Two-channel Model for Wavelike Convergence of Tail-biting SC-LDPC Codes

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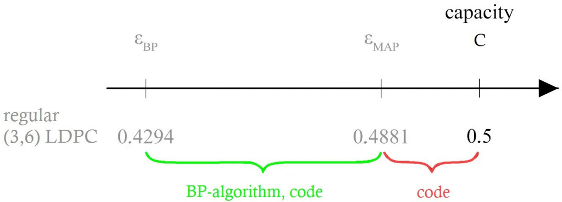
MCM, Munich, 2015

Universität Stuttgart

Motivation

- SC-LDPC codes can reach the capacity (MAP performance ε_{MAP}) under low-complexity BP decoding
- Termination is responsible for the excellent performance, but results in a rate loss for finite length codes
- Can we combine modulation with SC-LDPC codes to mitigate the rate loss?



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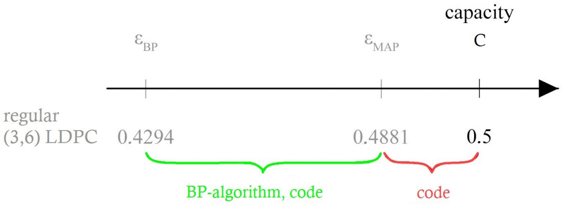

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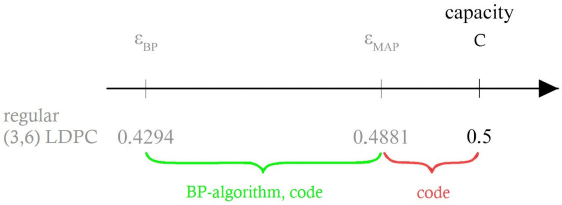

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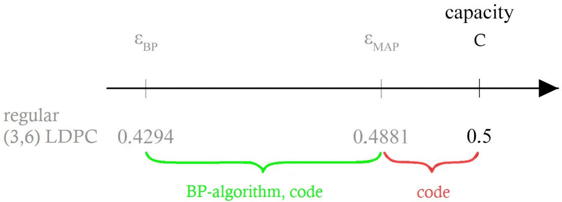

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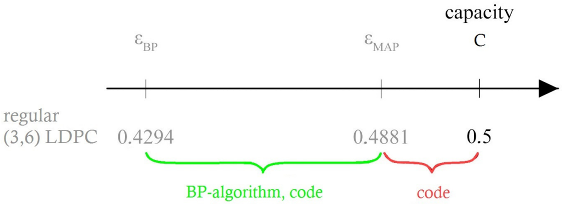
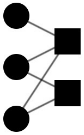
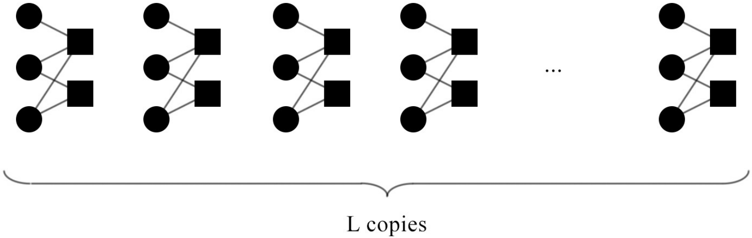
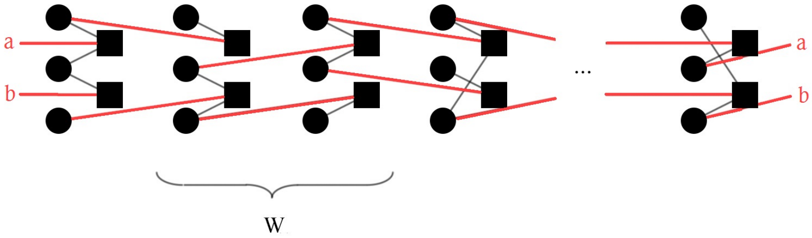


