UCLA Engineering
Electrical Engineering
Communication Systems Laboratory

Incremental Redundancy and Feedback at Finite Blocklengths

Richard Wesel, Kasra Vakilinia,
Adam Williamson

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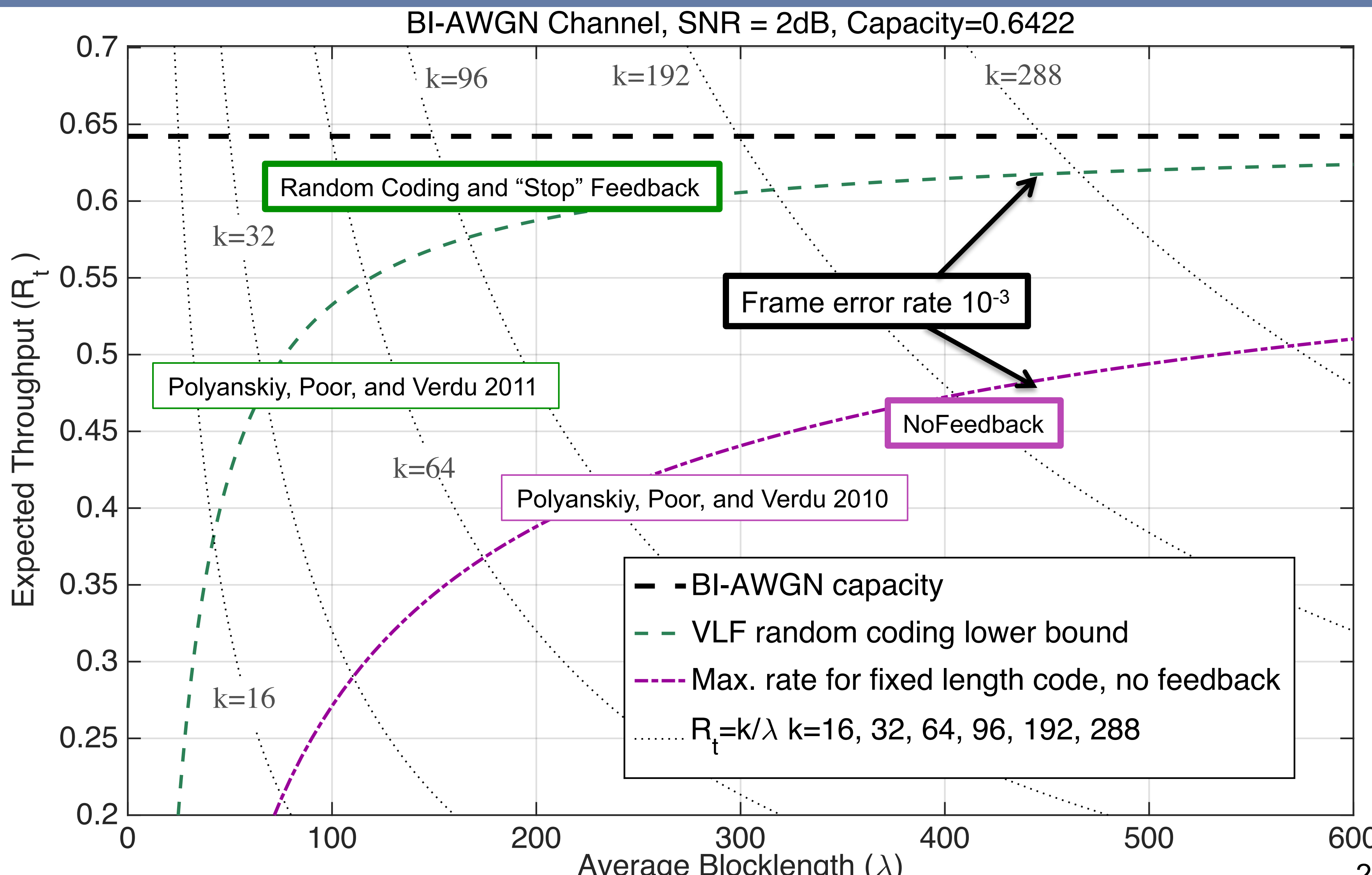
Lower Bound on Benefit of Feedback



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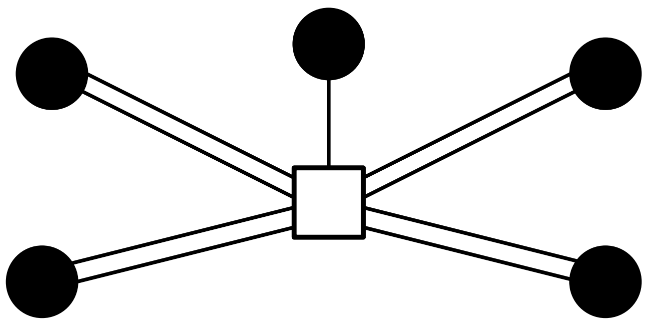
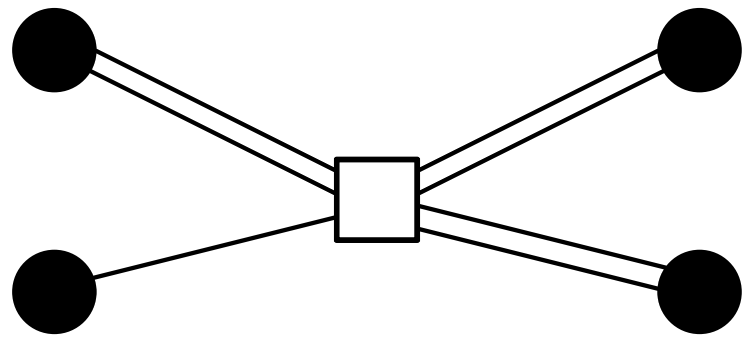
How well can a non-binary LDPC code do?



Rate-0.8 protograph



Rate-0.75 protograph

We will lift these protographs to produce GF(256) LDPC codes for 96 (rate-0.8), 192 and 288 (rate-0.75) input bits.

VERY incremental redundancy…

1. Send the initial NB-LDPC codeword
2. If the CRC checks, we are done
3. If not, request a transmission of a specific bit that helps the least reliable variable node.
4. Go to step 3

