Lecture Series, TU Munich
October 22, 29 & November 5, 2013

Glocal Control for Hierarchical Dynamical Systems

Theoretical Foundations with Applications in Energy Networks

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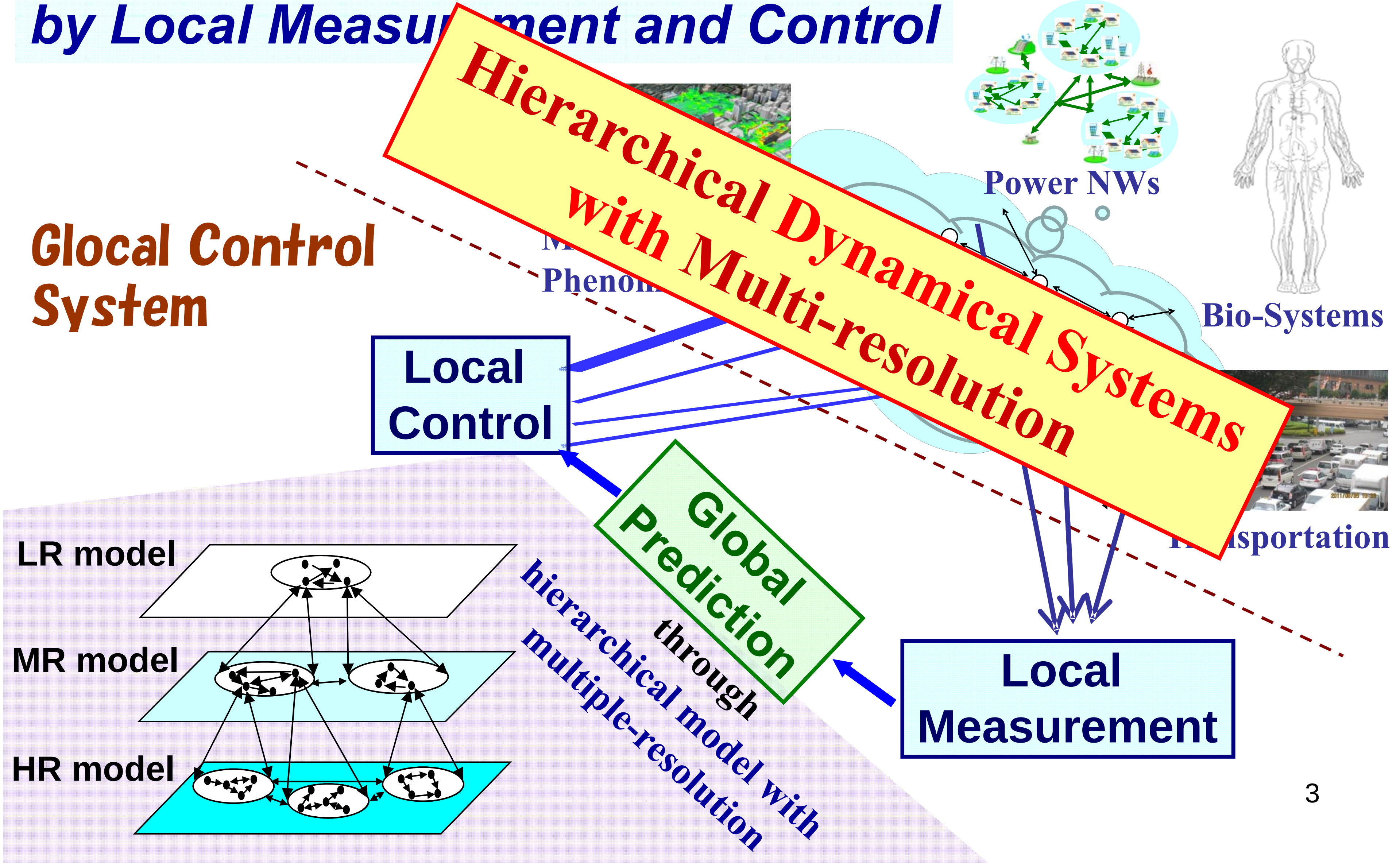
Outline

1. Glocal Control & Energy Networks
2. A Unified Framework for Networked Dynamical Systems with Stability Analysis
3. **From Homogeneous to Heterogeneous**
4. From Frat to Hierarchical
5. Decentralized Hierarchical Control Synthesis
6. Applications in Energy Networks

Framework for Glocal Control

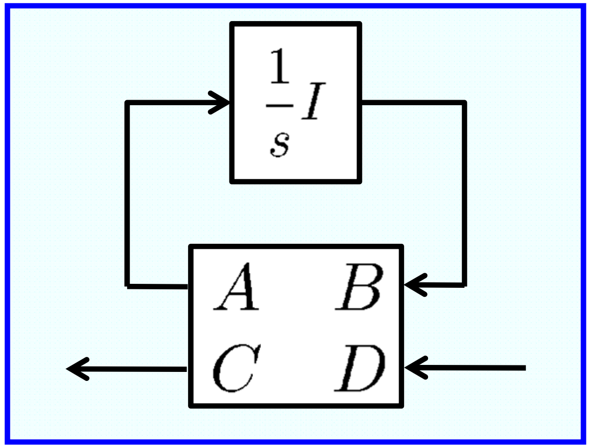
Realization of Global Functions
by Local Measur ment and Control



LTI System with Generalized Frequency Variable

A unified representation for multi-agent dynamical systems

$C\,(sI - A)^{-1}\,B + D$

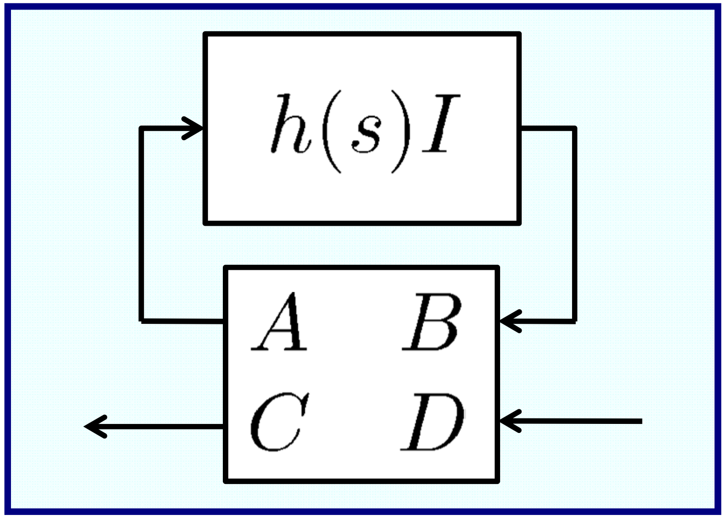


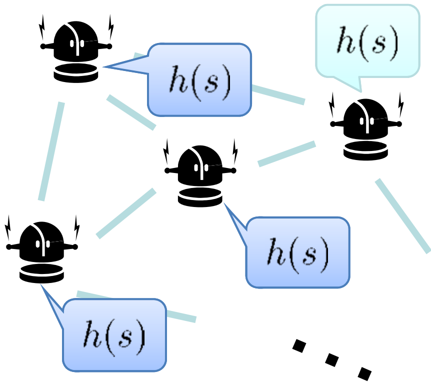
1/s → h(s)

Φ(s) = 1/h(s)

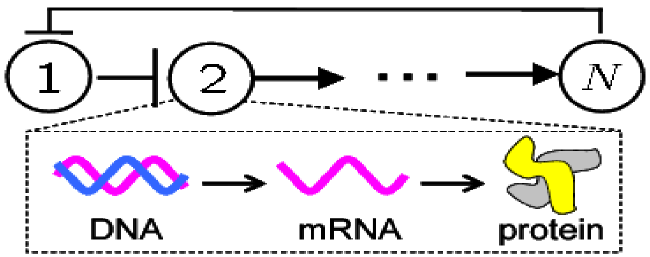
Generalized
Freq. Variable

$C\,(\phi(s)I - A)^{-1}\,B + D$





Group Robot



Gene Reg. Networks

Dynamics
+
Information Structure

Messages : A New Framework

① **LTI system with generalized freq. variable**

a proper class of homogeneous multi-agent dynamical systems

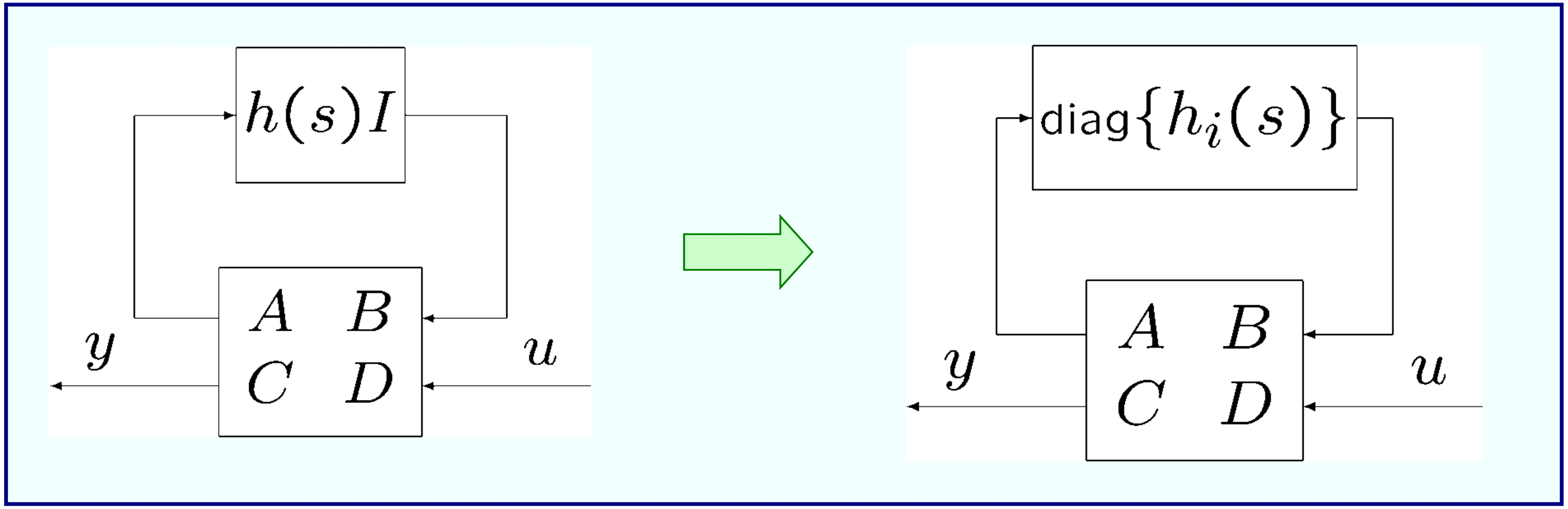
② **Three types of stability tests, namely graphical, algebraic, and numeric (LMI)**

