Lecture Series, TU Munich
October 22, 29 & November 5, 2013

Glocal Control for Hierarchical Dynamical Systems

Theoretical Foundations with Applications in Energy Networks

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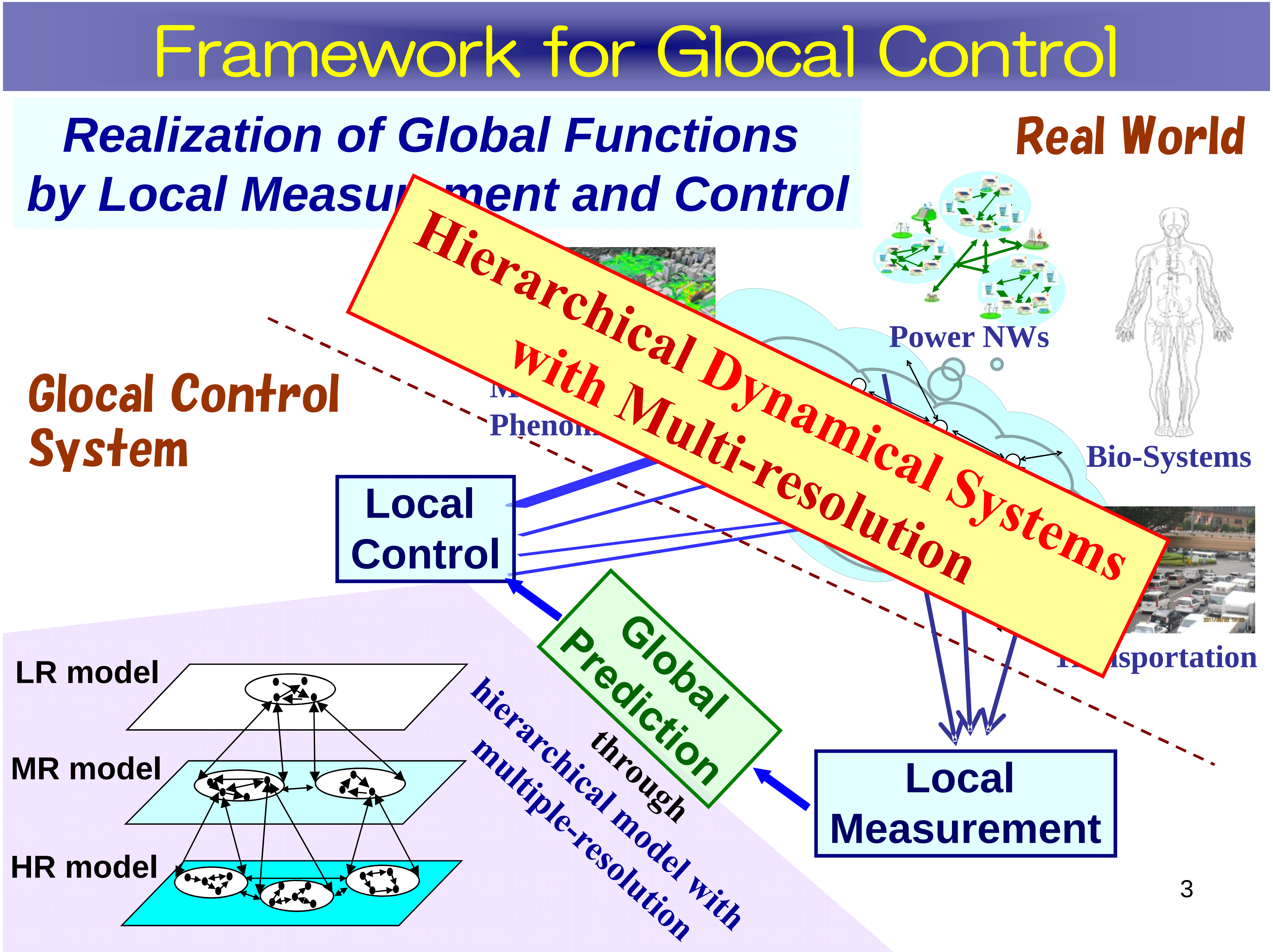
Outline


Framework for Glocal Control
Realization of Global Functions
by Local Measurement and Control
Real World
Hierarchical Dynamical Systems
with Multi-resolution
Power NWs
Bio-Systems
Glocal Control
System
Phenom
Local
Control
Local
Measurement
Global
Prediction
through
hierarchical model with
multiple-resolution
LR model
MR model
HR model
sportation

Messages : A New Framework

① **LTI system with generalized freq. variable**

a proper class of homogeneous multi-agent dynamical systems

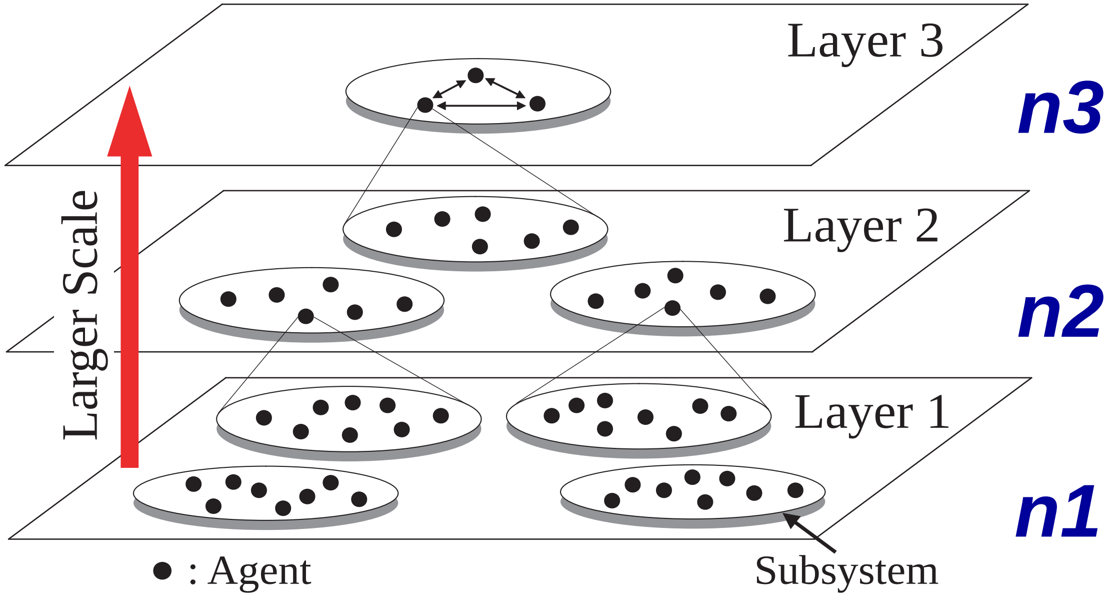
② **Three types of stability tests, namely graphical, algebraic, and numeric (LMI)**

powerful tools for analysis

Q3: from **Homogeneous**
to **Heterogeneous *?***

Q4: from **Flat Structure**
to **Hierarchical Structure *?***

From Frat to Hierarchical Structures



OUTLINE : Part 4

4. From Frat to Hierarchical

- Low-rank Interlayer Connections
- Hierarchical Consensus for Heterogeneous Networks
- Hierarchical Adaptive Consensus

OUTLINE : Part 4

4. From Frat to Hierarchical

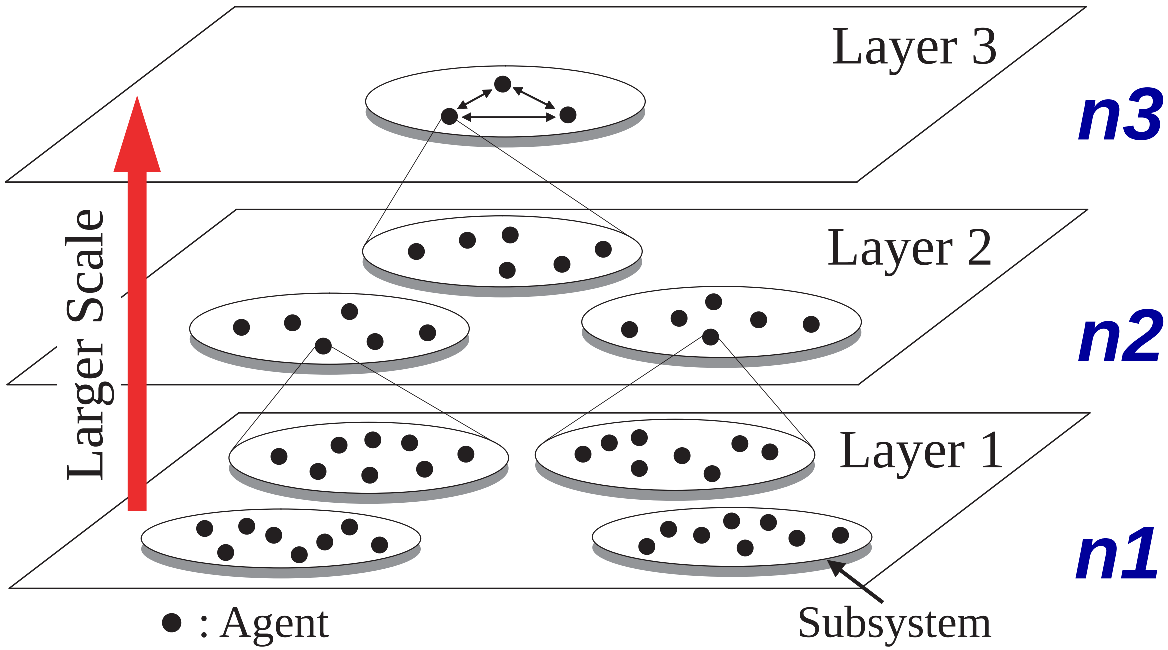
- **Low-rank Interlayer Connections**
- Hierarchical Consensus for Heterogeneous Networks
- Hierarchical Adaptive Consensus

(Shimizu, Hara: SICE2008)

Q9: What are Key Properties in Hierarchical Systems ?