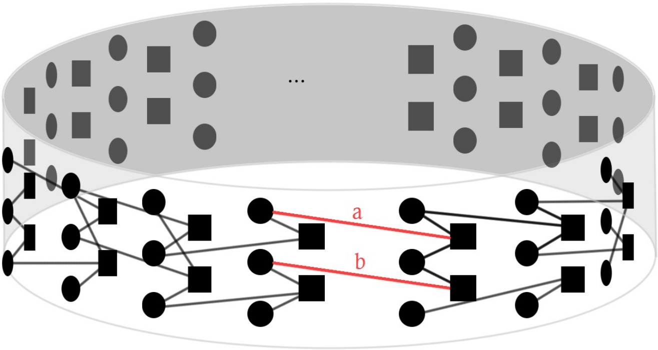
Spatial Coupling & Tail-Biting



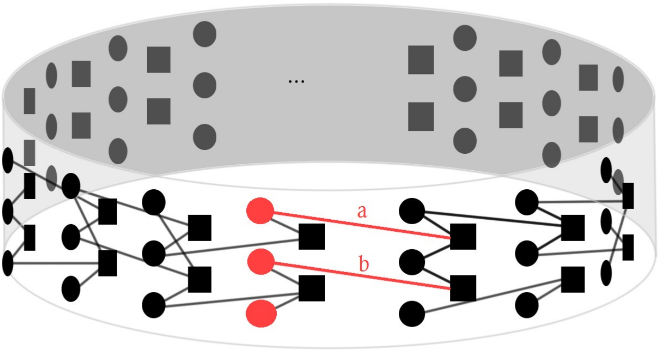

...
L copies


a
b
...
a
b
W

Tail-biting Structure

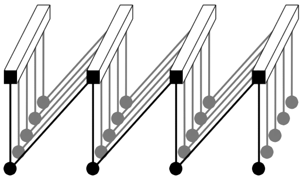


Terminated Structure



- $W - 1$ blocks are completly frozen (red nodes are set to 0)

Terminated SC [1]

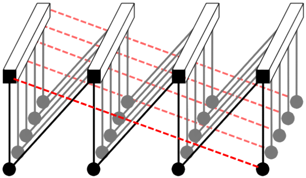


$R_D^{Term} = 1 - \frac{d_c}{d_v} \cdot \frac{(L+W-1)}{L} \leq 1 - \frac{d_c}{d_v} = R_D$

BP∼MAP performance

-rate-loss

Tail-biting SC



$R_D^{Tail} = 1 - \frac{d_c}{d_v} = R_D$

BP $\neq$ MAP performance

-no rate-loss (ring structure!)

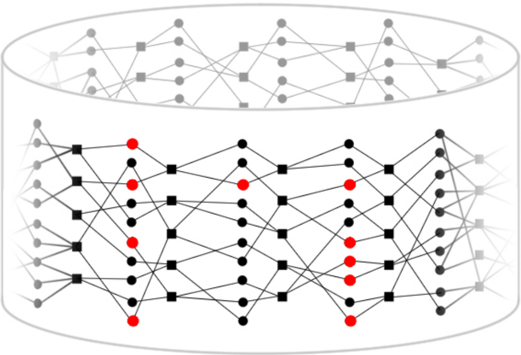
-performance loss

L...replication factor

W...coupling window size

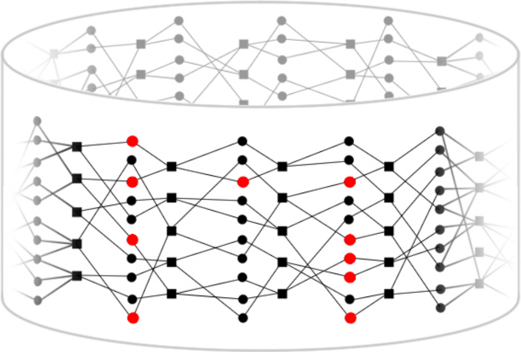
[1] S. Kudekar, T. Richardson, R. Urbanke, "Spatially coupled ensembles universally achieve capacity under belief propagation", IEEE Transactions on Information Theory, 7761-7813, 2013

Random Shortening



- Only freeze a fraction α_i of bits per block instead of freezing $W - 1$ blocks
- Choose the frozen bits randomly within a block

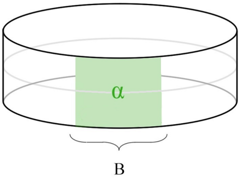
Random Shortening



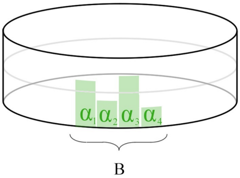
- Only freeze a fraction α_i of bits per block instead of freezing $W-1$ blocks
- Choose the frozen bits randomly within a block

- Introduce window B with fraction α of frozen bits
- Code rate depends on B and α
- Optimize α in terms of the code rate

uniform



non-uniform



Questions

- In terms of B, how large should α be taken to result the *threshold saturation*?
- Instead of uniform distribution, what is the best distribution for freezing bits on window B?

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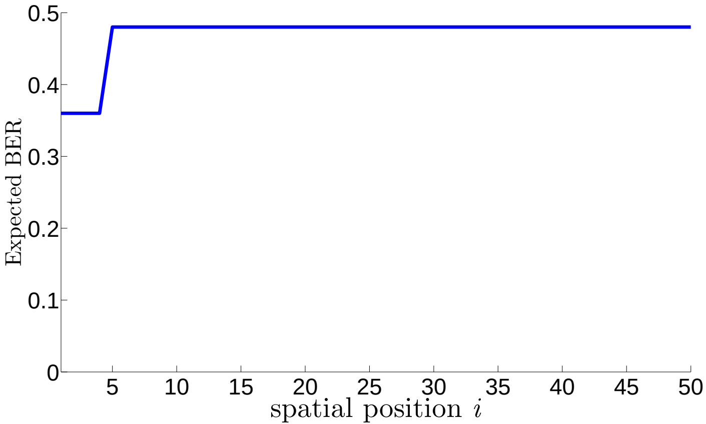
Expected BER after several iterations



