Lecture Series, TU Munich
October 22, 29 & November 5, 2013

Glocal Control for Hierarchical Dynamical Systems

Theoretical Foundations with Applications in Energy Networks

Shinji HARA
The University of Tokyo, Japan

OUTLINE

1. Glocal Control & Energy Networks
2. **A Unified Framework for Networked Dynamical Systems with Stability Analysis**
3. From Homogeneous to Heterogeneous
4. From Frat to Hierarchical
5. Decentralized Hierarchical Control Synthesis
6. Applications in Energy Networks

OUTLINE : Part 2

2. A Unified Framework for Networked Dynamical Systems with Stability Analysis

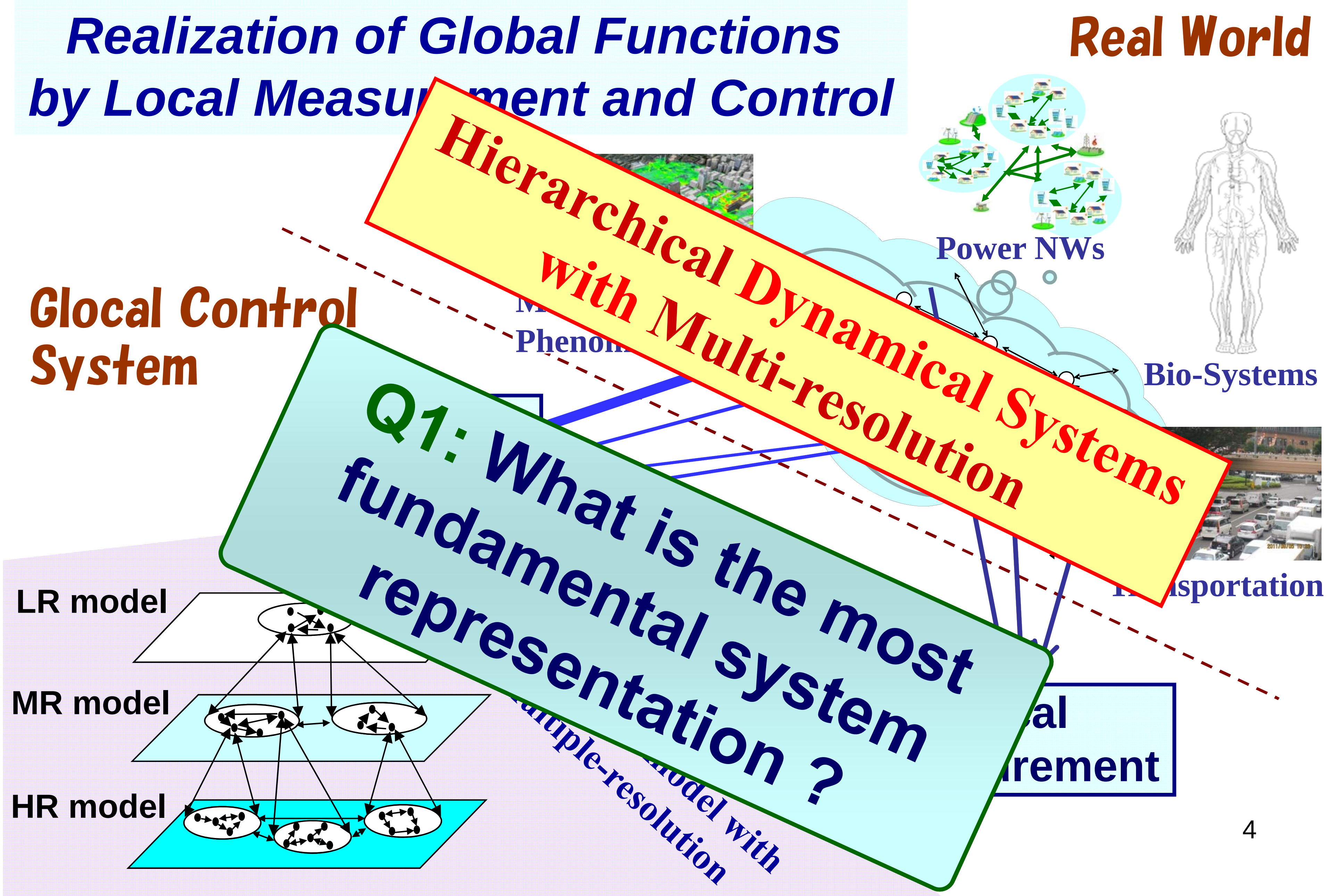
- **LTI System with Generalized Frequency Variables:**