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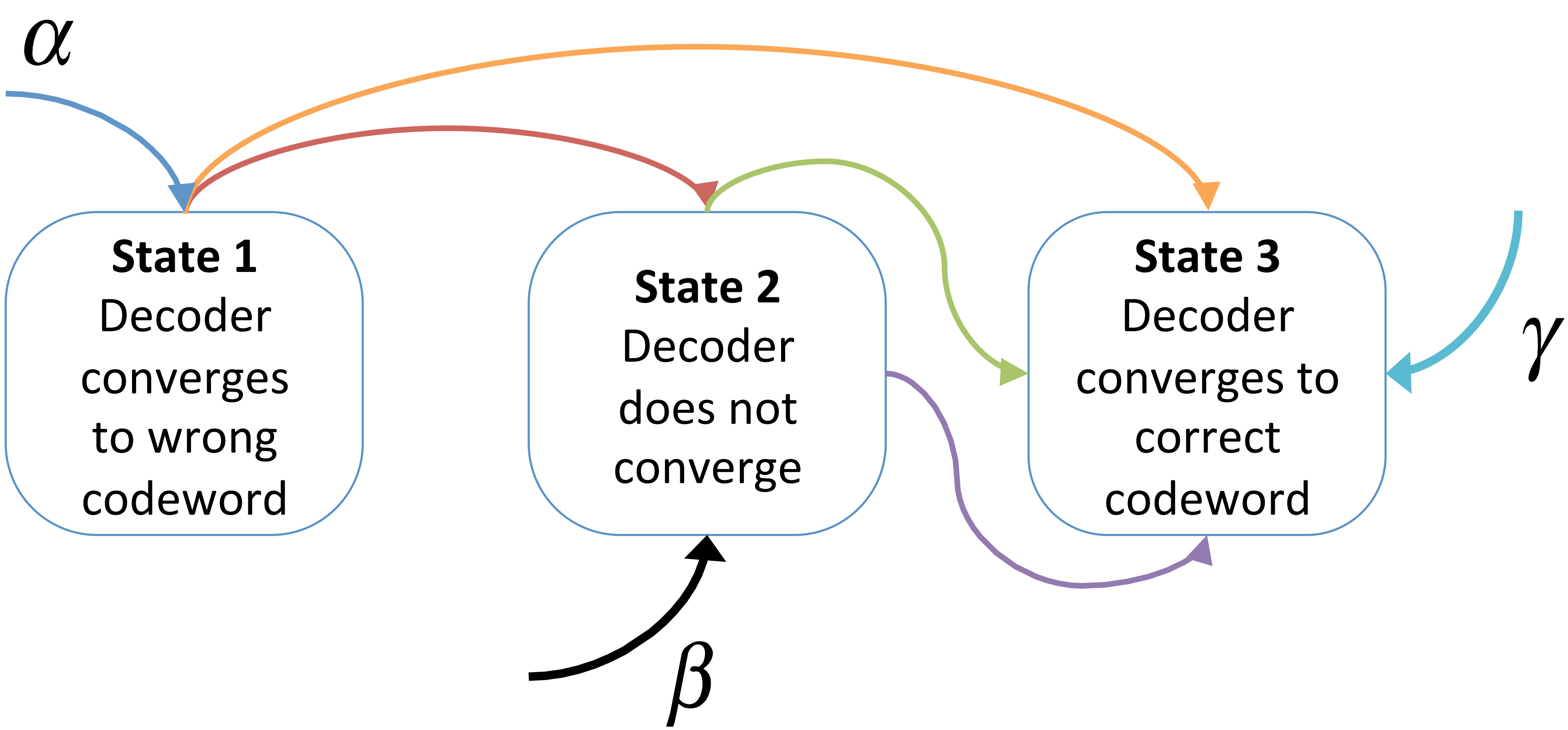
Observation:
NB-LDPC decoder never visits two wrong codewords.



$$\alpha + \beta + \gamma = 1$$

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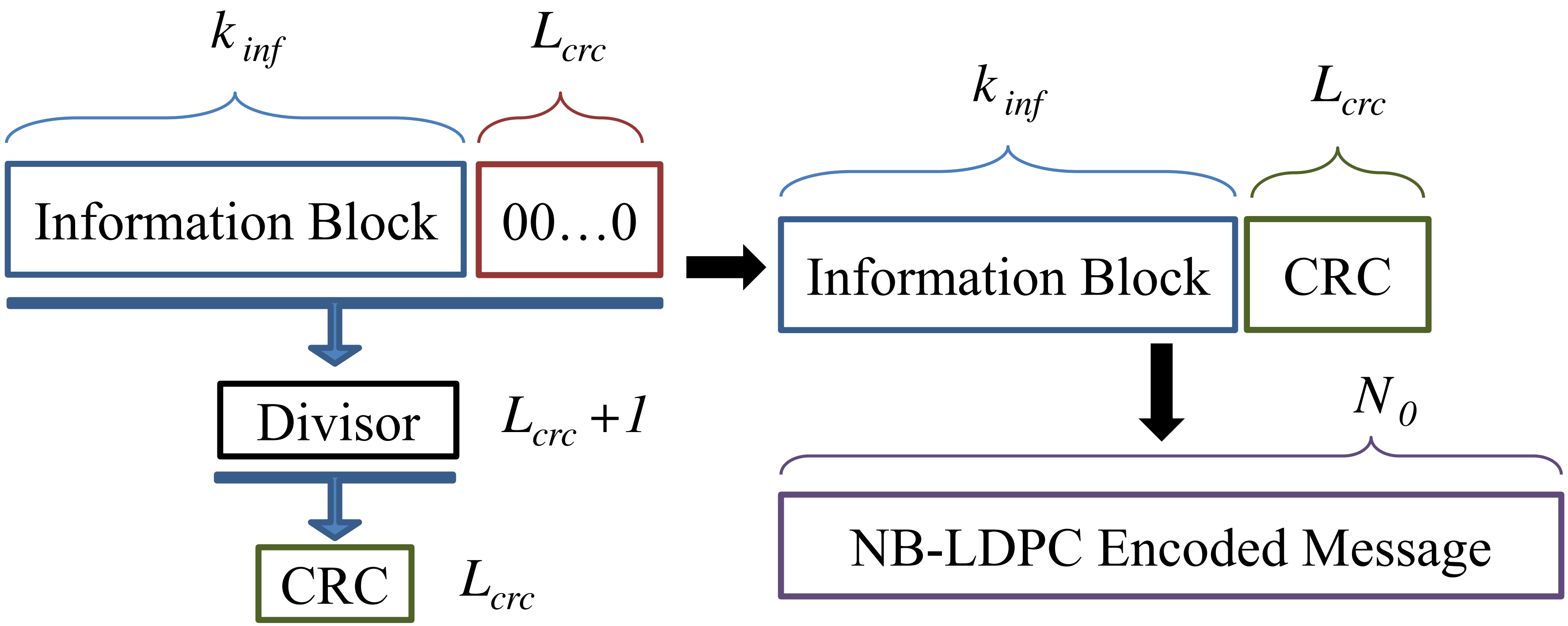
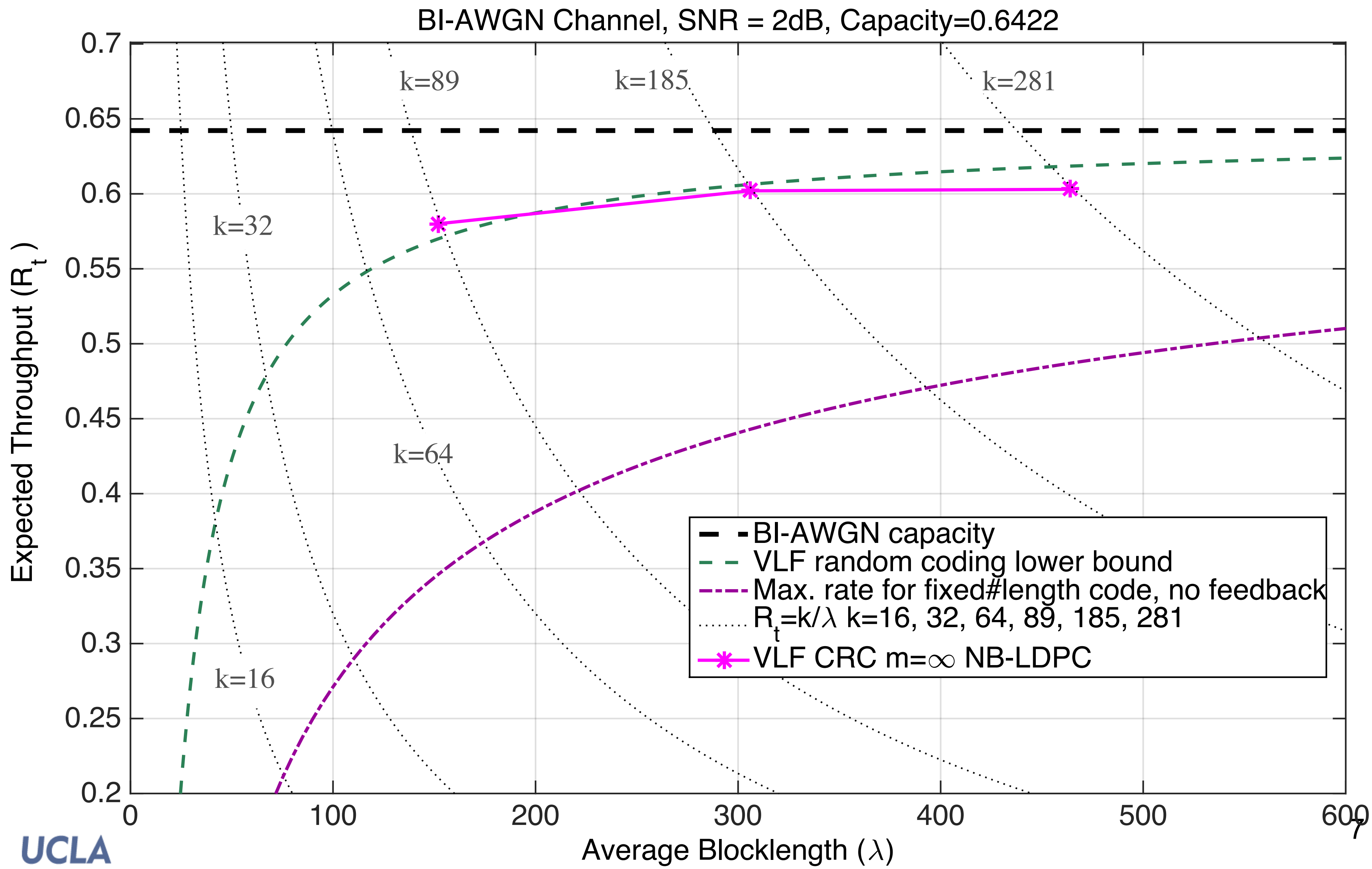
Choose CRC length to Guarantee FER=10^{-3}.