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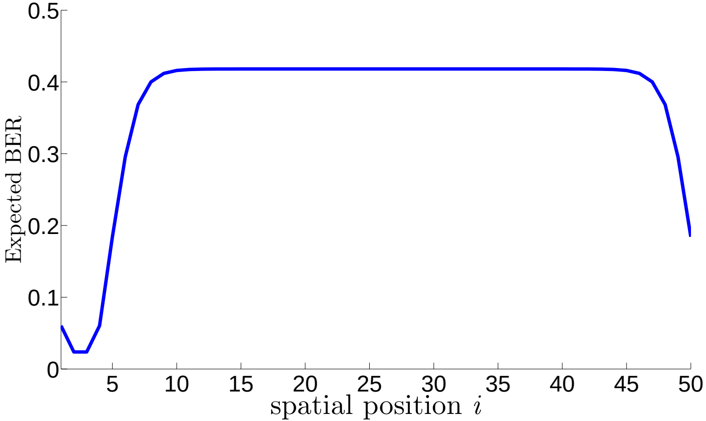

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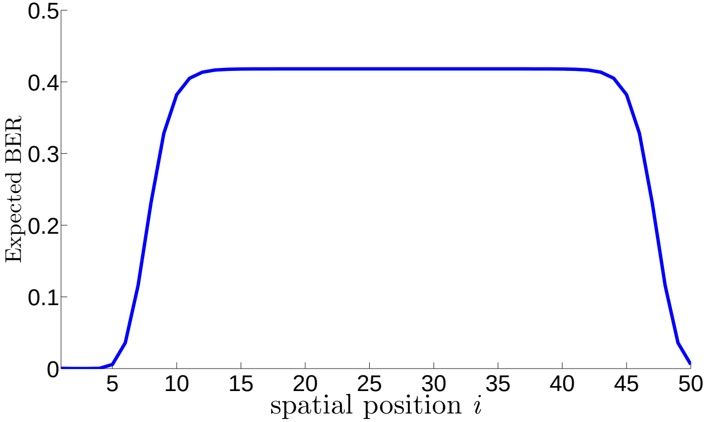

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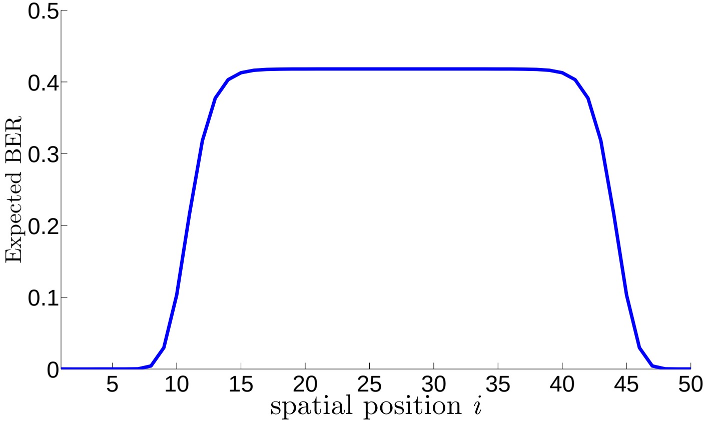

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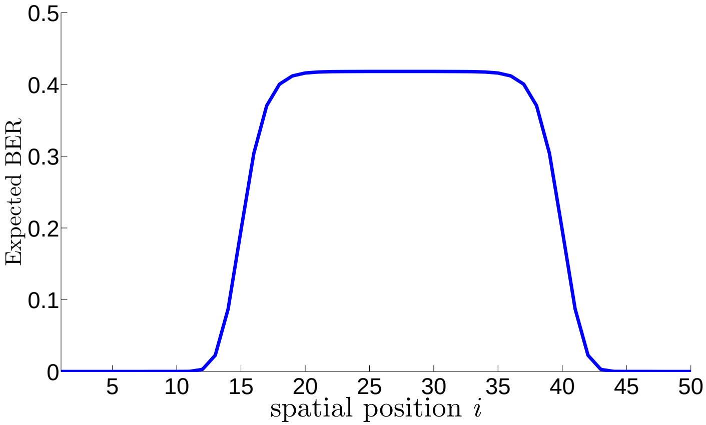

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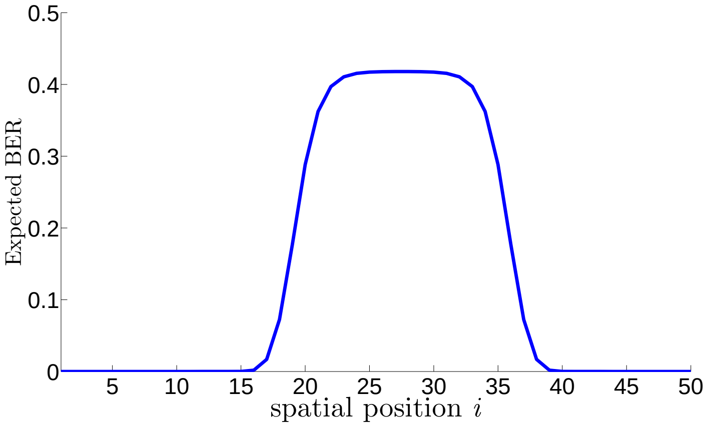

