Lecture Series, TU Munich

October 22, 29 & November 5, 2013

Glocal Control for Hierarchical Dynamical Systems

Theoretical Foundations with Applications in Energy Networks

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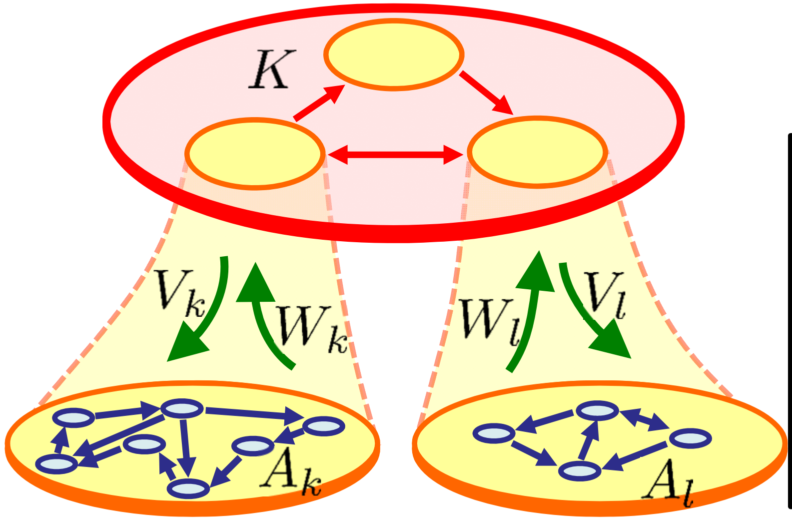
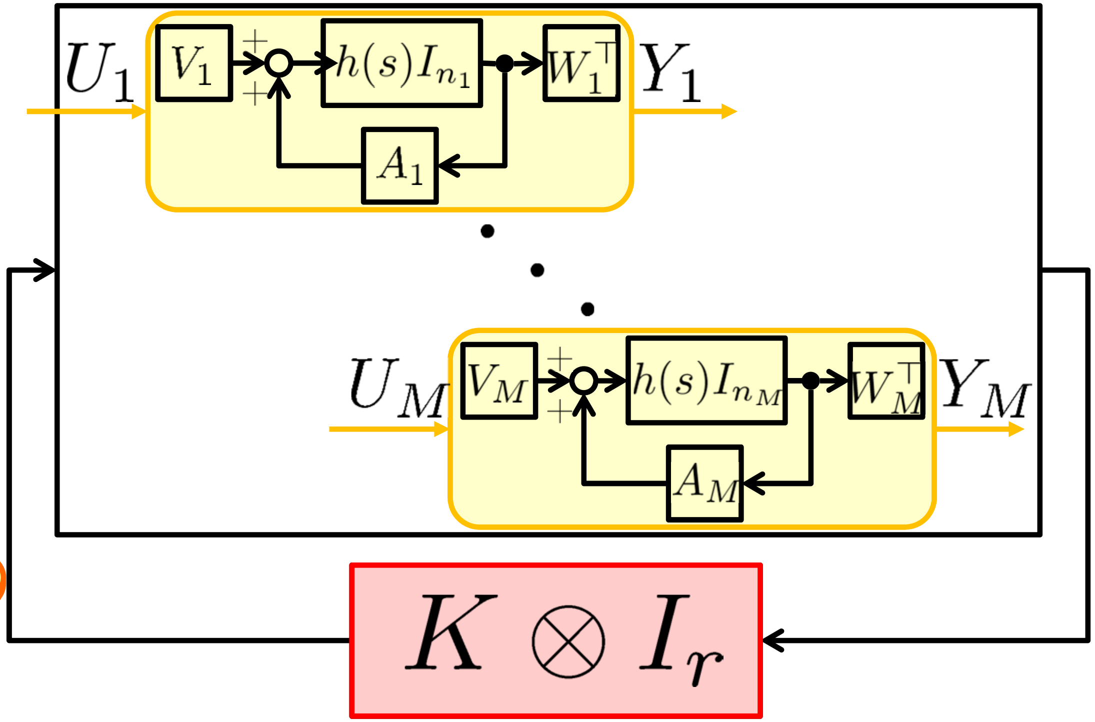
OUTLINE

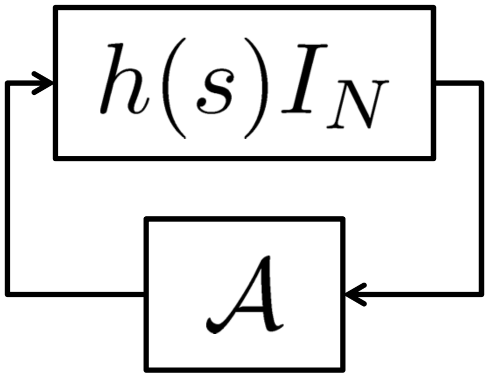
1. Glocal Control & Energy Networks
2. A Unified Framework for Networked Dynamical Systems with Stability Analysis
3. From Homogeneous to Heterogeneous
4. From Frat to Hierarchical
5. **Decentralized Hierarchical Control Synthesis**
6. Applications in Energy Networks

Hierarchical Decentralized Control

Design K, W_k and V_k







$$\mathcal{A} = \mathrm{diag}\{A_k\} + K \odot \Gamma$$

OUTLINE : Part 5

5. Decentralized Hierarchical Control Synthesis

- **Hierarchical LQR Synthesis**
- **Decentralized Hierarchical Control Synthesis via Decentralized Optimization**

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- Decentralized Hierarchical Control Synthesis via Decentralized Optimization

(Tsubakino et al.: ASCC2013)

Hierarchical Optimal Control Problem

(Tsubakino et al.: ASCC2013)

Optimal Control Problem

$$\dot{x} = A_L x + B_L u \quad Q_L \geq 0, \quad R_L = I$$

$$J(x_o, u) = \int_0^\infty \left(x(t)^\top Q_L x(t) + u(t)^\top R_L u(t) \right) dt$$

Optimal Control Law

$$u = Kx \qquad K = -R_L^{-1} B_L^\top P$$

$$A_L^\top P + P A_L - P B_L R_L^{-1} B_L^\top P + Q_L = 0$$

Q6: Under what condition, the optimal control gain *K*
- **preserves the hierarchical structure**
- **belongs to a desired decentralized structure ?**

An Example with 3 Subgroups

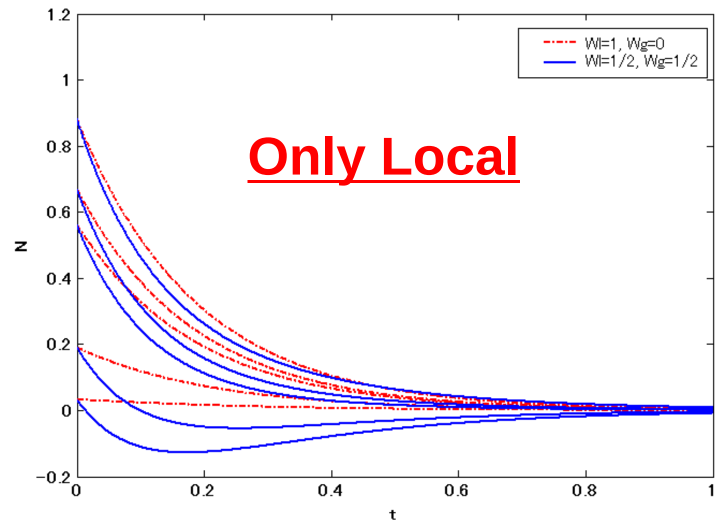
$$J = W_l J_l + W_g J_g,$$

$$J_l = \int_0^\infty \left(30\sum_{i=1}^{5}(N_i{}^2 + P_i{}^2 + Z_i{}^2) + \sum_i^{10}(u_i^2)\right)\mathrm{d}t,$$