$$\alpha \times 2^{-L_{CRC}} < 10^{-3}$$

NB-LDPC with single-bit increments performs as well as random coding lower bound, in a fair comparison.



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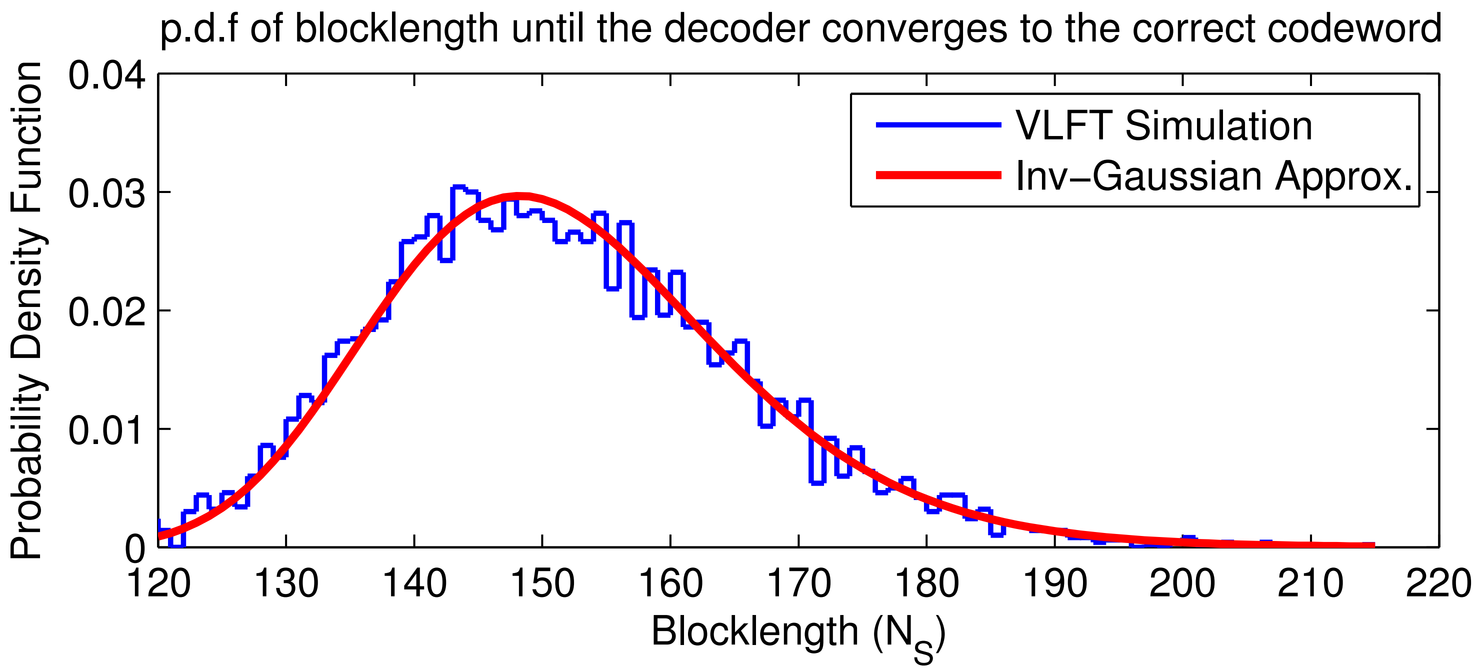
But what if I don't want to send feedback after every bit?

- Suppose my communication system can only tolerate m rounds of feedback.
- How many bits should I transmit in each increment?
- Can m incremental transmissions get close to the performance of an infinite number of single-bit increments?

How many bits does it take to decode successfully?



- From simulation of $k = 96$ system with 120-bit (rate-0.8) NB-LDPC code.
- N_S is the number of bits transmitted until the first successful decoding.
- $R_S = \dfrac{k}{N_S}$ is the instantaneous rate at the first successful decoding.

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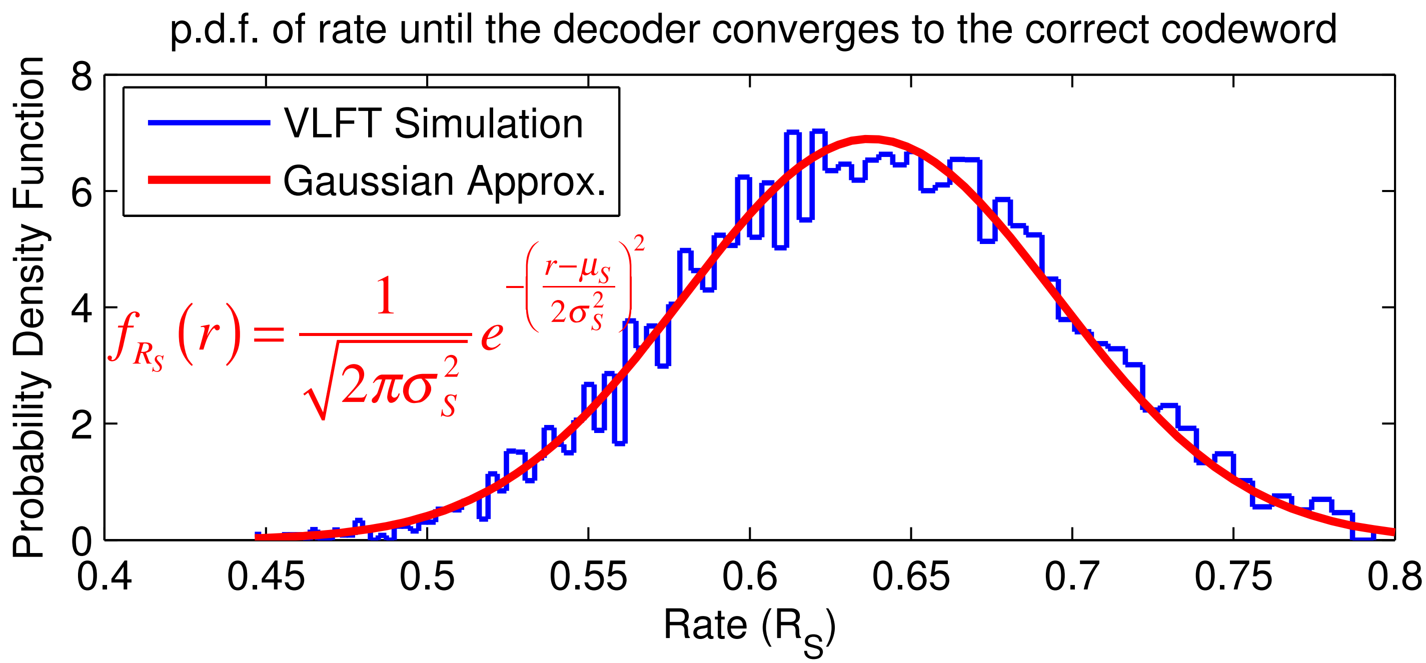
Rate histogram follows a Gaussian.