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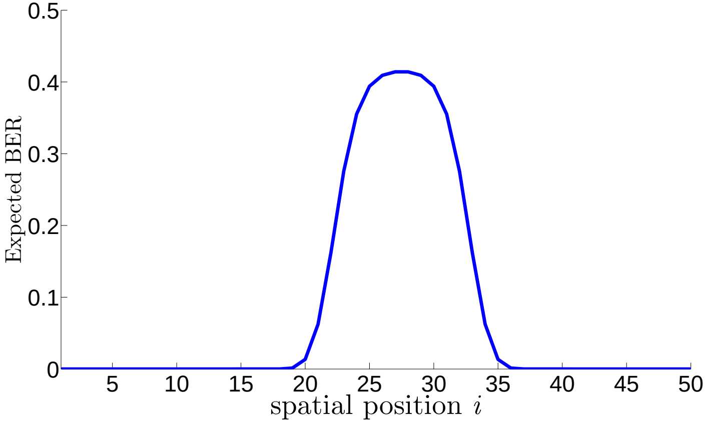

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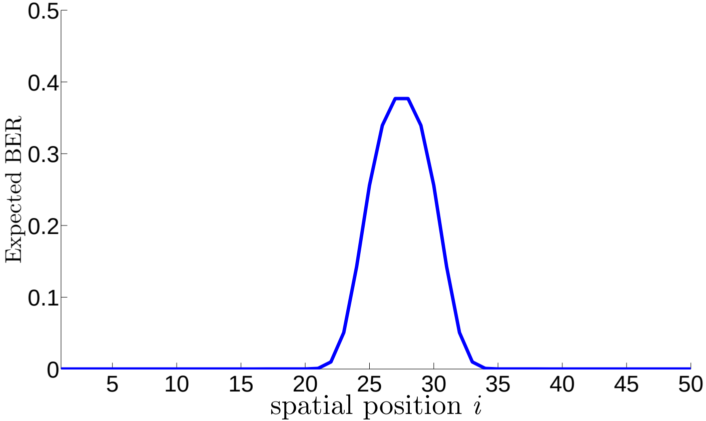

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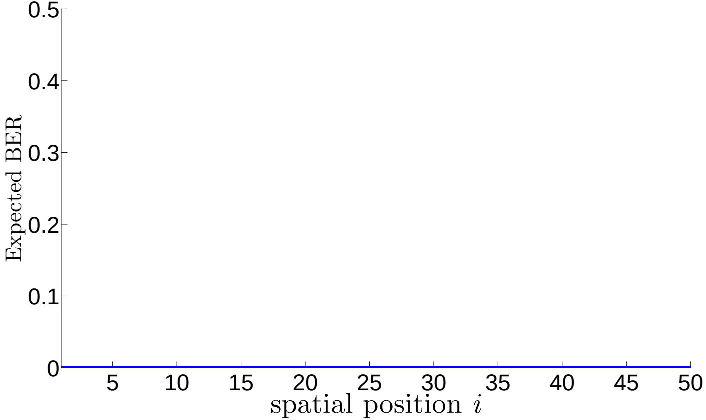
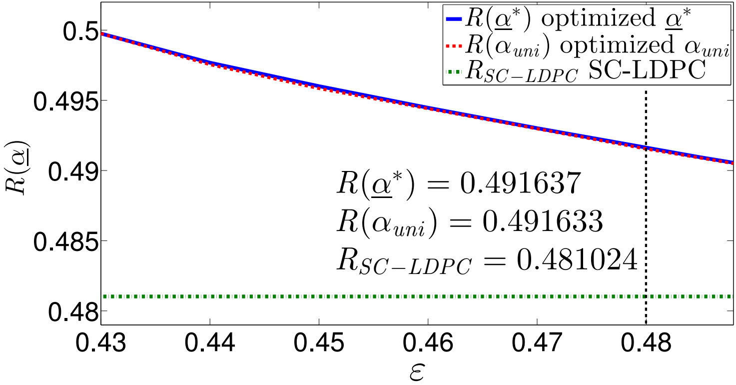
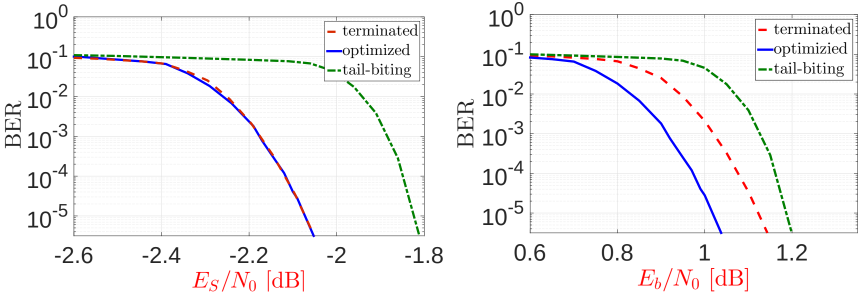