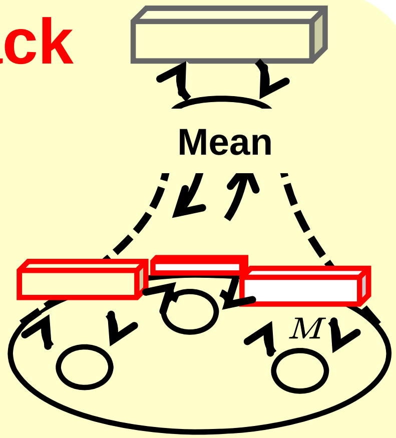
Local

$$J_g = \int_0^\infty \left(30(\bar{N}^2 + \bar{P}^2 + \bar{Z}^2) + \sum_i^{10}(u_i^2)\right)\mathrm{d}t$$

Global

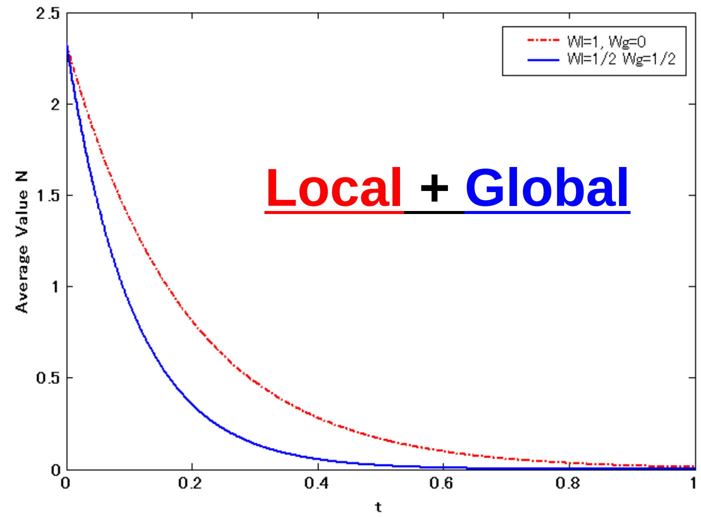


Structure of feedback control in *N*



$$u_i = k_l N_i + k_g \bar{N}$$

$\bar{N}$: N_i **(Average)**



How is the general case ?

Theorem: class of desired structure

$$A_L, B_L, Q_L \in \mathcal{H}_L$$

$\mathcal{G}_i = \{G_{ij}\}$: inter-layer interactions

$$\mathcal{H}_1 = \left\{ H_1 \in \mathbb{R}^{n_1 \times n_1} | H_1 = \sum_j^{N_1} a_{1j} G_{1j},\ a_{1j} \in \mathbb{C} \right\},$$

$$\mathcal{H}_L = \left\{ H_L \in \mathbb{R}^{n_L \cdots n_1 \times n_L \cdots n_1} | H_L = \sum_m^{N_L} a_{Lj} G_{Lj} \otimes H_{L-1,j}, \right.$$
$$\left. a_{Lj} \in \mathbb{R},\ H_{L-1,j} \in \mathcal{H}_{L-1} \right\}$$

Averaging, Circulant

Theorem

$\mathcal{G}_i$: a semi-group

$\Rightarrow$ $K \in \mathcal{H}_L,$

$$\mathcal{G}_i (i = 1, 2, \cdots, L)$$
$$= \left\{ G_{ij} \in \mathbb{C}^{n_i \times n_i} | \forall j, k,\ \exists l,\ G_{ij} G_{ik} = G_{il} \right\}$$

Same results for
Output Feedback & Hinf Control

Desired Hierarchical Structures

$\{I, \hat{I}\}$ $\quad \{I, \frac{1}{n}\mathbf{1}\mathbf{1}^\top\}$: averaging

$$\hat{I} = \begin{bmatrix} 0 & & 1 \\ & \iddots & \\ 1 & & 0 \end{bmatrix}$$

$\{I, L, L^2, \cdots, L^{n-1}\}$: Circulant

$$L = \left[\begin{array}{c|c} 0 & I \\ \hline 1 & 0 \end{array}\right]$$

$$\mathcal{T} = \{A | T(g_i)A = AT(g_i), \quad {}^\forall g_i \in G\}$$

Spatially Decay Operator

$$\mathcal{S}_\tau = \{\mathbb{R}^{n\times n} | \exists C, \exists \alpha \in \mathbb{R},\ 0 < \alpha < \tau,$$
$$A = [A_{ki}],\ \|A_{ki}\| \leq C \exp(-\alpha|k-i|)\}$$

Cooling in Iron Plate Production

$$\frac{\partial x}{\partial t} = \frac{\partial^2 x}{\partial y^2} + u, \quad x(0,t) = 0, \quad x(1,t) = 0,$$

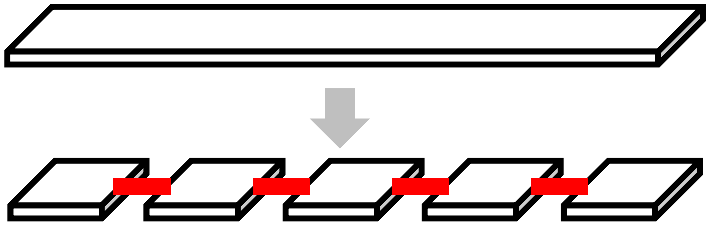
Discretization in space

$$x_i(t) = x(ih, t)$$

$$\dot{x} = \begin{bmatrix} -2 & 1 & & & & \\ 1 & -2 & 1 & & & \\ & \ddots & \ddots & \ddots & & \\ & & 1 & -2 & 1 & \\ & & & 1 & -2 & 1 \\ & & & & 1 & -2 \end{bmatrix} x + u$$

Performance Index

$$Q_c = 10 \begin{bmatrix} 2 & -1 & & & & \\ -1 & 3 & -1 & & & \\ & -1 & & \ddots & & \\ & & \ddots & & -1 & \\ & & & -1 & 3 & -1 \\ & & & & -1 & 2 \end{bmatrix}$$



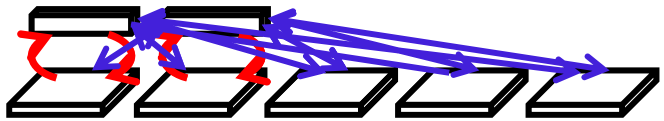
Optimal Control with SD Structure

Example : optimal feedback gain

$$(W_s, W_c) = (\frac{1}{2}, \frac{1}{2})$$

$$K = \begin{bmatrix} 1.7061 & -0.1245 & -0.0222 & -0.0045 & -0.0010 & -0.0002 & -0.0001 & -0.0000 & -0.0000 & -0.0000 \\ -0.1245 & 2.3181 & -0.0437 & -0.0107 & -0.0027 & -0.0007 & -0.0002 & -0.0000 & -0.0000 & -0.0000 \\ -0.0222 & -0.0437 & 2.3393 & -0.0394 & -0.0098 & -0.0025 & -0.0007 & -0.0002 & -0.0000 & -0.0000 \\ -0.0045 & -0.0107 & -0.0394 & 2.3405 & -0.0391 & -0.0098 & -0.0025 & -0.0007 & -0.0002 & -0.0001 \\ -0.0010 & -0.0027 & -0.0098 & -0.0391 & 2.3405 & -0.0391 & -0.0098 & -0.0025 & -0.0007 & -0.0002 \\ -0.0002 & -0.0007 & -0.0025 & -0.0098 & -0.0391 & 2.3405 & -0.0391 & -0.0098 & -0.0027 & -0.0010 \\ -0.0001 & -0.0002 & -0.0007 & -0.0025 & -0.0098 & -0.0391 & 2.3405 & -0.0394 & -0.0107 & -0.0045 \\ -0.0000 & -0.0000 & -0.0002 & -0.0007 & -0.0025 & -0.0098 & -0.0394 & 2.3393 & -0.0437 & -0.0222 \\ -0.0000 & -0.0000 & -0.0000 & -0.0002 & -0.0007 & -0.0027 & -0.0107 & -0.0437 & 2.3181 & -0.1245 \\ -0.0000 & -0.0000 & -0.0000 & -0.0001 & -0.0002 & -0.0010 & -0.0045 & -0.0222 & -0.1245 & 1.7061 \end{bmatrix}$$