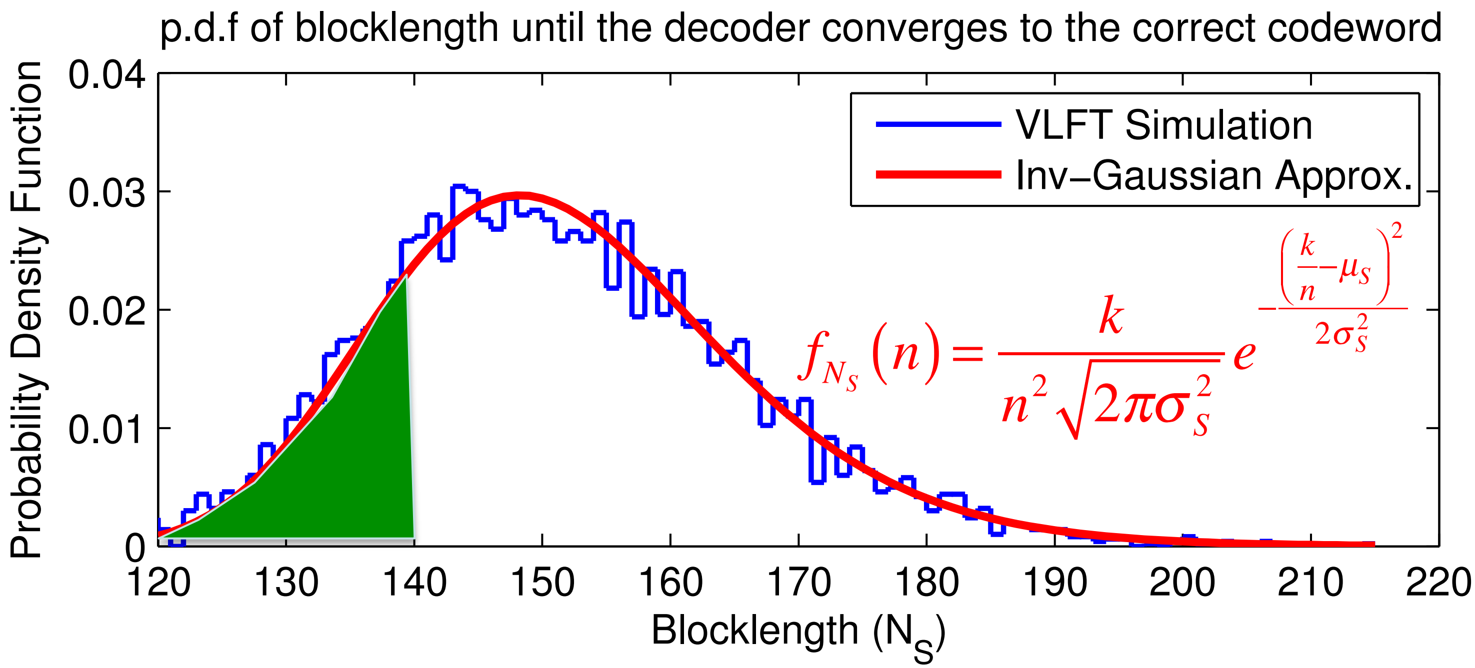


- Histogram of R_S, the highest rate supporting successful decoding.
- Polyanskiy's normal approximation is a practical reality!
- In this example $\mu_S = 0.63$ and $\sigma_S^2 = 0.057$.
- Note that capacity is $C = 0.6422 > \mu_S$.

m Transmissions: The Accumulation Cycle

- Let a maximum of m transmissions form an *accumulation cycle.*
- Performance depends on the cumulative blocklengths: $N_{1,} N_{2,} \cdots, N_m$.
- Note that since we end with a finite blocklength there will be some probability that the communication does not conclude.
- When that happens, the received transmissions are forgotten and we start over.

Computing probability of initial success…



- Suppose N_1, the initial blocklength, is 140 bits.
- The probability $P(n = N_1)$ of successful decoding is shown above.

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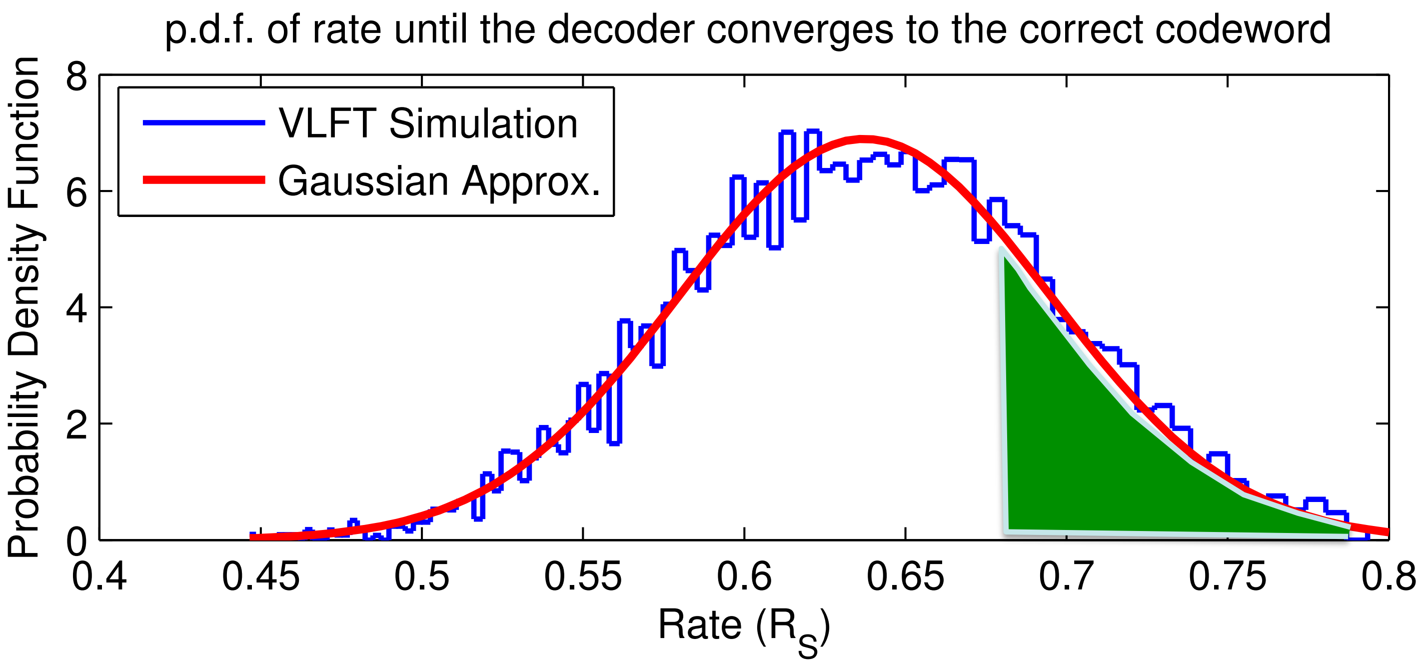
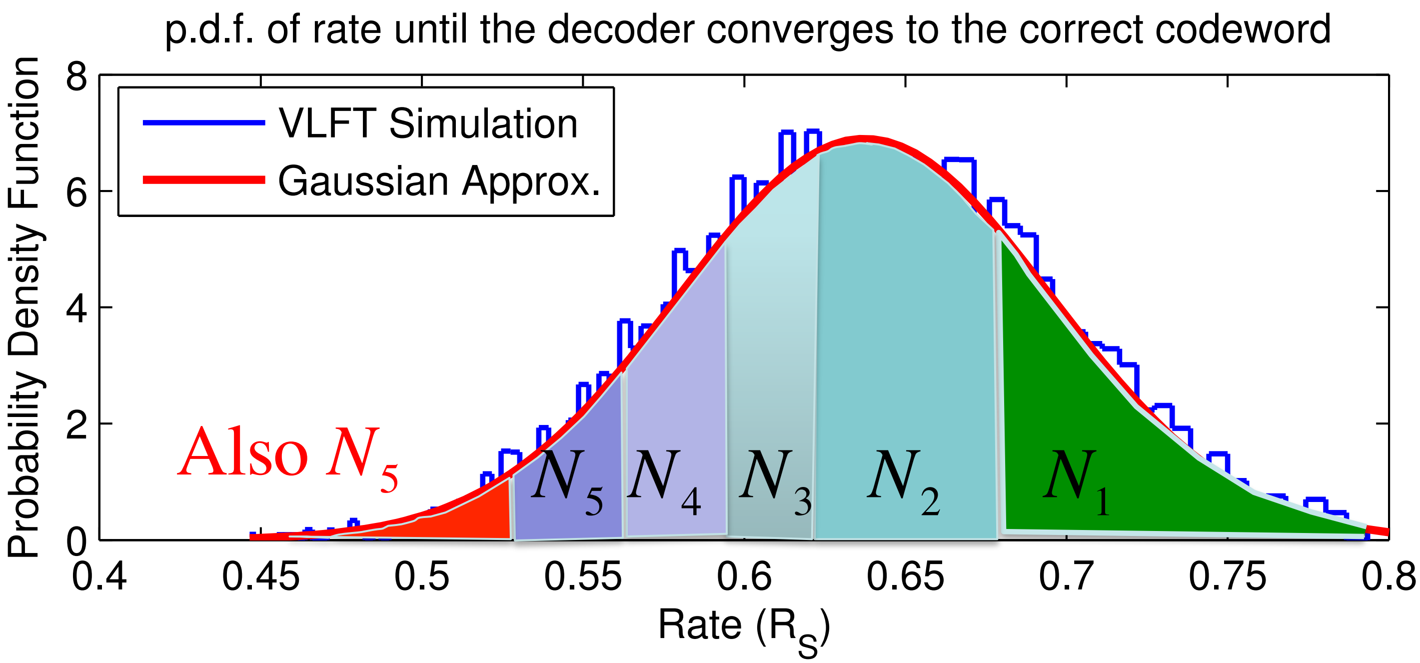
… is easier using the rate histogram.