Hierarchical NW Dynamical Systems

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) \qquad \exists \xi \ , \quad \lim_{t\to\infty} \boldsymbol{x}(t) = \xi \cdot \mathbf{1}$$



total agents : *n1*× *n2*× *n3*

Hierarchical Structure

$$A_l = \mathrm{diag}(A_{l-1} - I) + P \otimes \Delta$$

Homogeneous structure

Upper-layer structure

Property on Interactions

***Low Rank Interaction*:**

$$\Delta = 1 \cdot \zeta^T$$

Share an aggregated information
Control uniformly

weak interaction:

Sparse

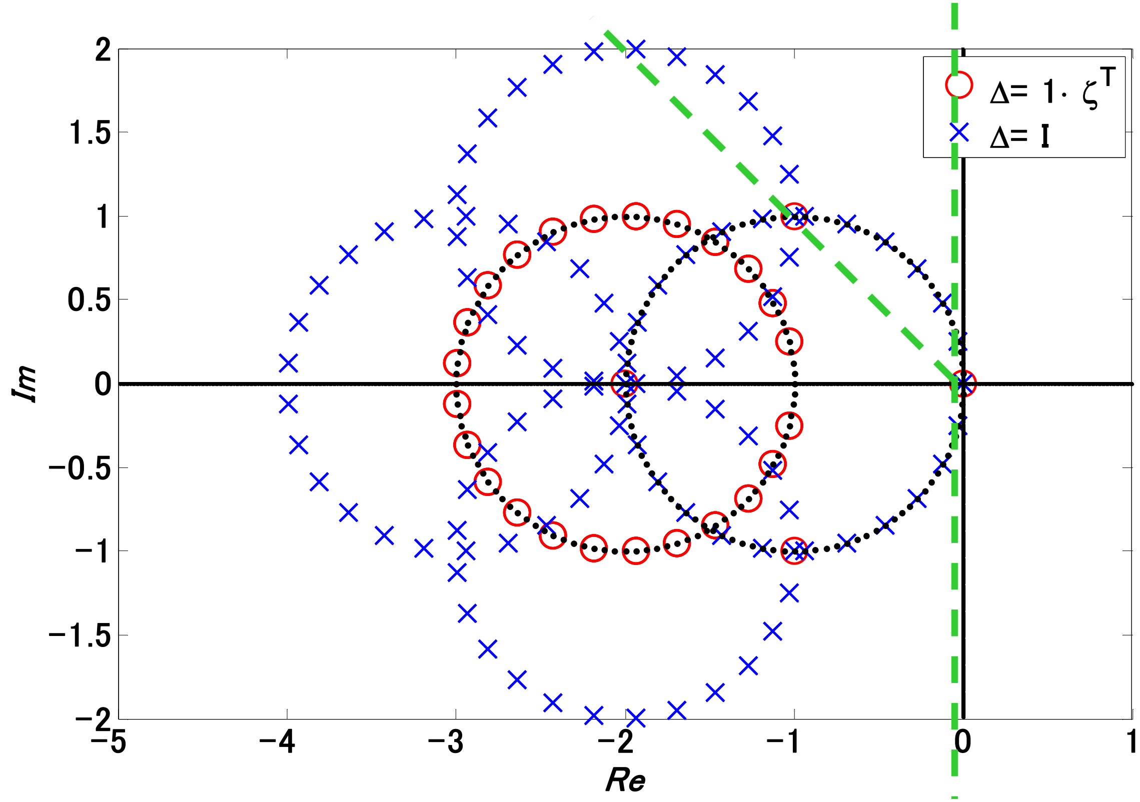
Small gain

Eigenvalue Distributions

⊿: Rank 1

$$\mathrm{eigs}(\boldsymbol{A}_1) = \bigcup_{r=1}^{n_1} \exp(2\pi j(r-1)/n_1) - 1$$

$$\mathrm{eigs}(\boldsymbol{A}_2) = \begin{cases} \bigcup_{r=1}^{n_2} \exp(2\pi j(r-1)/n_2) - 1 \\ \bigcup_{r=2}^{n_2} \exp(2\pi j(r-1)/n_1) - 2 \end{cases}$$

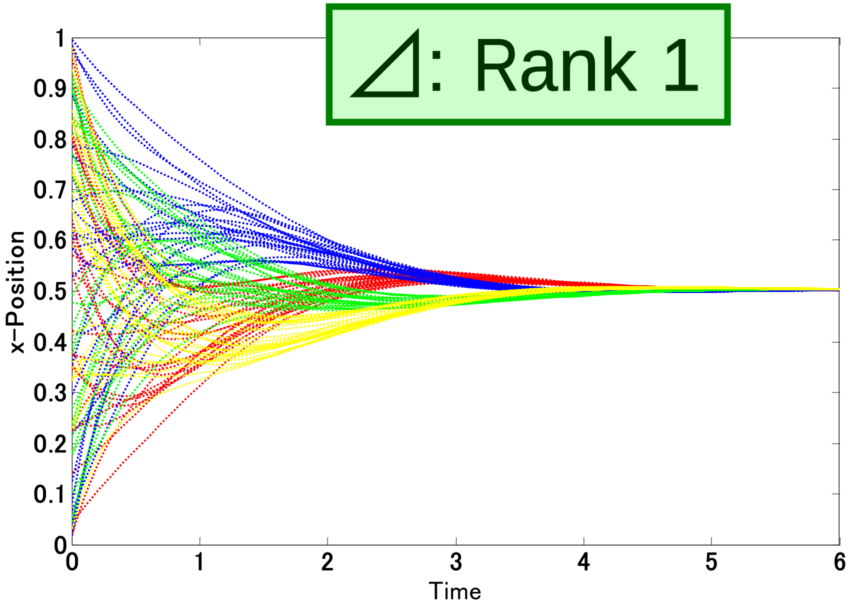


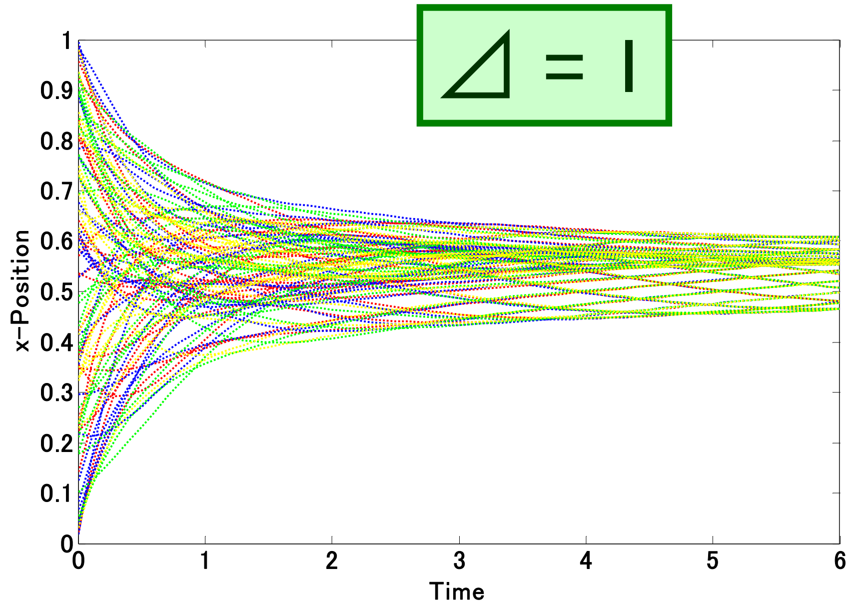
n1 = 25
> n2 = 4

○ : rank 1

× : Identity

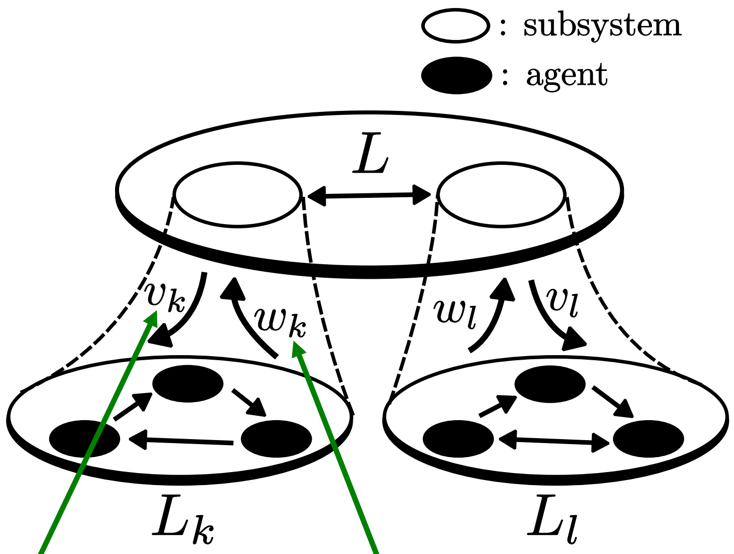
Time Responses (n1=25, n2=4)