powerful tools for analysis

Q3: from **Homogeneous**
to **Heterogeneous *?***

Q4: from **Flat Structure**
to **Hierarchical Structure *?***

New Framework for System Theory
from Homogeneous to Heterogeneous



OUTLINE : Part 3

3. From Homogeneous to Heterogeneous

- **Robust Stability Analysis**
- **Nonlinear Stability Analysis**

OUTLINE : Part 3

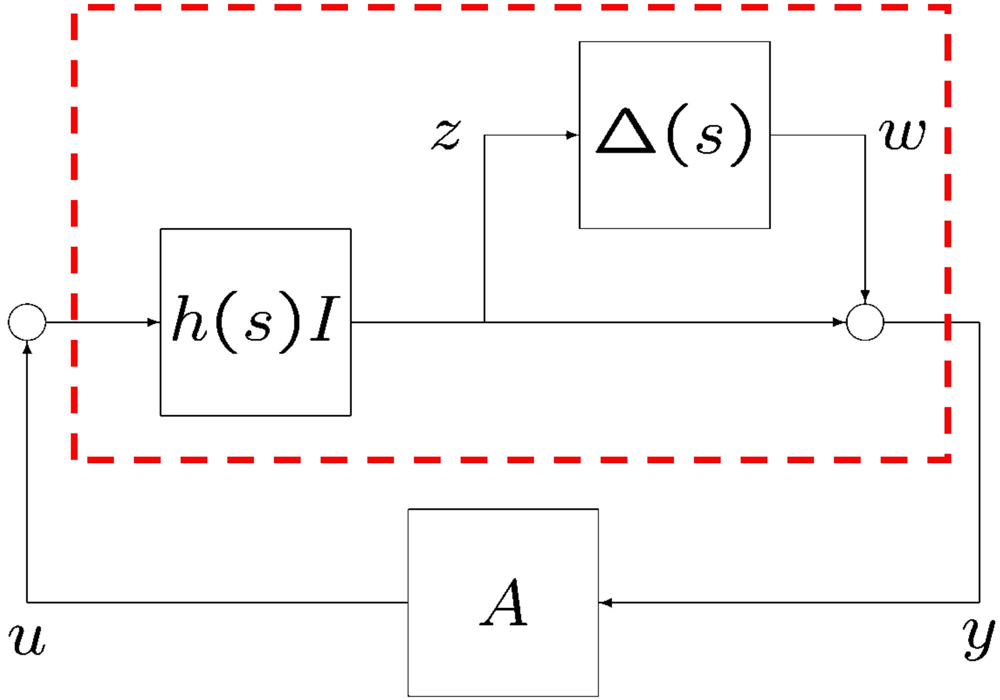
3. From Homogeneous to Heterogeneous

- **Robust Stability Analysis**
- Nonlinear Stability Analysis

(Hara et al.: CDC2010)

From Homogeneous to Heterogeneous

$$\tilde{H}(s) = (I + \Delta(s)) \cdot h(s)$$



Nominal system: homogeneous

$$h_i(s) = (1 + \delta_i(s))h(s)$$

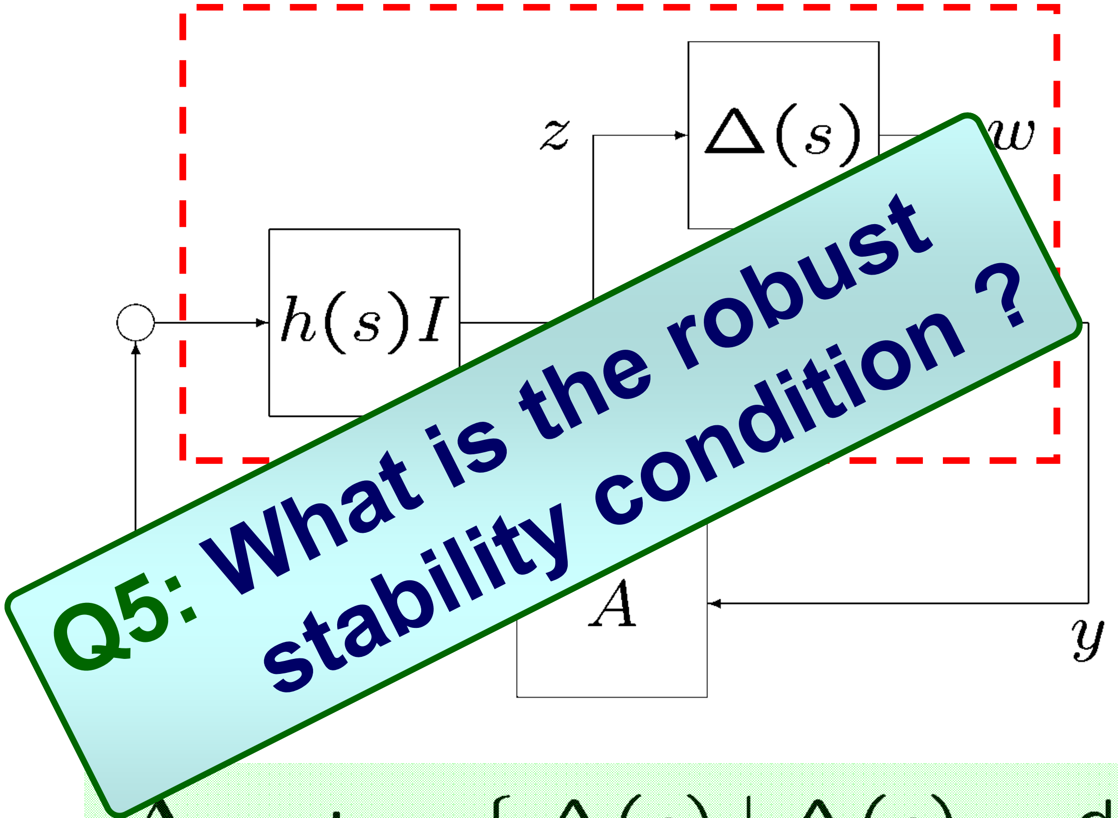
Independent perturbations

$$\Delta_{d\gamma} := \{ \Delta(s) \mid \Delta(s) = \text{diag}\{\delta_i(s)\}, \|\Delta(s)\|_\infty \leq 1/\gamma \}$$

From Homogeneous to Heterogeneous

$$\tilde{H}(s) = (I + \Delta(s)) \cdot h(s)$$

Nominal system: homogeneous



$$h_i(s) = (1 + \delta_i(s))h(s)$$

Independent perturbations

$$\Delta_{d\gamma} := \{ \Delta(s) \mid \Delta(s) = \mathrm{diag}\{\delta_i(s)\}, \|\Delta(s)\|_\infty \leq 1/\gamma \}$$

Three Classes of Perturbations

Multiplicative Perturbation:

$$\tilde{H}(s) = (I + \Delta(s)) \cdot h(s)$$

Three Classes:

Full perturbation:

$$\mathbf{\Delta}_\gamma \;:=\; \{\, \Delta(s) \in \mathbf{\Delta}_p \mid \; \|\Delta\|_\infty \le 1/\gamma \,\}$$

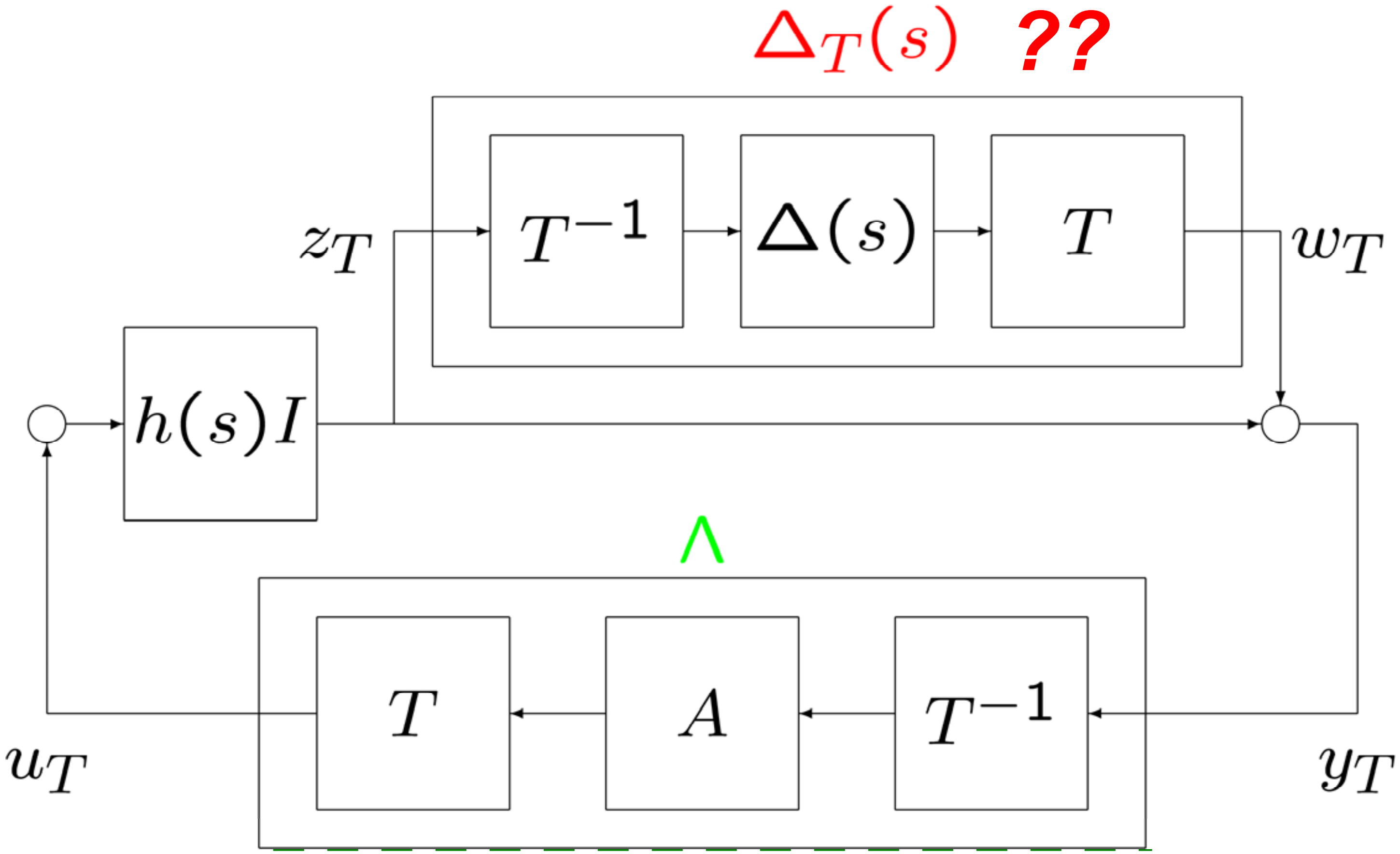
Heterogeneous:

$$\mathbf{\Delta}_{d\gamma} \;:=\; \{\, \Delta(s) \in \mathbf{\Delta}_\gamma \mid \Delta(s) \;:\; \text{diagonal} \,\}$$