  System representation & stability tests

- **Co-operative Stabilization**
- **D-stability Analysis**

Framework for Glocal Control

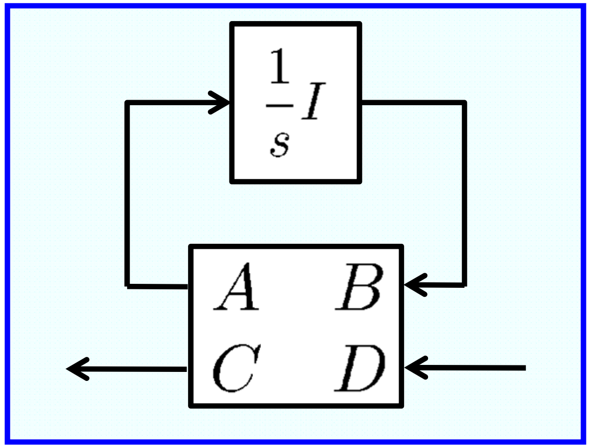
Realization of Global Functions by Local Measurement and Control



LTI System with Generalized Frequency Variable

A unified representation for multi-agent dynamical systems

$C\,(sI - A)^{-1}B + D$

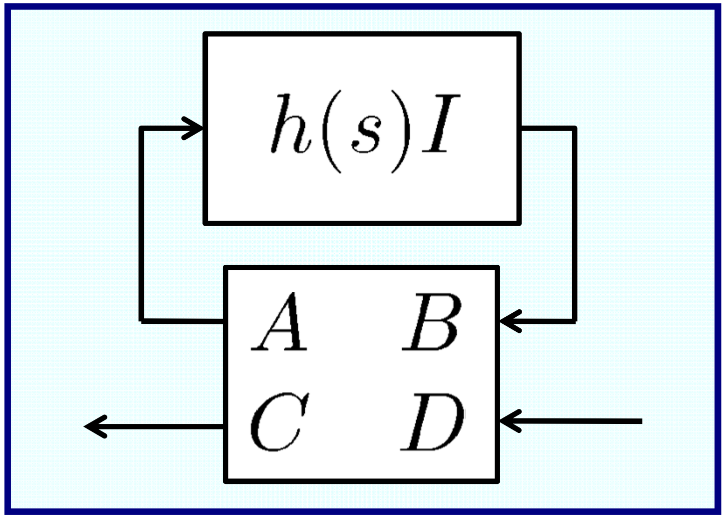


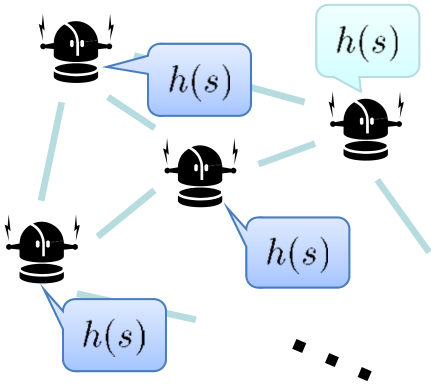
1/s → h(s)

Φ(s) = 1/h(s)