Rapid Consensus

n1 > n2



Distribution

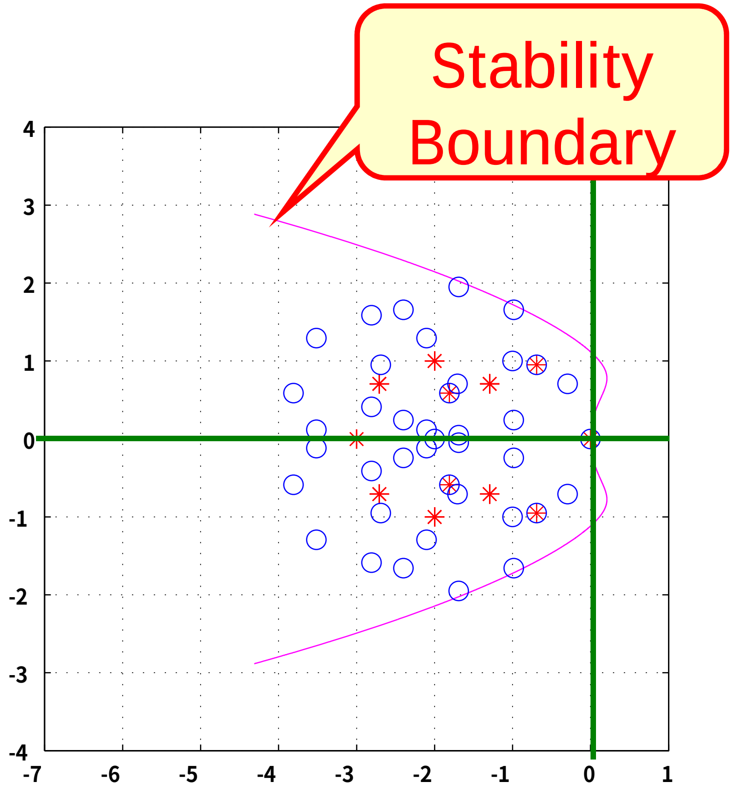
Aggregation

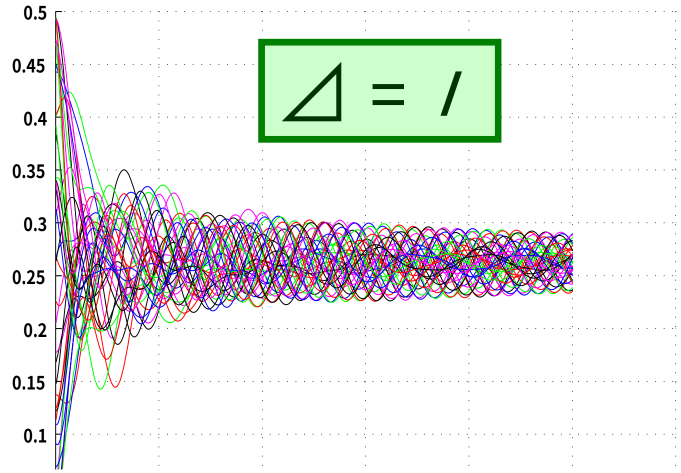
Low-rank Interlayer Interactions
→ Multiple resolution

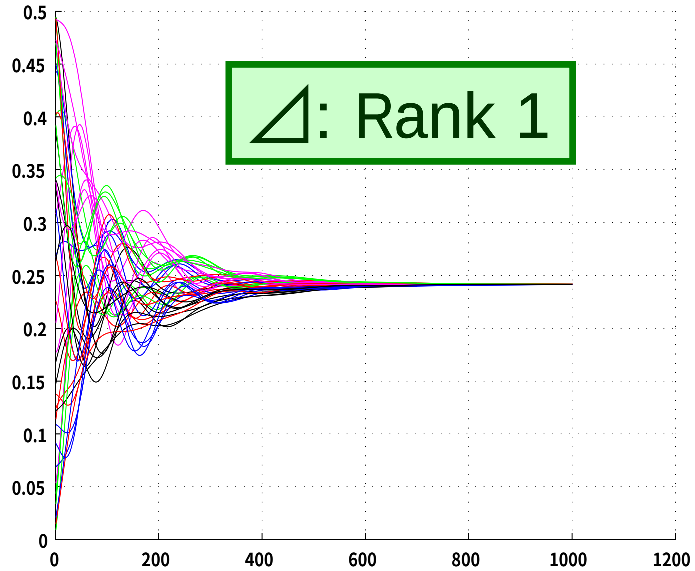
General Case with *h(s)*

$$h(s) = \frac{s+1}{s(0.1s^2+0.5s+1)}$$

$$n_1 = 8,\ n_2 = 5$$







○ : ⊿ = *I*

∗ : ⊿= Rank 1

OUTLINE : Part 4

4. From Frat to Hierarchical

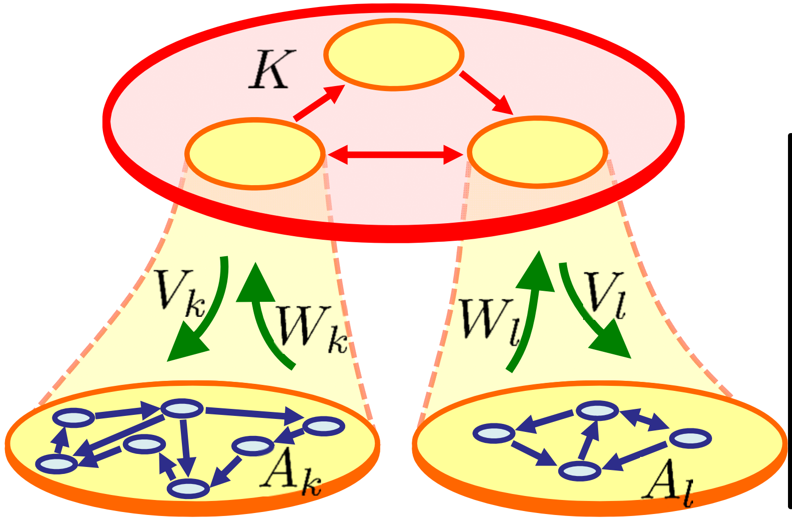
- Low-rank Interlayer Connections
- Hierarchical Consensus for Heterogeneous Networks
- Hierarchical Adaptive Consensus

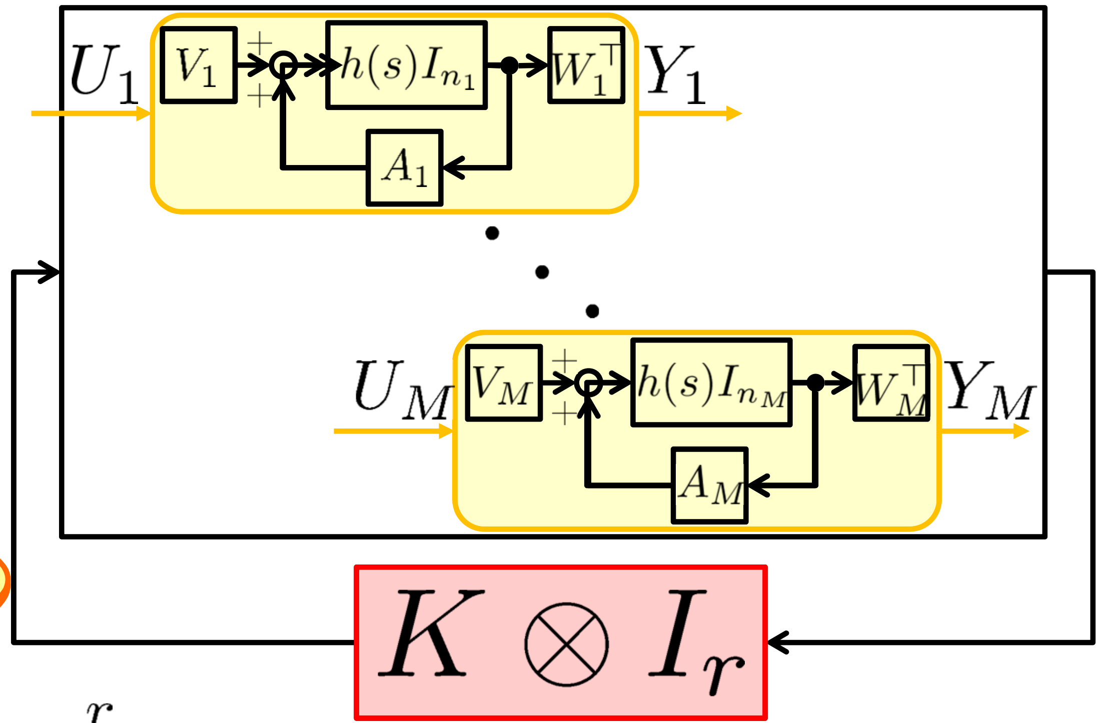
(Fujimori et. al., CDC2011)