Homogeneous:

$$\mathbf{\Delta}_{I\gamma} \;:=\; \{\, \Delta(s) \in \mathbf{\Delta}_\gamma \mid \Delta(s) = \delta(s) I \,\}$$

Basic Idea

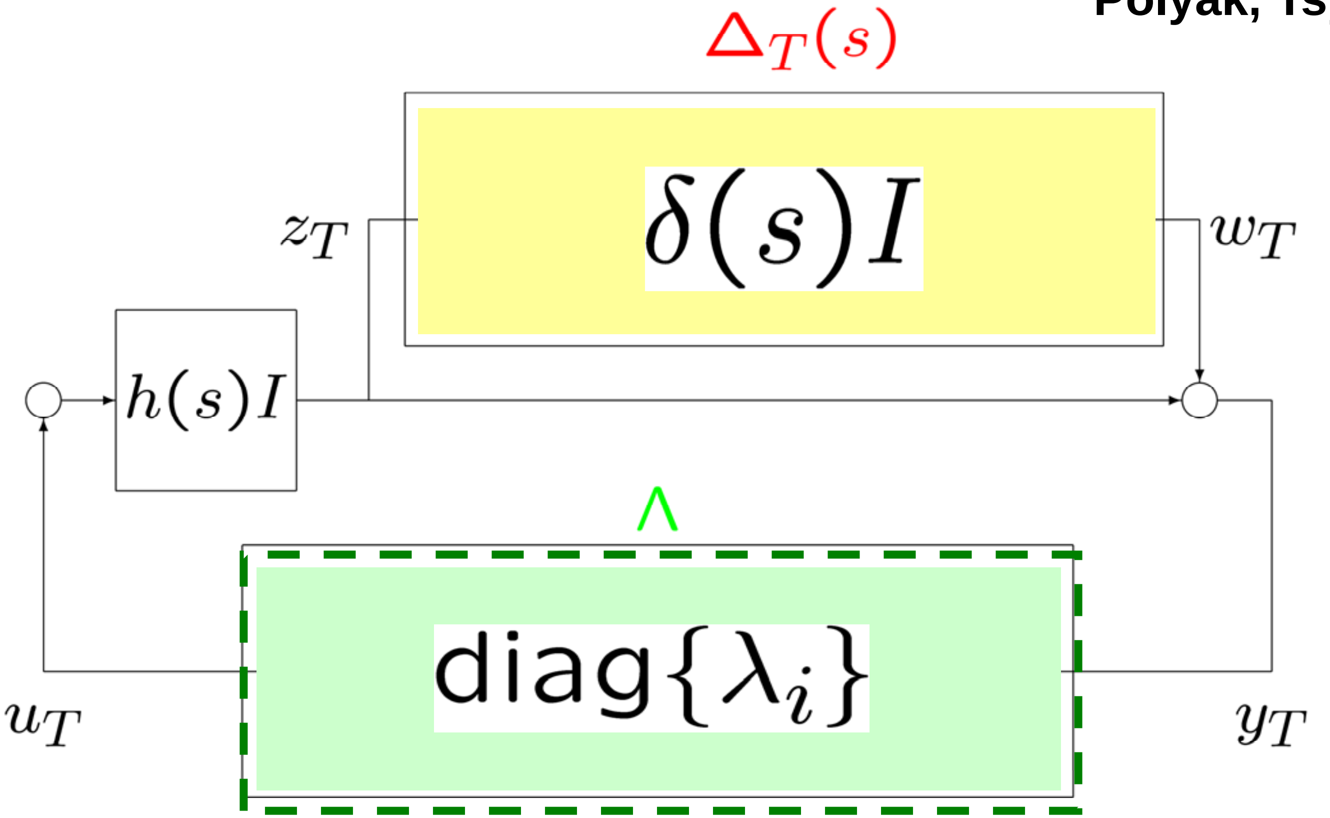


$$\Lambda := TAT^{-1} = \mathrm{diag}\{\lambda_i\}$$

***A*: diagonalizable**

Homogeneous Perturbations

Polyak, Tsypkin (1996)



$$w_T \to z_T \ : \ \text{diag}\{\frac{\lambda_i h(s)}{1-\lambda_i h(s)}\}$$

Complementary Sensitivity function (*h(s)*, *λi*)

Robust Stability Condition for Homogeneous Perturbations

$$\tilde{H}(s) = (1 + \delta(s)) \cdot h(s) I$$

⇓ Small Gain Criterion

Theorem: The following three conditions are equivalent.

(i) The system is robustly stable for $\mathbf{\Delta}_{I\gamma}$.

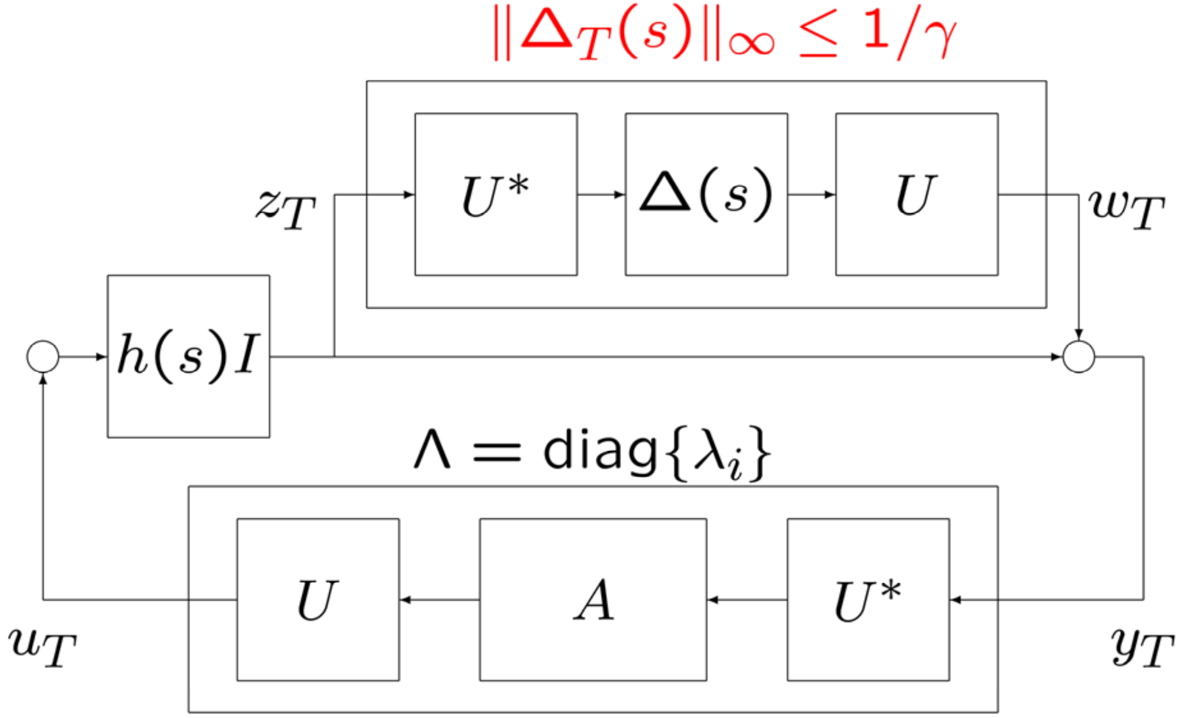
(ii) $\left\| \dfrac{\lambda h}{1 - \lambda h} \right\|_\infty < \gamma, \quad \forall\ \lambda \in \sigma(A)$

(iii) $\left| \dfrac{\lambda}{\phi - \lambda} \right| < \gamma, \quad \forall\ \lambda \in \sigma(A),$

$$\forall\ \phi \in \mathbf{\Phi} := \{1/h(j\omega) |\ \omega \in \mathbb{R}\ \}.$$

A : Normal ($T = U$: Unitary Matrix)

$A \in \mathbb{R}^{n \times n}$ is normal, i.e., $A^T A = A A^T$.



* Symmetric
* Skew - Symmetric
* Circulant

Sufficiency: small gain condition

Necessity: worst case $\Delta(s) = \delta(s)I$

Robust Stability Condition for Full Perturbations

Hara, Tanaka, Iwasaki (ACC2010)

Assumption

$A \in \mathbb{R}^{n \times n}$ is normal, i.e., $A^T A = A A^T$.

Theorem: The following three conditions are equivalent.

(i) The system is robustly stable for $\mathbf{\Delta}_\gamma$.

(ii) $\left\| \dfrac{\lambda h}{1 - \lambda h} \right\|_\infty < \gamma, \ \ \forall \ \lambda \in \sigma(A)$

(iii) $\left| \dfrac{\lambda}{\phi - \lambda} \right| < \gamma, \ \ \forall \ \lambda \in \sigma(A),$

$$\forall \ \phi \in \mathbf{\Phi} := \{ 1/h(j\omega) | \ \omega \in \mathbb{R} \ \}.$$

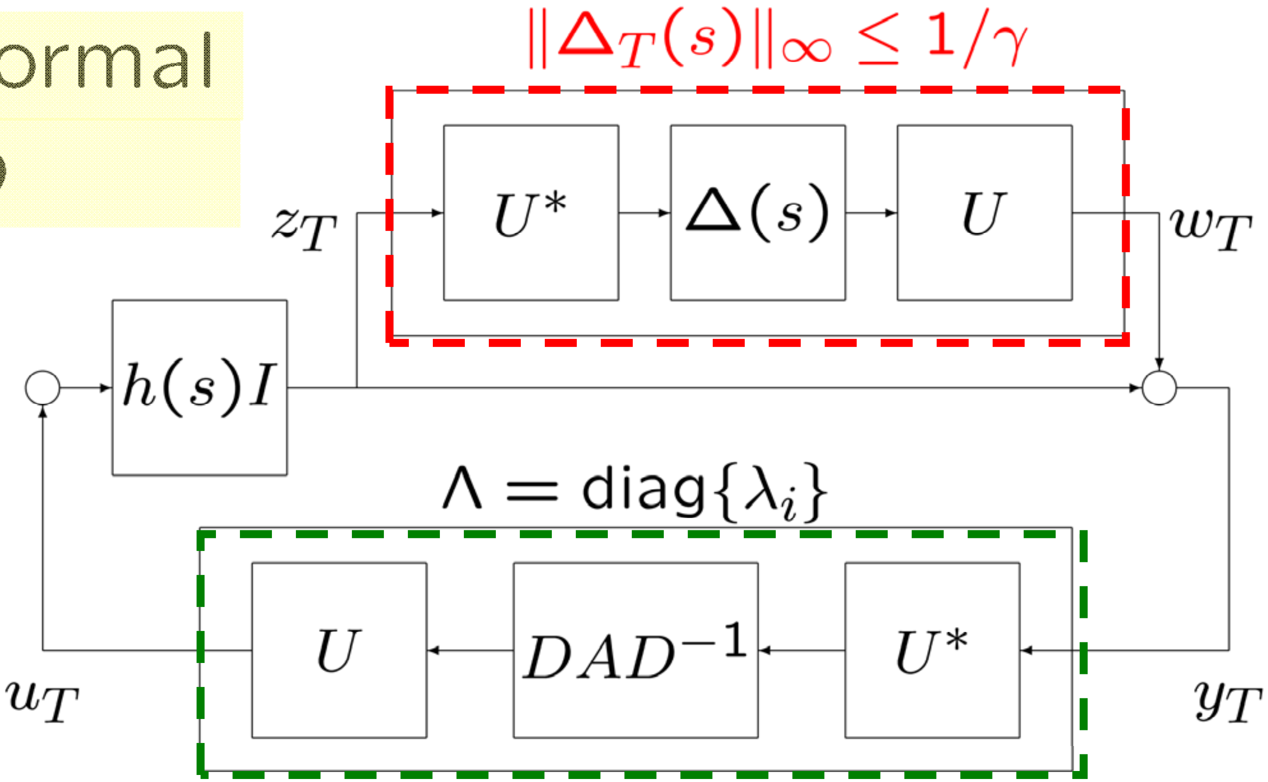
Heterogeneous Perturbations

$$\Delta(s) = \mathrm{diag}\{\delta_i(s)\}$$

$$^\forall D = \mathrm{diag}\{d_i\} > 0 \text{ s.t. } D\Delta(s)D^{-1} = \Delta(s)$$

DAD^{-1} is normal

$$T = UD$$



Robust Stability Condition for Heterogeneous Perturbations

Assumption (Hara et al.: CDC2010)

$^{\exists}D$: diagonal s.t. DAD^{-1} is normal

Symmetric
Circulant

Theorem: The following conditions are equivalent.

