# Coding and Modulation

## A Polar Coding Viewpoint

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# Outline

- Background
  - AWGN channel
  - Coding and modulation
- Polar coding and modulation
  - Direct polarization approach
  - Multi-level modulation and polar coding
  - Lattices and polar codes
  - BICM and polar coding

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# The AWGN Channel

The AWGN channel is a continuous-time channel

$$Y(t) = X(t) + N(t)$$

such that the input $X(t)$ is a random process bandlimited to $W$ subject to a power constraint $\overline{X^2(t)} \leq P$, and $N(t)$ is white Gaussian noise with power spectral density $N_0/2$.

# Capacity

Shannon's formula gives the capacity of the AWGN channel as

$$C_{[b/s]} = W \log_2(1 + P/WN_0) \quad (bits/s)$$

# Signal Design Problem

The continuous time and real-number interface of the AWGN channel is inconvenient for digital communications.

- Need to convert from continuous to discrete-time
- Need to convert from real numbers to a binary interface

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# Discrete Time Model

An AWGN channel of bandwidth $W$ gives rise to $2W$ independent discrete time channels per second with input-output mapping

$$Y = X + N$$

- $X$ is a random variable with mean 0 and energy $E[X^2] \leq P/2W$
- $N$ is Gaussian noise with 0-mean and energy $N_0/2$.
- It is customary to normalize the signal energies to joules per 2 dimensions and define

$$E_s = P/W \quad \text{Joules/2D}$$

  as signal energy (per two dimensions).
- One defines the the signal-to-noise ratio as $E_s/N_0$.

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# Capacity

The capacity of the discrete-time AWGN channel is given by

$$C = \frac{1}{2}\log_2(1 + E_s/N_0), \quad (bits/D),$$

achieved by i.i.d. Gaussian inputs $X \sim N(0, E_s/2)$ per dimension.

# Signal Design Problem

Now, we need a digital interface instead of real-valued inputs.

- Select a subset $\mathcal{A} \subset \mathcal{R}^n$ as the "signal set" or "modulation alphabet".
- Finding a signal set with good Euclidean distance properties and other desirable features is the "signal design" problem.
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# Separation of coding and modulation

- Each constellation $\mathcal{A}$ has a capacity $C_{\mathcal{A}}$ (bits/D) which is a function of $E_s/N_0$.
- The spectral efficiency $\rho$ (bits/D) has to satisfy

$$\rho < C_{\mathcal{A}}(E_s/N_0)$$

  at the operating $E_s/N_0$.
- The spectral efficiency is the product of two terms

$$\rho = R \times \frac{\log_2(|\mathcal{A}|)}{dim(\mathcal{A})}$$

  where $R$ (dimensionless) is the rate of the FEC.
- For a given $\rho$, there any many choices w.r.t. $R$ and $\mathcal{A}$.

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# Cutoff rate: A simple measure of reliability

Each constellation $\mathcal{A}$ has a cutoff rate $R_{0,\mathcal{A}}$ (bits/D) which is a function of $E_s/N_0$ such that through random coding one can guarantee the existence of coding and modulation schemes with probability of frame error

$$P_e < 2^{-N[R_{0,\mathcal{A}}(E_s/N_0)-\rho]}$$

where $N$ is the frame length in modulation symbols.

# Sequential decoding and cutoff rate

- Sequential decoding (Wozencraft, 1957) is a decoding algorithm for convolutional codes that can achieve spectral efficiencies as high as the cutoff rate at constant average complexity per decoded bit.
- The difference between cutoff rate and capacity at high $E_s/N_0$ is less than 3 dB.
- This was regarded as the solution of the coding and modulation problem in early 70s and interest in the problem waned. (See Forney 1995 Shannon Lecture for this story.)
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# $M$-ary Pulse Amplitude Modulation

A 1-D signal set with $\mathcal{A} = \{\pm\alpha, \pm 3\alpha, \ldots, \pm(M-1)\}$.