Q10: What is a General Framework for Heterogeneous Network Synthesis *?*

Hierarchical Structure

Design K, W_k and V_k





$$\underbrace{\boldsymbol{u}^k}_{n_k} = A_k \boldsymbol{y}^k + \underbrace{V_k}_{r} U_k \qquad \underbrace{Y_k}_{r} = W_k^\top \boldsymbol{y}^k$$

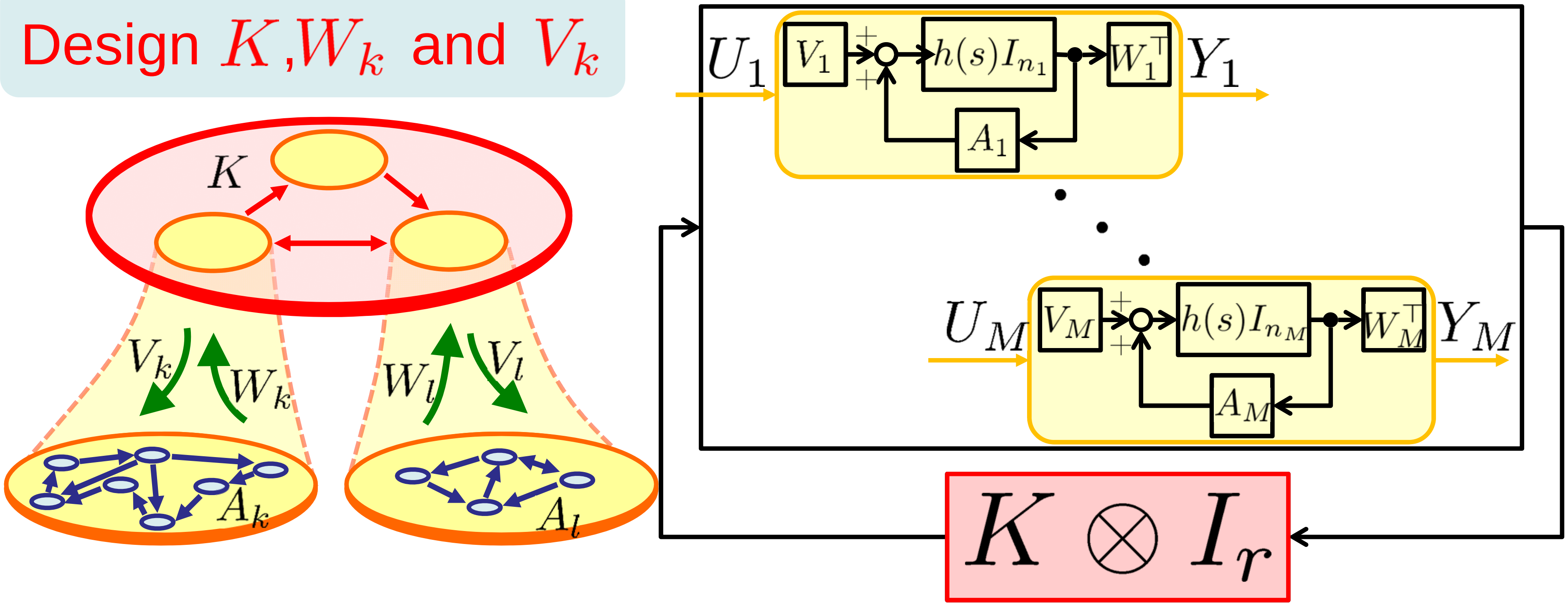
Local connection (A_k)

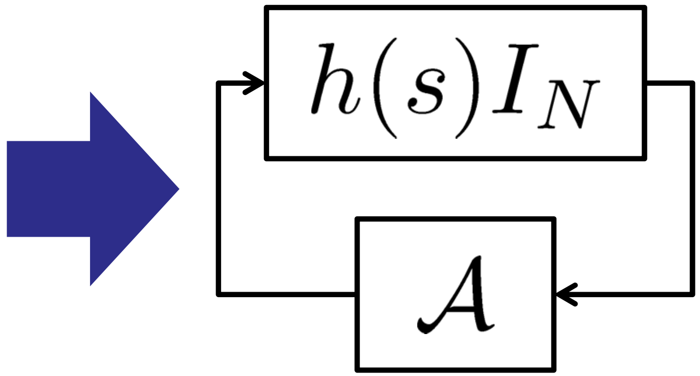
Distribution of information (V_k)

Aggregation of information ($W_k^\top$)

Hierarchical Structure

Design K, W_k and V_k

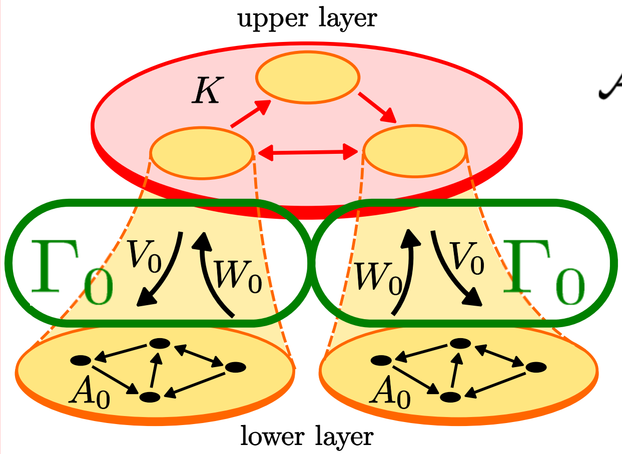




Hierarchical structure is compressed into matrix $\mathcal{A}$

Homogeneous vs Heterogeneous

Homogeneous

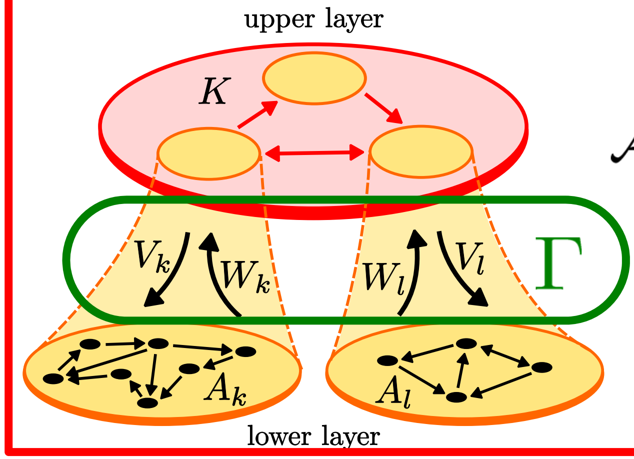


$$\mathcal{A} = \begin{bmatrix} A_0 & & \\ & \ddots & \\ & & A_0 \end{bmatrix} + \begin{bmatrix} k_{11}\Gamma_0 & \cdots & k_{1M}\Gamma_0 \\ \vdots & \ddots & \vdots \\ k_{M1}\Gamma_0 & \cdots & k_{MM}\Gamma_0 \end{bmatrix}$$

$$= I_M \otimes A_0 + K \otimes \Gamma_0 \;\; ; \;\; \Gamma_0 = V_0 W_0^\top$$

$\mathcal{A}$ can be written using Kronecker product $\otimes$.

Heterogeneous



$$\mathcal{A} = \begin{bmatrix} A_1 & & \\ & \ddots & \\ & & A_M \end{bmatrix} + \begin{bmatrix} k_{11}\Gamma_{11} & \cdots & k_{1M}\Gamma_{1M} \\ \vdots & \ddots & \vdots \\ k_{M1}\Gamma_{M1} & \cdots & k_{MM}\Gamma_{MM} \end{bmatrix}$$

$$= \mathrm{diag}\{A_k\} + K \odot \Gamma$$

We need Khatori-Rao product $\odot$.

Eigen-connection Matrix

$$\Gamma = [\Gamma_{kl}] \quad ; \quad \Gamma_{kl} := v_k w_l^\top \quad (k, l = 1, \ldots, M)$$

is *right eigen-connection matrix* of $\{A_k\}$ associated with eigenvalue $\{\lambda_{k1}\}$

Def

$\forall k = 1, \ldots, M$

$\exists \lambda_{k1} \quad \text{s.t.} \quad A_k v_k = \lambda_{k1} v_k$

(v_k is the right eigenvector associated with λ_{k1})

Analogously, *left eigen-connection matrix* can also be defined by using left eigenvector.