→ **Error-free decoding due to *decoding wave***

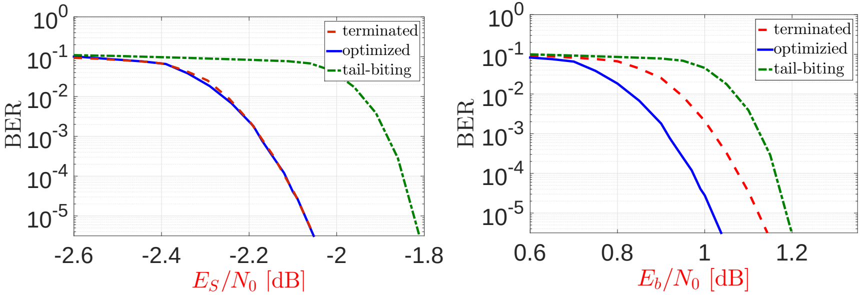
- Optimization between $\varepsilon_{BP} = 0.4294$ and $\varepsilon_{MAP} = 0.4881$
- results for $(d_v = 3,\ d_c = 6,\ L = 50,\ W = 3)$ SC-LDPC ensemble with $R_{C,term} = 0.481024$



$\rightarrow$ **Rate loss decreases by over 50%**



- $(d_v = 3,\ d_c = 6,\ L = 50,\ W = 3)$ code with $n = 2000$ bits per underlying block ($\rightarrow$ total block length of $N = 100\,000$ bits!)
- left $\left(\frac{E_S}{N_0}\right)$: similar behavior for random shortening and terminated SC-LDPC
- right $\left(\frac{E_b}{N_0}\right)$: additional net coding gain of $\sim 0.1\,dB$



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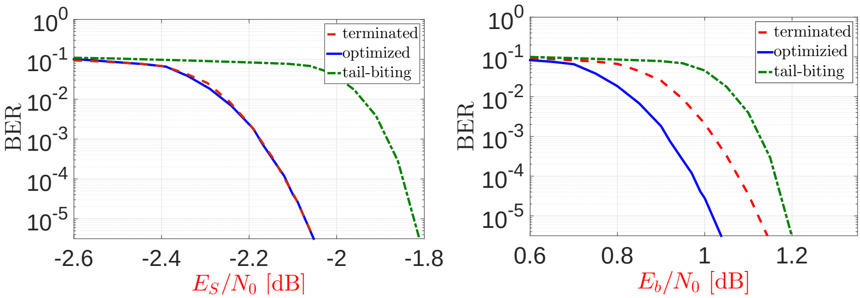

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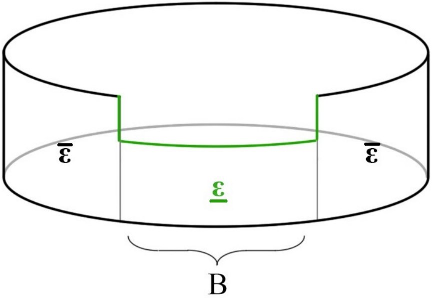
Motivation for a two channel setup

- Random Shortening results in a locally decreased error probability
- Idea: Replace random shortening by bits from the more reliable channel
- Avoid the rate loss of terminated SC-LDPC codes



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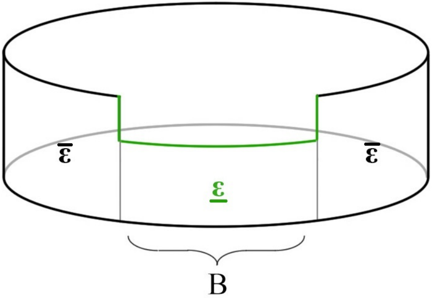

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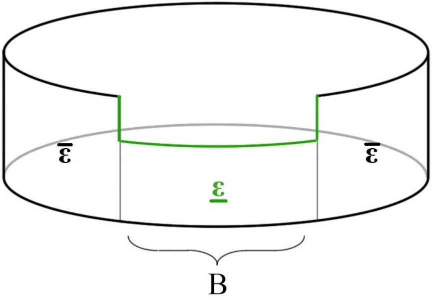

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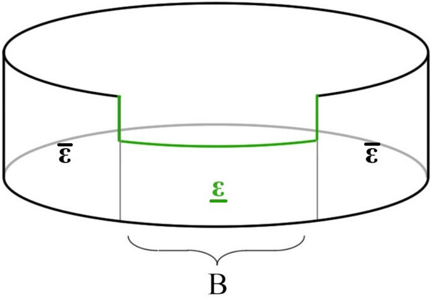

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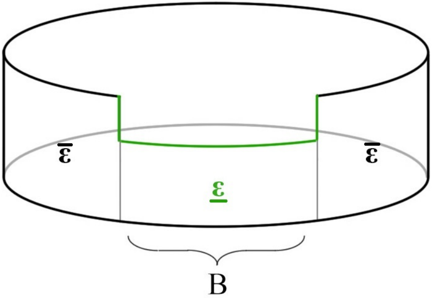
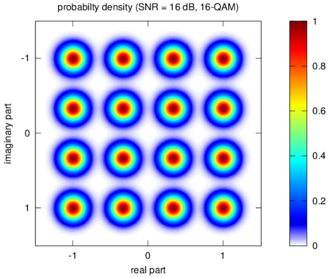
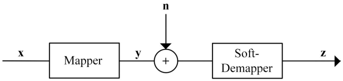