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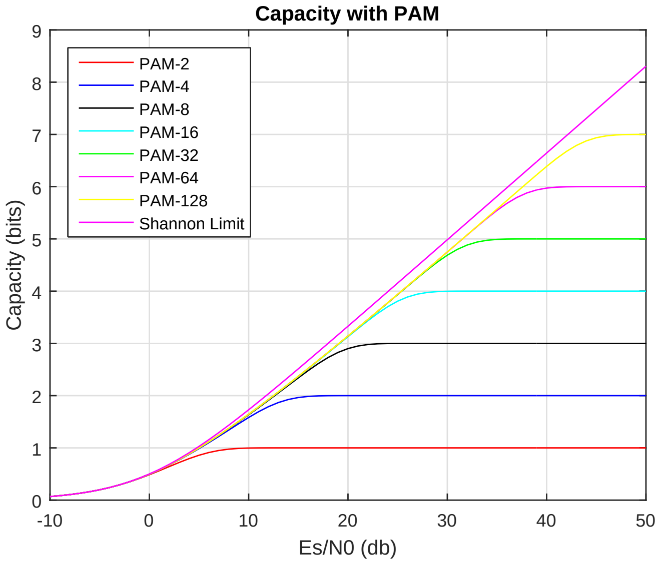
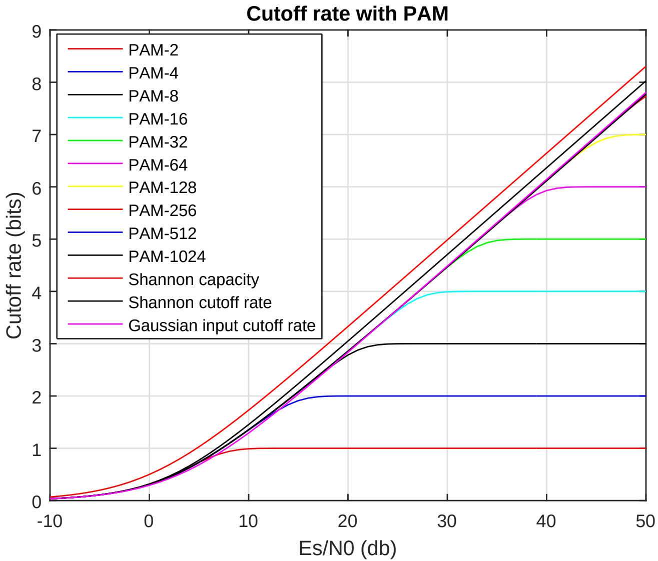
# Capacity of *M*-PAM



# Cutoff rate of $M$-PAM



# Conventional approach

Given a target spectral efficiency $\rho$ and a target error rate $P_e$ at a specific $E_s/N_o$,

- select $M$ large enough so that $M$-PAM capacity is close enough to the Shannon capacity at the given $E_s/N_o$
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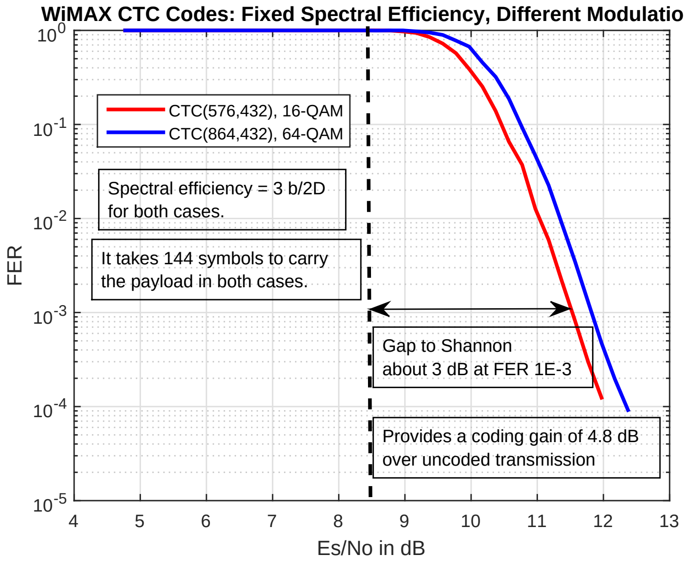
However, with the advent of powerful codes at affordable complexity, there is a return to the conventional design methodology.