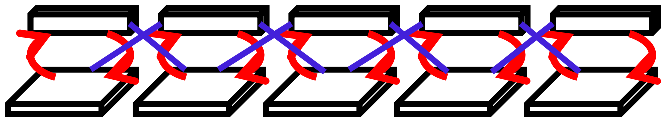
Centralized



approximation **Error:** 9.4081×10^{-7}

$$K_t = \begin{bmatrix} 1.7061 & -0.1245 & -0.0222 & -0.0045 & & & & & & \\ -0.1245 & 2.3181 & -0.0437 & -0.0107 & -0.0027 & & & & & \\ -0.0222 & -0.0437 & 2.3393 & -0.0394 & -0.0098 & -0.0025 & & & & \\ -0.0045 & -0.0107 & -0.0394 & 2.3405 & -0.0391 & -0.0098 & -0.0025 & & & \\ & -0.0027 & -0.0098 & -0.0391 & 2.3405 & -0.0391 & -0.0098 & -0.0025 & & \\ & & -0.0025 & -0.0098 & -0.0391 & 2.3405 & -0.0391 & -0.0098 & -0.0027 & \\ & & & -0.0025 & -0.0098 & -0.0391 & 2.3405 & -0.0394 & -0.0107 & -0.0045 \\ & & & & -0.0025 & -0.0098 & -0.0394 & 2.3393 & -0.0437 & -0.0222 \\ & & & & & -0.0027 & -0.0107 & -0.0437 & 2.3181 & -0.1245 \\ & & & & & & -0.0045 & -0.0222 & -0.1245 & 1.7061 \end{bmatrix}$$

Decentralized



Example : simulation result

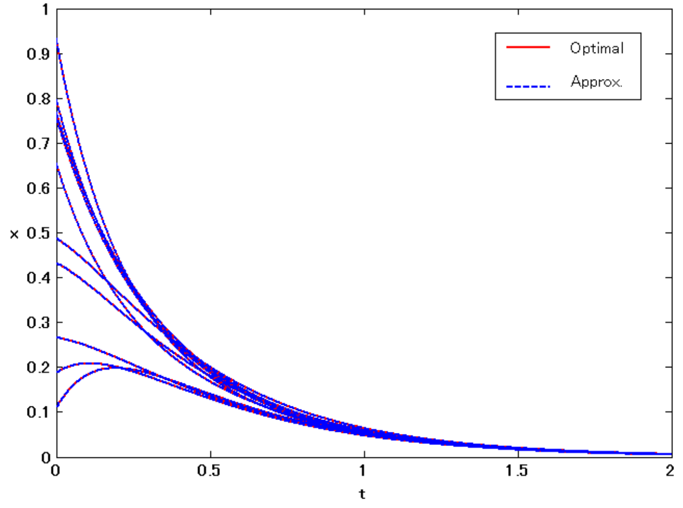
$$J = W_s J_s + W_c J_c,$$

$$J_s = \int_0^\infty \left(10 \sum_{i=1}^{10} (x_i{}^2) + \sum_{i}^{10} (u_i^2)\right) \mathrm{d}t$$

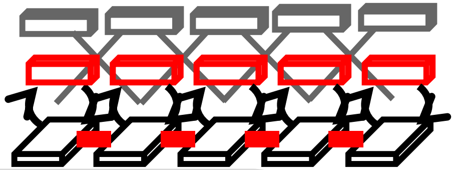
Local

$$J_c = \int_0^\infty \left(10 \sum_{i=1}^{9} (x_i - x_{i+1})^2 + \sum_{i}^{10} (u_i^2)\right) \mathrm{d}t$$