Theorem : Rank 1 Case

Assumption

$\forall k = 1, \dots, M$

- A_k has at least one simple eigenvalue λ_{k1}
- Γ is a right eigen-connection matrix of $\{A_k\}$ associated with eigenvalue $\{\lambda_{k1}\}$

Theorem: Rank 1

For any K, the set of all the eigenvalues of $\mathcal{A}$ is given by

$$\sigma(\mathcal{A}) = \bigcup_{k=1}^{M} \Big(\sigma(A_k) \setminus \{\lambda_{k1}\}\Big) \cup \sigma\Big(KD + \Lambda\Big)$$

$$D = \mathrm{diag}\left\{v_k^\top w_k\right\}$$

$$\Lambda = \mathrm{diag}\left\{\lambda_{k1}\right\}$$

The eigenvalues of local interconnection

The eigenvalues determined by hierarchical structure

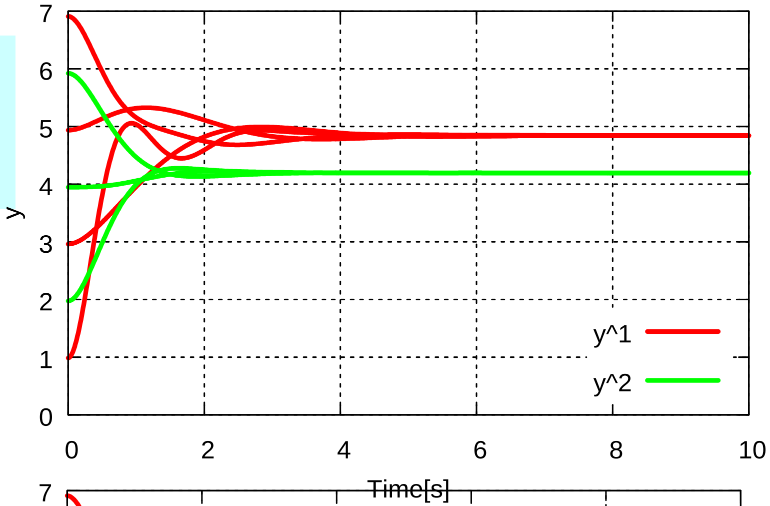
An analogous result is obtained for left eigen-connection matrices.

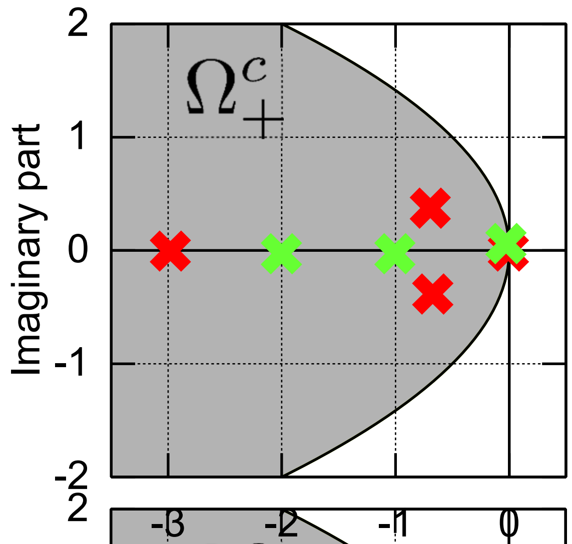
Numerical Examples (1/2)

$$h(s) = \frac{b}{s(s+a)} e^{-\tau s}\ ; \quad a = \pi,\ b = \frac{\pi^2}{2}\ ; \quad n_1 = 4, n_2 = 3$$

