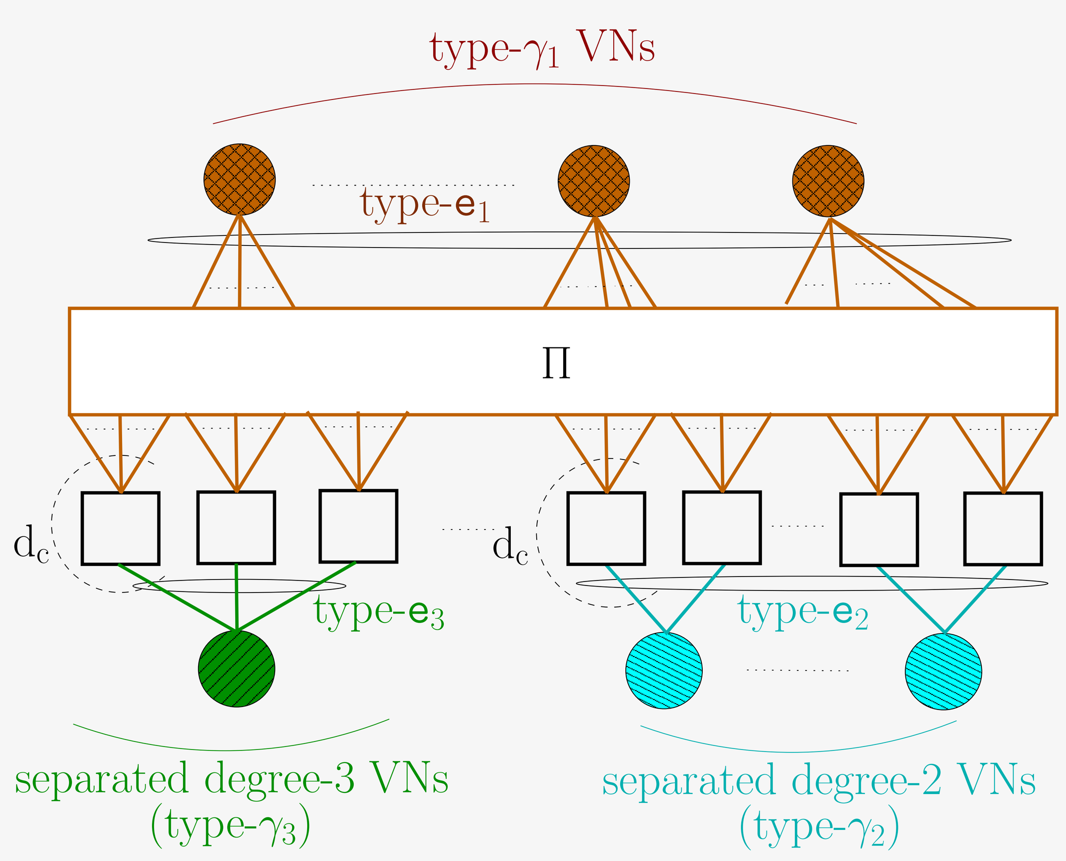
Abstract

We analyze a partially-structured ensemble of LDPC (erasure) codes: asymptotic thresholds and weight distribution. We design finite-length codes from this ensemble and we analyze them in terms of performance and decoding complexity.

