Grassmannian Product Codebooks for Limited Feedback FD-MIMO

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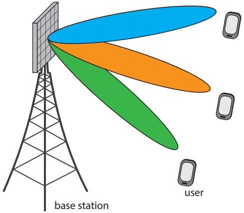
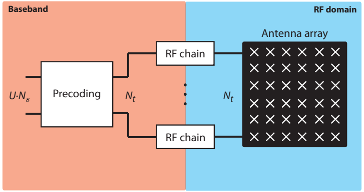
Full-Dimension MIMO Systems





- Utilize two-dimensional active antenna arrays to enable 2D-beamforming (azimuth, elevation)
- Allows to improve spectral efficiency, energy efficiency and network capacity
- We consider downlink linear multi-user MIMO transmission

$$\mathbf{y}_u = \mathbf{H}_u^{\mathrm{H}} \mathbf{F}_u \mathbf{x}_u + \mathbf{H}_u^{\mathrm{H}} \sum_{j=1, j \neq u}^{U} \mathbf{F}_j \mathbf{x}_j + \mathbf{z}_u$$

- $\mathbf{H}_u \in \mathbb{C}^{N_t \times N_r}$, $\mathbf{F}_u \in \mathbb{C}^{N_t \times N_s}$: N_t, N_r transmit and receive antennas, N_s streams per user

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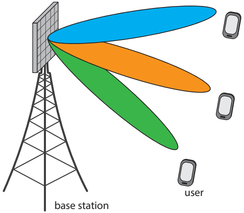

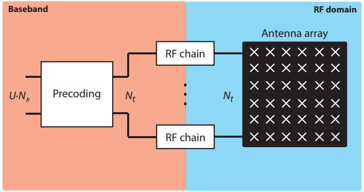

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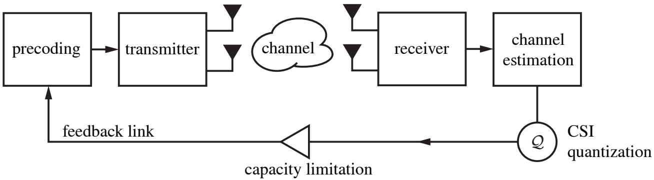
Limited Feedback Operation in FDD Systems



- Linear precoding requires channel state information (CSI) at the transmitter (CSIT)
- Time division duplex (TDD): uplink channel estimation can be employed
- Frequency division duplex (FDD): explicit limited feedback of CSI from the users

  $\Rightarrow$ Downlink channel estimation and CSI quantization/feedback required

  $\Rightarrow$ Downlink pilot overhead proportional to N_t

  $\Rightarrow$ Uplink CSI overhead proportional to $N_t \cdot N_r$

- Hardly feasible for large N_t

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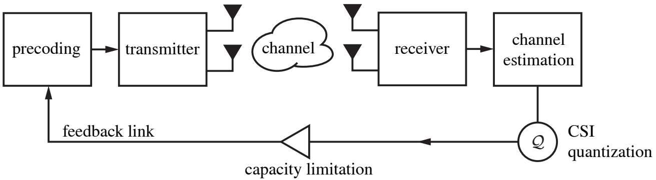

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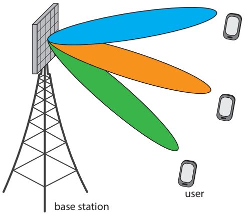
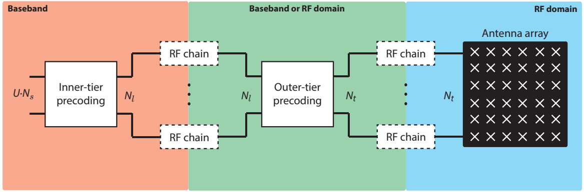
Two-Tier Precoding





- Split-up precoding into two-tiers:

$$\mathbf{F}_u = \mathbf{F}_o \mathbf{F}_{i,u}$$

1. User-group-specific outer precoding $\mathbf{F}_o \in \mathbb{C}^{N_t \times N_\ell}$
2. User-specific inner precoding $\mathbf{F}_{i,u} \in \mathbb{C}^{N_\ell \times N_s}$

