# Low-Density Lattice Coded Relaying with Joint Iterative Decoding

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- Low density lattice codes (LDLCs) were introduced in [Sommer et al., 08], where it was shown that LDLCs can perform close to AWGN capacity with low decoding complexity.
- Lattice codes also naturally support higher-order modulation as well as signal shaping.

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- Recently, lattice codes have been considered for use in networks with relays:
- Classical relay channel: [Ferdinand et al. '14]
- Two-way relay channel (TWRC): [Baik et al. '08], [Wilson et al. '10], [Song et al. '13]
- Multiple access relay channel (MARC): [Song Devroye '13]

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- Question: Can lattice codes be used for joint network-channel coding in the MARC?
- Can we do this with low-complexity joint iterative decoding at the destination?

Propose a scheme for LDLC cooperative transmission in Multiple Access Relay

Channel (MARC) uplink.



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- Present two relaying methods, one having lower decoding complexity and one having a greater power efficiency.
- Propose an efficient joint iterative channel-network decoding algorithm.
- Present a soft symbol relaying technique to mitigate decoding error propagation from the relay.



- Consider a Multiple Access Relay Channel (MARC) where Users (Sources) S<sub>1</sub> and S<sub>2</sub> each have their own information packets to transmit to the same destination via a shared relay.
- The relay node decodes and forwards a combination message to the destination.
- Three transmission phases (half duplex relay).

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■ Messages: Integer (*L*-ary PAM) information vectors: each source  $S_i$  wishes to transmit  $\mathbf{b}_i \in \{0, 1, 2, ..., L-1\}^n$ 

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- Encoding with hypercube shaping: Map each integer information vector  $\mathbf{b}_i$  to another integer vector  $\mathbf{b}'_i = \mathbf{b}_i L\mathbf{a}$ , where  $\mathbf{a} \in \mathbb{Z}^n$

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- Form LDLC codeword  $\mathbf{x}'_i = \mathbf{G}\mathbf{b}'_i$ , which lies in  $\Lambda$ ; use the lower-triangular *parity-check matrix* to encode via  $\mathbf{b}'_i = \mathbf{H}\mathbf{x}'_i$



First two phases (i = 1, 2):

$$\mathbf{y}_i^R = \sqrt{P_i} \alpha_i^R \mathbf{x}_i' + \mathbf{n}_i^R \; ,$$

$$\mathbf{y}_i^D = \sqrt{P_i} \alpha_i^D \mathbf{x}_i' + \mathbf{n}_i^D \; ,$$

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Decoding at relay:

$$\hat{\mathbf{x}}_i = \text{LDLCdecoder}\left(rac{\mathbf{y}_i^R}{\alpha_i^R \sqrt{P_i}}
ight), \ \ i = 1, 2$$

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 Superposition LDLC (S-LDLC): In this method, the relay simply adds the lattice codewords to form

$$\mathbf{x}_3' = \hat{\mathbf{x}}_1 + \hat{\mathbf{x}}_2$$

which is equivalent to addition of the underlying information vectors, since

$$\mathbf{x}_3' = \hat{\mathbf{x}}_1 + \hat{\mathbf{x}}_2 = \mathbf{G} \cdot (\hat{\mathbf{b}}_1 + \hat{\mathbf{b}}_2)$$

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 Modulo-Addition LDLC (MA-LDLC): In this method, in order to improve the power efficiency of the relay, the LDLC codeword is generated via

$$\mathbf{x}_3' = \mathbf{G} \cdot \mathbf{b}_3'$$

where

$$\textbf{b}_3'=\hat{\textbf{b}}_1+\hat{\textbf{b}}_2-\textit{L}\textbf{a}$$

and hypercube shaping is used to choose the integer vector **a**.

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MA-LDLC provides better power efficiency, but requires more complex decoding at the destination.