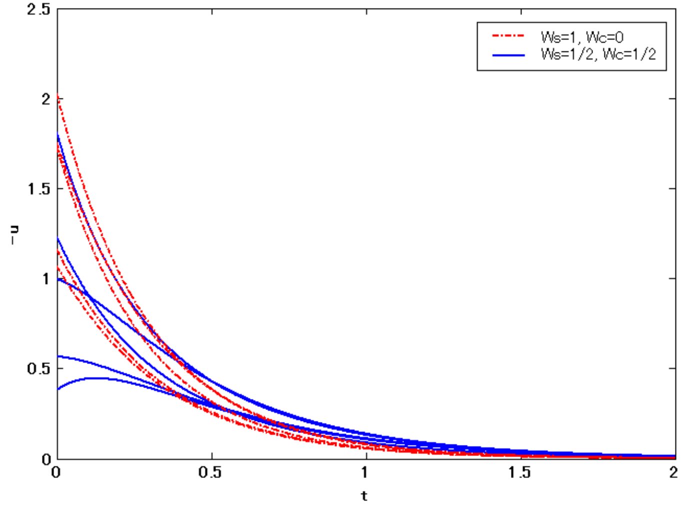
Cooperation



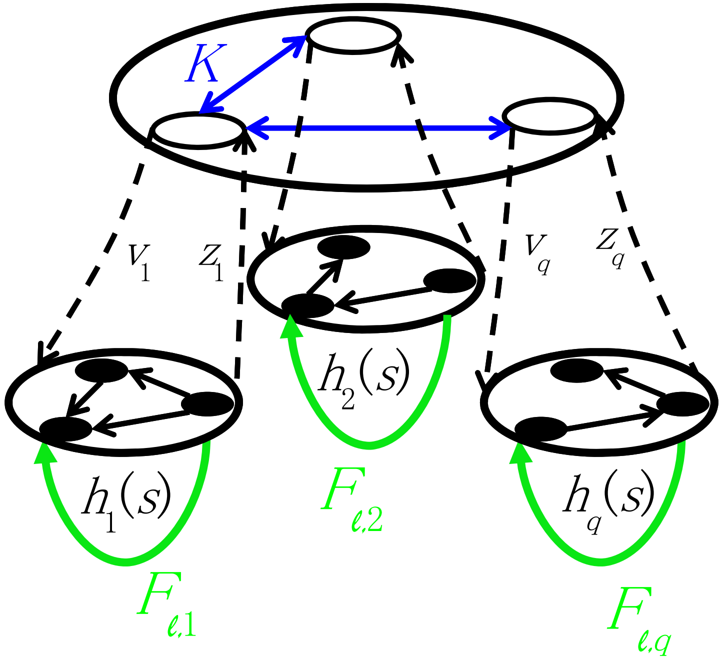
Control Structure



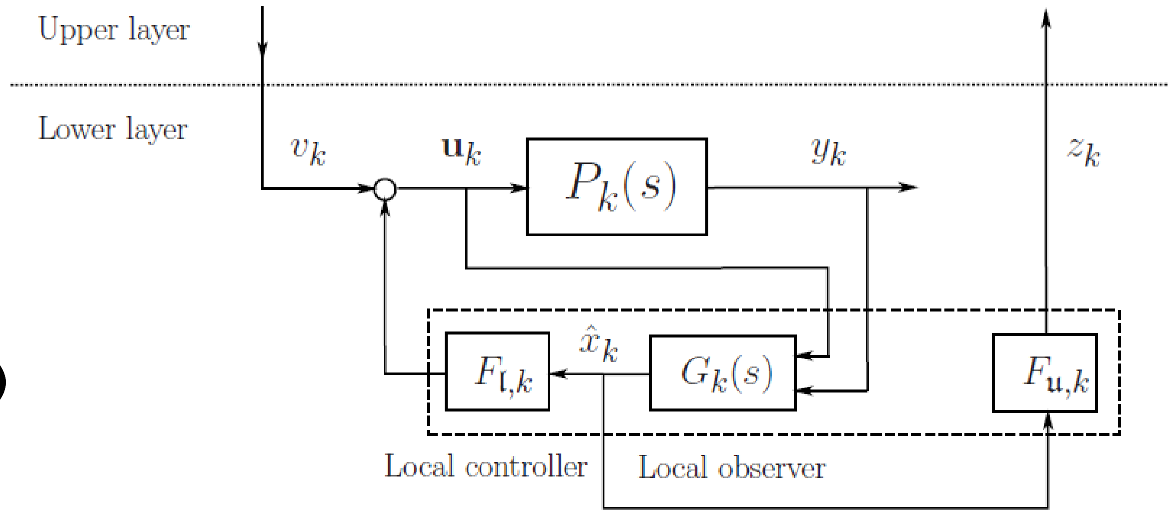
$$u_i = k_s u_i + \sum_{j=1}^{3} k_{cj} (x_{i-j} + x_{i+j})$$

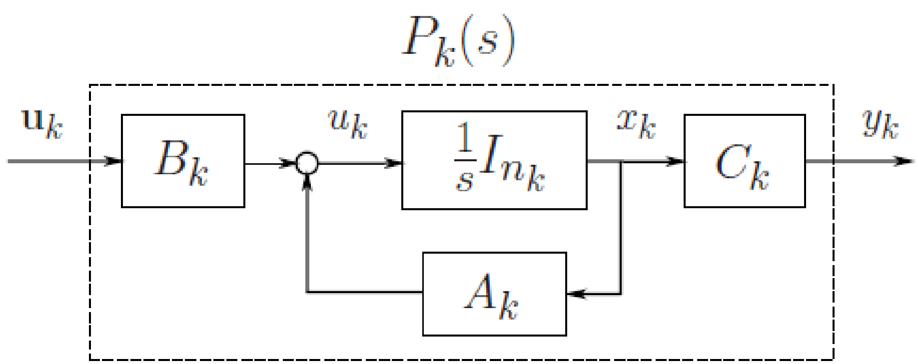


LQR Synthesis + Local Observer

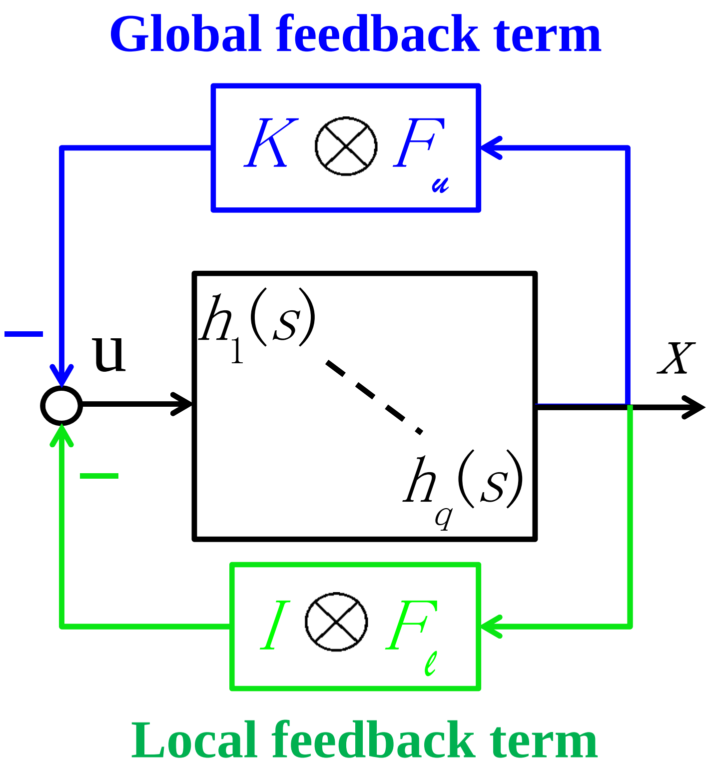


k-th subsystem





LQR Setting : State Feedback



K : symmetric

$$u = -Fx,\ F = I \otimes F_{\ell} + K \otimes F_{g}$$

$$\text{s.t. } J = \int_0^{\infty} \left(x^T Q x + u^T R u \right) dt \to \min$$

Performance Index

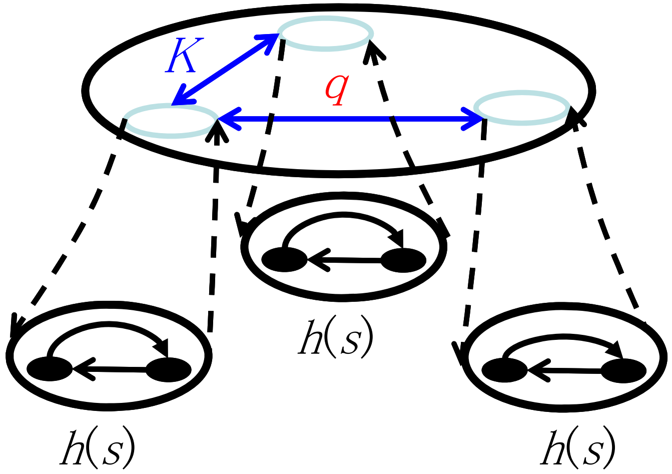
$$J = J_{x,\ell} + J_{x,g} + J_u$$

$$J_{x,\ell} = \int_0^{\infty} x^T \left(I_q \otimes Q_1 \right) x dt,$$

$$J_{x,g} = \int_0^{\infty} x^T \left(K \otimes Q_2 \right) x dt,$$

$$J_u = \int_0^{\infty} u^T R u dt$$

Example：Role of Higher Level Control

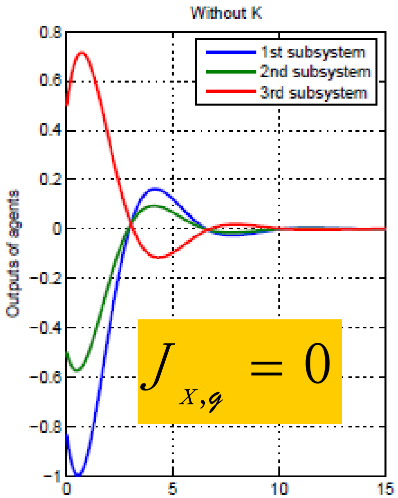


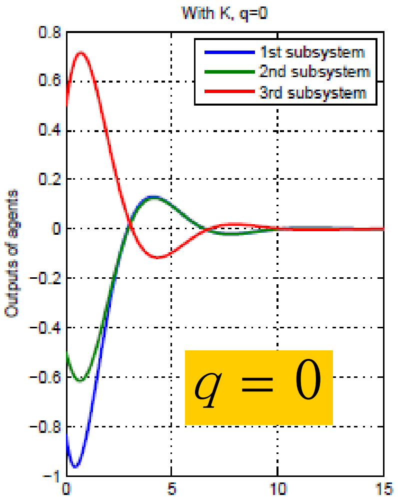
<u>Subsystem</u>: 2nd-order unstable
<u>Local Objective</u>: quick convergence
<u>Global Objective</u>:
minimizing energy
+ additional requirement