- $\mathbf{F}_o$ often considered as wideband precoder (adapted based on channel statistics)
- $\mathbf{F}_{i,u}$ often considered as subband precoder (adapted based on instantaneous CSI)

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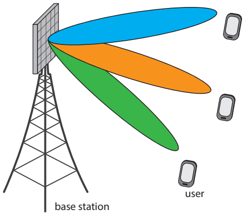

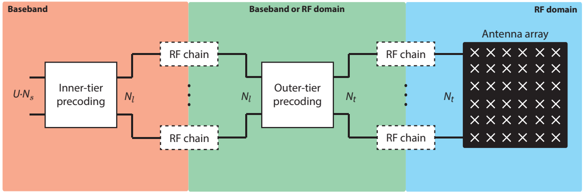

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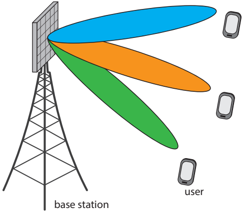
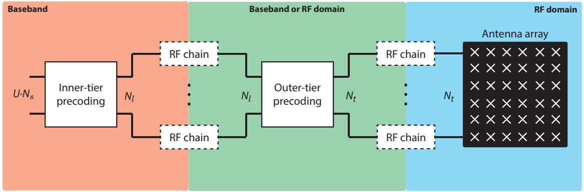
Two-Tier Precoding – Advantages/Disadvantages





- Reduced downlink pilot overhead and CSI feedback overhead
  - Infrequent estimation and feedback of the full channel matrix $\mathbf{H}_u$
  - Per TTI estimation and feedback of $\mathbf{H}_{o,u} = \mathbf{F}_o^{\mathrm{H}}\mathbf{H}_u \in \mathbb{C}^{N_\ell \times N_r}$, $N_\ell \ll N_t$
- Reduced number of RF chains if $\mathbf{F}_o$ is implemented in the RF domain (hybrid precoding)
- Spatial multiplexing capabilities restricted to N_ℓ
- Suboptimal structure imposed on precoders

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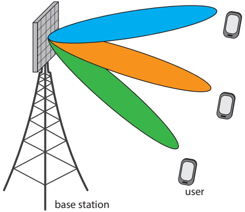

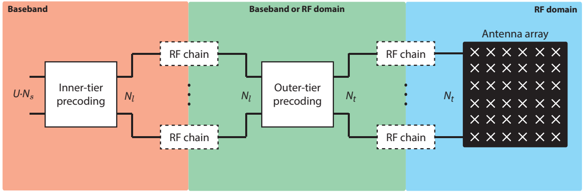

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Considered Two-Tier Precoding Strategy

- Outer-tier maximum eigenmode transmission (MET)
  - Maximum eigenmodes of the channel:

$$\mathbf{H}_u \mathbf{H}_u^{\mathrm{H}} = \mathbf{U}_u \boldsymbol{\Sigma}_u \mathbf{U}_u^{\mathrm{H}},$$
$$\mathbf{U}_u^{\mathrm{H}} \mathbf{U}_u = \mathbf{I}_{N_r}, \quad \boldsymbol{\Sigma}_u = \mathrm{Diag}\left(\sigma_u^{(1)}, \ldots, \sigma_u^{(N_r)}\right)$$

  - Multi-user MET precoder:

$$\mathbf{F}_o = \left[\mathbf{U}_1^{(N_s)}, \mathbf{U}_2^{(N_s)}, \ldots, \mathbf{U}_U^{(N_s)}\right],$$
$$\mathbf{U}_j^{(N_s)} = \left[\mathbf{u}_j^{(1)}, \ldots, \mathbf{u}_j^{(N_s)}\right]$$

- Inner-tier block-diagonalization (BD) precoding

$$\left(\mathbf{U}_{o,j}^{(N_s)}\right)^{\mathrm{H}} \mathbf{F}_{i,u} \stackrel{!}{=} \mathbf{0}, \ \forall j \neq u,$$
$$\mathrm{rank}\left(\left(\mathbf{U}_{o,j}^{(N_s)}\right)^{\mathrm{H}} \mathbf{F}_{i,u}\right) = N_s$$