# How does it work in practice?



# Why change modulation instead of just the code rate?

- Suppose we fix the modulation as 64-QAM and wish to deliver data at spectral efficiencies 1, 2, 3, 4, 5 b/2D.
- We would need a coding scheme that works well at rates 1/6, 1/3, 1/2, 2/3, 5/6.
- The inability of delivering high quality coding over a wide range of rates forces one to change the order of modulation.
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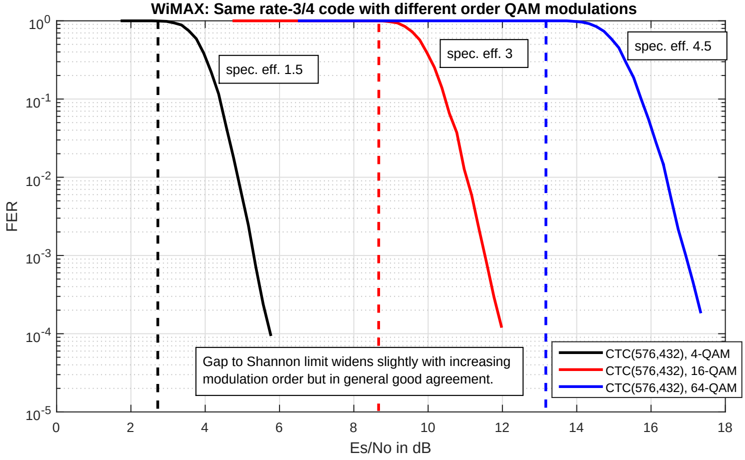
# Alternative: Fixed code, variable modulation



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# Polar coding and modulation

Polar codes can be applied to modulation in at least three different ways.

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- Multi-level techniques
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# Direct Method

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Theory of $q$-ary polarization exists.

- Şasoğlu, E., E. Telatar, and E. Arıkan. "Polarization for arbitrary discrete memoryless channels." IEEE ITW 2009.
- Sahebi, A. G. and S. S. Pradhan, "Multilevel polarization of polar codes over arbitrary discrete memoryless channels." IEEE Allerton, 2011.
- Park, W.-C. and A. Barg. "Polar codes for q-ary channels," IEEE Trans. Inform. Theory, 2013.
- ...

# Direct Method

The difficulty with the direct approach is complexity of decoding.

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G. Montorsi's ADBP is a promising approach for reducing the complexity here.

# Multi-Level Modulation (Imai and Hirakawa, 1977)

Represent (if possible) each channel input symbol as a vector $X = (X_1, X_2, \ldots, X_r)$; then the capacity can be written as a sum of capacities of smaller channels by the chain rule:

$$
\begin{aligned}
I(X;Y) &= I(X_1, X_2, \ldots, X_r; Y) \\
&= \sum_{i=1}^{r} I(X_i; Y | X_1, \ldots, X_{i-1}).
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Polarization is a natural complement to MLM.

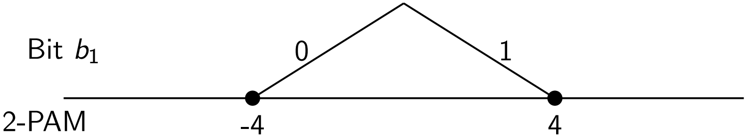
# Polar coding with multi-level modulation

Already a well-studied subject:

- Arıkan, E., "Polar Coding," Plenary Talk, ISIT 2011.
- Seidl, M., Schenk, A., Stierstorfer, C., and Huber, J. B. "Polar-coded modulation," IEEE Trans. Comm. 2013.
- Seidl, M., Schenk, A., Stierstorfer, C., and Huber, J. B. "Multilevel polar-coded modulation‘," IEEE ISIT 2013
- Ionita, Corina, et al. "On the design of binary polar codes for high-order modulation." IEEE GLOBECOM, 2014.
- Beygi, L., Agrell, E., Kahn, J. M., and Karlsson, M., "Coded modulation for fiber-optic networks," IEEE Sig. Proc. Mag., 2014.
- ...