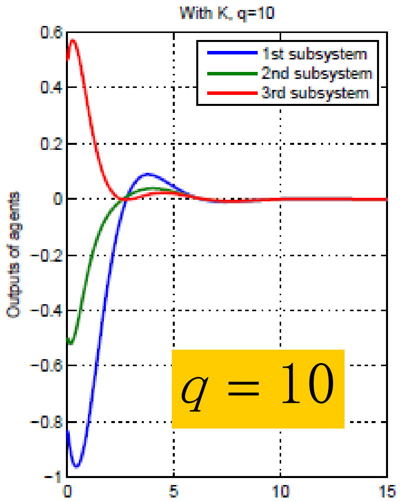
$$K = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 1+q & -q \\ 0 & -q & q \end{bmatrix}$$

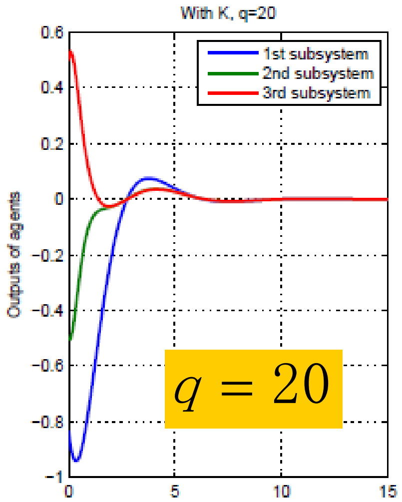
Additional Global Objective

$$J_{x,q} = \left(x_1 - x_2\right)^T Q_2 \left(x_1 - x_2\right) + q\left(x_2 - x_3\right)^T Q_2 \left(x_2 - x_3\right), q \geq 0$$









Systematic Way of Synthesis

Key idea : Select weighting matrices with proper hierarchical structures

Theorem

With

$$Q = I_q \otimes Q_1 + K \otimes Q_2, R^{-1} = I_q \otimes R_1 + K \otimes R_2$$

$$Q_2 = P_1 \mathrm{B} R_2 \mathrm{B} P_1, R_2 \succ 0, K \succeq 0$$

where $P_1 = \mathrm{diag}\{P_{1k}\}_{k=1,\ldots,q}$, P_{1k} satisfies

$$P_{1k} A_k + A_k^T P_{1k} + Q_{1k} - P_{1k} B_k R_{1k} B_k^T P_{1k} = 0,$$

then the hierarchical LQR optimal controller is

$$F = I_q \otimes \underbrace{\left(R_1 \mathrm{B}^T P_1\right)}_{F_\ell} + K \otimes \underbrace{\left(R_2 \mathrm{B}^T P_1\right)}_{F_u}.$$

Furthermore,

$$A = I_q \otimes \left(\mathrm{A} - \mathrm{B} R_1 \mathrm{B}^T P_1\right) - K \otimes \left(\mathrm{B} R_2 \mathrm{B}^T P_1\right)$$

Global optimality is obtained based on local optimality

Hierarchical structure is preserved

Messages：Hierarchical Control

① **Proper ways of aggregation and distribution are important to achieve rapid consensus.**

② **Low rankness of interlayer connection captures them properly.**

③ **Heterogeneous agents:** ***Khatri-Rao Product***

hierarchical network synthesis based on left eigenvectors

④ **LQR optimal control with desired hierarchical structure**

certain semi-group property

local observer + optimal SF

OUTLINE : Part 5

5. Decentralized Hierarchical Control Synthesis

- Hierarchical LQR Synthesis
- **Decentralized Hierarchical Control Synthesis via Decentralized Optimization**

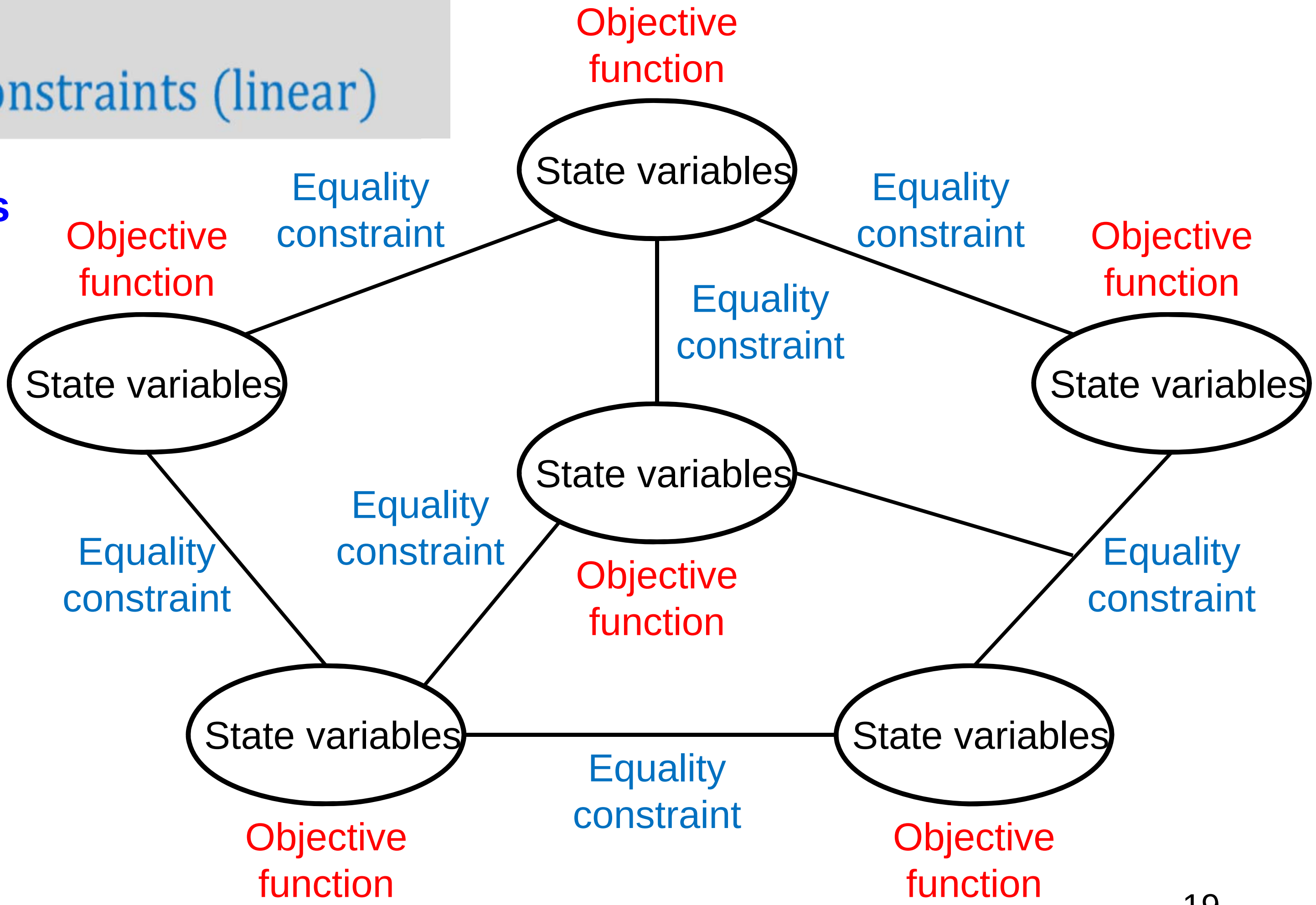
Decentralized Control：Optimization

$$\max_{\text{State variables}} \sum \text{Objective function}$$

s.t. Equality constraints (linear)