- $P(n = N_1)$ is shown can also be seen on the rate histogram.
- $P(n = N_1) = Q\left(\dfrac{\frac{k}{N_1} - \mu_S}{\sigma_S} \right).$

Expected blocklength $E[N]$



$$E[N] = N_1 Q\left(\frac{\frac{k}{N_1} - \mu_S}{\sigma_S}\right) + \sum_{i=2}^{m} N_i \left[Q\left(\frac{\frac{k}{N_i} - \mu_S}{\sigma_S}\right) - Q\left(\frac{\frac{k}{N_{i-1}} - \mu_S}{\sigma_S}\right)\right] + N_m \left[1 - Q\left(\frac{\frac{k}{N_m} - \mu_S}{\sigma_S}\right)\right]$$

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Expected throughput per block $E[K]$

$$E[K] = kQ\left(\frac{\frac{k}{N_m} - \mu_S}{\sigma_S}\right) \approx k$$

…since we will always choose N_m large enough that the probability of a lost accumulation cycle is small.

Optimizing the throughput rate R_T

$$R_T = \frac{E[K]}{E[N]} \approx \frac{k}{E[N]}$$

So we just need to choose $N_{1,} N_{2,} \cdots, N_m$ to minimize $E[N]$.

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Lets take some derivatives

$$\frac{\partial E[N]}{\partial N_1} = Q\left(\frac{\frac{k}{N_1} - \mu_S}{\sigma_S}\right) + (N_1 - N_2)Q'\left(\frac{\frac{k}{N_1} - \mu_S}{\sigma_S}\right)$$

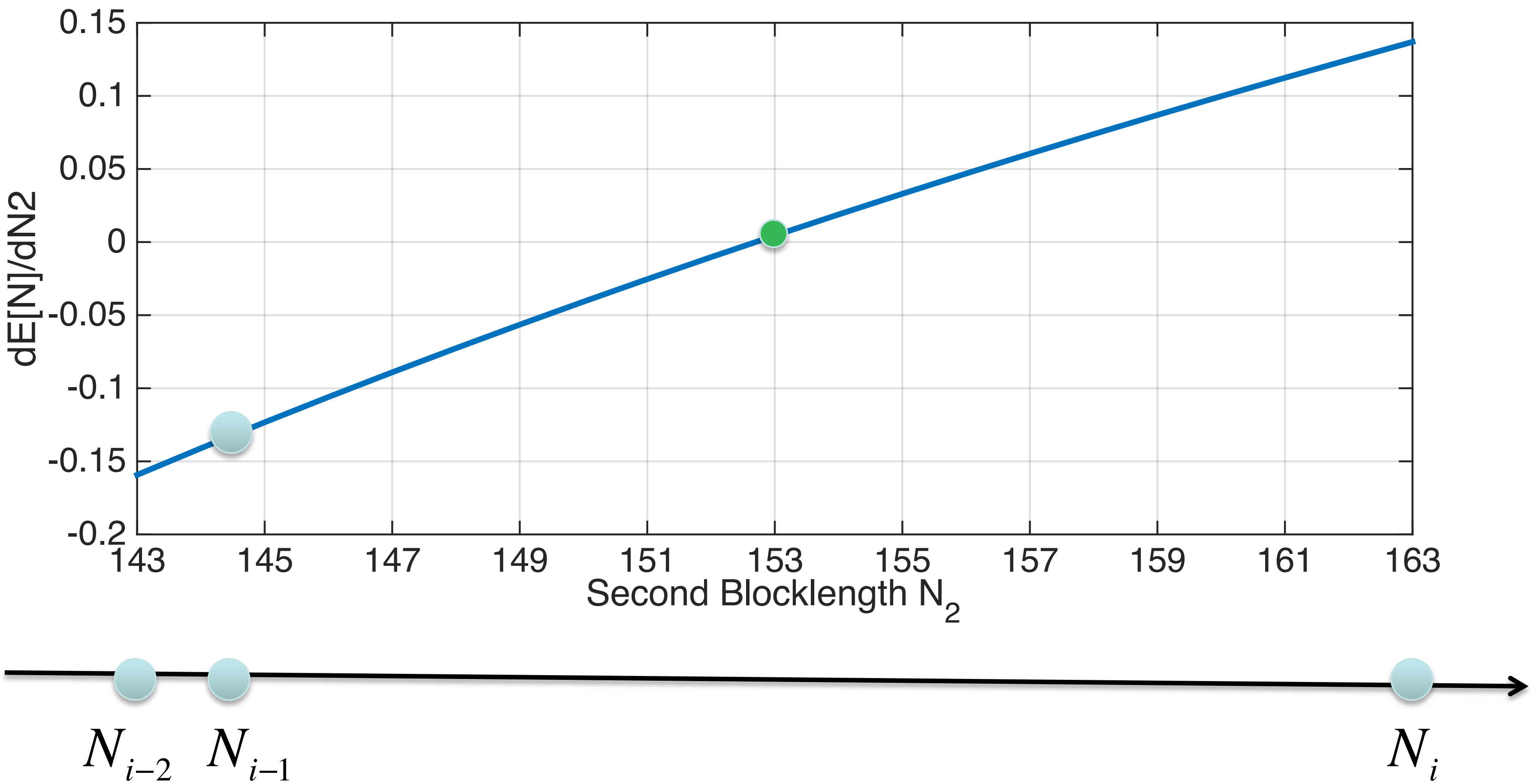
Setting $\frac{\partial E[N]}{\partial N_1} = 0 \ldots$

$$N_2 = \frac{Q\left(\frac{\frac{k}{N_1} - \mu_S}{\sigma_S}\right) + N_1 Q'\left(\frac{\frac{k}{N_1} - \mu_S}{\sigma_S}\right)}{Q'\left(\frac{\frac{k}{N_1} - \mu_S}{\sigma_S}\right)}$$