# Example: 8-PAM as 3 bit channels

- PAM signals selected by three bits $(b_1, b_2, b_3)$
- Three layers of binary channels created
- Each layer encoded independently
- Layers decoded in the order $b_3$, $b_2$, $b_1$



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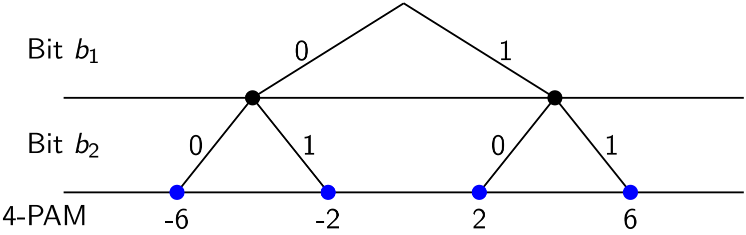

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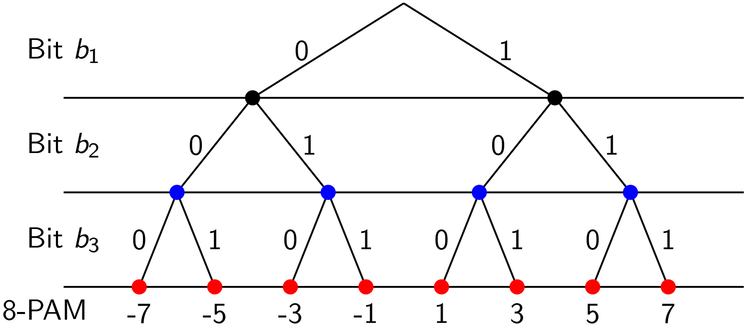

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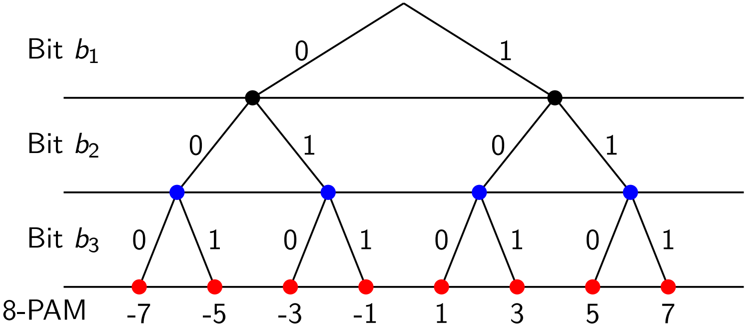

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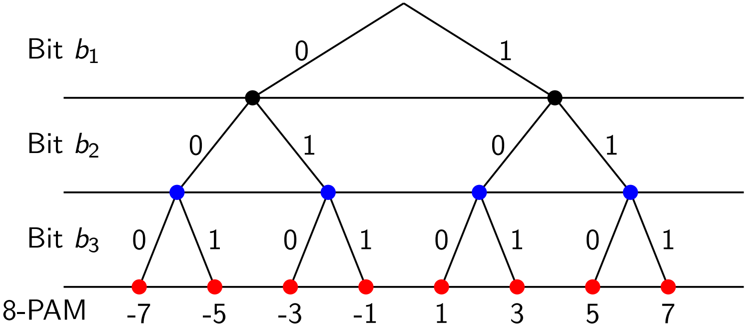