- $\mathbf{U}_{o,j}^{(N_s)}$ obtained from an SVD of $\mathbf{H}_{o,j} \in \mathbb{C}^{N_\ell \times N_r}$

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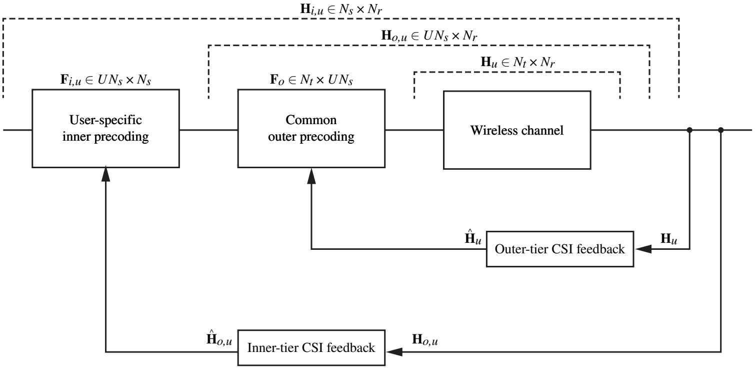
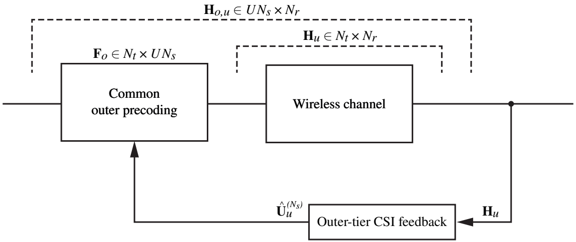


- Many efficient CSI feedback methods are available for small-scale MIMO systems
- These can be employed for the quantization and feedback of $\mathbf{H}_{o,u}$

  $\Rightarrow$ We focus on computationally and rate-distortion efficient outer-tier CSI feedback of $\mathbf{H}_u$

Outer-Tier CSI Feedback



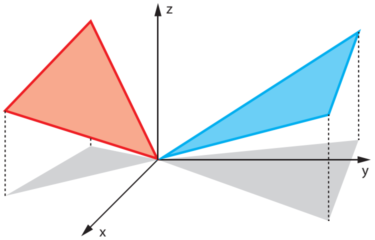
- What information does the transmitter need about $\mathbf{H}_u$ to enable MET precoding?

$$\mathbf{F}_o = \left[\mathbf{U}_1^{(N_s)}, \ldots, \mathbf{U}_U^{(N_s)}\right] \equiv \left[\mathbf{U}_1^{(N_s)}\mathbf{Q}_1, \ldots, \mathbf{U}_U^{(N_s)}\mathbf{Q}_U\right],$$
$$\mathbf{Q}_i^{\mathrm{H}}\mathbf{Q}_i = \mathbf{Q}_i\mathbf{Q}_i^{\mathrm{H}} = \mathbf{I}_{N_s}$$

$\Rightarrow$ Subspace information is sufficient, since we do not consider power-loading over the eigenmodes

$\Rightarrow$ We apply Grassmannian quantization to $\mathrm{span}\left(\mathbf{U}_1^{(N_s)}\right)$ on the Grassmann manifold $\mathcal{G}_{N_s}^{(N_t)}$

Grassmann Manifold Basics



- $\mathcal{G}_{N_s}^{(N_t)}$ manifold of N_s-dimensional subspaces of the N_t-dimensional real/complex Euclidean space
- Chordal distance between two subspaces defined by their orthogonal bases $\mathbf{U}_1, \mathbf{U}_2 \in \mathbb{C}^{N_t \times N_s}$

$$d_{\mathcal{G}}^2(\mathbf{U}_1, \mathbf{U}_2) = N_s - \mathrm{tr}\left(\mathbf{U}_1^{\mathrm{H}}\mathbf{U}_2\mathbf{U}_2^{\mathrm{H}}\mathbf{U}_1\right) = \sum_{i=1}^{N_s} \sin\varphi_i^2$$

- φ_i are the principal angles between the subspaces

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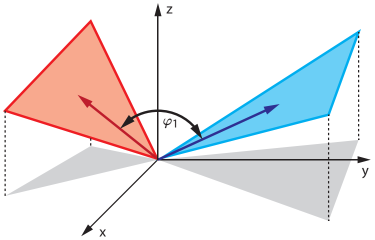