(i) The system is robustly stable for $\mathbf{\Delta}_{d\gamma}$.

(ii) $$\left\| \frac{\lambda h}{1 - \lambda h} \right\|_{\infty} < \gamma, \ \forall \ \lambda \in \sigma(A)$$

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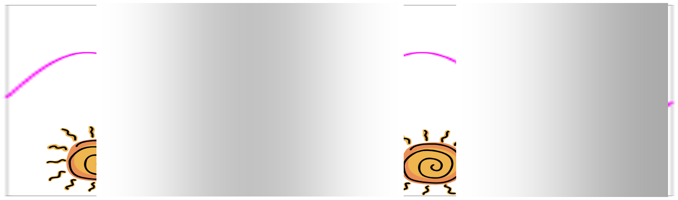
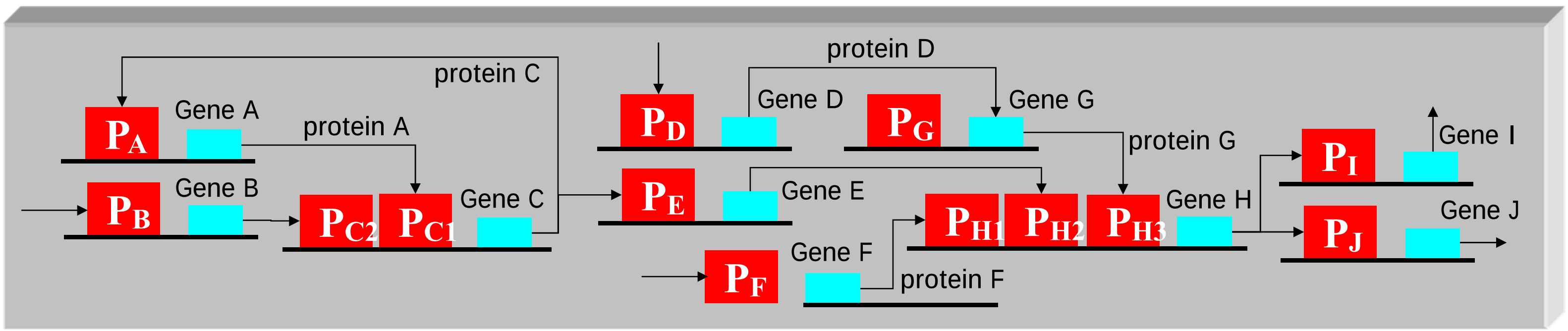
$$\forall \ \phi \in \mathbf{\Phi} := \{1/h(j\omega) | \ \omega \in \mathbb{R} \ \}.$$

An Application : Biological rhythms

Motivation

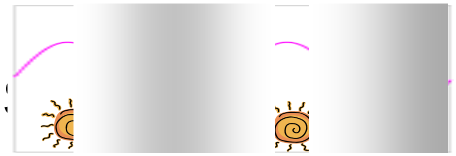


- **Biological rhythms**
  – 24h-cycle, heart beat, sleep cycle etc.
  – caused by periodic oscillations of protein concentrations in ***Gene Regulatory Networks***
- **Medical and engineering applications**
  – Artificially engineered biological oscillators
    (e.g.) Repressilator [Elowitz & Leibler, *Nature*, 2000]

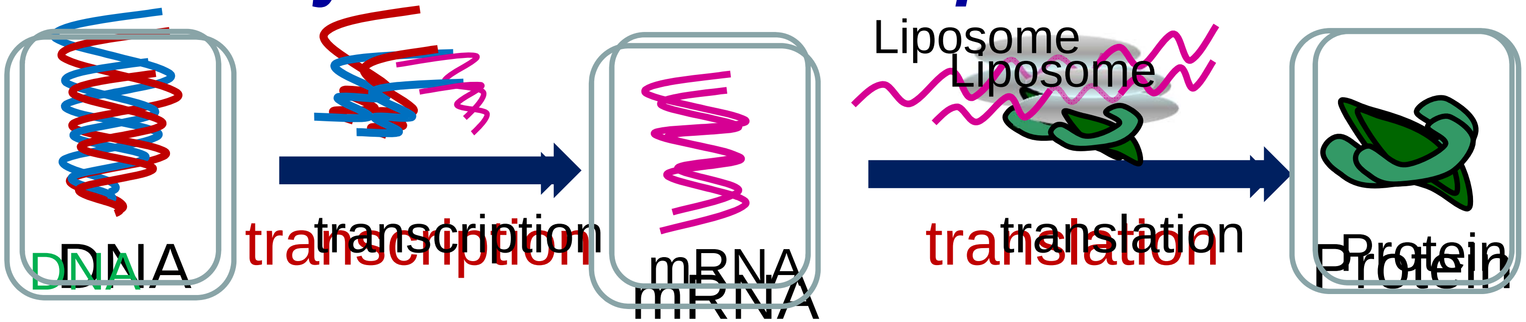


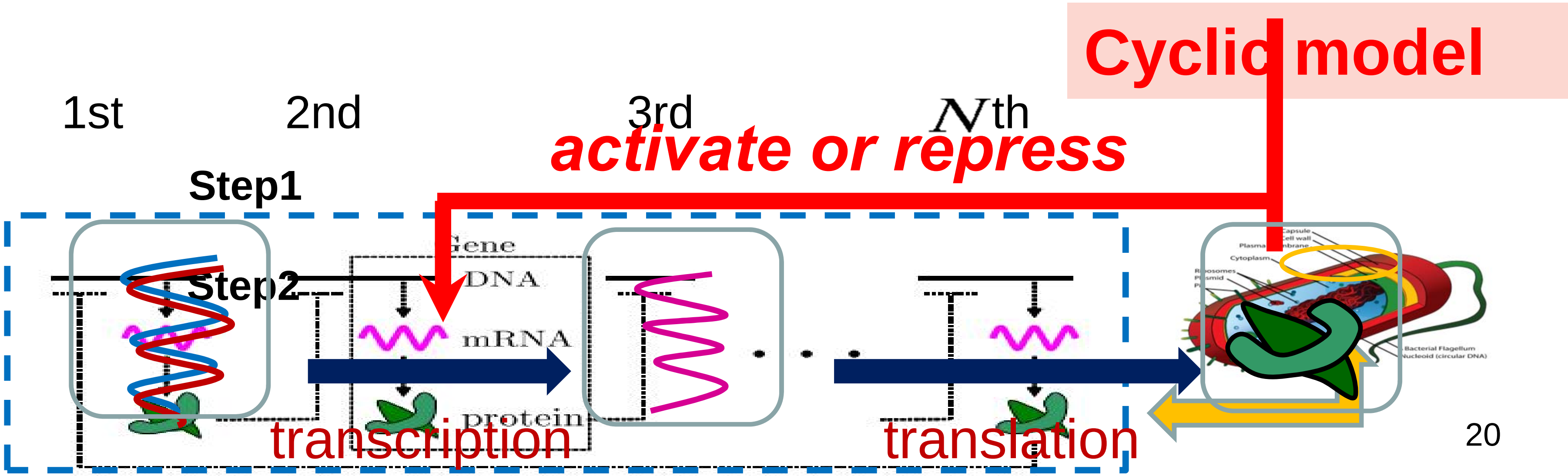
Gene Regulatory Network Systems

- **Biological rhythms:** 24h-cycle, heart beat
  periodic oscillations of protein concentrations
  in ***Gene Regulatory Networks***



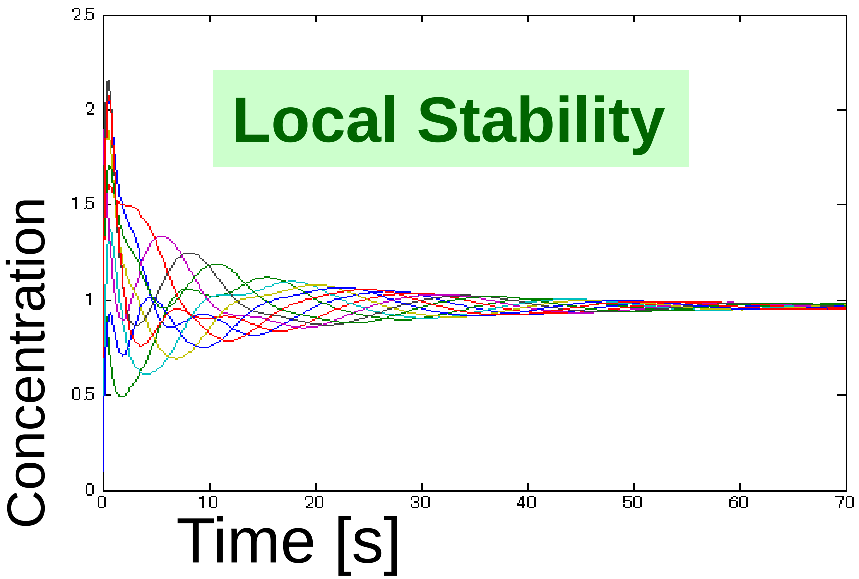
- ***Protein synthesis : transcription & translation***

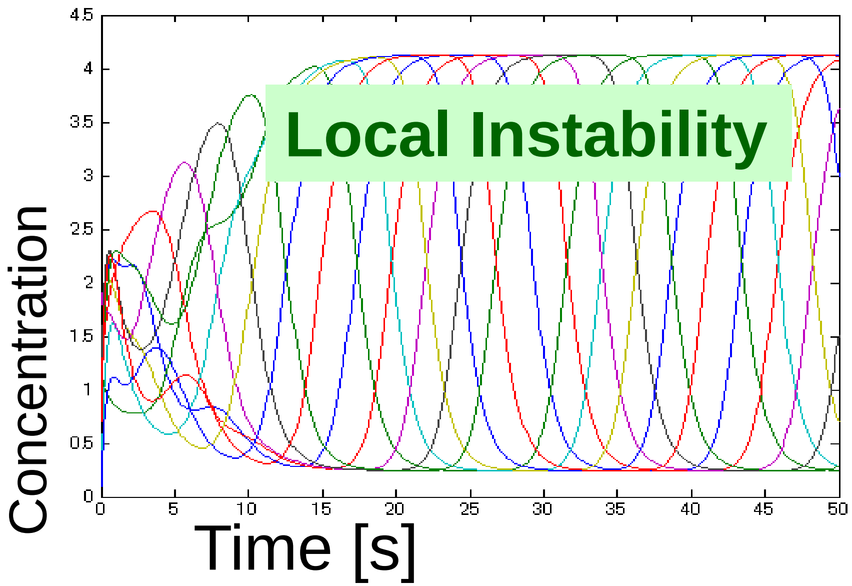




Convergence or Oscillations ?