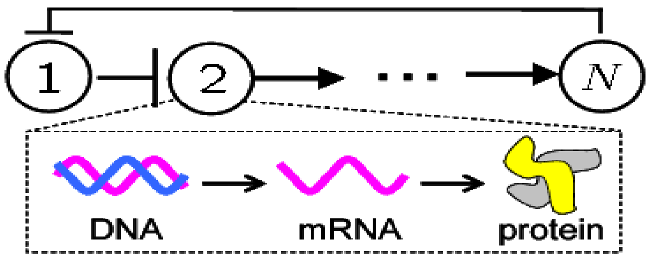
Generalized Freq. Variable

$C\,(\phi(s)I - A)^{-1}B + D$





Group Robot



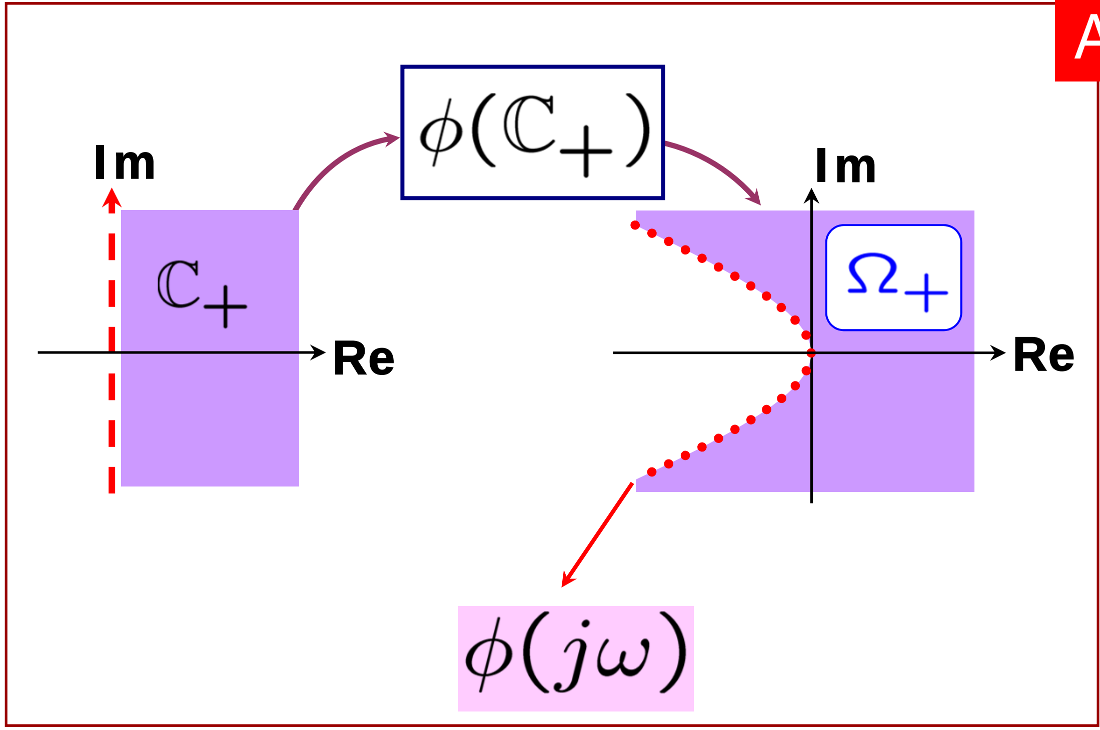
Gene Reg. Networks

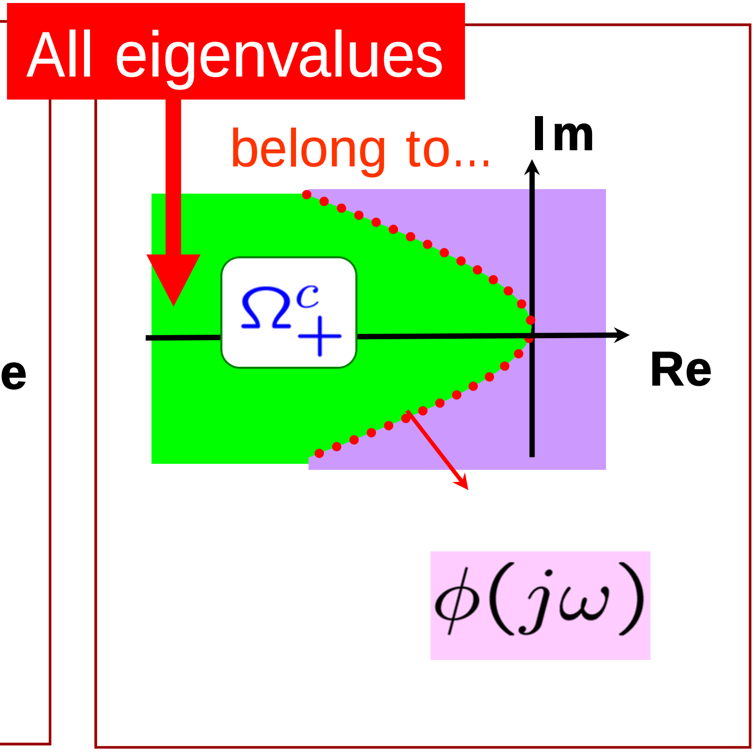
Dynamics + Information Structure

Stability Region for LTISwGFV

(Hara et al. IEEE CDC, 2007)

- <u>Define</u>: Domains $\Omega_+ := \phi(\mathbb{C}_+)$, $\Omega_+^c := \mathbb{C} \backslash \Omega_+$





Q2A: How to characterize the region ?