For $i > 2$

$$\frac{\partial E[N]}{\partial N_{i-1}} = Q\left(\frac{\frac{k}{N_{i-1}} - \mu_S}{\sigma_S}\right) + \left(N_{i-1} - N_i\right)Q'\left(\frac{\frac{k}{N_{i-1}} - \mu_S}{\sigma_S}\right) - Q\left(\frac{\frac{k}{N_{i-2}} - \mu_S}{\sigma_S}\right)$$

The sweet spot



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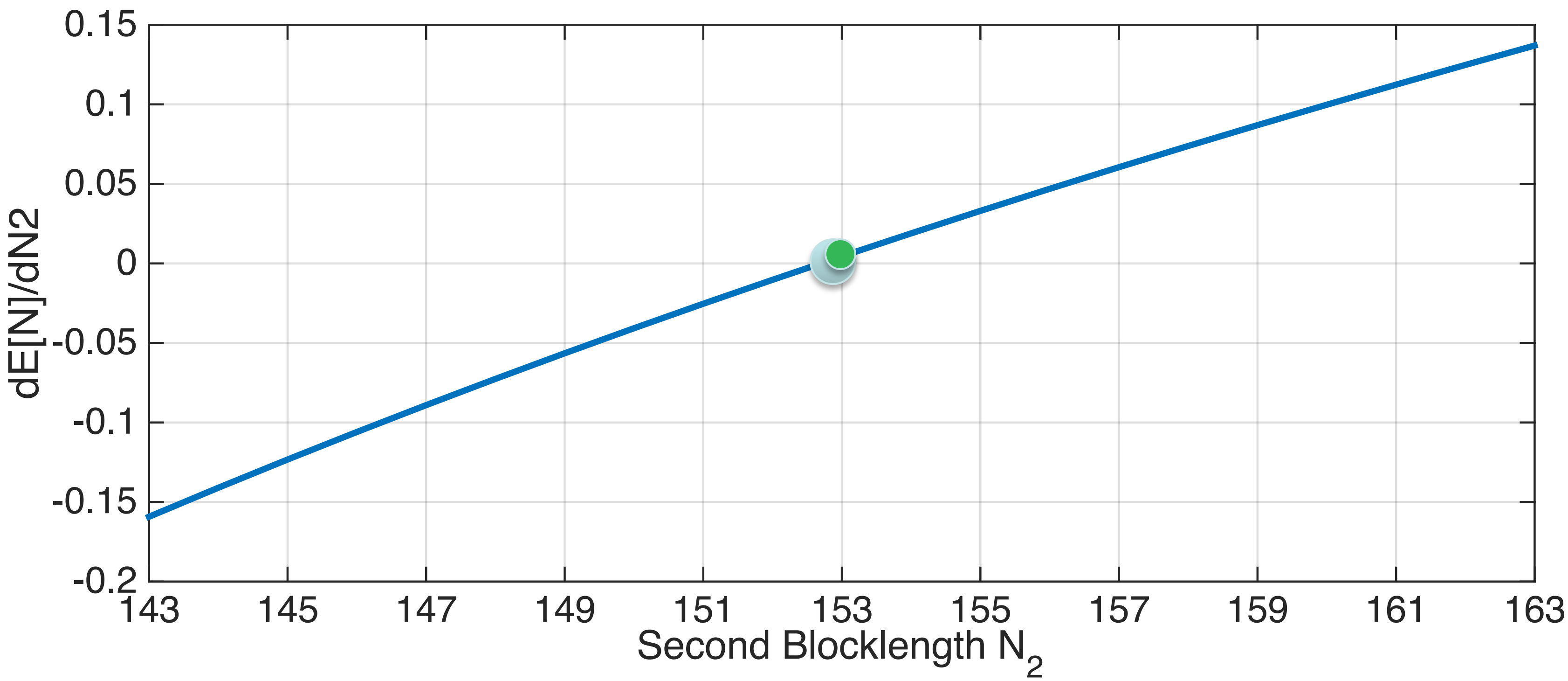
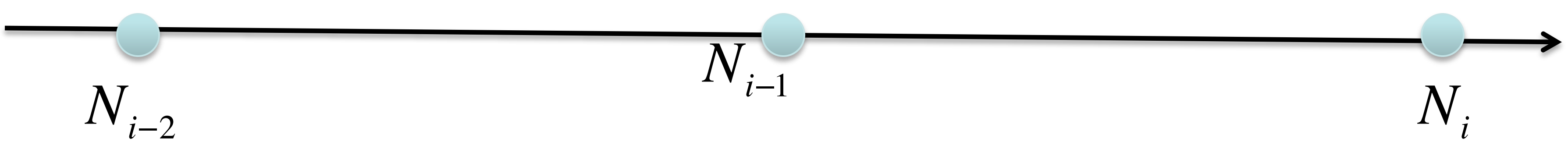
We actually choose N_i to make N_{i-1} optimal





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For $i > 2$

$$\frac{\partial E[N]}{\partial N_{i-1}} = Q\left(\frac{\frac{k}{N_{i-1}} - \mu_S}{\sigma_S}\right) + \left(N_{i-1} - N_i\right)Q'\left(\frac{\frac{k}{N_{i-1}} - \mu_S}{\sigma_S}\right) - Q\left(\frac{\frac{k}{N_{i-2}} - \mu_S}{\sigma_S}\right)$$

Setting $\frac{\partial E[N]}{\partial N_{i-1}} = 0 \ldots$

$$N_i = \frac{Q\left(\frac{\frac{k}{N_{i-1}} - \mu_S}{\sigma_S}\right) + N_{i-1}Q'\left(\frac{\frac{k}{N_1} - \mu_S}{\sigma_S}\right) - Q\left(\frac{\frac{k}{N_{i-2}} - \mu_S}{\sigma_S}\right)}{Q'\left(\frac{\frac{k}{N_{i-1}} - \mu_S}{\sigma_S}\right)}$$