- **Numerical simulations**
  – Changing chemical parameters





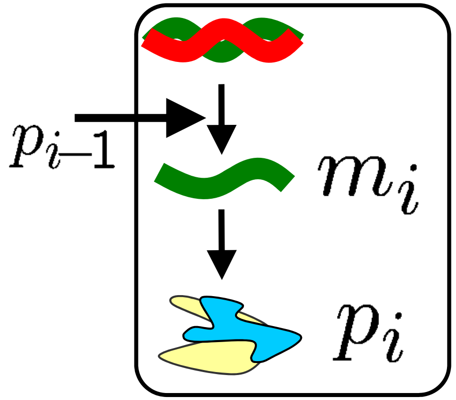
What are the conditions for convergence and the existence of oscillations ?

Nonlinear Analysis

Gene Regulatory Network Model

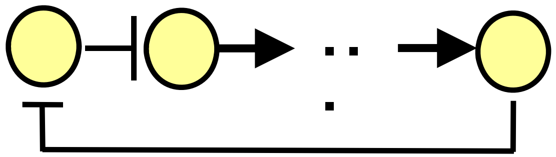
gene model $(i = 1, \cdots, N)$

$$\frac{d}{dt}\begin{bmatrix} r_i \\ p_i \end{bmatrix} = \begin{bmatrix} -a_i & 0 \\ c_i & -b_i \end{bmatrix}\begin{bmatrix} r_i \\ p_i \end{bmatrix} + \begin{bmatrix} \beta_i \\ 0 \end{bmatrix} f_i(p_{i-1})$$



$a_i, b_i > 0$: Degradation rates (1/Time constants)

$c_i, \beta_i > 0$: Production rates



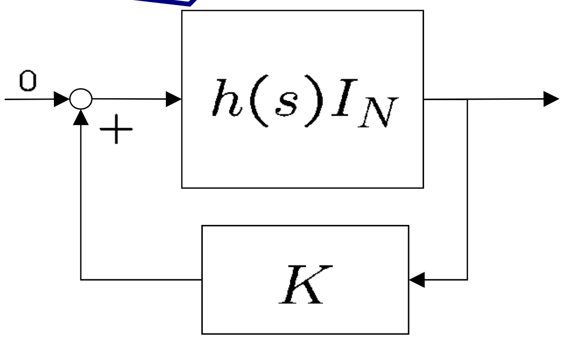
$f_i(p_{i-1})$: Hill function

$$f_i(p_{i-1}) := \begin{cases} \dfrac{p_{i-1}^{\nu}}{1 + p_{i-1}^{\nu}} & \text{(Mono. increasing for activation)} \\ \dfrac{1}{1 + p_{i-1}^{\nu}} & \text{(Mono. decreasing for repression)} \end{cases}$$

Linearized Gene Network Model

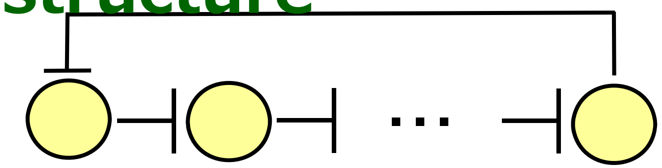
Each gene's dynamics

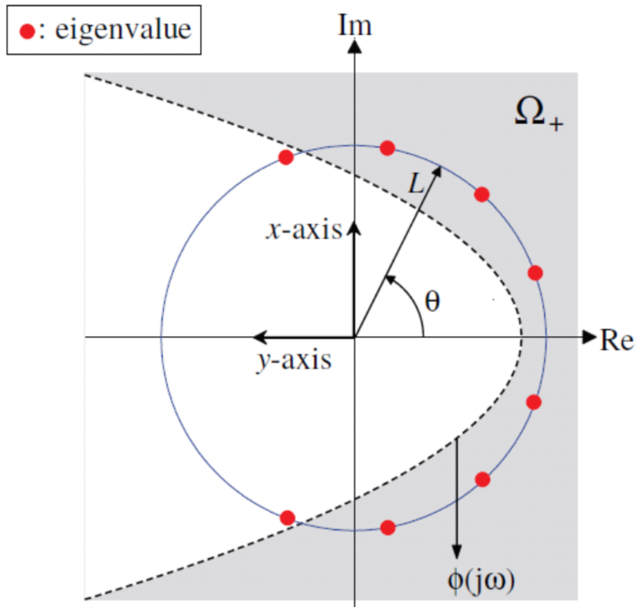
$$h(s) := \frac{1}{(T_a s + 1)(T_b s + 1)}$$

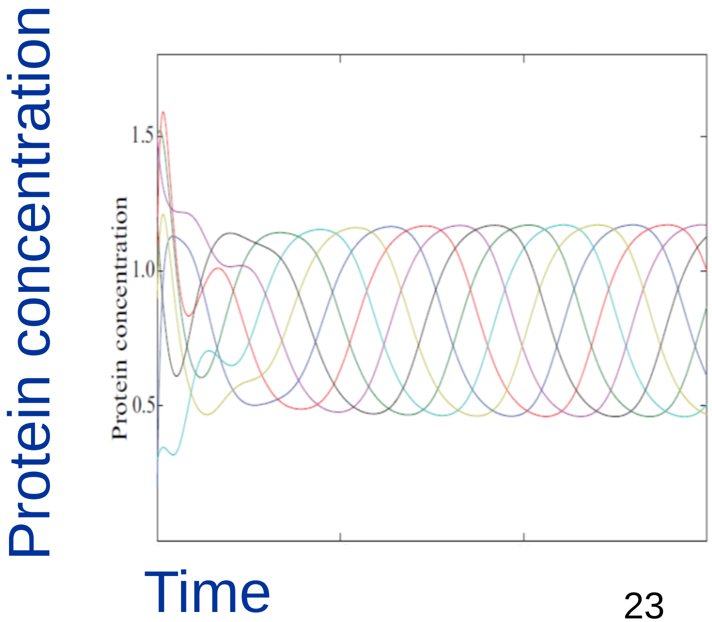


$$\begin{bmatrix} 0 & 0 & 0 & \cdots & R_1^2 f_1'(p_N^*) \\ R_2^2 f_2'(p_1^*) & 0 & 0 & \cdots & 0 \\ 0 & R_3^2 f_3'(p_2^*) & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & R_N^2 f_N'(p_{N-1}^*) & 0 \end{bmatrix}$$

Interaction structure

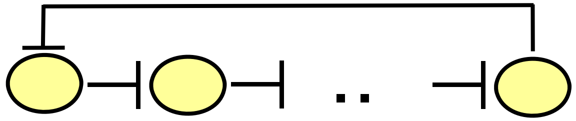






Analytic Criteria

- **Assumptions:**
  - All interactions are repressive



$$R := \frac{\sqrt{c\beta}}{\sqrt{ab}}$$

$$Q := \frac{\sqrt{T_a T_b}}{(T_a + T_b)/2}$$

Theorem [Hori et al., 09]

The cyclic GRN has periodic oscillations, if $\nu > \frac{2W(Q,N)}{Q(1+\cos(\frac{\pi}{N}))}$

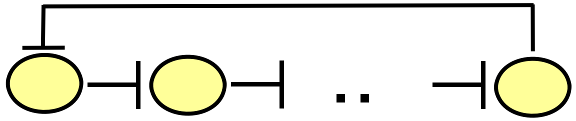
and $R^2 > \left(\frac{2W(Q,N)}{-2W(Q,N)+\nu Q(1+\cos(\frac{\pi}{N}))}\right)^{1/\nu}\left(\frac{\nu Q(1+\cos(\frac{\pi}{N}))}{-2W(N,Q)+\nu Q(1+\cos(\frac{\pi}{N}))}\right)$

where $W(Q,N) := \frac{\sqrt{\cos^2(\pi/N)+Q^2\sin^2(\pi/N)}-\cos(\pi/N)}{1-\cos(\pi/N)}$

- Four essential parameters (N, ν, R, Q) which determine the existence of periodic oscillations
- This coincides with [H. E. Samad *et al.*, 05] $N=3$, $Q = 1$

Analytic Criteria

- **Assumptions:**



  – All interactions are repressive

$R = \frac{\sqrt{c\beta}}{\sqrt{ab}}$

$\sqrt{T_a T_b}$ … $T_b)/2$

Theorem [Hori et al., 09]

The cyclic GR…

a… $+\cos(\frac{\pi}{N}))$ … $(N,Q)+\nu Q(1+\cos(\frac{\pi}{N}))$

whe… $(\pi/N) - \cos(\pi/N)$ … $1-\cos(\pi/N)$

Time-delay case

Non-homogeneous case

Robustness analysis

Experiments with RIKEN

- Four …ntial parameters (N, ν, R, Q) which determine the existence of periodic oscillations
- This coincides with [H. E. Samad *et al.*, 05] N=3 , $Q = 1$