Q2B: How to check the condition ?

Stability Tests for LTISwGFV

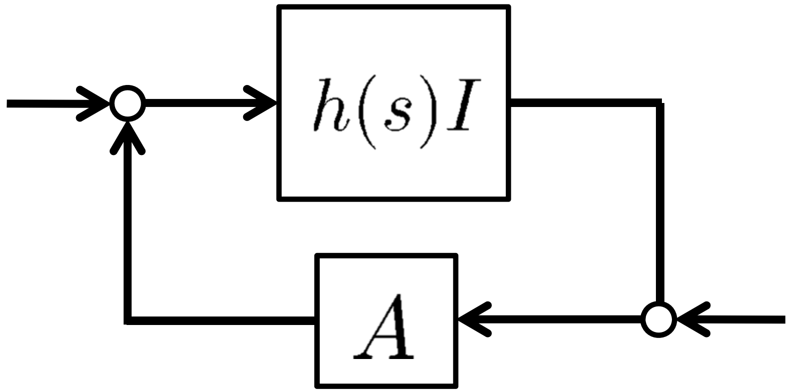
(Tanaka et al., ASCC, 2009)

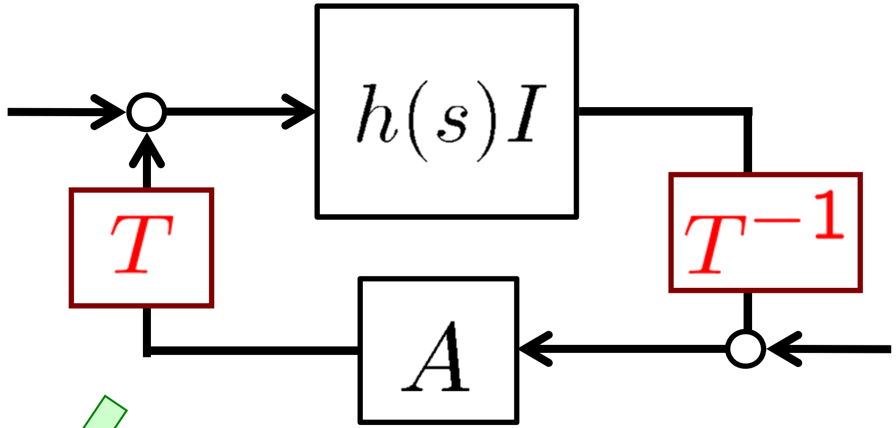
Graphical	Algebraic	Numeric (LMI)
Nyquist – type	**Hurwitz – type**	**Lyapunov – type**
Fax & Murray (2004) Hara et al. (2007)	**Tanaka, Hara, Iwasaki (2009)**	**Tanaka, Hara, Iwasaki (2009)**
$h(s)$ and $\sigma(A)$	$h(s)$ and $\sigma(A)$	$h(s)$ and A
	Hurwitz test for complex coefficients	Generalized Lyapunov Ineq.