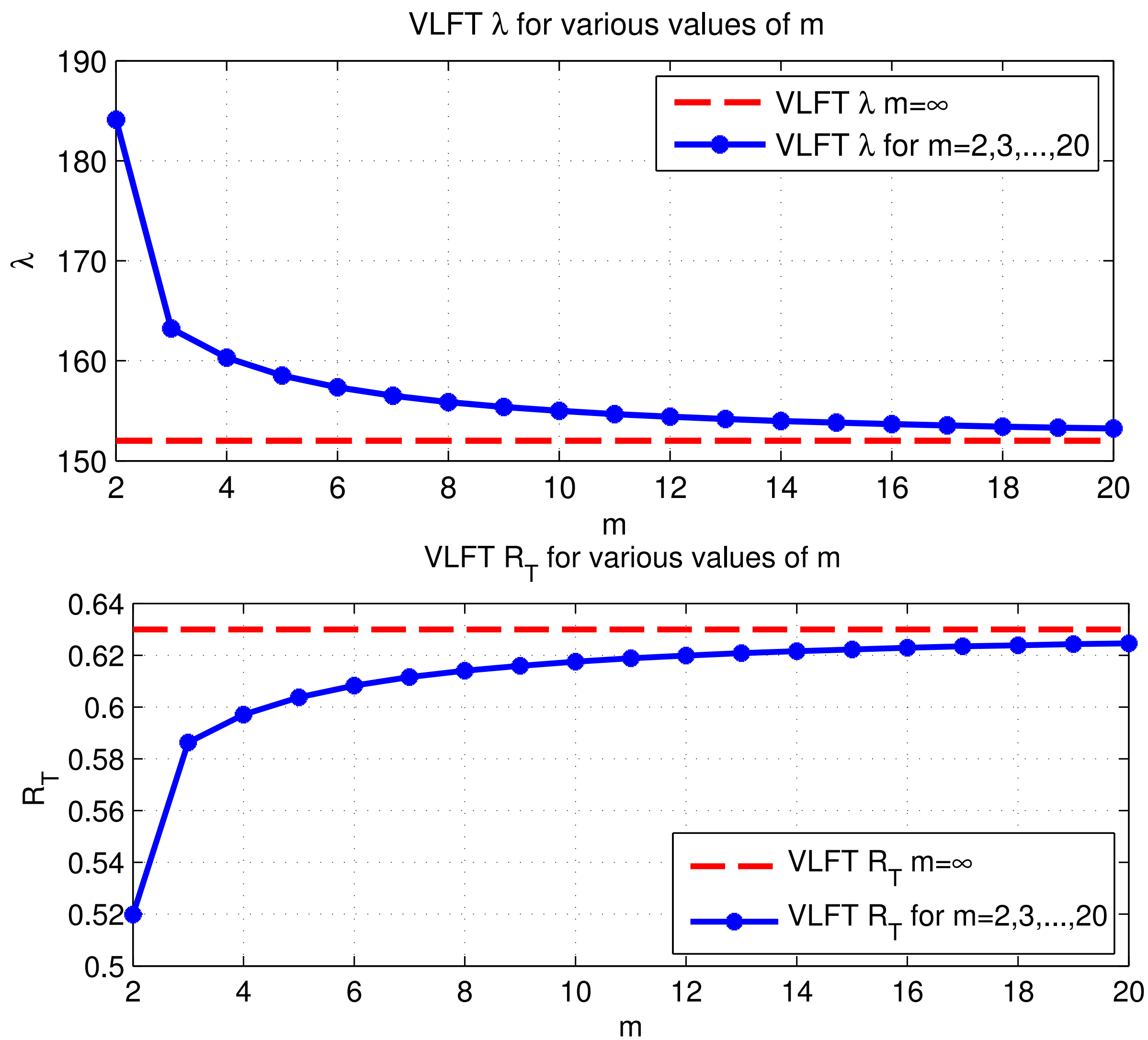
Sequential Differential Approximation

- So, for each choice of N_1 the remaining blocklengths can be selected so that

$$\frac{\partial E[N]}{\partial N_i} = 0 \quad \text{for } i \in \{1,\ldots,m-1\}$$

- This gives the same set of blocklengths as exhaustive search (within one bit) and essentially the same optimal throughputs.
- It is easy to check the set of interesting N_1 values.

Looks like 10 (maybe 20) = ∞.



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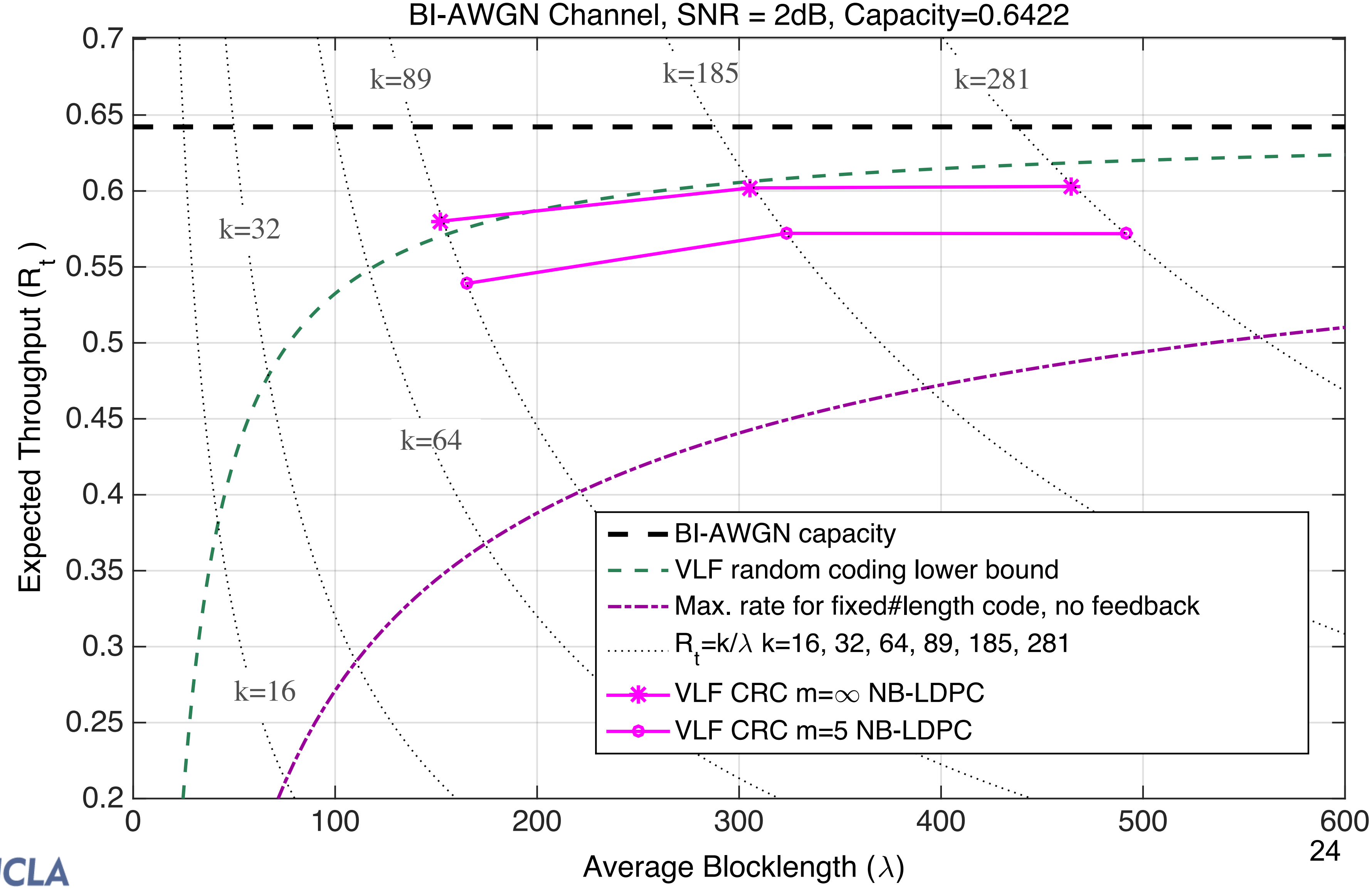
CRC-based Feedback with Non-binary LDPC Code