Robust Stability Condition

$$h(s) = \frac{1}{(T_a s + 1)(T_b s + 1)}$$

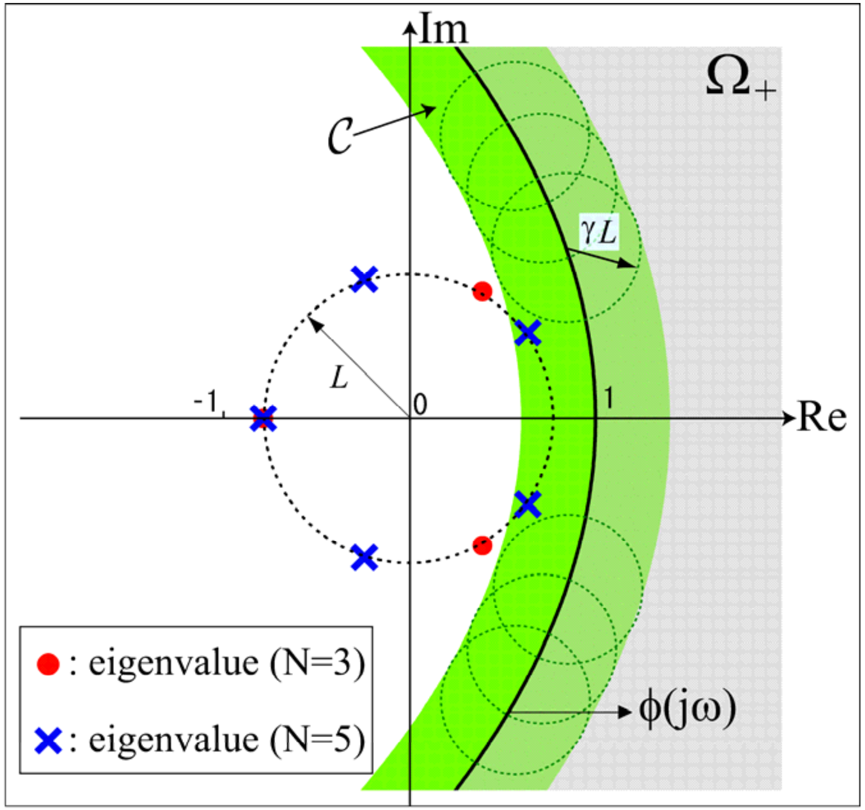
$$A = R^2 \begin{bmatrix} 0 & 0 & 0 & \cdots & \kappa_1 \\ \kappa_2 & 0 & 0 & \cdots & 0 \\ 0 & \kappa_3 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & \kappa_N & 0 \end{bmatrix}$$

$$\left\| \frac{h}{1 - \lambda h} \right\|_\infty < \gamma, \quad \forall \lambda \in \sigma(A)$$

$^\exists D$: diagonal s.t.
DAD^{-1} is normal

$$Q := \frac{\sqrt{T_a T_b}}{(T_a + T_b)/2} \quad R := \frac{\sqrt{c\beta}}{\sqrt{ab}}$$

$$L := \prod_{k=1}^{N} \left| \frac{df_i}{dp}\Big|_{p^*} \right|^{\frac{1}{N}}$$



More Robust as N, R^2, Q, L decrease.

Robust Stability Condition for Heterogeneous Perturbations

Assumption (Hara et al.: CDC2010)

$^{\exists}D$: diagonal s.t. DAD^{-1} is normal

Symmetric Circulant

Theorem: The following conditions are equivalent.

(i) The system is robustly stable for $\mathbf{\Delta}_{d\gamma}$.

(ii) $$\left\| \frac{\lambda h}{1-\lambda h} \right\|_{\infty} < \gamma, \ \forall \ \lambda \in \sigma(A)$$

(iii) $$\left| \frac{\lambda}{\phi - \lambda} \right| < \gamma, \ \forall \ \lambda \in \sigma(A),$$

$$\forall \ \phi \in \mathbf{\Phi} := \{1/h(j\omega) | \ \omega \in \mathbb{R} \ \}.$$

Same results for MIMO general classes of perturbations

Coprime Factor Perturbations (1/2)

$$G(s) := \begin{bmatrix} A \\ I \end{bmatrix} (I - h(s)A)^{-1} \begin{bmatrix} h(s)I & I \end{bmatrix}$$

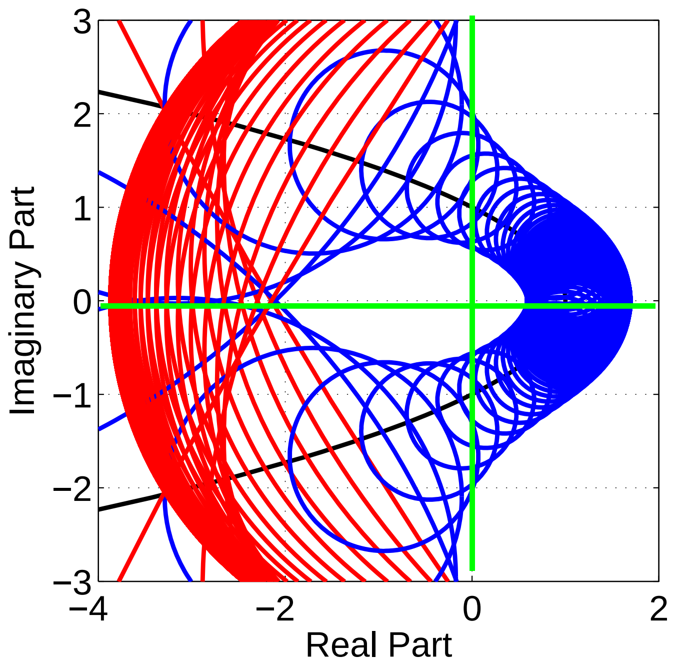
$$A = U^* \Lambda U$$

$$G(s) = \begin{bmatrix} U^* & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \Lambda \\ I \end{bmatrix} (I - h(s)\Lambda)^{-1} \begin{bmatrix} h(s)I & I \end{bmatrix} \begin{bmatrix} U & 0 \\ 0 & I \end{bmatrix}$$

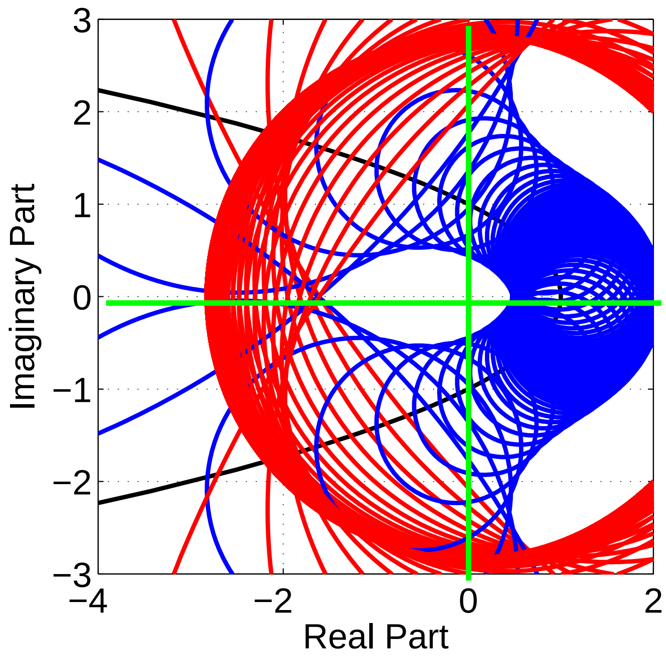
$$\|G\|_\infty < \gamma \quad \Leftrightarrow \quad \left\| \begin{bmatrix} \lambda \\ 1 \end{bmatrix} (1 - h\lambda)^{-1} \begin{bmatrix} h & 1 \end{bmatrix} \right\|_\infty < \gamma, \quad \forall \lambda \in \sigma(A)$$

Coprime Factor Perturbations (2/2)

$$h(s) = \frac{1}{s^2 + s + 1}$$



$\gamma = 4.0$

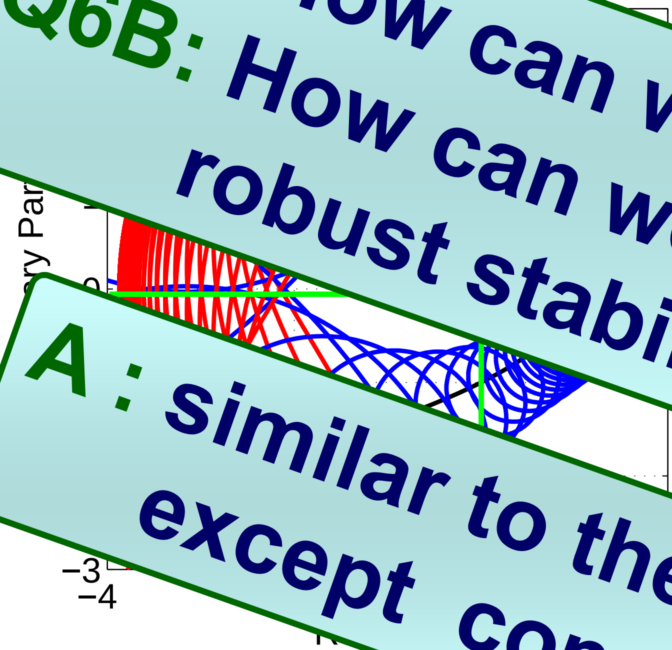


$\gamma = 3.0$

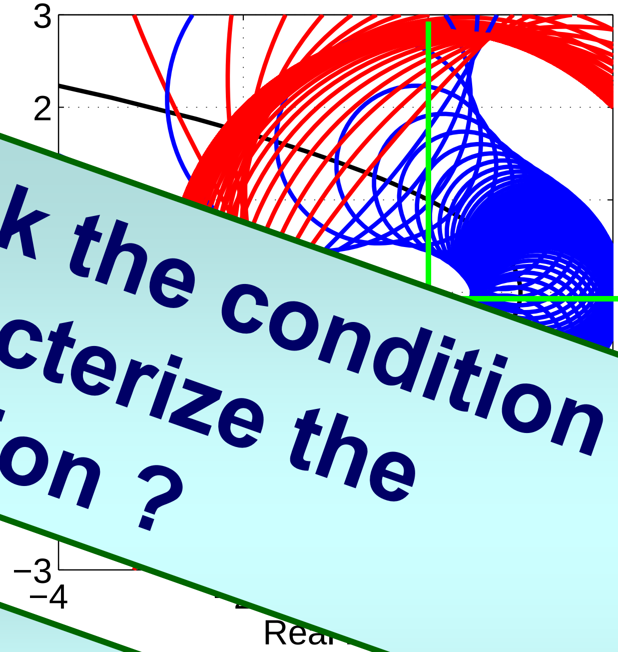
Outside of Blue Circle & Inside of Red Circle

Coprime Factor Perturbations (2/2)

$$h(s) = \frac{1}{s^2 + s + 1}$$



$\gamma = 4.0$



$= 3.0$

Q6A: How can we check the condition ?

Q6B: How can we characterize the robust stability region ?