Strictly Concave

Conservative Laws
Dynamics



Decentralized Control：Optimization

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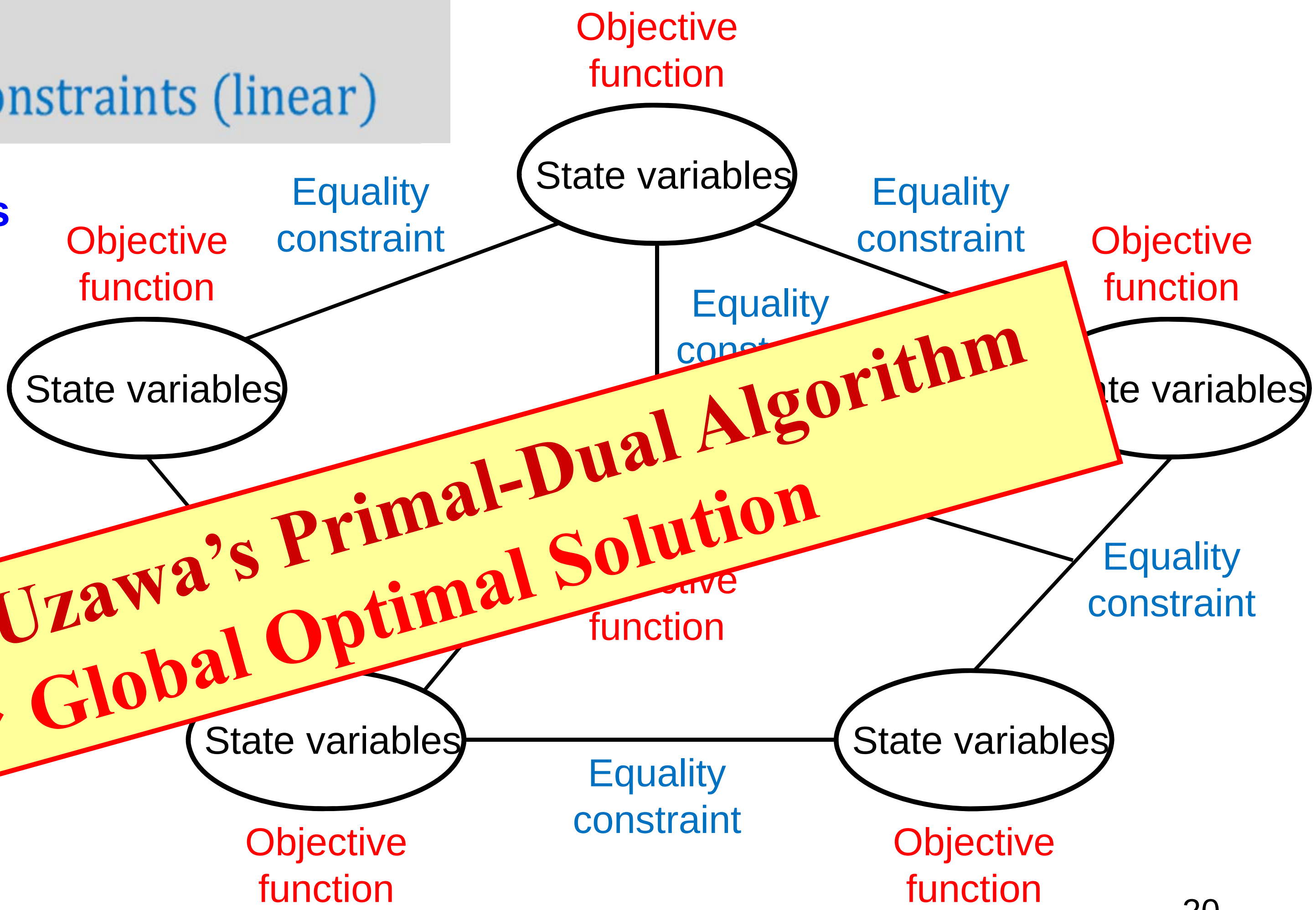


To Apply Uzawa's Primal-Dual Algorithm
→ Global Optimal Solution

Uzawa's Primal-Dual Algorithm

(Arrow, Hurwicz, Uzawa, 1958)

$\boldsymbol{x} \in \mathbb{R}^n$

$f: \mathbb{R}^n \to \mathbb{R}$, C2 class , strictly concave function

$$\max_{\boldsymbol{x}} f(\boldsymbol{x}) \quad \text{s.t.} \quad R\boldsymbol{x} = \boldsymbol{0} \qquad \text{(P)}$$

$\mathcal{L}(\boldsymbol{x}, \boldsymbol{\lambda}) = f(\boldsymbol{x}) + \boldsymbol{\lambda}^{\mathrm{T}} R\boldsymbol{x}$: Lagrangian

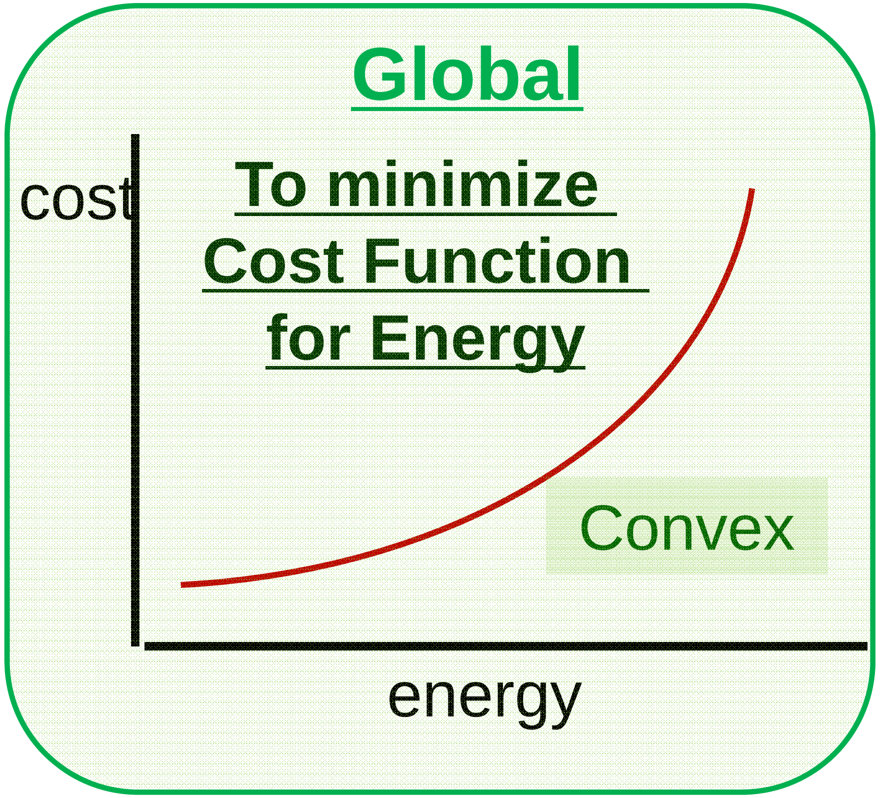
$$\dot{\boldsymbol{x}} = \frac{\partial \mathcal{L}}{\partial \boldsymbol{x}}(\boldsymbol{x}, \boldsymbol{\lambda}) = \frac{\partial f}{\partial \boldsymbol{x}}(\boldsymbol{x}) + R^{\mathrm{T}} \boldsymbol{\lambda}$$