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Single-Stage Outer-Tier CSI Feedback

- The SNR-loss of MET-BD precoding with imperfect CSIT w.r.t. perfect CSIT is determined by the chordal distance CSI error $\Rightarrow$ minimum chordal distance quantization:

$$\hat{\mathbf{U}}_{j,\text{single}}^{(N_s)} = \underset{\mathbf{W}_\ell \in \mathcal{Q}_{N_s}^{(N_t)}}{\arg\min}\ \mathrm{d}_{\mathcal{G}}^2\left(\mathbf{U}_j^{(N_s)}, \mathbf{W}_\ell\right)$$

- Quantization codebook $\mathcal{Q}_{N_s}^{(N_t)} = \left\{\mathbf{W}_\ell \in \mathbb{C}^{N_t \times N_s} | \mathbf{W}_\ell^{\mathrm{H}} \mathbf{W}_\ell = \mathbf{I}_{N_s}, \forall \ell\right\}$ of size 2^b

- Rayleigh fading channels: random subspace quantization is asymptotically efficient

$$\bar{\mathrm{d}}_{c,\text{single}}^2 = \mathbb{E}\left(\mathrm{d}_{\mathcal{G}}^2\left(\mathbf{U}_j^{(N_s)}, \hat{\mathbf{U}}_{j,\text{single}}^{(N_s)}\right)\right) \propto 2^{-b/(N_s(N_t - N_s))}$$

- Problem: huge codebooks are required for large N_t $\rightarrow$ computationally infeasible

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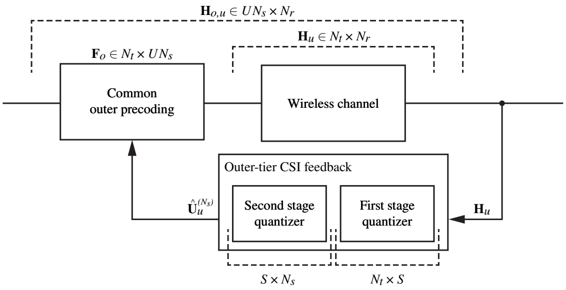

- Employ a product-codebook construction to reduce the quantization complexity

$$\hat{\mathbf{U}}_1 \in \mathcal{Q}_{1,S}^{(N_t)} \subset \mathbb{C}^{N_t \times S}, \qquad \hat{\mathbf{U}}_2 \in \mathcal{Q}_{2,N_s}^{(S)} \subset \mathbb{C}^{S \times N_s}$$

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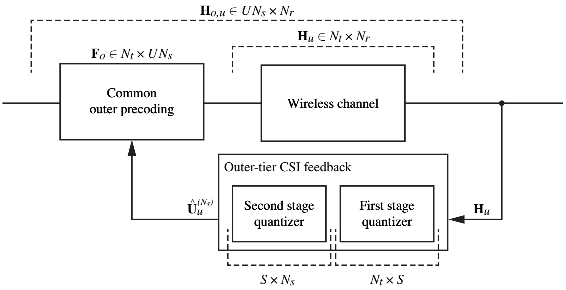

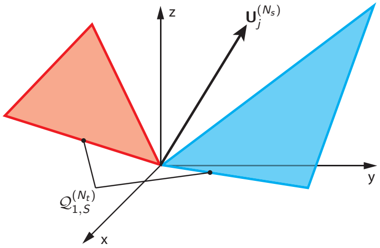

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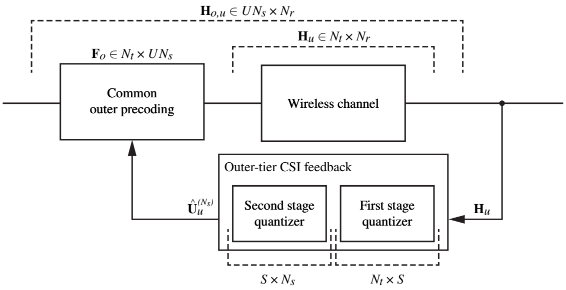