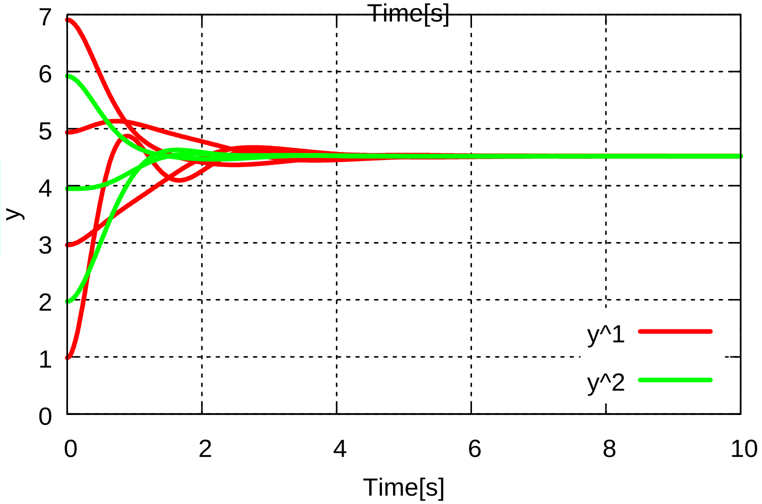
$\tau = 0$

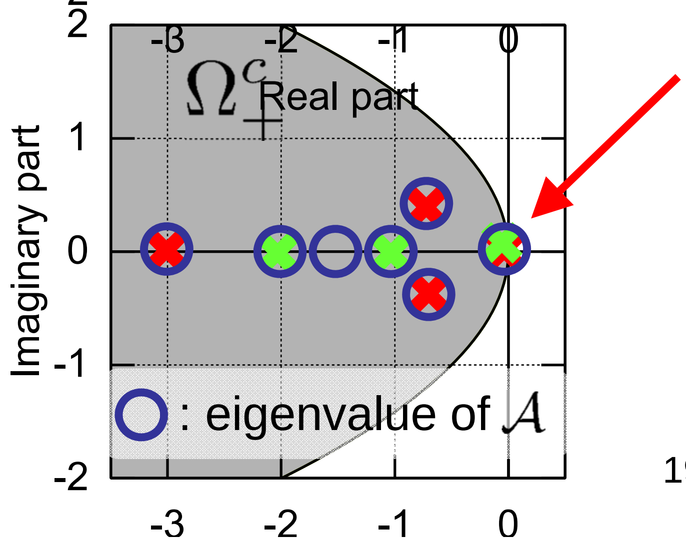
Without Control





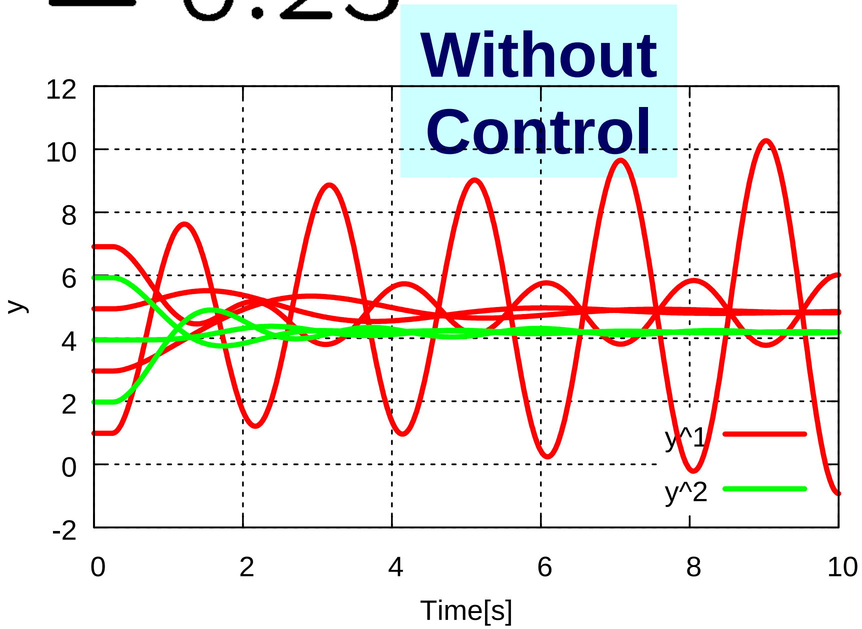
Rank 1

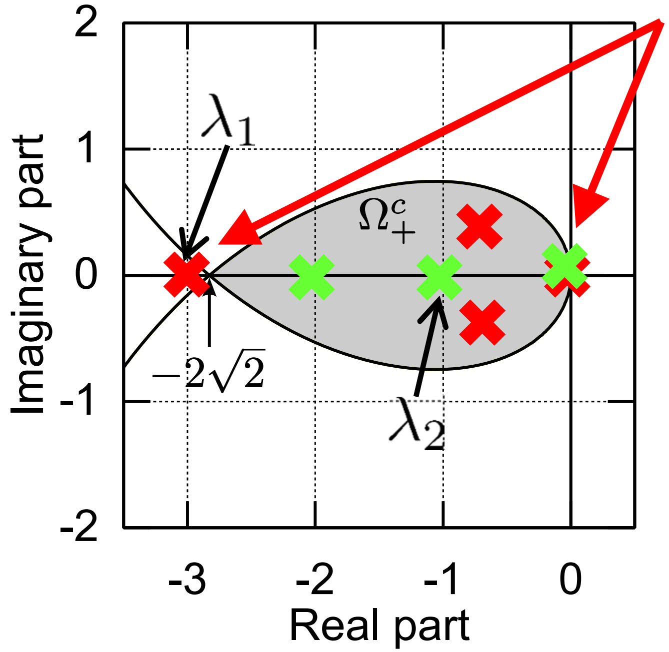


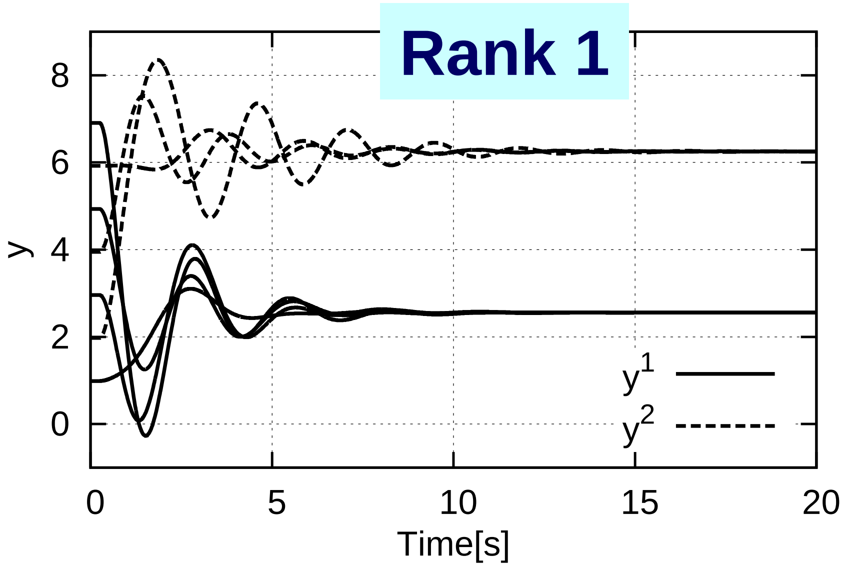


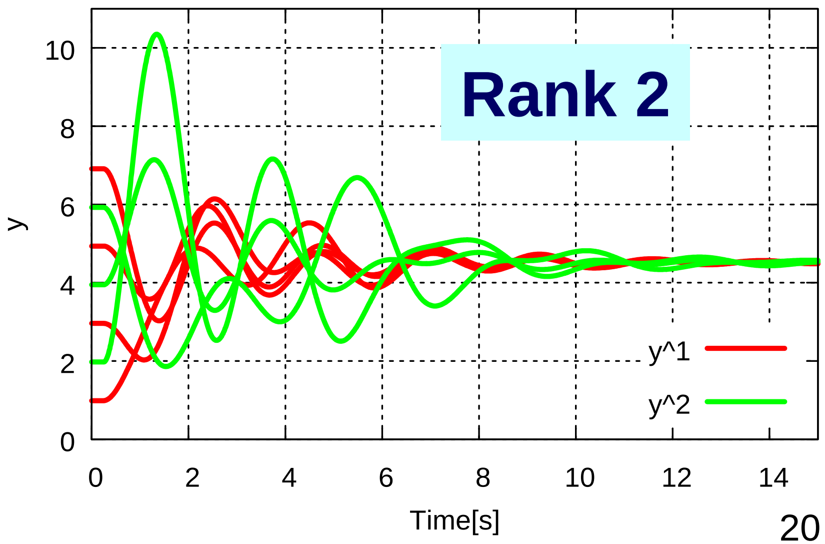
Numerical Examples (2/2)

$\tau = 0.25$









Theorem : Rank 2 Case

Assumption

$\forall k = 1, \ldots, M$

- A_k has at least two simple eigenvalues $\lambda_{k1}, \lambda_{k2}$
- Γ is a right eigen-connection matrix of $\{A_k\}$ associated with eigenvalue $\{\lambda_{k1}\}, \{\lambda_{k2}\}$

Theorem: Rank 2

For any K, the set of all the eigenvalues of $\mathcal{A}$ is given by

$$\sigma(\mathcal{A}) = \bigcup_{k=1}^{M} \Big(\sigma(A_k) \setminus \{\lambda_{k1}, \lambda_{k2}\} \Big) \cup \sigma\Big(S(K \otimes I_2)\Phi + \Lambda \Big)$$

$$S = \mathrm{diag}\{S_k\} \quad \Lambda = \mathrm{diag}\left\{ \begin{bmatrix} \lambda_{k1} & 0 \\ 0 & \lambda_{k2} \end{bmatrix} \right\} \quad \Phi = \mathrm{diag}\left\{ [w_{k1}\ w_{k2}]^\top [v_{k1}\ v_{k2}] \right\}$$

An analogous result is obtained for left eigen-connection matrices.

OUTLINE : Part 4

4. From Frat to Hierarchical

- Low-rank Interlayer Connections
- Hierarchical Consensus for Heterogeneous Networks
- **Hierarchical Adaptive Consensus**

(Fujimori et.al., SICE2011)

Stability for Dissipative Agents

Agent Dynamics — SISO (Q, S, R)-dissipative

$$\dot{x}_i = f_i(x_i) + g_i(x_i)u_i$$
$$y_i = h_i(x_i)$$

$Q = \mathrm{diag}\{Q_i\} \leq 0$,
$S = \mathrm{diag}\{S_i\}$,
$R = \mathrm{diag}\{R_i\} \geq 0$.

$V := \sum_{i=1}^{N} d_i \cdot V_i$

Theorem (LMI)

If $\exists$ a diagonal matrix $D > 0$ such that

$$A^T DRA + DSA + A^T S^T D + DQ < 0$$

holds, then the network of N interconnected (Q_i, S_i, R_i)-dissipative agents is asymptotically stable.

(Hirsch, Hara: IFAC2008)

Passive Systems：Non-hierarchical Case

Nonlinear Dynamics

$$\dot{x}_i = f_i(x_i, u_i)$$
$$y_i = h_i(x_i, u_i) \qquad i = 1, \cdots, N$$

Goal of Consensus

Conformation of Outputs

$$\forall i, j = 1, \cdots, N$$
$$\lim_{t \to \infty} (y_i(t) - y_j(t)) = 0$$

$$u = -Ly \ (L : \text{Graph Laplacian})$$

Assumption

- Passivity
- Graph -*L* is **Strongly Connected** with positive edges

⇒

Theorem

All agents achieve consensus robustly for unknown positive weights.

Passive Systems：Non-hierarchical Case

Nonlinear Dynamics

$$\dot{x}_i = f_i(\dots u_i)$$

$$y_i = h_i \dots \quad i = 1, \cdots, N$$

$$u = -Ly \;(\dots$$ …cian)

Goal of Consensus

Conformation of Outputs

$$\forall i, j = 1, \cdots, N$$

$$\lim_{t \to \infty} (y_i(t) - y_j(t)) = 0$$

Q: Does this still hold for the hierarchical case?

Assumption

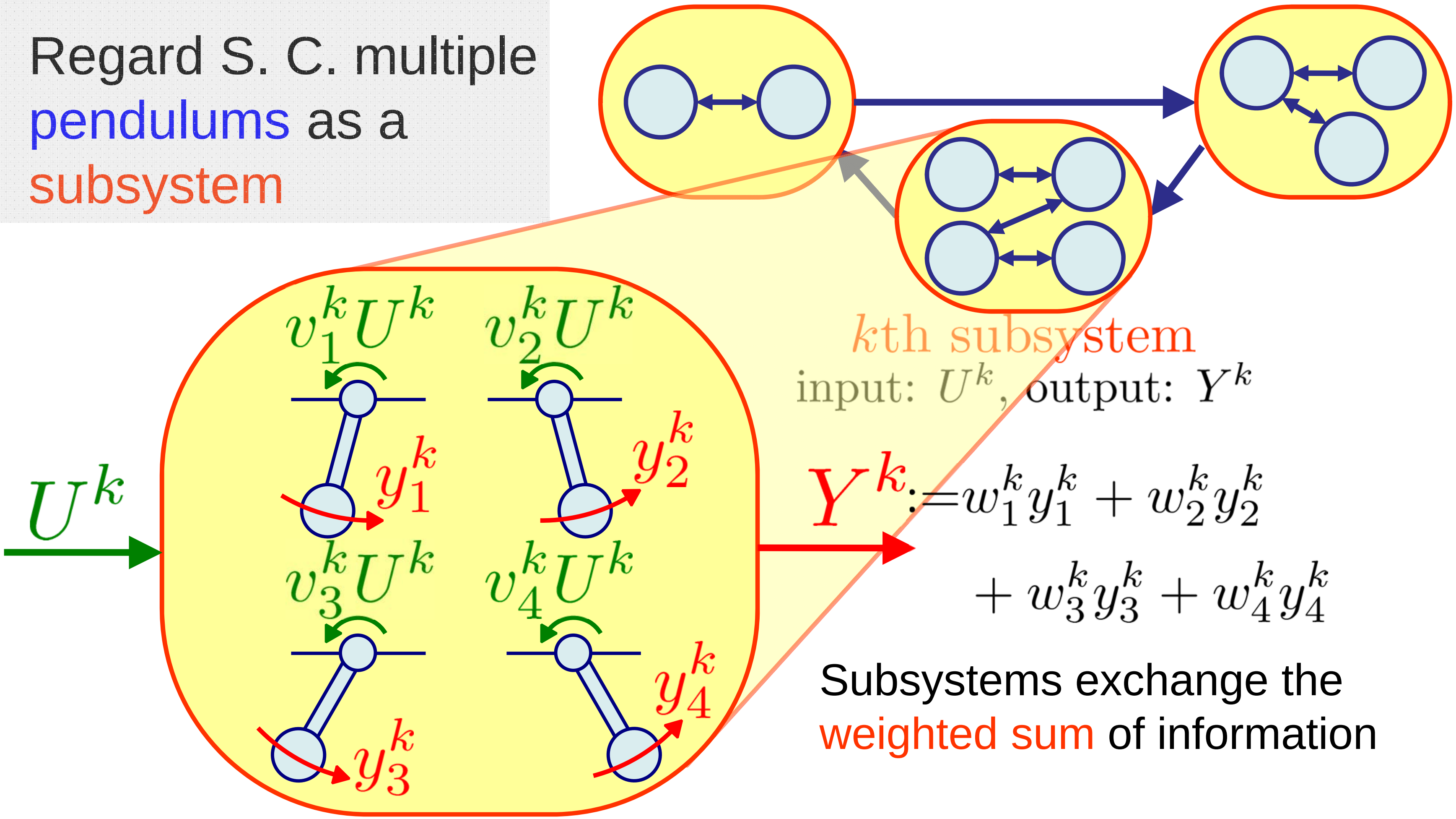
- Passivity
- Graph -*L* is **Strongly Connected** with positive edges

Theorem

All agents achieve consensus robustly for unknown positive weights.