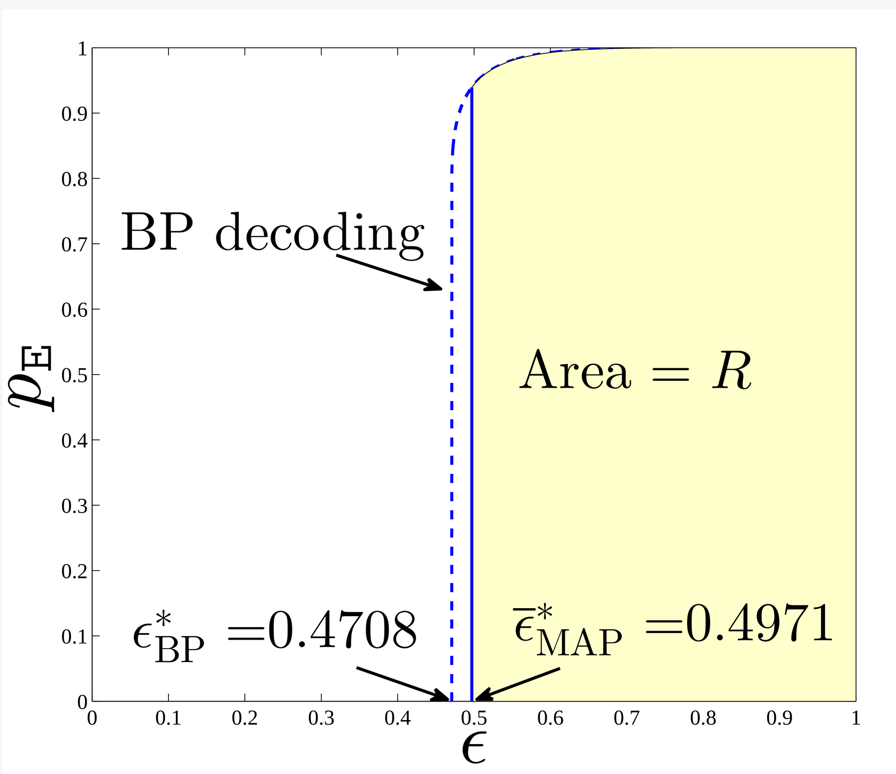
Separated Variable Nodes (SVN) Ensemble



- The degree-2 VNs are all separated (type-e_2 edges).
- Some of the degree-**3** VNs are separated (type-e_3 edges).
- All possible type-e_1 (brown) edge permutations Π and all possible edge labelings from $\mathbb{F}_q \setminus \{0\}$ (uniform probability).
- Notation: V_2: number of degree-**2** VNs (type γ_2); V_3^S: number of separated degree-**3** VNs (type γ_3); $\tilde{V}_j$: number of degree-j VNs of type γ_1 (brown).

BP and MAP Thresholds of SVN Ensembles

$p_\mathrm{E}(\epsilon)$: average extrinsic symbol erasure probability at the output of a decoder.



Weight Distribution of SVN Ensembles

The expected number of weight-l codewords for a code $\mathcal{C}$ from a SVN ensemble is

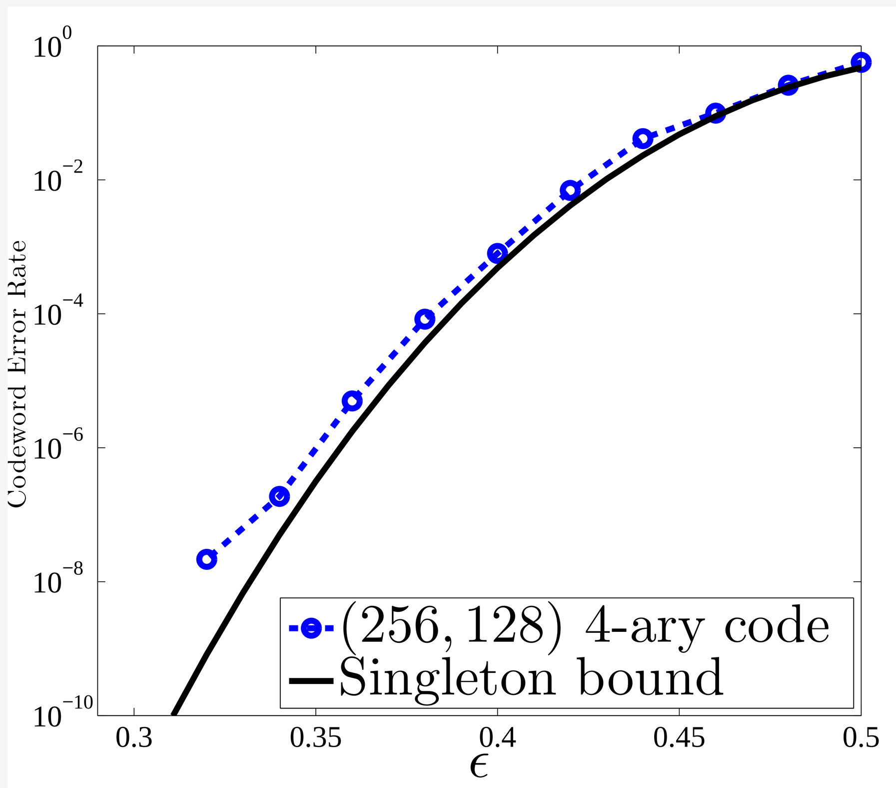
$$\mathbb{E}[A(\mathcal{C},l)] = \sum_{l: l_{\gamma_2}+l_{\gamma_3}+\sum_j \tilde{l}_j = l} \binom{V_2}{l_{\gamma_2}}\binom{V_3^S}{l_{\gamma_3}} \prod_j \binom{\tilde{V}_j}{\tilde{l}_j} \times \frac{\mathrm{Coeff}\left((N^-(z))^{2l_{\gamma_2}+3l_{\gamma_3}}(N^+(z))^{m-2l_{\gamma_2}-3l_{\gamma_3}}, z^{\sum_j \tilde{l}_j j}\right)}{(q-1)^{-(l_{\gamma_2}+l_{\gamma_3}+\sum_j\tilde{l}_j)}\binom{m(\mathrm{d_c}-1)}{\sum_j \tilde{l}_j j}(q-1)^{\sum_j \tilde{l}_j j+2l_{\gamma_2}+3l_{\gamma_3}}}$$