Characteristic Polynomial

$$p(\lambda, s) := d(s) - \lambda\, n(s) \qquad \lambda \in \sigma(A)$$ **(complex)**

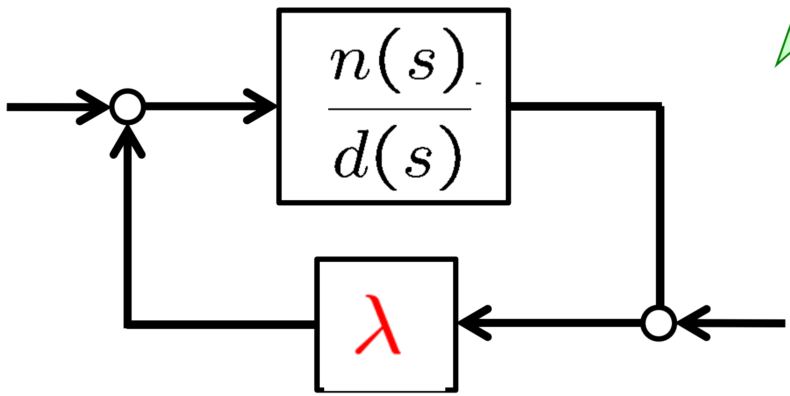
Key for Stability Conditions





$$h(s) = \frac{n(s)}{d(s)}$$

$$TAT^{-1} = \mathrm{diag}\{\lambda_i\}$$



$\lambda \in \sigma(A)$ **(complex)**

Closed-loop Characteristic Polynomial

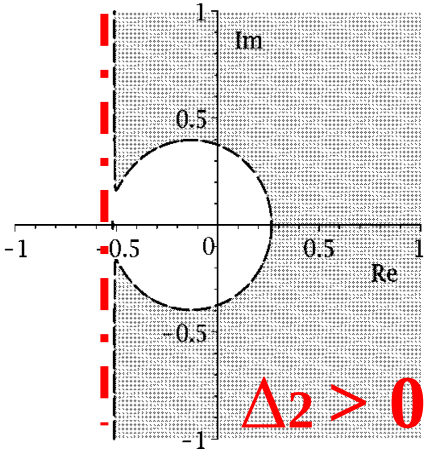
$$p(\lambda, s) := d(s) - \lambda\, n(s)$$

Numerical Example : 4th order

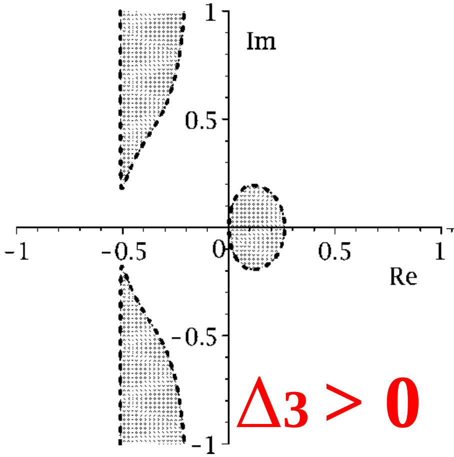
$$h(s) = \frac{100(s+2)(\frac{19}{10}s^2 - \frac{1}{500000}s + \frac{21}{10})}{(s-1)^2(s+1)(s+100)}$$

