Operator Learning of Dynamical Systems for Control

Max Beier,

joined work with

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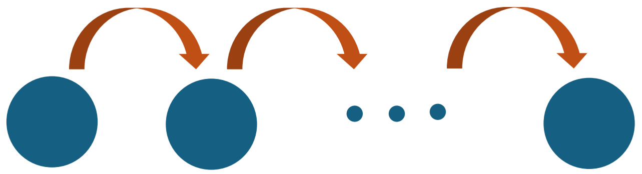
Chair of Information-oriented Control
Technical University of Munich

ITR Lab Tour

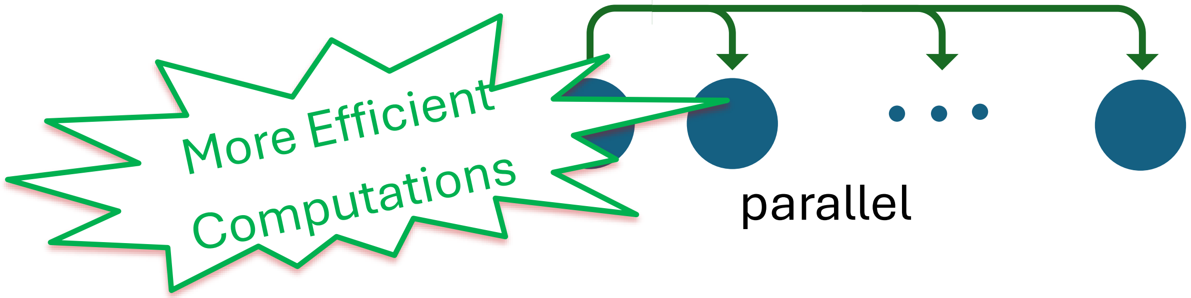
Juli 11 2025

Why Operator Models?

Training and Inference

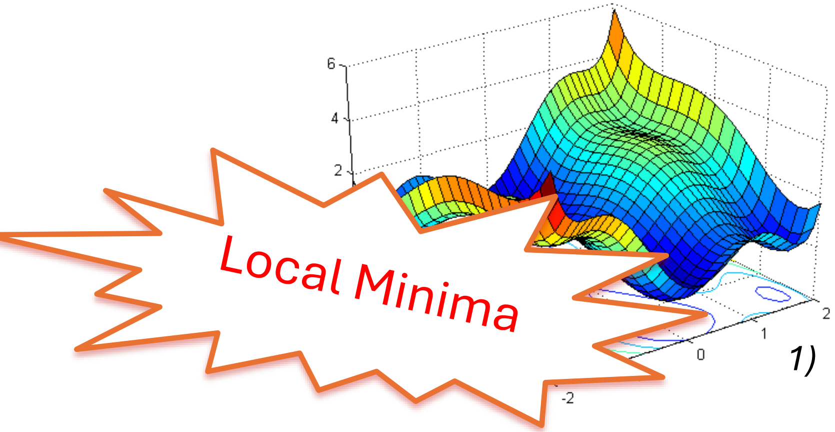


sequential

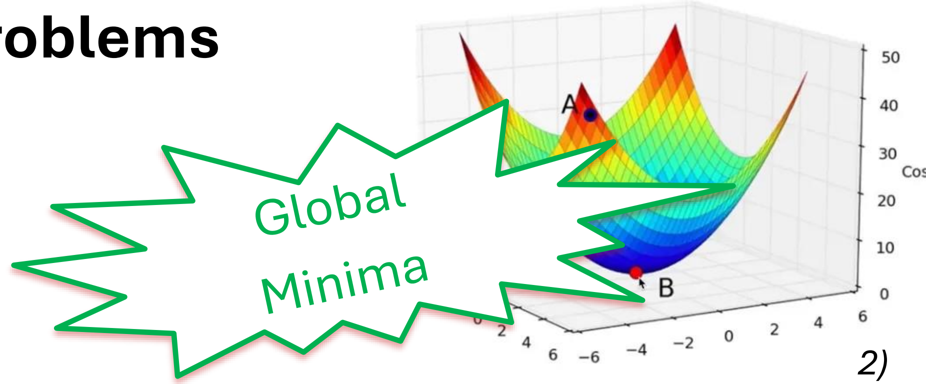


parallel

Optimization Problems



1)



2)

Linear Models have the Properties we want!