- Different equivalent channels for higher order modulations (e.g. 4-ASK, 16-QAM,...) or OFDM
- BICM: separate the coding from the modulation by an interleaver





[1] Michael Bernhard, "QAM," webdemo, Institute of Telecommunications, University of Stuttgart, Germany, Jul. 2015. Available: http://webdemo.inue.uni-stuttgart.de

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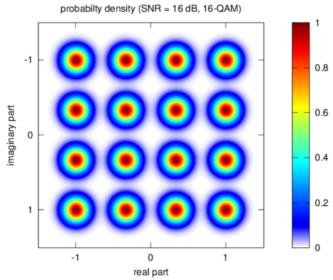

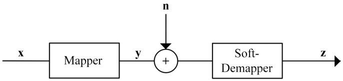

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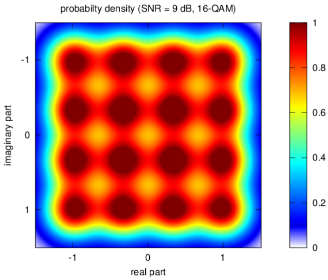
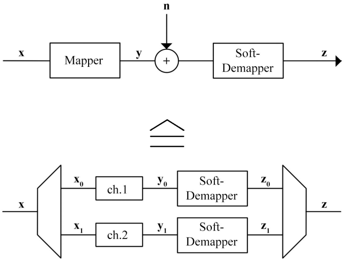
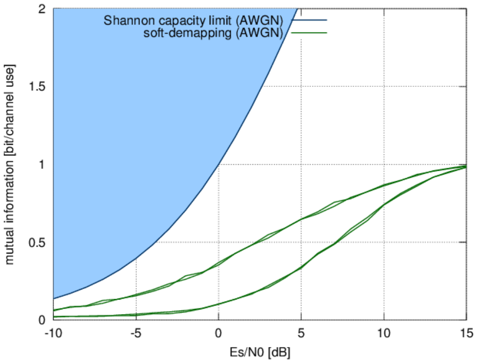




$\Longrightarrow$ two bit channels for low SNR (for 16-QAM)

[1] Michael Bernhard, "QAM," webdemo, Institute of Telecommunications, University of Stuttgart, Germany, Jul. 2015. Available: http://webdemo.inue.uni-stuttgart.de

Mutual Information for 16-QAM [1]

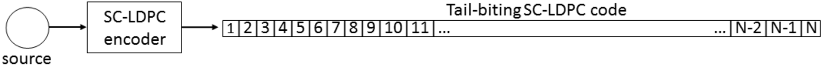


- different mutual information per bit (for small SNR)

$\Longrightarrow$ each bit per symbol experiences a different SNR

[1] Klaus Oestreich, "Physical Layer Performance Measures," webdemo, Institute of Telecommunications, University of Stuttgart, Germany, Jul. 2015. [Online] Available: http://webdemo.inue.uni-stuttgart.de

Reallocate the Channel Uses



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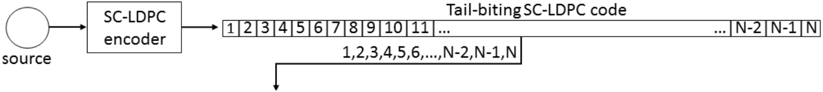

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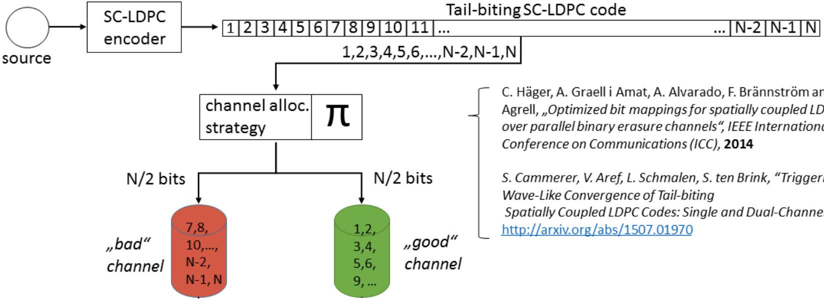

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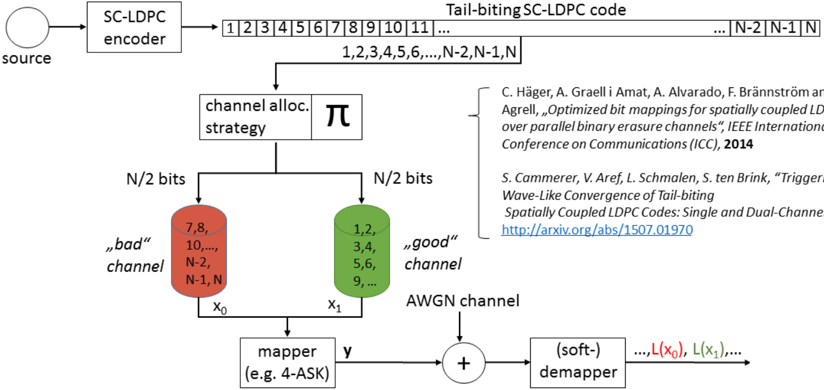