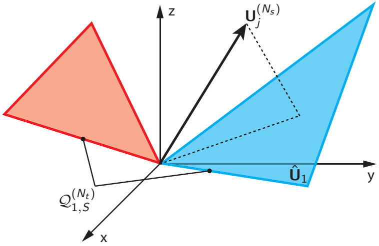

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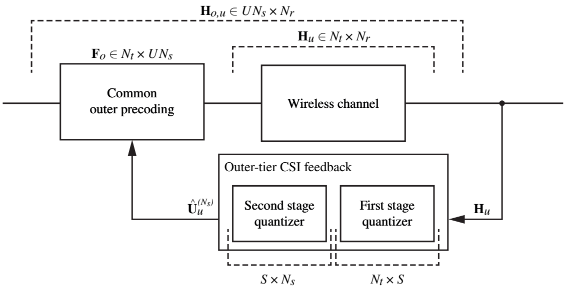

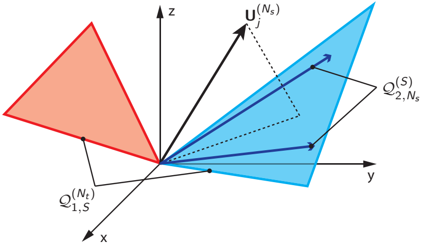

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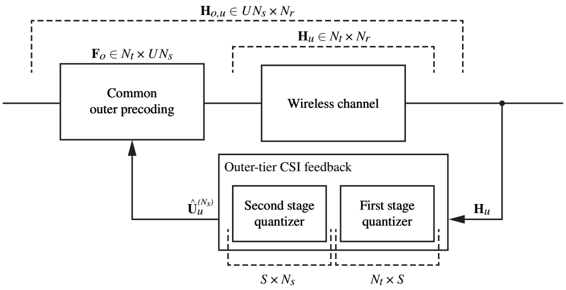

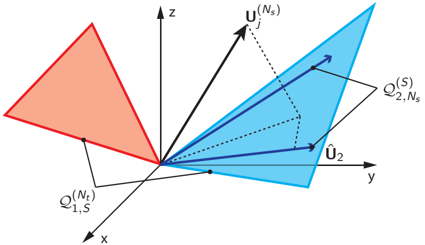

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Multi-Stage Outer-Tier CSI Feedback (II)

- First-stage quantizer:

$$\hat{\mathbf{U}}_1 = \underset{\mathbf{W}_\ell \in \mathcal{Q}_{1,S}^{(N_t)}}{\arg\min}\; \mathrm{d}_{\mathcal{G}}^2\left(\mathbf{U}_j^{(N_s)}, \mathbf{W}_\ell\right) \in \mathbb{C}^{N_t \times S}$$

- Subspace-quantization based combining

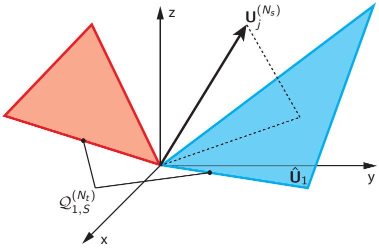
$$\mathbf{B} = \hat{\mathbf{U}}_1^{\mathrm{H}} \mathbf{U}_j^{(N_s)} \left(\left(\mathbf{U}_j^{(N_s)}\right)^{\mathrm{H}} \hat{\mathbf{U}}_1 \hat{\mathbf{U}}_1^{\mathrm{H}} \mathbf{U}_j^{(N_s)} \right)^{-\frac{1}{2}} \in \mathbb{C}^{S \times N_s},$$

$$\mathbf{B}^{\mathrm{H}}\mathbf{B} = \mathbf{I}_{N_s}, \quad \mathrm{d}_{\mathcal{G}}^2\left(\hat{\mathbf{U}}_1, \mathbf{U}_j^{(N_s)}\right) = \mathrm{d}_{\mathcal{G}}^2\left(\hat{\mathbf{U}}_1\mathbf{B}, \mathbf{U}_j^{(N_s)}\right)$$

- Second-stage quantizer:

$$\hat{\mathbf{U}}_2 = \underset{\mathbf{W}_\ell \in \mathcal{Q}_{2,N_s}^{(S)}}{\arg\min}\; \mathrm{d}_{\mathcal{G}}^2\left(\mathbf{B}, \mathbf{W}_\ell\right)$$