with $l = (\tilde{l}_3, \ldots, \tilde{l}_{\mathrm{d_{v,max}}}, l_{\gamma_2}, l_{\gamma_3})$, $0 \le l_{\gamma_2} \le V_2, 0 \le l_{\gamma_3} \le V_3^S, 0 \le \tilde{l}_j \le \tilde{V}_j$. $N^+(z)$ and $N^-(z)$ are univariate polynomials.

- The SVN ensemble has always a strictly positive typical minimum distance (larger than its unstructured counterpart).

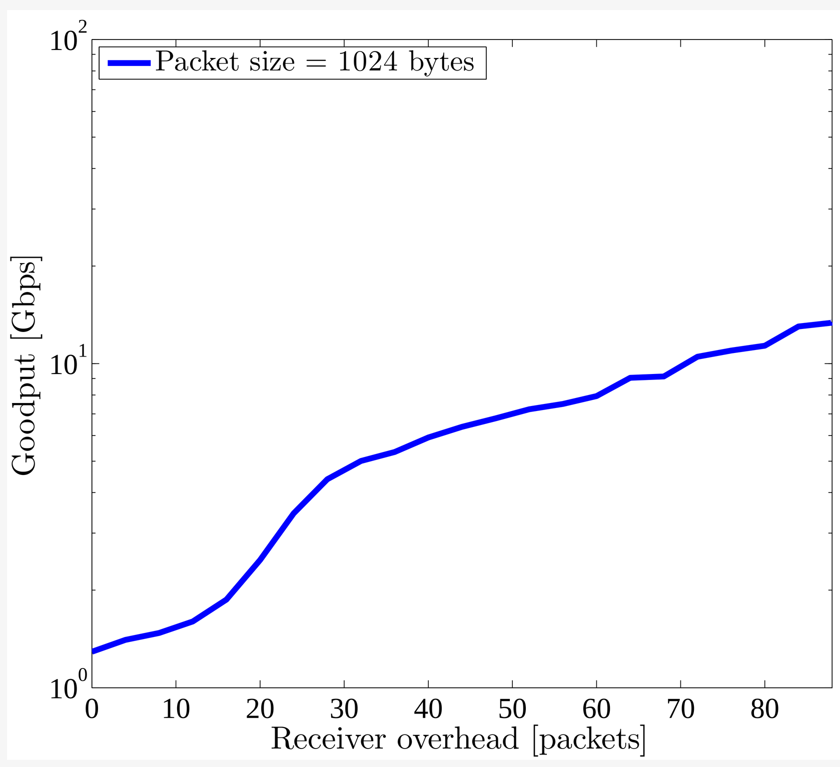
Finite-Length Design: Performance

Short **4**-ary **(256, 128)** LDPC code. $n = 256$ symbols of $\mathbb{F}_4$.



Finite-Length Design: Decoding Speed

(256, 128) code on $\mathbb{F}_4$ over the PEC. $n = 256$ packets.



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