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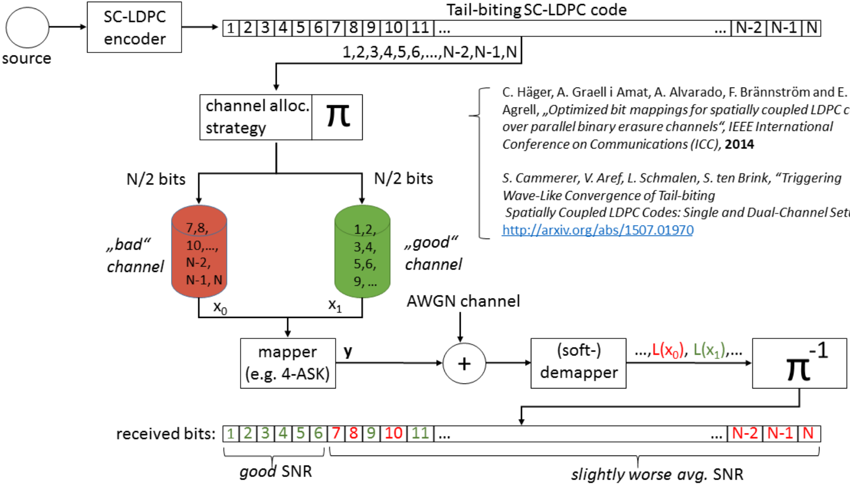

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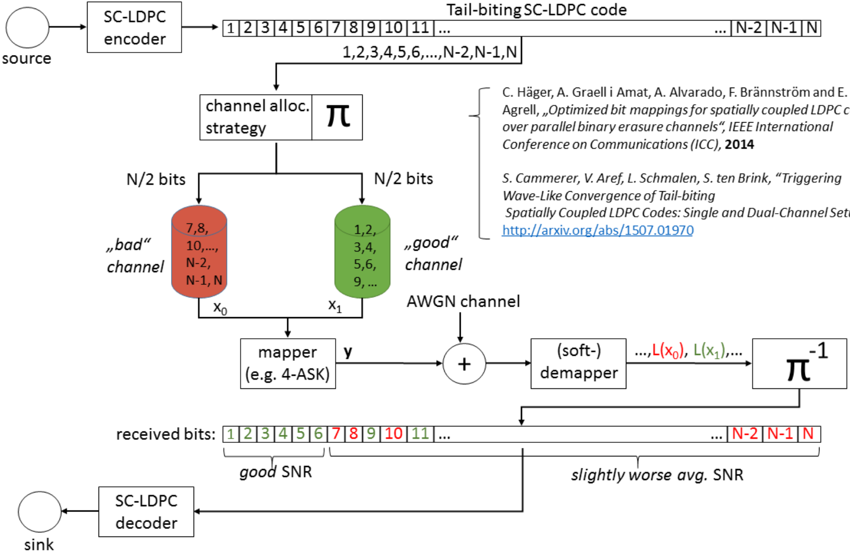

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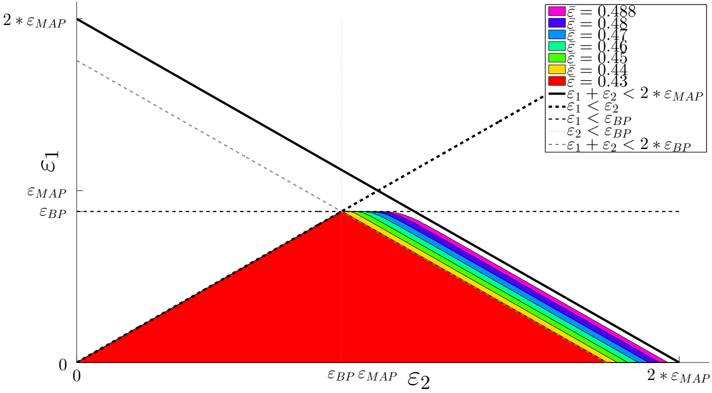
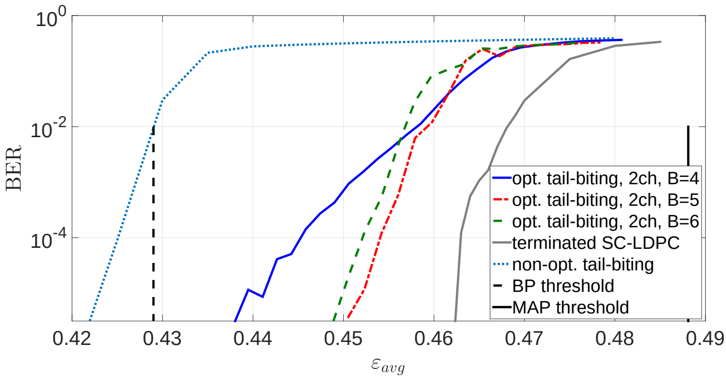
Reachable Areas