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The lattice point  $x'_3$ , which can be considered as a network coded component, will be transmitted from the relay to the destination, i.e.,

$$\mathbf{y}_3^D = \sqrt{P_3}\alpha_3^D\mathbf{x}_3' + \mathbf{n}_3^D$$

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$$\mathbf{y}_3^D = \sqrt{P_3}\alpha_3^D\mathbf{x}_3' + \mathbf{n}_3^D$$

Finally, the destination uses the signals  $\mathbf{y}_1^D$ ,  $\mathbf{y}_2^D$  and  $\mathbf{y}_3^D$  to jointly decode  $\mathbf{b}_1$  and  $\mathbf{b}_2$ .

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- Key problem: how to efficiently recover the information symbols at the destination based on the signals received.
- Therefore, we next focus on the joint iterative decoding structure.

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 At destination, the LDLC decoders receive three packets from three independent channels.

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Inner iterations: Each LDLC decoder performs *M* inner iterations.

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Outer iteration: After every *M* inner iterations of LDLC decoding, the network coding nodes implement one outer iteration to exchange extrinsic information between the LDLC decoders.

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Outer iteration: The extrinsic information R<sup>a</sup>(x) will be considered as a priori information for each decoder in the next iteration.

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 Final decision: After several inner and outer iterations, the final variable node messages are calculated to make decisions for information symbol vector b<sub>i</sub>.

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# Factor graph





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**Decoder** *i*: Initially each variable node  $x_{i,k}$  sends the message  $f_{i,k,j}(x) = \frac{1}{\sqrt{\pi N_0}} e^{-\frac{\left(y_{i,k}^D - \sqrt{P_i \alpha_i^D x}\right)^2}{N_0}}$ to each neighboring check node  $c_{i,j}$ 

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■ Inner iterations: each LDLC decoder performs *M* inner iterations.

- Check-to-variable messages
- Variable-to-check messages

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**Check node message at decoder** *i*: each check node  $c_{i,j}$  sends a PDF function  $Q_{i,j,k}(x)$  to each of its neighboring variable nodes  $x_{i,k}$ .

a. Convolution step: All messages, except  $f_{i,k,j}(x)$ , are convolved (after expansion of each  $f_{i,l,j}(x)$  by the factor  $h_{i,l}$ ):

$$\widetilde{p}_{i,j,k}(x) = \bigotimes_{\substack{l \in \mathcal{A}_j \\ l \neq k}} f_{i,l,j}\left(\frac{x}{h_{j,l}}\right)$$

where the set  $A_j$  denotes the set of indices *I* for which  $h_{j,l} \neq 0$ .

b. Stretching step:

$$p_{i,j,k}(x) = \widetilde{p}_{i,j,k}(-h_{j,k}x)$$

c. Periodic extension step: extend with period  $1/|h_{j,k}|$ 

$$Q_{i,j,k}(x) = \sum_{a \in \mathbb{Z}} p_{i,j,k}\left(x - \frac{a}{h_{j,k}}\right)$$

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Variable node message at decoder *i*: each variable node  $x_{i,k}$  sends a message  $f_{i,k,j}(x)$  to each of its neighboring check nodes  $c_{i,j}$ .

a. Variable node update rule:

$$\widetilde{f}_{i,k,j}(x) = e^{-\frac{(v_{i,k}^D - \sqrt{\mathcal{P}_i \alpha_i^D x)^2}}{N_0}} R_{i,k}^a(x) \prod_{\substack{l \in \mathcal{B}_k \\ l \neq j}} Q_{i,l,k}(x)$$

where the set  $\mathcal{B}_k$  denotes the set of indices *I* for which  $h_{I,k} \neq 0$ .

b. Normalization step:

$$f_{i,k,j}(x) = \frac{\widetilde{f}_{i,k,j}(x)}{\int_{-\infty}^{\infty} \widetilde{f}_{i,k,j}(x) dx}$$

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Outer iteration: After M inner iterations, each variable node x<sub>i,k</sub> will send the extrinsic message R<sup>i</sup><sub>i,k</sub>(x) to the corresponding network coding node n<sub>k</sub>:

$$R_{i,k}^{e}(x) = e^{-\frac{(y_{i,k}^{D} - \sqrt{P_{i}}\alpha_{i}^{D}x)^{2}}{N_{0}}} \prod_{l \in \mathcal{B}_{k}} Q_{i,l,k}(x)$$