A : similar to the standard cases except complex number λ

Outside of Blue Circle & Inside o… Circle

Analytic Expressions by QE

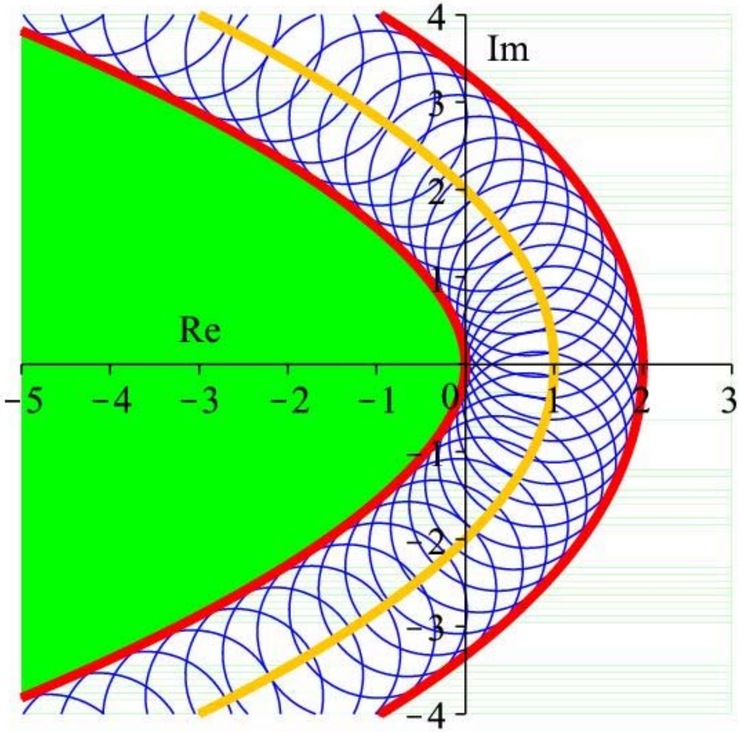
$$h(s) = 1/(s^2+2s+1)$$

$$\downarrow \quad \lambda = x + jy$$

$$(\omega^2 + x - 1)^2 + (2\omega - y)^2 > 1/\gamma^2 \quad ; \quad \forall \omega > 0$$

$$\downarrow \quad \textbf{QE} \quad (\gamma = 1.0)$$

$$y^6 + (x^2+8x-11)y^4 + (8x^3+6x^2-30x-1)y^2 + (16x^3-24x^2-15x-2)x > 0$$

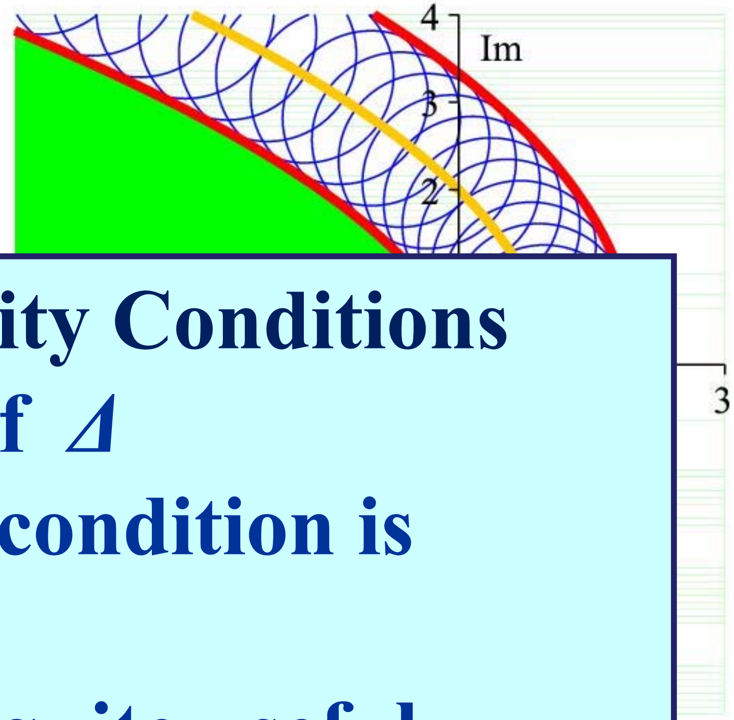


$$\Phi = \begin{bmatrix} 0 & 0 & 0 & 0 & \frac{1}{16} & 0 \\ 0 & 0 & 0 & -\frac{1}{4} & -1 & -\frac{1}{16} \\ 0 & 0 & \frac{3}{8} & 1 & \frac{27}{4} & \frac{3}{4} \\ 0 & -\frac{1}{4} & 1 & \frac{21}{8} & -\frac{51}{4} & -\frac{7}{2} \\ \frac{1}{16} & -1 & \frac{27}{4} & -\frac{51}{4} & -8 & -1 \\ 0 & -\frac{1}{16} & \frac{3}{4} & -\frac{7}{2} & -1 & 0 \end{bmatrix}$$

Analytic Expressions by QE

$$h(s) = 1/(s^2 + 2s + 1)$$

$$\lambda = x + jy$$



Summary : Robust Stability Conditions

- Class of *A* ⇔ Class of *Δ*
- Small gain (H∞-norm) condition is necessary & sufficient
- *D*-scaling technique is quite useful
- Some applications
  - bio-systems

$$+ \quad (8x^3 + 6x^2 - 30x - 1)y^2$$
$$+ \quad (16x^3 - 24x^2 - 15x - 2)x > 0$$

$$\begin{bmatrix} 0 & -\frac{1}{4} & 1 & \frac{21}{8} & -\frac{51}{4} & -\frac{7}{2} \\ \frac{1}{16} & -1 & \frac{27}{4} & -\frac{51}{4} & -8 & -1 \\ 0 & -\frac{1}{16} & \frac{3}{4} & -\frac{7}{2} & -1 & 0 \end{bmatrix}$$

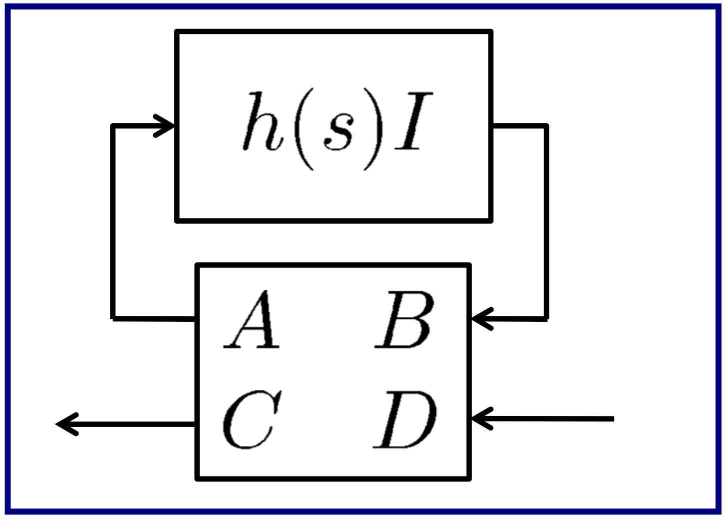
OUTLINE : Part 3

3. From Homogeneous to Heterogeneous

- Robust Stability Analysis
- **Nonlinear Stability Analysis**

(Hirsch, Hara: IFAC2008)

Linear → Nonlinear



Homogeneous
→ Heterogeneous

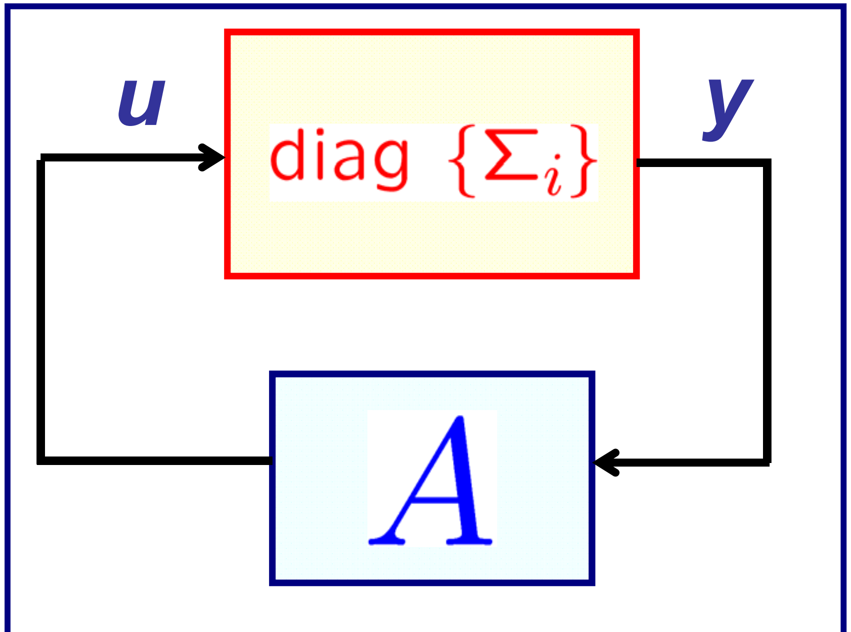
Robust Stability Analysis

$$\Sigma_i \ : \ h_i(s)$$

Linear → Nonlinear

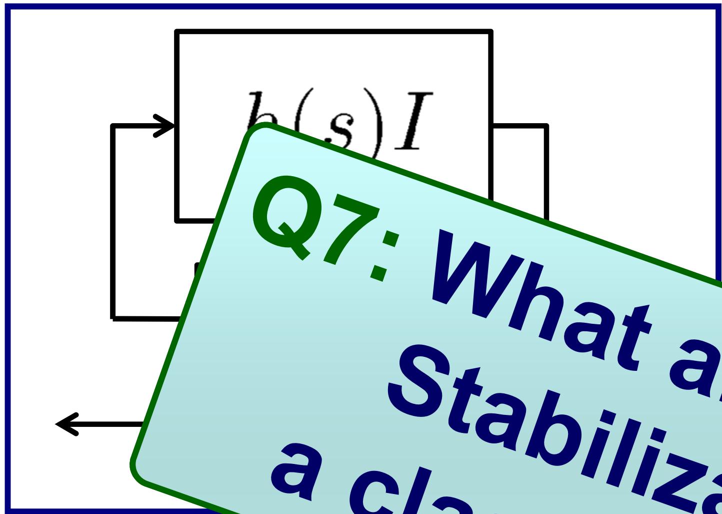
Nonlinear Analysis

$$\Sigma_i \ : \ \mathcal{N}_i$$



Interconnection Matrix

Linear → Nonlinear



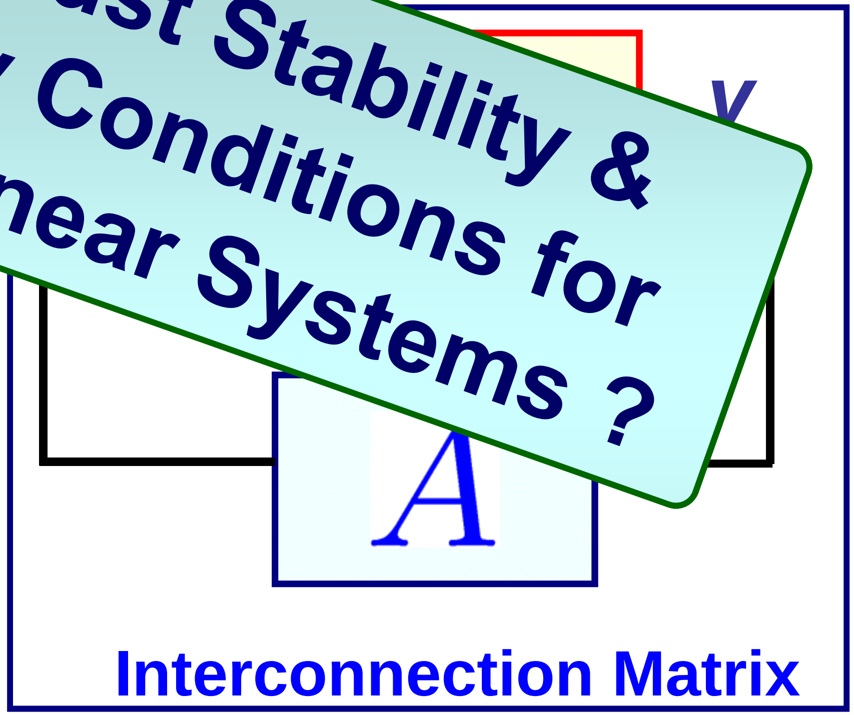
Homogeneous
→ Heterogeneous
Robust Stability Analysis

$$\Sigma_i \; : \; h_i(s)$$

Q7: What are Robust Stability & Stabilizability Conditions for a class of Nonlinear Systems ?