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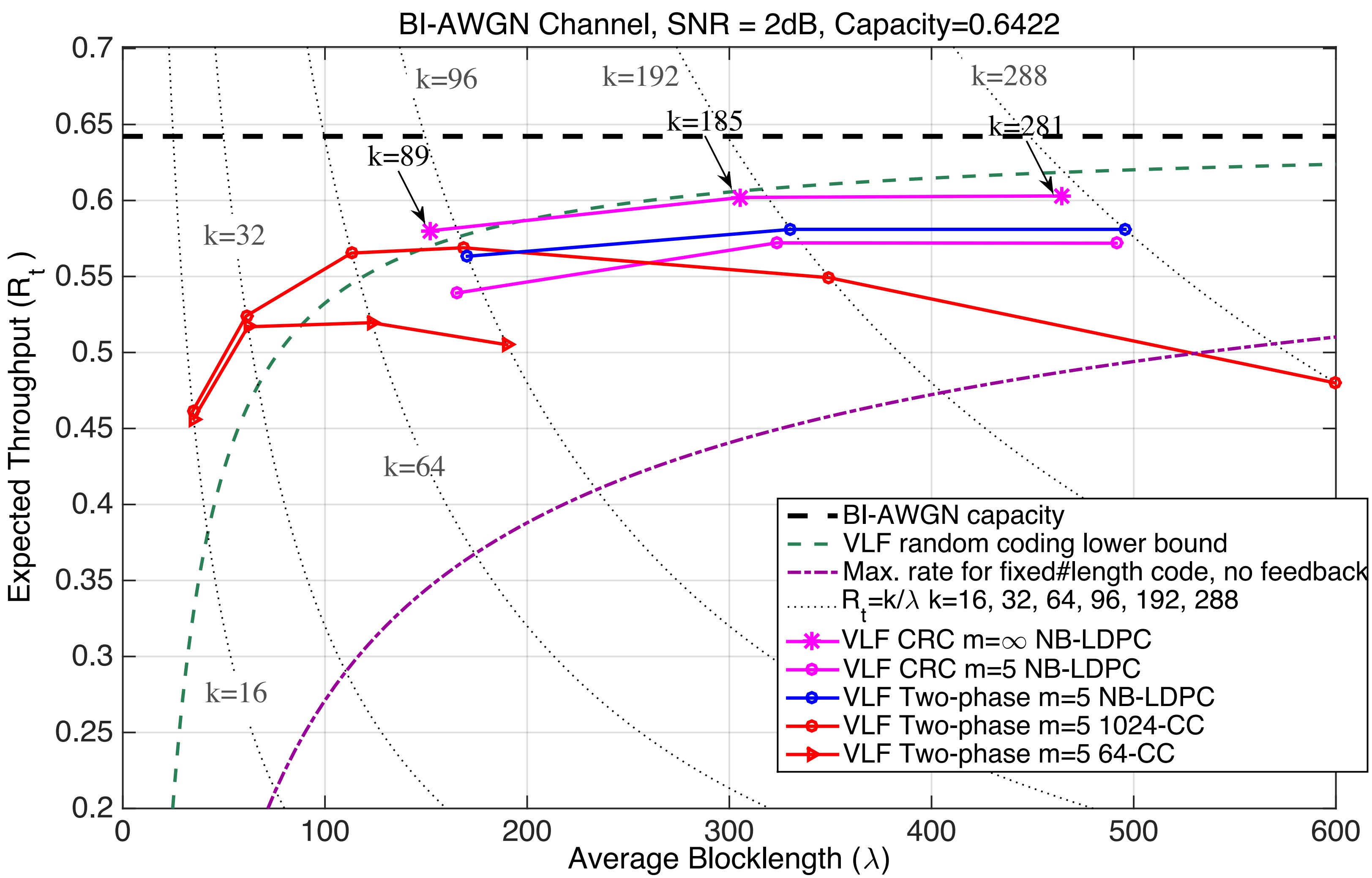
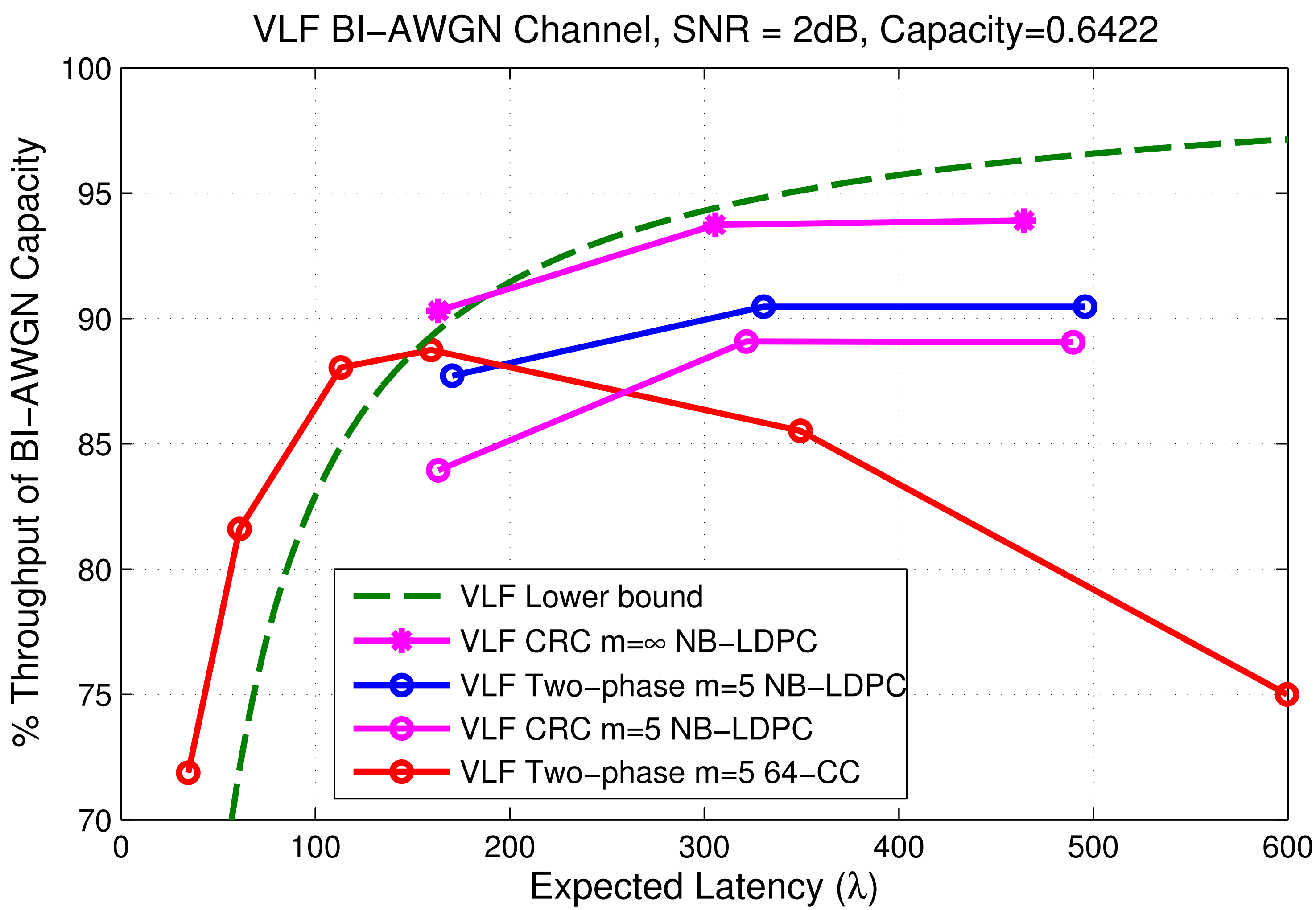
Some “two-phase” feedback schemes



Percentage of Capacity Perspective



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Conclusions

- Sequential Differential Approximation (SDA) is a general tool that can find the optimal incremental transmission sizes for a wide class of incremental redundancy feedback schemes.
- Today we used it to show that short NB-LDPC codes with a CRC and incremental redundancy can achieve 89% of capacity with 5 transmissions. (94% with infinite transmissions).
- We expect a significant portion of that gap to be closed by using 10 transmissions.