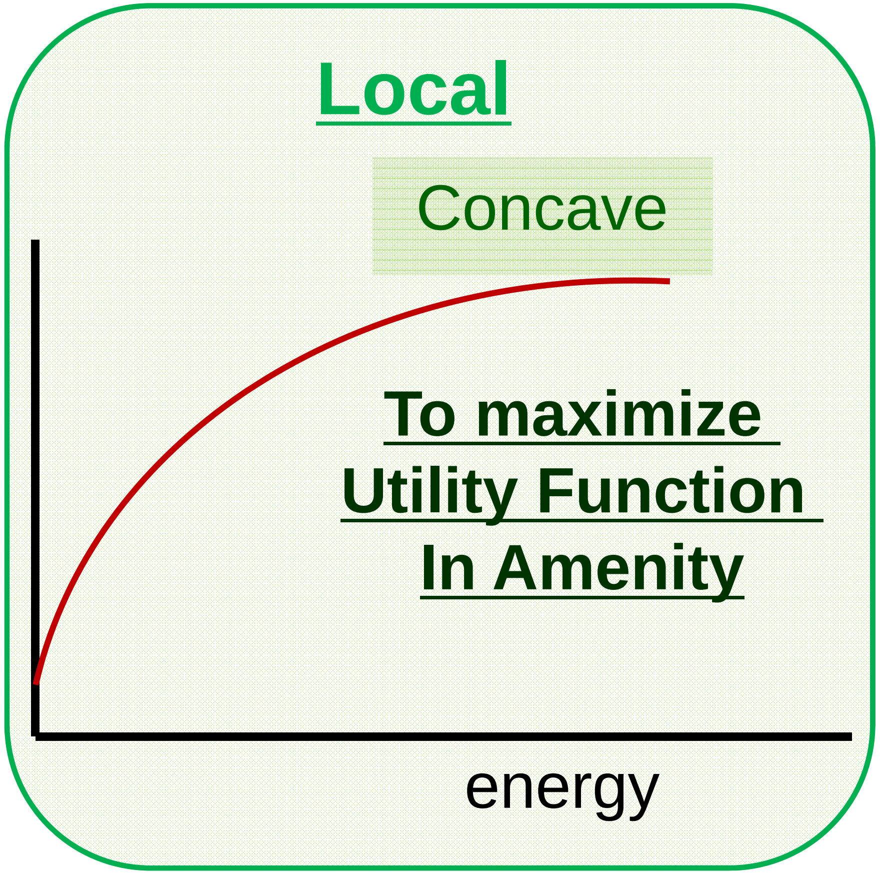
$$\dot{\boldsymbol{\lambda}} = -\frac{\partial \mathcal{L}}{\partial \boldsymbol{\lambda}}(\boldsymbol{x}, \boldsymbol{\lambda}) = -R\boldsymbol{x}$$

$(\boldsymbol{x}^*, \boldsymbol{\lambda}^*)$

A simple gradient method guarantees the convergence to the unique optimal

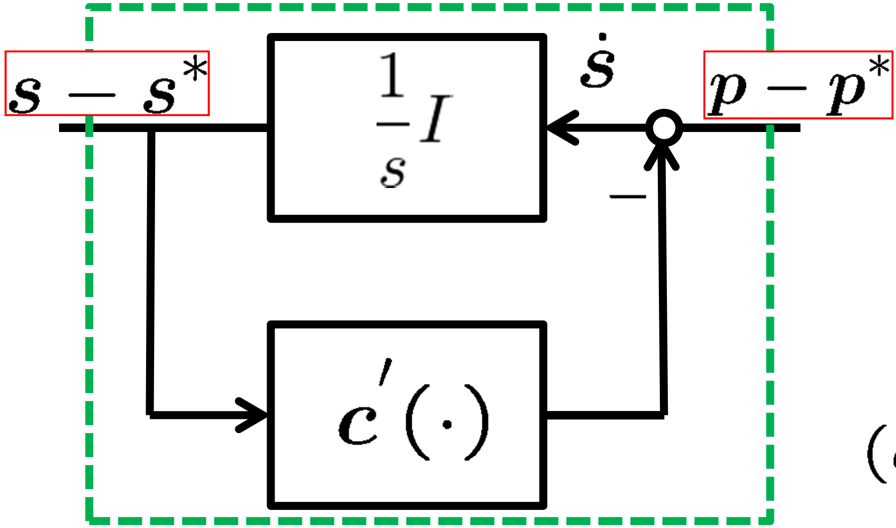
Properties of Objective Functions





Control Theoretic Interpretation

(Yamamoto, Tsumura: METR 2012)