1) https://stats.stackexchange.com/questions/279292/non-convex-loss-function

2) https://poulami98bakshi.medium.com

How to get Operator Models**?**

Nonlinear State Space:

$$\dot{x} = f(x) + g(x)u,$$
$$y_t = h(x(t)), \qquad x_0 = x(0),$$

$$x \in R^n$$
$$f, g \in C(R^n)$$

Minimal

Nonlinear

→ **Rewrite Model** →

Linear Operator:

$$\dot{h} = \nabla \cdot f + u\nabla \cdot g = Ah + uBh$$
$$y_t = h(t)(x), \qquad h(t) = T(t)h_0$$

$$h \in C(R^n)$$
$$A, B: C(R^n) \to C(R^n)$$

Infinite Dimensional

Linear

If we can handle infinite dimensions, we get linear models!

What Research does ITR do**?**

Koopman-Equivariant Gaussian Processes

Petar Bevanda* TU Munich · Max Beier* TU Munich · Alex Capone CMU Robotics Institute
Stefan Sosnowski TU Munich · Sandra Hirche TU Munich · Armin Lederer ETH Zürich

Abstract

Credible forecasting and representation learning of dynamical systems are of ever-increasing importance for reliable decision-

Table 1: Nonlinear dynamics modeling from data

Approach	LTI forecast	End-to-end	Bayesian
GPs	✗	✓	✓
Koopman	✓	✗	✗
this work	✓	✓	✓

Fundamental research on learning with operators of dynamical systems from data

Koopman Kernel Regression

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Abstract

Many machine learning approaches for decision making, such as reinforcement learning, rely on simulators or predictive models to forecast the time-evolution of quantities of interest, e.g., the state of an agent or the reward of a policy. Forecasts of such complex phenomena are commonly described by highly nonlinear dynamical systems, making their use in optimization-based decision-making challenging.

Nonparametric Control Koopman Operators

Petar Bevanda, Bas Driessen, Lucian Cristian Iacob, Stefan Sosnowski, Roland Tóth and Sandra Hirche

Abstract—This paper presents a novel Koopman (composition) operator representation framework for control systems in reproducing kernel Hilbert spaces (RKHSs) that is free of explicit dictionary or input parametrizations. By establishing fundamental equivalences between different model representations, we are able to close the gap of control system operator learning and infinite-dimensional regression, enabling various empirical estimators and the connection to well-understood learning theory in RKHSs under one unified framework. As a consequence,

set of input values and describing a switched model [18] or analytically deriving the lifted representation [19]. It has become established that control-affine systems can be written as bilinear lifted models under certain conditions, at least in continuous-time. The authors of [20] show that for both continuous- and discrete-time systems with inputs, an invariant Koopman form can be analytically derived, granted that the

Dynamics-informed **sequence modelling**

Optimal control via convex optimization

ol vvv:1–14, 2025

Control: An Infinitesimal Generator Approach

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Abstract

This paper presents a novel approach for optimal control of nonlinear stochastic systems using infinitesimal generator learning within infinite-dimensional reproducing kernel Hilbert spaces. Our learning framework leverages data samples of system dynamics and stage cost functions, with only

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Diffeomorphically Learning Stable Koopman Operators

Petar Bevanda, *Member, IEEE*, Max Beier, *Graduate Student Member, IEEE*, Sebastian Kerz, *Graduate Student Member, IEEE*, Armin Lederer, *Graduate Student Member, IEEE*, Stefan Sosnowski, and Sandra Hirche, *Fellow, IEEE*

Abstract—System representations inspired by the infinite-dimensional Koopman operator (generator) are increasingly considered for predictive modeling. Due to the operator's linearity, a range of nonlinear systems admit linear predictor representations – allowing for simplified prediction, analysis and control. However, finding meaningful finite-dimensional representations for prediction is

are compatible with linear techniques for prediction, analysis and control [6].

However, obtaining long-term accurate predictive models using finite-dimensional Koopman operator dynamical models is challenging, as it generally incorporates an unsupervised learning problem. The latter involves learning a linear pre-

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What's Next?

Probabilistic Perspective and Modelling

Dynamics Representation Learning

Developing Optimal Control Solvers

Fulfilling Safety Constraints

Learn Different Operators

Happy to Talk!

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Always open for FP, MA

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