 $\bigotimes^{\text{convolution}} \rightarrow R^{\mathfrak{s}}_{3,k}(x')$   $R^{e}_{2,k}(x') \nearrow$ 

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MA-LDLC:

$$\begin{array}{c} R^{e}_{1,k}(x') \xrightarrow{\mathbf{H}} R^{e}_{1,k}(b') \searrow \\ & \bigotimes^{\text{convolution}} \xrightarrow{\text{extension}} R^{a}_{3,k}(b') \rightarrow R^{a}_{3,k}(x') \\ \\ R^{e}_{2,k}(x') \xrightarrow{\mathbf{H}} R^{e}_{2,k}(b') \nearrow \end{array}$$

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# Details: MA-LDLC

**I** According to the relationship between  $\mathbf{b}'_i = \mathbf{H}\mathbf{x}'_i$ , we can calculate the corresponding extrinsic PDFs of  $\mathbf{b}'_i$  by convolution:

$$R^e_{i,l}(b') = \bigotimes_{r \in \mathcal{A}_l} R^e_{i,r}\left(rac{x'}{h_{l,r}}
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### Details: MA-LDLC

According to the relationship between b'<sub>i</sub> = Hx'<sub>i</sub>, we can calculate the corresponding extrinsic PDFs of b'<sub>i</sub> by convolution:

$$R_{i,l}^{e}(b') = \bigotimes_{r \in \mathcal{A}_{l}} R_{i,r}^{e}\left(\frac{x'}{h_{l,r}}\right) \ .$$

2 According to the modulo addition relationship, for each *I* we can write  $b'_{1,l} + b'_{2,l} - b'_{3,l} = a_l L, a_l \in \mathbb{Z}$ . We first calculate the PDF message for the sum of  $b'_{1,l}$  and  $b'_{2,l}$  using convolution:

$$r_{3,l}^{a}(b') = R_{1,l}^{e}(b') \otimes R_{2,l}^{e}(b')$$

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$$r^{a}_{3,l}(b') = R^{e}_{1,l}(b') \otimes R^{e}_{2,l}(b')$$

**I** Then, we periodically extend the appropriate PDFs  $r_{3,l}^a(b')$  via

$$R^{a}_{3,l}(b') = \sum_{a \in \mathbb{Z}} r^{a}_{3,l}(b' - aL)$$

and combine to calculate the *a-priori* information for the LDLC codeword element  $x'_{3,k}$  (using  $x'_3 = \mathbf{Gb}'_3$ ):

$$R_{3,k}^{a}(x') = \bigotimes_{l \in \mathcal{C}_{k}} R_{3,l}^{a}\left(\frac{b'}{g_{k,l}}\right)$$

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- The resulting messages  $R_{i,k}^a(x)$  are generated from the network coding nodes.
- Each LDLC decoder will repeat the inner and outer iterations until the maximum outer iteration number N is achieved or until all message variances lie below a preset threshold value ξ.



**Final decision:** after a maximum of *MN* LDLC iterations, the final variable node PDF is calculated via

$$\widetilde{f}_{i,k,j} = e^{-\frac{(v_{i,k}^D/(\alpha_i^D\sqrt{P_i})-x)^2}{2\sigma^2}} R_{i,k}^{\mathfrak{s}}(x) \prod_{l \in \mathcal{B}_k} Q_{i,l,k}(x) .$$

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The integer vector  $\mathbf{b}_i$  is then estimated as

$$\hat{\mathbf{x}}_{i,k} = rg\max\widetilde{f}_{i,k,j}(x), \ \ i=1,2$$

and

$$\hat{\mathbf{b}}_i = \lfloor \mathbf{H} \hat{\mathbf{x}}_i 
ceil \mod L, \quad i = 1, 2$$

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# Soft Symbol Relaying



- Hard decision relaying can cause error propagation to the destination under poor SR-link SNR conditions (in case of incorrect decoding at the relay).
- Soft symbol relaying chooses as symbol estimate that real vector x'<sub>3</sub> which maximizes the multidimensional PDF F<sub>3</sub>(x').