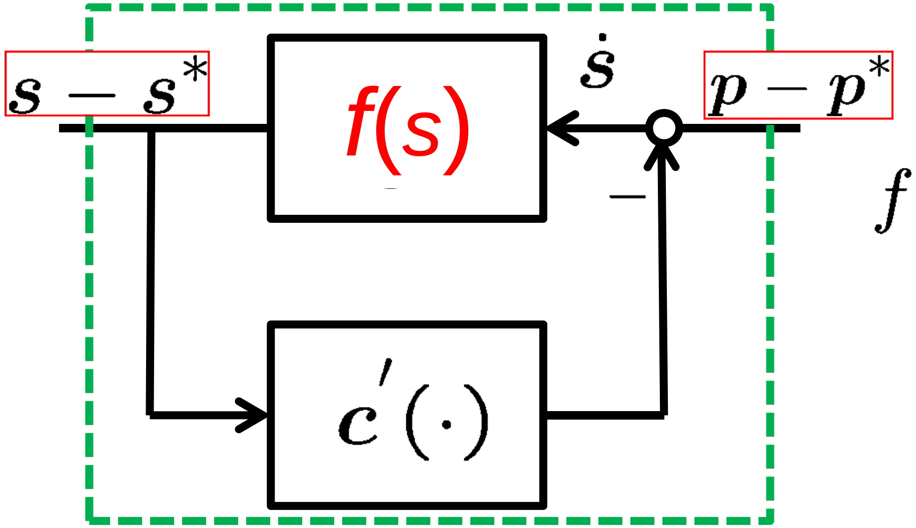


Incremental passive

Storage Function

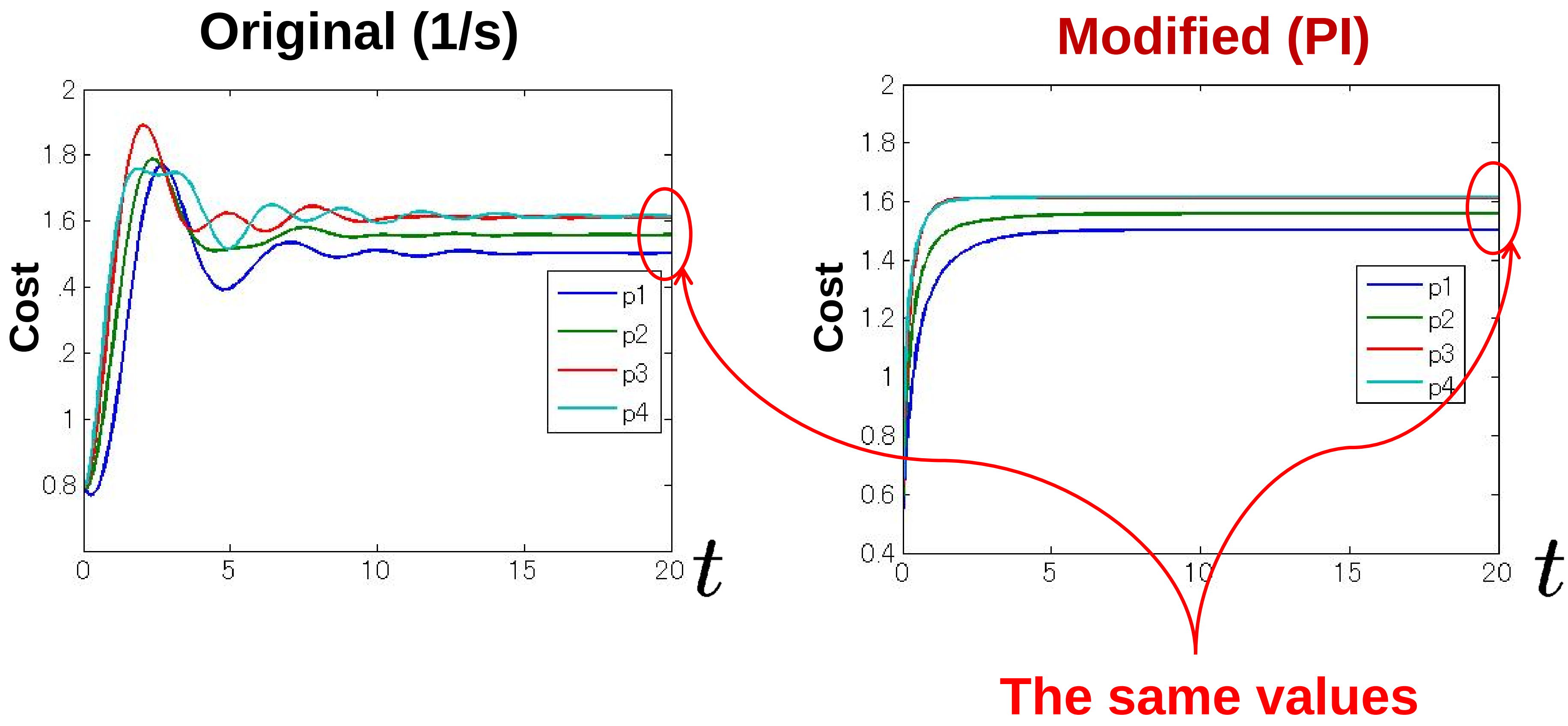
$$S_{s_i} := \frac{1}{2}(s_i - s_i^*)^2$$

$(c_i''(\xi_{s_i}) > 0$, strictly convex)



$f(s)$: any passive system
e.g. PI-type

A Numerical Example



- **Reduction of Computational Cost**
- **Possibility of Receding Horizon Strategy**

On Going Research Directions

① Low rank interlayer connections are quite helpful for rapid consensus. *aggregation*

Robustness, Control Performances
Internal Model Principle ?

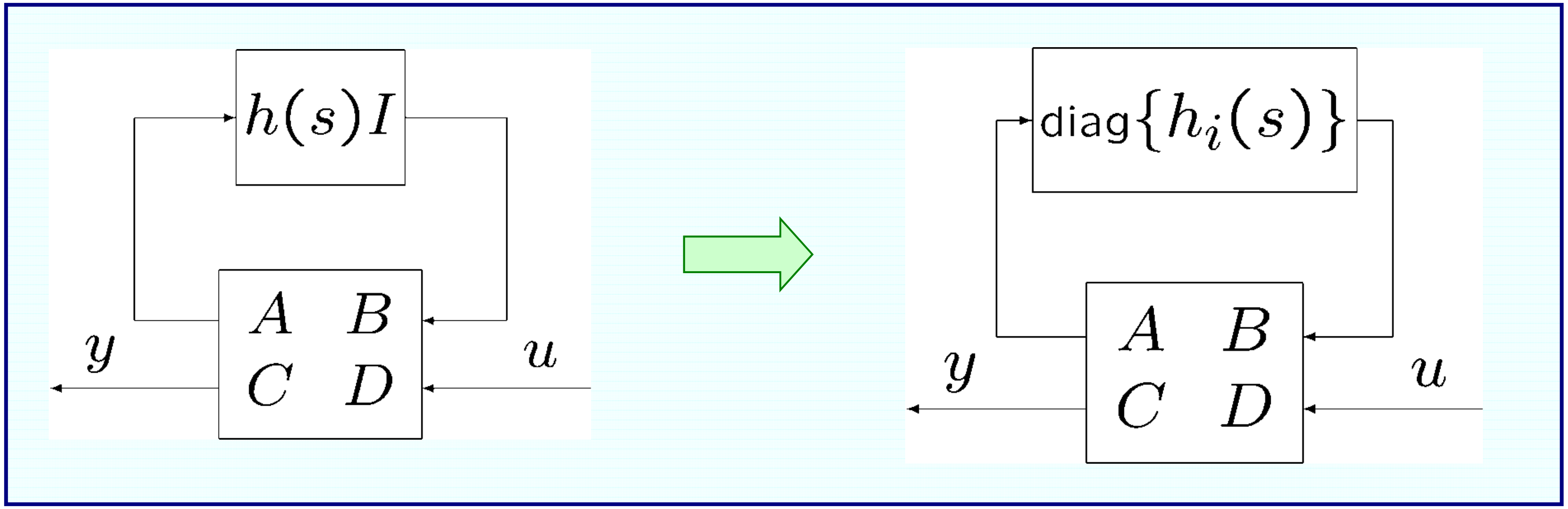
② Nonlinear agents: *left eigenvector*
strongly connected graph in the upper layer
\+ subsystems which can be passive

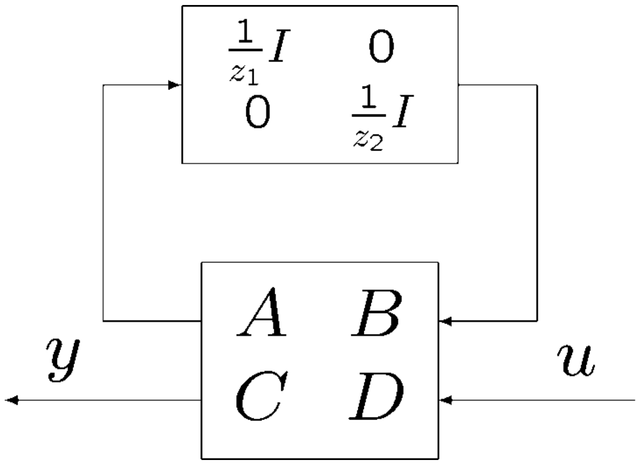
Beyond passivity ?

③ Heterogeneous agents: *Khatri-Rao Product*
hierarchical network synthesis based on left eigenvectors

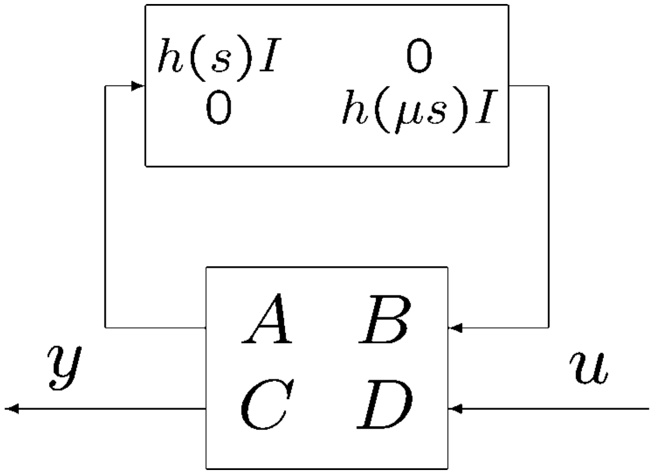
Generalization: Systematic design procedure
Dynamic control synthesis ?

New Framework for System Theory





2D System



Singular Perturbed System

Multi-resolved Systems