Unstable
& NMP

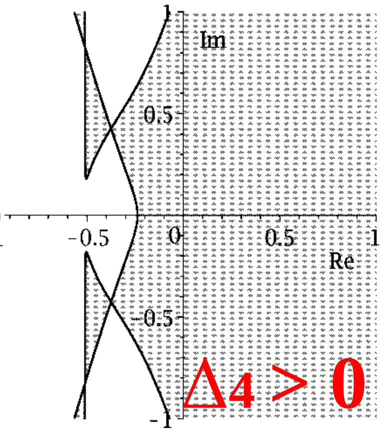
$\Delta_1 > 0$



$\Delta_2 > 0$

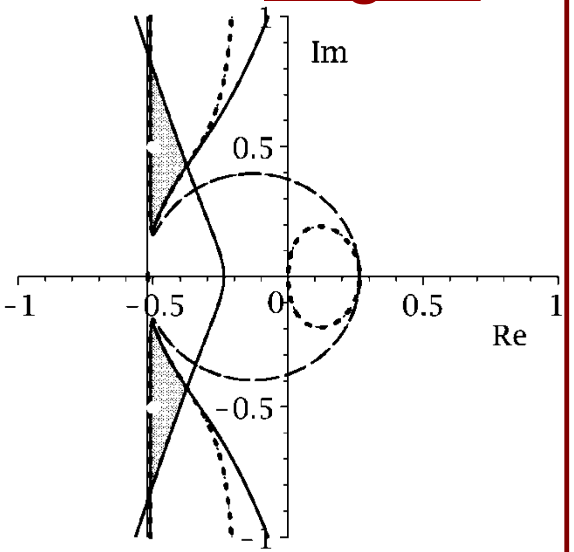


$\Delta_3 > 0$



$\Delta_4 > 0$

Stability Region



$$\Delta_k := l_k(\lambda)^* \Phi_k l_k(\lambda) > 0 \qquad l_\ell(\lambda) := \begin{bmatrix} 1 \\ \lambda \\ \vdots \\ \lambda^\ell \end{bmatrix}$$

Algorithm

Given Data: all coefficients of numerator and denominator of *h(s)*

Algorithm h2Phi($h(s)$) :

Input : $h(s) = \dfrac{b_1 s^{\nu-1} + \cdots + b_\nu}{s^\nu + a_1 s^{\nu-1} + \cdots + a_\nu}$

Output : ℓ_k and Φ_k

1. $p_0 \leftarrow 1,\ q_0 \leftarrow 0$
   for $i \leftarrow 1$ **until** $2\nu - 1$ **do**
   if $i \le \nu$ **then**
   $p_i \leftarrow a_i - b_i x,\ q_i \leftarrow -b_i y$
   else
   $p_i \leftarrow 0,\ q_i \leftarrow 0$

2 $\Delta_1 \leftarrow p_1$
  for $k \leftarrow 2$ **until** 2ν **do**
  $M \leftarrow O^{(2k-1)\times(2k-1)}$
  for $i \leftarrow 1$ **until** k **do**
  if i is odd **then**
  for $m \leftarrow 1$ **until** $k - (i-1)/2$ **do**
  $M(i, k-m+1) \leftarrow p_{2k-i-2(m-1)}$
  for $m \leftarrow 1$ **until** $k - (i-1)/2 - 1$ **do**
  $M(i, 2k-m) \leftarrow -q_{2k-i-1-2(m-1)}$
  else
  for $m \leftarrow 1$ **until** $k - (i-2)/2$ **do**
  $M(i, k-m+1) \leftarrow p_{2k-i-2(m-1)}$
  for $m \leftarrow 1$ **until** $k - (i-2)/2 - 1$ **do**
  $M(i, 2k-m) \leftarrow p_{2k-i-1-2(m-1)}$
  $M^{k+1,k+2,\ldots,2k-1}_{2,3,\ldots,k} \leftarrow -M^{1,2,\ldots,k-1}_{k+1,k+2,\ldots,2k-1}$
  $M^{k+1,k+2,\ldots,2k-1}_{k+1,k+2,\ldots,2k-1} \leftarrow M^{1,2,\ldots,k-1}_{1,2,\ldots,k-1}$
  $\Delta_k(x,y) \leftarrow |M|$

3 **for** $k \leftarrow 1$ **until** ν **do**
  $\Delta'_k(\lambda, \bar\lambda) \leftarrow \Delta_k((\lambda + \bar\lambda)/2, (\lambda - \bar\lambda)/2j)$

4 **for** $k \leftarrow 1$ **until** ν **do**
  $\ell_k \leftarrow$ the maximum of the degree of λ in $\Delta'_k(\lambda, \bar\lambda)$ -1
  $\Phi_k \leftarrow O^{\ell_k \times \ell_k}$
  for $m \leftarrow 0$ **until** $\ell_k - 1$ **do**
  for $l \leftarrow 0$ **until** $\ell_k - 1$ **do**
  $\Phi_k(m+1, l+1)$
  $\leftarrow$ the coefficient of $\lambda^m \bar\lambda^l$ in $\Delta'_k(\lambda, \bar\lambda)$

5 **rerurn** ℓ_k and Φ_k.