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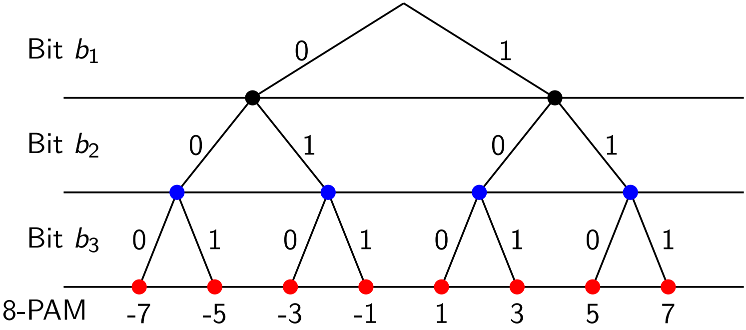
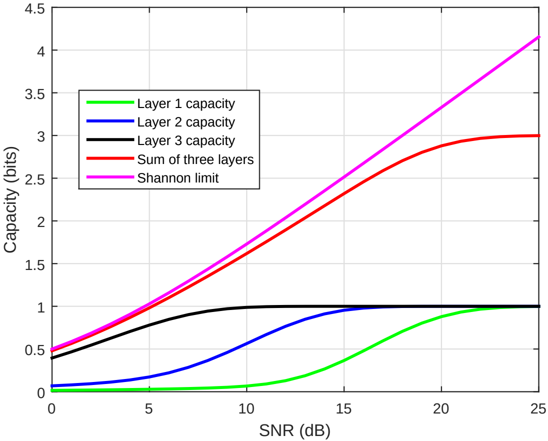


# Polarization across layers by natural labeling




Most coding work needs to be done at the least significant bits.

# Lattices and polar coding

Yan, Cong, and Liu explored the connection between lattices and polar coding.

- Yan, Yanfei, and L. Cong, "A construction of lattices from polar codes." IEEE 2012 ITW.
- Yan, Yanfei, Ling Liu, Cong Ling, and Xiaofu Wu. "Construction of capacity-achieving lattice codes: Polar lattices." arXiv preprint arXiv:1411.0187 (2014)

# Lattices and polar coding

Yan *et al* used the Barnes-Wall lattice contructions such as

$$\mathsf{BW}_{16} = \mathsf{RM}(1,4) + 2\mathsf{RM}(3,4) + 4(\mathbb{Z}^{16})$$

as a template for constructing polar lattices of the type

$$\mathsf{P}_{16} = \mathsf{P}(1,4) + 2\mathsf{P}(3,4) + 4(\mathbb{Z}^{16})$$

and demonstrated by simulations that polar lattices perform better.

# BICM

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As in MLM, BICM splits the channel input symbols into a vector $X = (X_1, X_2, \ldots, X_r)$ but strives to do so such that

$$\begin{aligned} I(X;Y) &= I(X_1, X_2, \ldots, X_r; Y) \\ &= \sum_{i=1}^{r} I(X_i; Y | X_1, \ldots, X_{i-1}) \\ &\approx \sum_{i=1}^{r} I(X_i; Y). \end{aligned}$$

# BICM vs Multi Level Modulation

Why has BICM won over MLM and other techniques in practice?

- MLM is provably capacity-achieving; BICM is suboptimal but the rate penalty is tolerable.
- MLM has to do delicate rate-matching at individual layers, which is difficult with turbo and LDPC codes.
- BICM is well-matched to iterative decoding methods used with turbo and LDPC codes.
- MLM suffers extra latency due to multi-stage decoding (mitigated in part by the lack of need for protecting the upper layers by long codes)
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# BICM and Polar Coding

This subject, too, has been studied in connection with polar codes.

- Mahdavifar, H. and El-Khamy, M. and Lee, J. and Kang, I., "Polar Coding for Bit-Interleaved Coded Modulation," IEEE Trans. Veh. Tech., 2015.
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# Thank you!

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