Linear → Nonlinear
Nonlinear Analysis

$$\Sigma_i \; : \; \mathcal{N}_i$$



(Q, S, R) Dissipativity

Definition

(Q, S, R)-*dissipative* : The system is dissipative with respect to the quadratic supply rate

$$w(u, y) = y^T Q y + 2 y^T S u + u^T R u,$$

with $R \in \mathbb{R}^{m \times m}$, $S \in \mathbb{R}^{p \times m}$, $Q \in \mathbb{R}^{p \times p}$, constant matrices and $Q = Q^T$, $R = R^T$ symmetric.

Dissipative : $\exists$ a positive definite function $V(x)$ called *storage function*, such that for all $x \in \mathcal{X}$

$$V(x(T)) - V(x(0)) \leq \int_0^T w(u(t), y(t))\, dt$$

Stability for Dissipative Agents

(Hirsch, Hara: IFAC2008)

Agent Dynamics — SISO (Q, S, R)-dissipative

$$\dot{x}_i = f_i(x_i) + g_i(x_i)u_i$$
$$y_i = h_i(x_i)$$

$Q = \text{diag}\{Q_i\} \leq 0$,
$S = \text{diag}\{S_i\}$,
$R = \text{diag}\{R_i\} \geq 0$.

Theorem (LMI)

$V := \sum_{i=1}^{N} d_i \cdot V_i$

If $\exists$ a diagonal matrix $D > 0$ such that

$$A^T DRA + DSA + A^T S^T D + DQ < 0$$

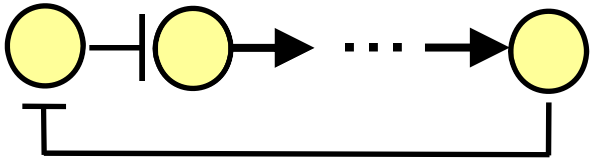
holds, then the networked system is asymptotically stable.

If $R = 0$ and $S > 0$, then
$A + S^{-1}Q/2$: diagonally stable

Stability Condition for GRNs

Cyclic Structure with Negative Feedback

$$A = \begin{bmatrix} 0 & 0 & 0 & \cdots & -1 \\ 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix}$$



$$A + S^{-1}Q/2 = \begin{bmatrix} -1/\gamma_1 & 0 & 0 & \cdots & -1 \\ 1 & -1/\gamma_2 & 0 & \cdots & 0 \\ 0 & 1 & -1/\gamma_3 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 & -1/\gamma_N \end{bmatrix}$$

$$\text{diag}\{1/\gamma_i\} \begin{bmatrix} -1 & 0 & 0 & \cdots & -\gamma_1 \\ \gamma_2 & -1 & 0 & \cdots & 0 \\ 0 & \gamma_3 & -1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & \gamma_N & -1 \end{bmatrix}$$

Secant Criterion

$$\gamma_1 \cdots \gamma_N < \sec(\pi/N)^N$$

⇔ **Diagonally Stable**

(Arcak & Sontag. Automatica, 2006)

Stabilization

Theorem (LMI)

If there exist a solution (X, Y) … ep-
resented by

$Y > 0,$

XY^{-1}

networked system.

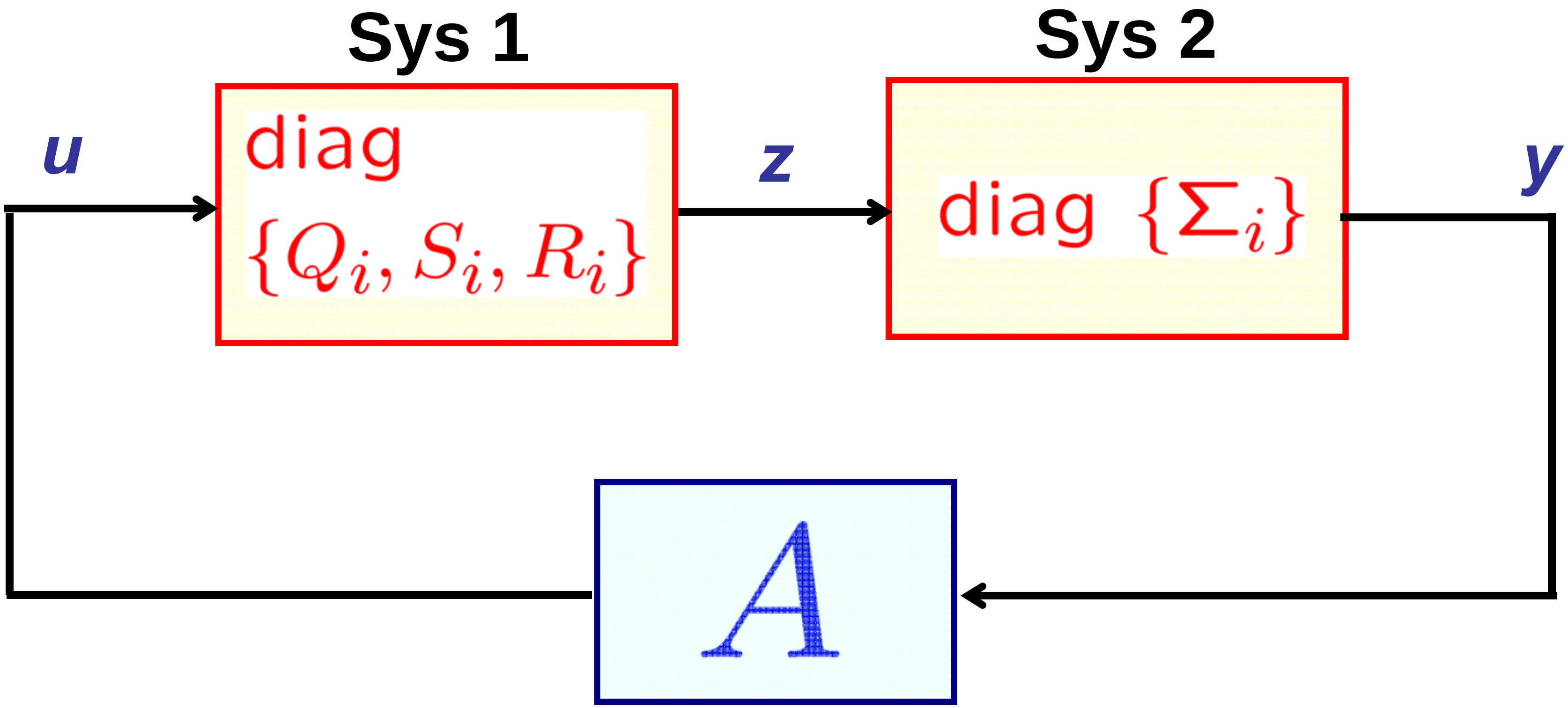
Q8 : Dissipative (Passive) → beyond Dissipative (Passive) ?

Can we apply the results to non-dissipative systems ?

Note: *X* preserves the network structure of *A*, since *Y* is diagonal.

MADS with Cascaded Dissipative Systems

A class of multi-agent dynamical systems based on dissipative properties



Interconnection Matrix

Two Cascaded Dissipative Systems

Sys 2

$$\text{diag}\ \{Q_i, S_i, R_i\} \Leftrightarrow A = \begin{bmatrix} 0 & I_N \\ A_2 & 0 \end{bmatrix} \in \mathbb{R}^{2N \times 2N}$$

$$Q = \text{diag}\{Q_1, Q_2\},$$
$$S = \text{diag}\{S_1, S_2\},$$
$$R = \text{diag}\{R_1, R_2\}$$

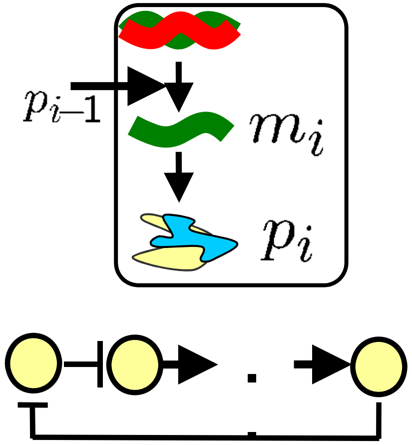
Stability Condition (LMI)

$$\begin{bmatrix} D_1 Q_1 & D_1 S_1 + A_2^T S_2 D_2 & 0 & A_2^T R_2 D_2 \\ D_1 S_1 + D_2 S_2 A_2 & D_2 Q_2 & D_1 R_1 & 0 \\ 0 & D_1 R_1 & -D_1 R_1 & 0 \\ D_2 R_2 A_2 & 0 & 0 & -D_2 R_2 \end{bmatrix} < 0$$

$$D_1 > 0,\ D_2 > 0 \ : \ \text{diagonal}$$

Gene Regulatory Network

$$Q = \text{diag}\{Q_1, Q_2\},$$
$$S = \text{diag}\{S_1, S_2\},\ R = 0$$



Stability Condition (LMI)

$$\begin{bmatrix} D_1 Q_1 & D_1 S_1 + A_2^T D_2 S_2 \\ S_1 D_1 + S_2 D_2 A_2 & D_2 Q_2 \end{bmatrix} < 0$$

Stabilization Condition (LMI)

$$\begin{bmatrix} Y_1 Q_1 & Y_1 S_1 + {X_2}^T S_2 \\ S_1 Y_1 + S_2 X_2 & Y_2 Q_2 \end{bmatrix} < 0 \qquad A_2 = X_2 {Y_2}^{-1}$$

Dissipative + Integrator

Sys 2

$$\frac{1}{s} \cdot I$$

Theorem

Suppose $R = 0$ and $Q < 0$.

The MADS is stable, if

A_2 is diagonally stable.

Proposition

Suppose A_2 is Diagonally normal, i.e.,

$^\exists D$: positive diagonal s.t.

$A_{2D} := D A_2 D^{-1}$ is normal. Then,

A_2 is stable $\Leftrightarrow$ A_2 is diagonally stable .

Summary

<u>Summary</u> : Dissipative Networked Systems

- **LMI stability & stabilizability conditions**
- **Stability and robust stability conditions**
- ***D*-scaling technique is also useful**
- **Dissipative → Non-dissipative**