# Complexity of Joint Decoding

- Gaussian mixture model (GMM) usually used for representing PDFs (messages)
- Complexity dominated by Gaussian mixture reduction algorithm; using GMM with *T* Gaussians, complexity of GMR is proportional to *T*<sup>4</sup> [Kurkoski Dauwels '08]
- Let  $E_h$  = number of edges in the LDLC Tanner graph;  $N_d$  = number of outer iterations
- Check node operations: Complexity is approximately proportional to  $MN_dE_hT^4$
- Variable node operations: Complexity is approximately proportional to  $MN_d(E_h + n)T^4$
- Network coding nodes: Complexity is approximately proportional to  $N_d(2E_h + n)T^4$
- Therefore, total complexity is proportional to  $N_d[(2M+2)E_h + (M+1)n]T^4$ .
- We use the single-Gaussian approximation method (T = 1, [Kurkoski *et al.* '09]) for large block size LDLCs (n = 1000).

#### Simulation scenario

We compare S-LDLC and MA-LDLC with three competing schemes in a quasi-static Rayleigh fading environment

- Non-cooperative LDLC
- Network coded 4-PAM cooperation without channel coding (NC)
- Network-Turbo-coded 4-PAM cooperation (NTC) similar to [Hausl. et al., 06], but with 4-PAM modulation.

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We set the link SNRs as follows:

$$\gamma_{S_1D} = \gamma_{S_2D} = \gamma_{SD}$$

• 
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•  $\gamma_{SR} = \gamma_{RD} = \gamma_{SD} + 6 \text{ dB}$ ; we vary the SD link SNR.

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All compared schemes were normalized to have the same overall transmitted power and code rate (4/3 bits/symbol).

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- MA-LDLC scheme outperforms all other cooperative schemes
- At SER of 10<sup>-5</sup>, gain is 6.2dB over NC 4-PAM, and 2.5dB over NTC 4-PAM.

# Performance variation with block length



For fixed block length, MA-LDLC outperforms S-LDLC.

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Low-Density Lattice Coded Relaying with Joint Iterative Decoding

# Outage Probability and FER



At an FER of  $10^{-4}$ , MA-LDLC outperforms NC 4-PAM by about 4dB, and NTC 4-PAM by about 2dB.

# Soft symbol relaying



• We fix  $\gamma_{SD} = \gamma_{RD} - 3 \text{ dB} = 25 \text{ dB}$  and vary  $\gamma_{SR}$ .

- Soft symbol relaying outperforms hard decision relaying when  $\gamma_{SR}$  is worse than  $\gamma_{SD}$  (the maximum gain is 2.5dB at an SER of  $10^{-3}$ ).
- As  $\gamma_{SR}$  increases, hard decision relaying outperforms soft symbol relaying due to the forwarding of the correct LDLC codeword almost every time by the relay.

# Soft symbol relaying



• Here  $\gamma_{SD} = \gamma_{SR} + 5dB = \gamma_{RD} - 3dB$ , and we vary  $\gamma_{SD}$ .

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# Choosing the inner and outer iteration numbers



- Inner iteration number M is chosen as 5 to balance the convergence rate and decoding complexity.
- It can be seen that there is marginal benefit for increasing outer iterations N beyond 15.

# Investigation of the average required number of iterations



- Plot shown for n = 1000, single-Gaussian approximation decoding.
- Average required number of iterations per LDLC decoder decreases quickly with increasing SNR.

# Summary

- We have proposed a new scheme for coded cooperation, based on joint network coding and low-density lattice coding
- Outlined two approaches for relay processing: S-LDLC and MA-LDLC
  - S-LDLC has low decoding complexity due to the simple superposition of codewords;
  - MA-LDLC has better performance (shaping gain) due to the usage of modulo-addition which can improve the power efficiency of the relay.
- Designed an efficient joint iterative decoding structure at the destination node.
- Proposed a soft symbol relaying method to mitigate the error propagation.
- The proposed scheme provides 2.5dB SER gain over network-turbo-coded 4-PAM and may be easily extended to the case of multiple sources.

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# Thank You

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