Passive Systems : Hierarchical Case

- Regard S. C. multiple pendulums as a subsystem



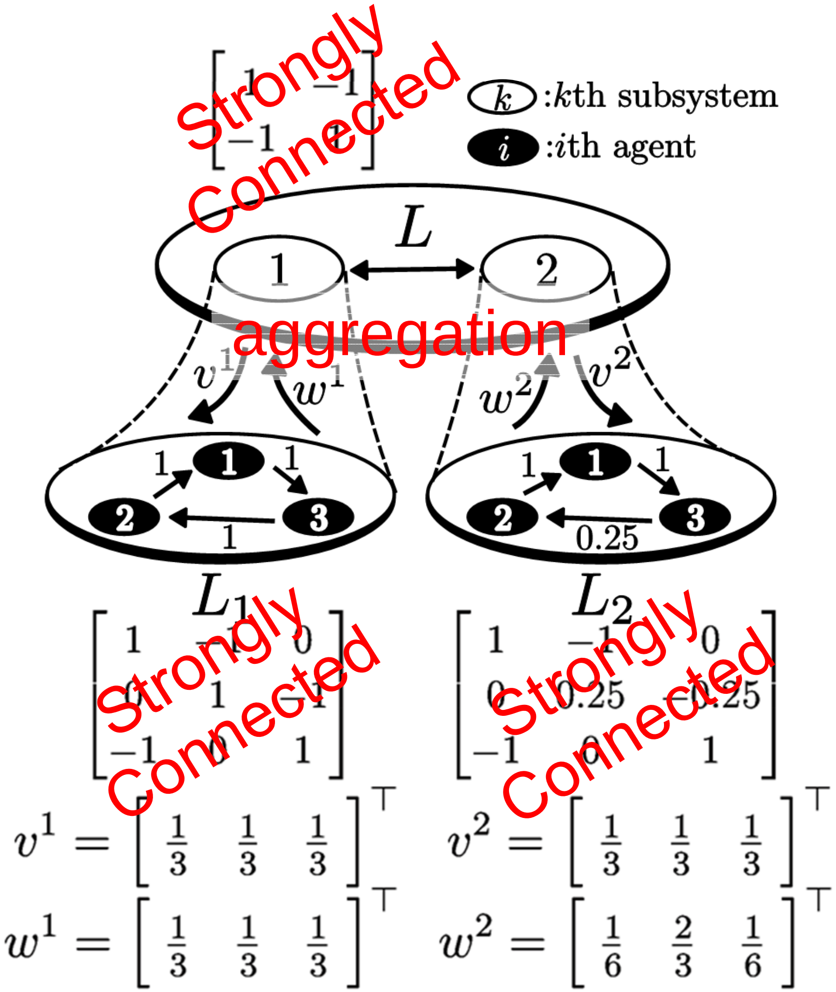
$$Y^k := w_1^k y_1^k + w_2^k y_2^k + w_3^k y_3^k + w_4^k y_4^k$$

Subsystems exchange the weighted sum of information

$\boldsymbol{v}^k = [v_1^k, v_2^k, v_3^k, v_4^k]^\top$: input weight

$\boldsymbol{w}^k = [w_1^k, w_2^k, w_3^k, w_4^k]^\top$: output weight

An Example of Hierarchical Structure



Whole Graph Laplacian

$$\begin{bmatrix} L_1 & O_3 \\ O_3 & L_2 \end{bmatrix} + \begin{bmatrix} v^1 w^{1\top} & -v^1 w^{2\top} \\ -v^2 w^{1\top} & v^2 w^{2\top} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{10}{9} & -\frac{8}{9} & \frac{1}{9} & -\frac{1}{18} & -\frac{2}{9} & -\frac{1}{18} \\ \frac{1}{9} & \frac{10}{9} & -\frac{8}{9} & -\frac{1}{18} & -\frac{2}{9} & -\frac{1}{18} \\ -\frac{8}{9} & \frac{1}{9} & \frac{10}{9} & -\frac{1}{18} & -\frac{2}{9} & -\frac{1}{18} \\ -\frac{1}{9} & -\frac{1}{9} & -\frac{1}{9} & \frac{19}{18} & -\frac{7}{9} & \frac{1}{18} \\ -\frac{1}{9} & -\frac{1}{9} & -\frac{1}{9} & \frac{1}{18} & \frac{17}{36} & -\frac{7}{36} \\ -\frac{1}{9} & -\frac{1}{9} & -\frac{1}{9} & -\frac{17}{18} & \frac{2}{9} & \frac{19}{18} \end{bmatrix}$$

There are Negative Edge!

We need another criterion

Q: How can we decide the weights $\boldsymbol{v}^k, \boldsymbol{w}^k$
so that the subsystem is passive ?

Consensus of Hierarchical Structure

Assumption

- Passivity
- subsystem: S. C.
- $\boldsymbol{v}^k, \boldsymbol{w}^k$ satisfy

$$w_i^k = \mu_i^k v_i^k$$
$$\forall i = 1, \ldots, N_k$$

Proposition

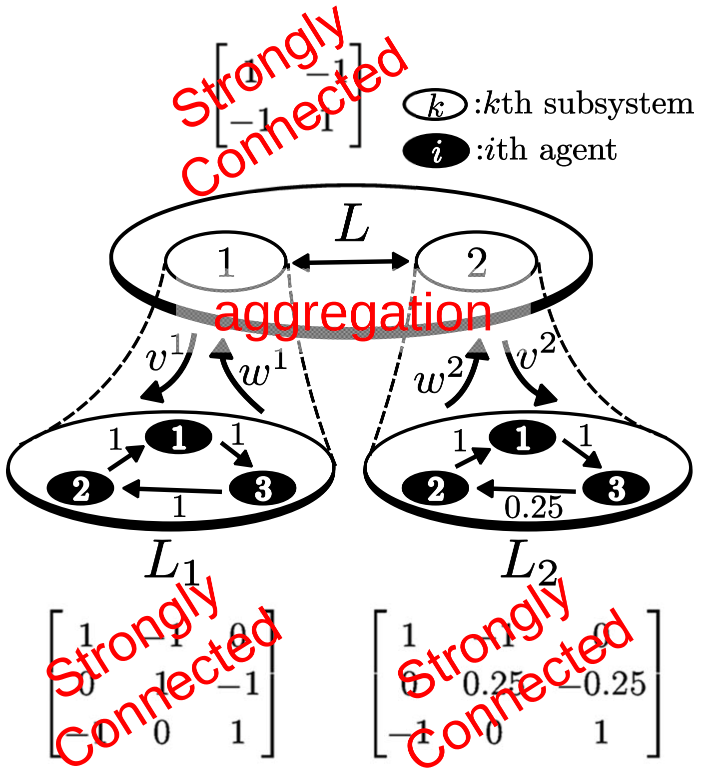
All the subsystems are Passive

The positive left eigenvector of the graph Laplacian L_k representing the connection inside the subsystem associated with the eigenvalue 0.

- Outputs of subsystems achieve consensus
- Agents in each subsystem achieve consensus

When sums of output weights are coincident, all the agents achieve consensus.