- Total quantized CSI: $\hat{\mathbf{U}}_{j,\mathrm{dual}}^{(N_s)} = \hat{\mathbf{U}}_1 \hat{\mathbf{U}}_2 \in \mathbb{C}^{N_t \times N_s}$



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$$\hat{\mathbf{U}}_1 = \underset{\mathbf{W}_\ell \in \mathcal{Q}_{1,S}^{(N_t)}}{\arg\min}\ \mathrm{d}_{\mathcal{G}}^2\left(\mathbf{U}_j^{(N_s)}, \mathbf{W}_\ell\right) \in \mathbb{C}^{N_t \times S}$$

- Subspace-quantization based combining

$$\mathbf{B} = \hat{\mathbf{U}}_1^{\mathrm{H}} \mathbf{U}_j^{(N_s)} \left(\left(\mathbf{U}_j^{(N_s)}\right)^{\mathrm{H}} \hat{\mathbf{U}}_1 \hat{\mathbf{U}}_1^{\mathrm{H}} \mathbf{U}_j^{(N_s)}\right)^{-\frac{1}{2}} \in \mathbb{C}^{S \times N_s},$$

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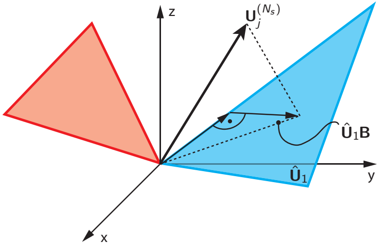
- Second-stage quantizer:

$$\hat{\mathbf{U}}_2 = \underset{\mathbf{W}_\ell \in \mathcal{Q}_{2,N_s}^{(S)}}{\arg\min}\ \mathrm{d}_{\mathcal{G}}^2\left(\mathbf{B}, \mathbf{W}_\ell\right)$$

- Total quantized CSI: $\hat{\mathbf{U}}_{j,\mathrm{dual}}^{(N_s)} = \hat{\mathbf{U}}_1 \hat{\mathbf{U}}_2 \in \mathbb{C}^{N_t \times N_s}$



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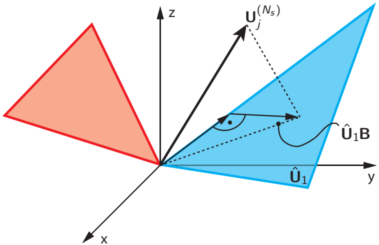

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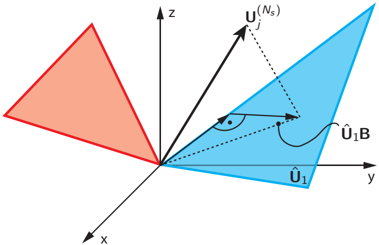

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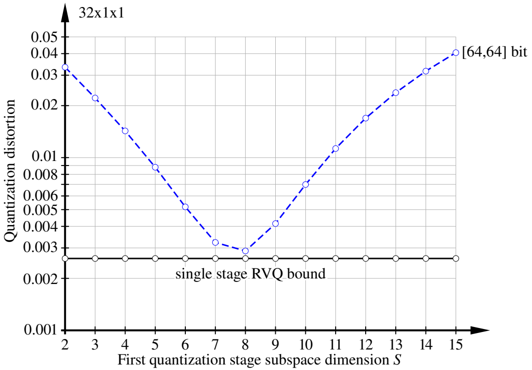
Single-Stage vs. Multi-Stage Outer-Tier CSI Feedback – Rayleigh Fading



- The product codebook quantization error is determined by:

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- Notice $2^{64} + 2^{64} = 2^{65} \ll 2^{128} \rightarrow$ significant complexity reduction

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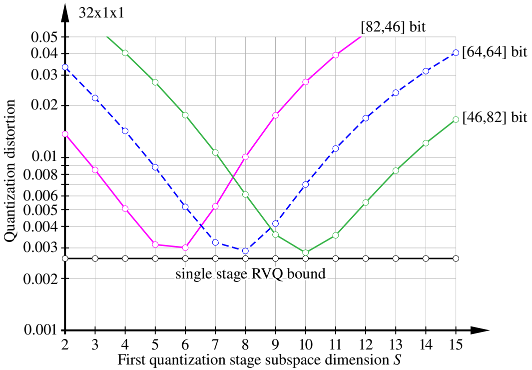