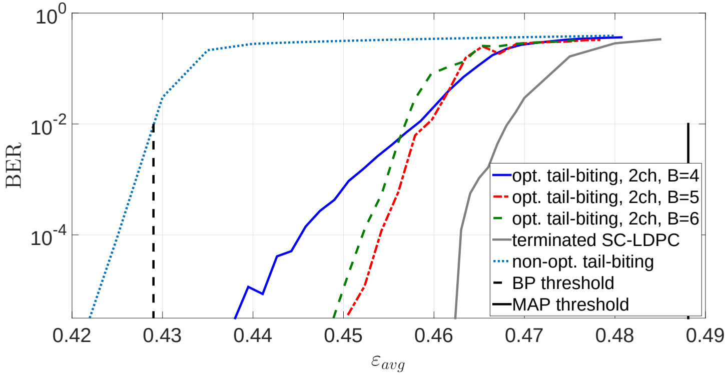
→ We can reach almost all possible points (except small “white” area) without rate loss



- Reliable transmission above the BP-threshold (proof-of-concept, channels modeled as two BECs)
- still under research for the AWGN channel (see next workshop)

Simulations based on the ($d_v = 3$, $d_c = 6$, $L = 50$, $W = 3$, $n_{underlying\ block} = 2000$) SC-LDPC Code

Simulation Results for a Two-Channel BEC Setup



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Simulations based on the ($d_v = 3$, $d_c = 6$, $L = 50$, $W = 3$, $n_{underlying\ block} = 2000$) SC-LDPC Code

Comparing the Different Strategies

strategy	Terminated SC-LDPC	Random Shortening	Two Channel Setup
BEC: ε (@10^{-5})	**0.462**	**0.462**	0.4521
AWGN: Es/N0 (@10^{-5})	**-2.056 dB**	**-2.045 dB**	to be done
AWGN: Eb/N0 (@10^{-5})	1.122 dB	**1.021 dB**	to be done
achievable code rate R_C	0.481	0.491	**0.500**
requires two bit channels	**no**	**no**	yes

results based on the ($d_v = 3$, $d_c = 6$, $L = 50$, $W = 3$, $n_{underlying\ block} = 2000$) SC-LDPC Code

Results

- The Performance of tail-biting SC LDPC codes can be improved by random shortening the code.
- Higher-order modulation results in parallel equivalent channels.
- Combining SC-LDPC Codes with a two channel setup can help to avoid the rate loss.
- Outlook
  - Optimize for the AWGN channel
  - Consider finite length code in optimization
  - see http://arxiv.org/abs/1507.01970 for details

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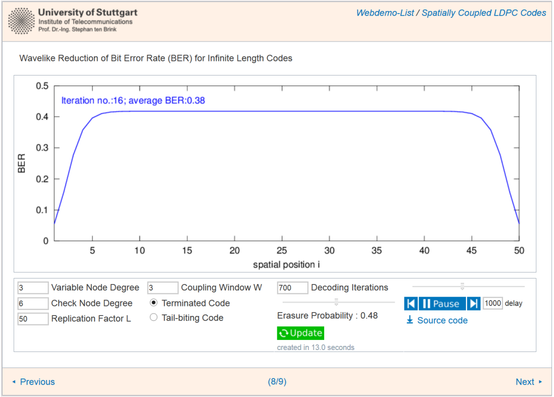
Thank you!

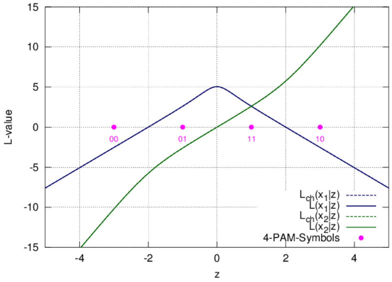
and a little bit of advertisment

http://www.inue.uni-stuttgart.de/lehre/demo.html

→ SC-LDPC Codes



APPENDIX - Example: LLRs for 4-ASK



$$L\left(x_0 \mid z\right) = \ln \frac{\exp\left(-\frac{(z-1)^2}{2\sigma_n^2}\right) + \exp\left(-\frac{(z+1)^2}{2\sigma_n^2}\right)}{\exp\left(-\frac{(z-3)^2}{2\sigma_n^2}\right) + \exp\left(-\frac{(z+3)^2}{2\sigma_n^2}\right)}$$

$$L\left(x_1 \mid z\right) = \ln \frac{\exp\left(-\frac{(z-1)^2}{2\sigma_n^2}\right) + \exp\left(-\frac{(z-3)^2}{2\sigma_n^2}\right)}{\exp\left(-\frac{(z+1)^2}{2\sigma_n^2}\right) + \exp\left(-\frac{(z+3)^2}{2\sigma_n^2}\right)}$$