Numerical Simulation (1/2)



Case1)

The Assumption is satisfied

$$v^1 = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix}^\top \quad v^2 = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix}^\top$$

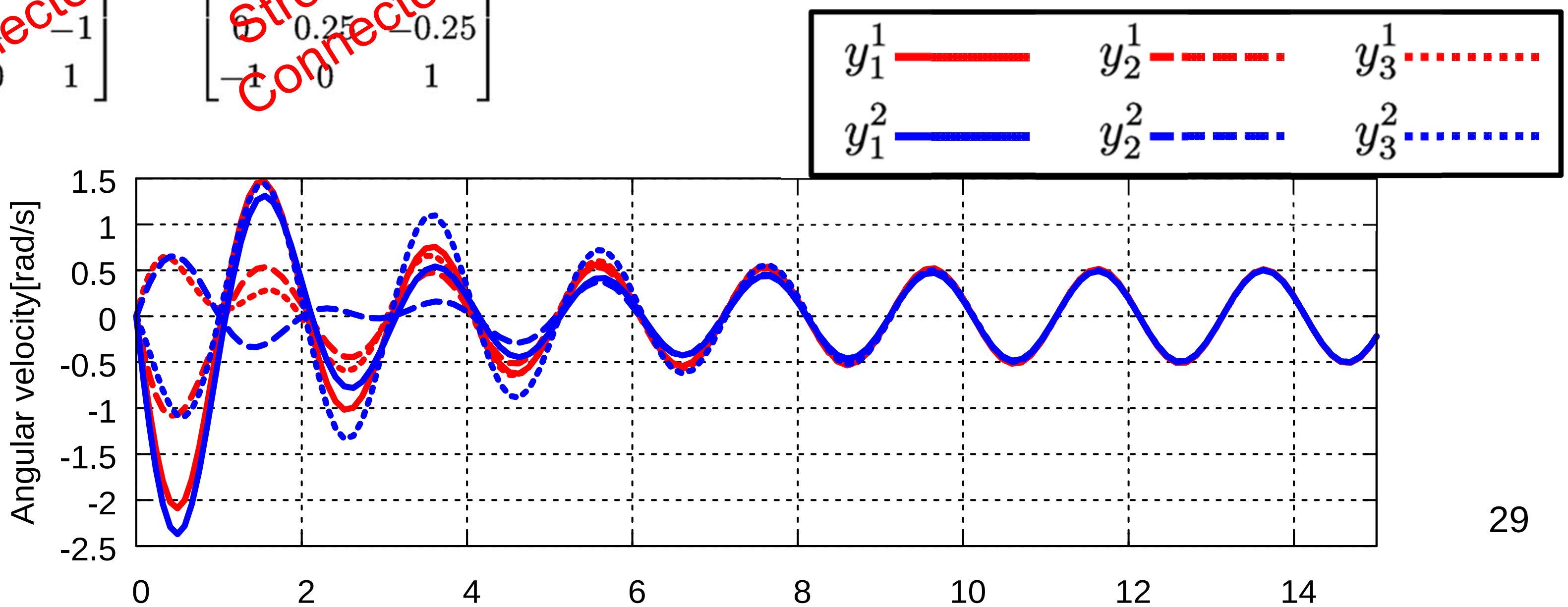
$$\times \quad \times \quad \times$$

$$\mu^1 = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}^\top \quad \mu^2 = \begin{bmatrix} \frac{1}{2} & 2 & \frac{1}{2} \end{bmatrix}^\top$$

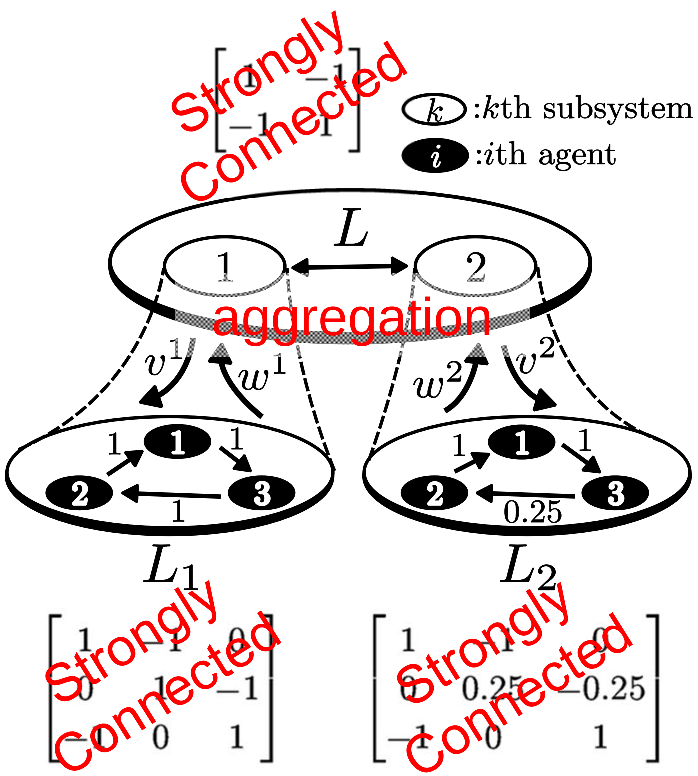
$$\| \quad \| \quad \|$$

$$w^1 = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix}^\top \quad w^2 = \begin{bmatrix} \frac{1}{6} & \frac{2}{3} & \frac{1}{6} \end{bmatrix}^\top$$

Sum of output weight is equivalent



Numerical Simulation (2/2)



Case2)

$$\boldsymbol{v}^1 = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix}^\top$$

$$\times \quad \times \quad \times$$

$$\boldsymbol{\mu}^1 = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}^\top$$

$$\parallel \quad \parallel \quad \parallel$$

$$\boldsymbol{w}^1 = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix}^\top$$

The Assumption is not satisfied

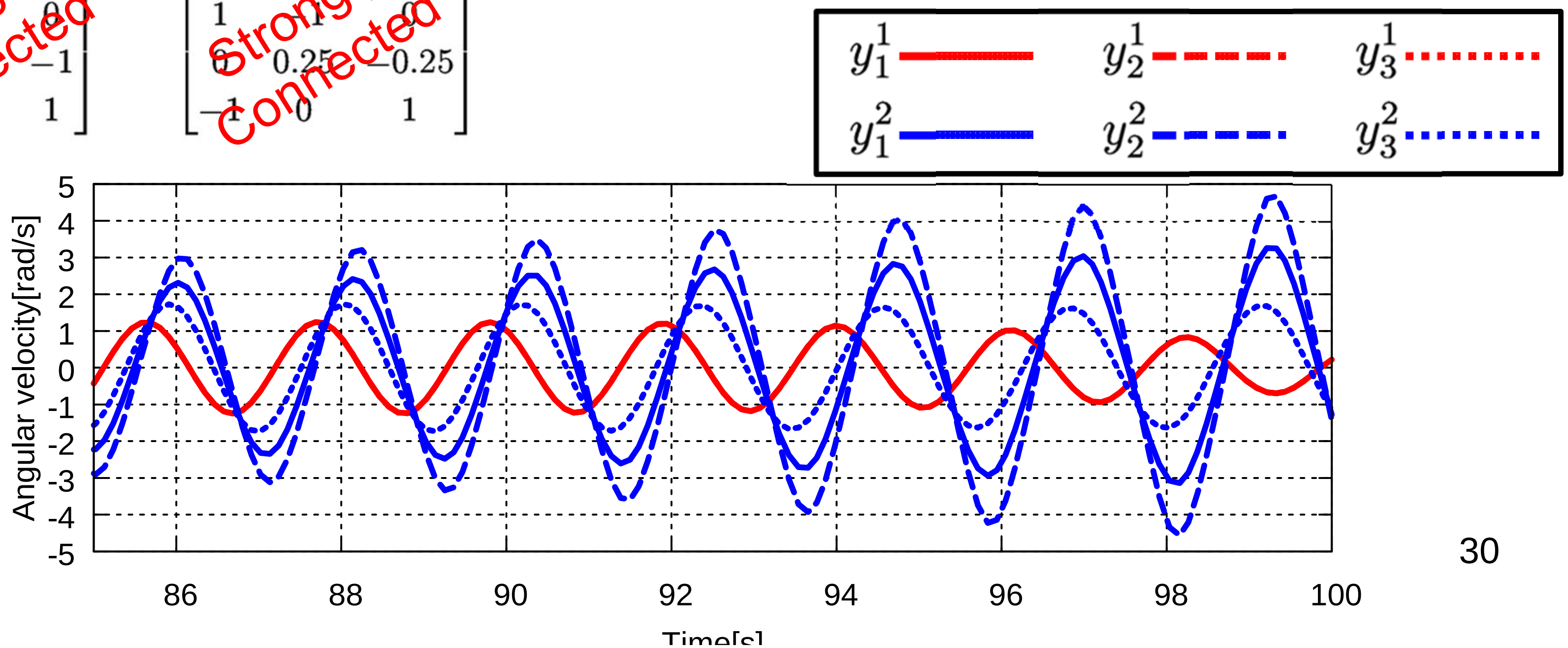
$$\boldsymbol{v}^2 = \begin{bmatrix} 1 & -1 & 1 \end{bmatrix}^\top$$

$$\times \quad \times \quad \times$$

$$\boldsymbol{\mu}^2 = \begin{bmatrix} \frac{1}{2} & 2 & \frac{1}{2} \end{bmatrix}^\top$$

$$\nparallel \quad \nparallel \quad \nparallel$$

$$\boldsymbol{w}^2 = \begin{bmatrix} 1 & -1 & 1 \end{bmatrix}^\top$$



Three Messages

① **Low rank interlayer connections are quite helpful for rapid consensus.** ***aggregation***

② **Heterogeneous agents:** ***Khatri-Rao Product***
hierarchical network synthesis based on left eigenvectors

③ **Nonlinear agents:** ***left eigenvector***
strongly connected graph in the upper layer
+ subsystems which can be passive

Messages：A New Framework

① **LTI system with generalized freq. variable**

a proper class of homogeneous multi-agent systems

② **Three types of stability tests, namely graphical, algebraic, and numeric (LMI)**

powerful tools for analysis

③ **From Homogeneous to Heterogeneous**

robust stability analysis (Hinf norm condition)

④ **From Flat to Hierarchical Structure**

low-rank interlayer connection (aggregation & distribution)

How to Design Decentralized Control Systems Systematically ?