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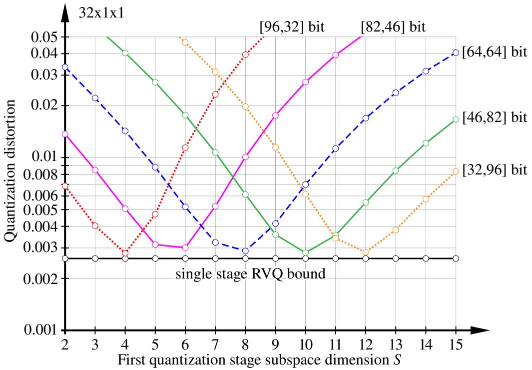

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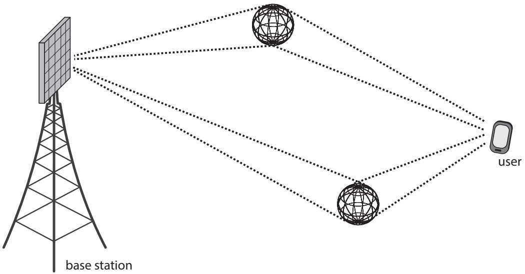
Multi-Scattering Directional Channels



- Multi-scattering directional channel model:

$$\mathbf{H}_u = \sqrt{N_t N_r} \sum_{p=1}^{N_p} \alpha_{p,u} \mathbf{a}_t \left(\phi_{p,u}\right) \mathbf{a}_r \left(\boldsymbol{\theta}_{p,u}\right)^{\mathrm{T}}$$

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$$\left[\mathbf{a}_t^{\mathrm{ULA}}(\phi)\right]_k = \frac{g_e(\phi)}{\sqrt{N_t}} \exp\left(j\frac{2\pi d}{\lambda}(k-1)\sin(\phi)\right)$$

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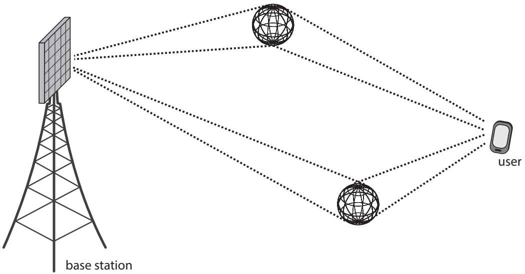

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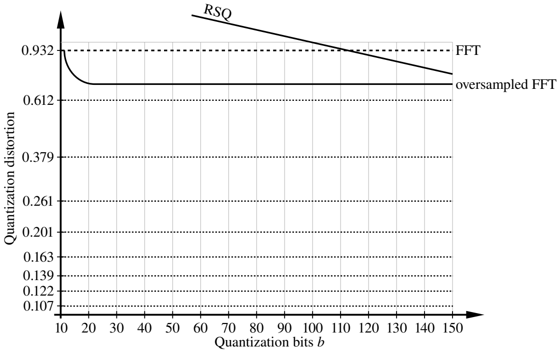

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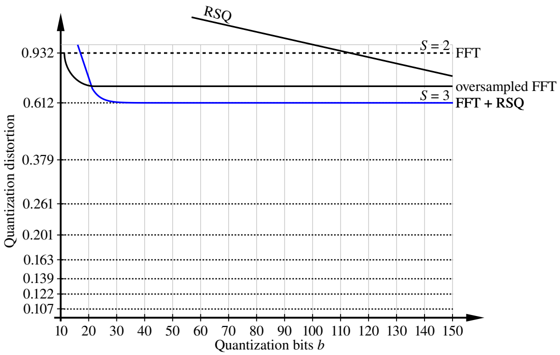

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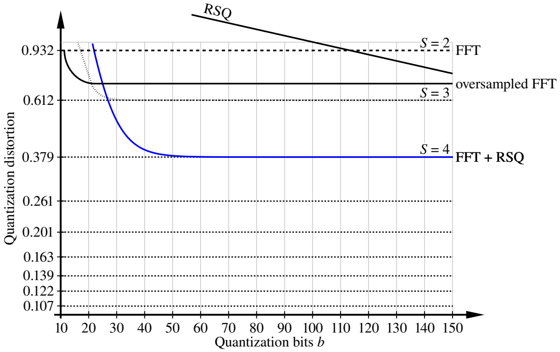

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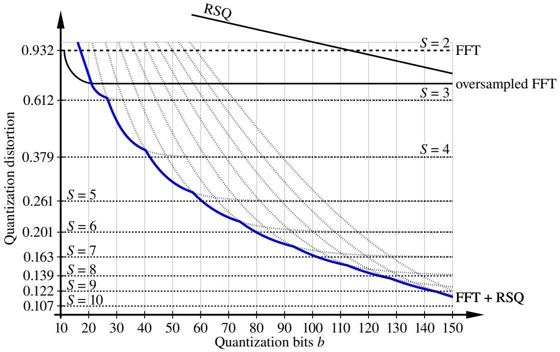

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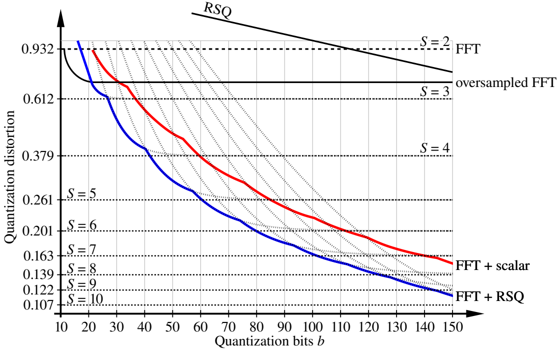

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Throughput Performance

- Measured channel traces of Nokia Bell Labs, Stuttgart
- $N_t = 64$, $N_r = N_s = 2$, $U = 6$ users with velocities between 15 and 25 km/h @2 GHz center frequency

$$\left(\log_2\left(\binom{N_{\mathrm{DFT}}}{S}\right) + S\,N_s\,b_s\right) R_f\,R_t \approx 5.36\,\mathrm{bit/ms/MHz}.$$

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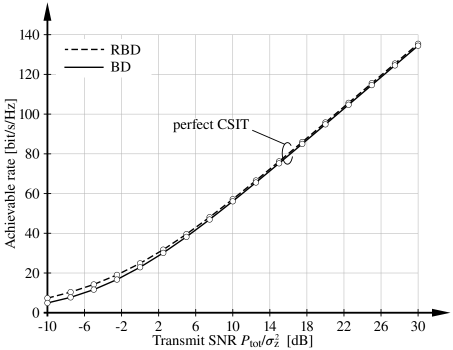

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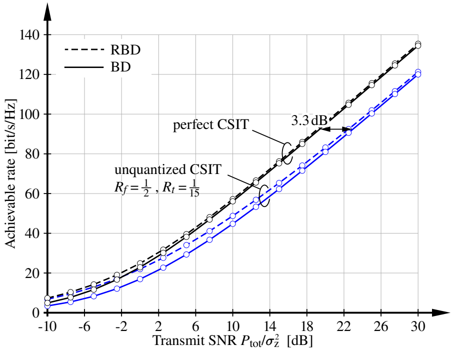

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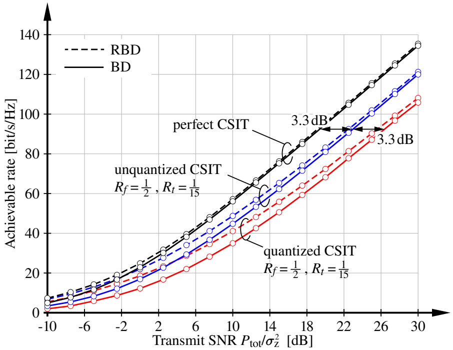

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Contents

Summary and Conclusion

- Grassmannian dual-stage product codebooks can perform very close to single-stage quantization when employing RVQ/RSQ
- Significant complexity reduction by utilizing two relatively small codebooks rather than a single large codebook
- In multi-scattering channels, the product codebook can additionally mitigate the error floor of DFT codebooks
- Future work: further extension to multi-stage product codebooks?