Result:

$$\Phi_k \quad (k = 1, 2, \cdots, \nu)$$

Algorithm

Given Data: all coefficients of numerator and denominator of *h(s)*

Algorithm h2Phi($h(s)$) :

Input : $h(s) = \dfrac{b_1 s^{\nu-1} + \cdots + b_\nu}{s^\nu + a_1 s^{\nu-1} + \cdots + a_\nu}$

Output : ℓ_k and Φ_k

1. $p_0 \leftarrow 1,\ q_0 \leftarrow 0$
   for $i \leftarrow 1$ **until** $2\nu - 1$ **do**
   if $i \le \nu$ **then**
   $p_i \leftarrow a_i - b_i x,\ q_i \leftarrow -b_i y$
   else
   $p_i \leftarrow 0,\ q_i \leftarrow 0$

2 $\Delta_1 \leftarrow p_1$
for $k \leftarrow 2$ **until** 2ν **do**
$M \leftarrow O^{(2k-1)\times(2k-1)}$
for $i \leftarrow 1$ **unti**...
if i is ...

... $-2)/2$ **do**
... $\leftarrow p_{2k-i-2(m-1)}$
... **until** $k-(i-2)/2-1$ **do**
... $(i, 2k-m) \leftarrow p_{2k-i-1-2(m-1)}$
$M^{k+1,k+2,\ldots,2k-1}_{2,3,\ldots,k} \leftarrow -M^{1,2,\ldots,k-1}_{k+1,k+2,\ldots,2k-1}$
$M^{k+1,k+2,\ldots,2k-1}_{k+1,k+2,\ldots,2k-1} \leftarrow M^{1,2,\ldots,k-1}_{1,2,\ldots,k-1}$
$\Delta_k(x,y) \leftarrow |M|$

... 1 **until** ν **do**
... $\leftarrow \Delta_k((\lambda+\bar\lambda)/2, (\lambda-\bar\lambda)/2j)$
... **til** ν **do**
... imum of the degree of λ in $\Delta'_k(\lambda,\bar\lambda)$ -1

... **ntil** $\ell_k - 1$ **do**
... $\leftarrow 0$ **until** $\ell_k - 1$ **do**
$\Phi_k(m+1, l+1)$
$\leftarrow$ the coefficient of $\lambda^m \bar\lambda^l$ in $\Delta'_k(\lambda,\bar\lambda)$

5 **rerurn** ℓ_k and Φ_k.

Systematic methods for stability analysis
Hurwitz-type & LMI

Result:

$$\Phi_k \quad (k = 1, 2, \cdots, \nu)$$

Stability Conditions

(Tanaka et al., ASCC, 2009)

Given $h(s) = n(s)/d(s),\ A$

$\mathcal{H}_A(s)$ is stable

$\Longleftrightarrow$

$\sigma(A) \subset \Lambda := \{\ \lambda \in \mathbb{C} \mid d(s) - \lambda n(s)$ is Hurwitz stable $\}$

Key lemma

Extended Routh-Hurwitz Criterion [Frank,1946]

Algebraic condition

$$\sigma(A) \subset \bigcap_{k=1}^{\nu} \Sigma_k$$

$$\Sigma_k := \{\ \lambda \in \mathbb{C} \mid l_k(\lambda)^* \Phi_k l_k(\lambda) > 0\ \}$$

$(k = 1, 2, \ldots, \nu)$

Generalized Lyapunov inequality

$$l_\ell(\lambda) := \begin{bmatrix} 1 \\ \lambda \\ \vdots \\ \lambda^\ell \end{bmatrix},\ L_\ell(A) := \begin{bmatrix} I \\ A \\ \vdots \\ A^\ell \end{bmatrix}$$

LMI feasibility problem

$X_k = X_k^T > 0$ s.t. $L_k(A)^T (\Phi_k \otimes X_k) L_k(A) > 0$

for each $k = 1, 2, \ldots, \nu$

Numerical Example : 2nd order (1/2)

Given $h(s) = \dfrac{2s+1}{s^2+s+1}$, $A \in \mathbb{R}^{n\times n}$

$$\sigma(A) \subset \Lambda := \{\ \lambda \in \mathbb{C} \mid (s^2+s+1) - \lambda(2s+1) \text{ is Hurwitz stable }\}$$

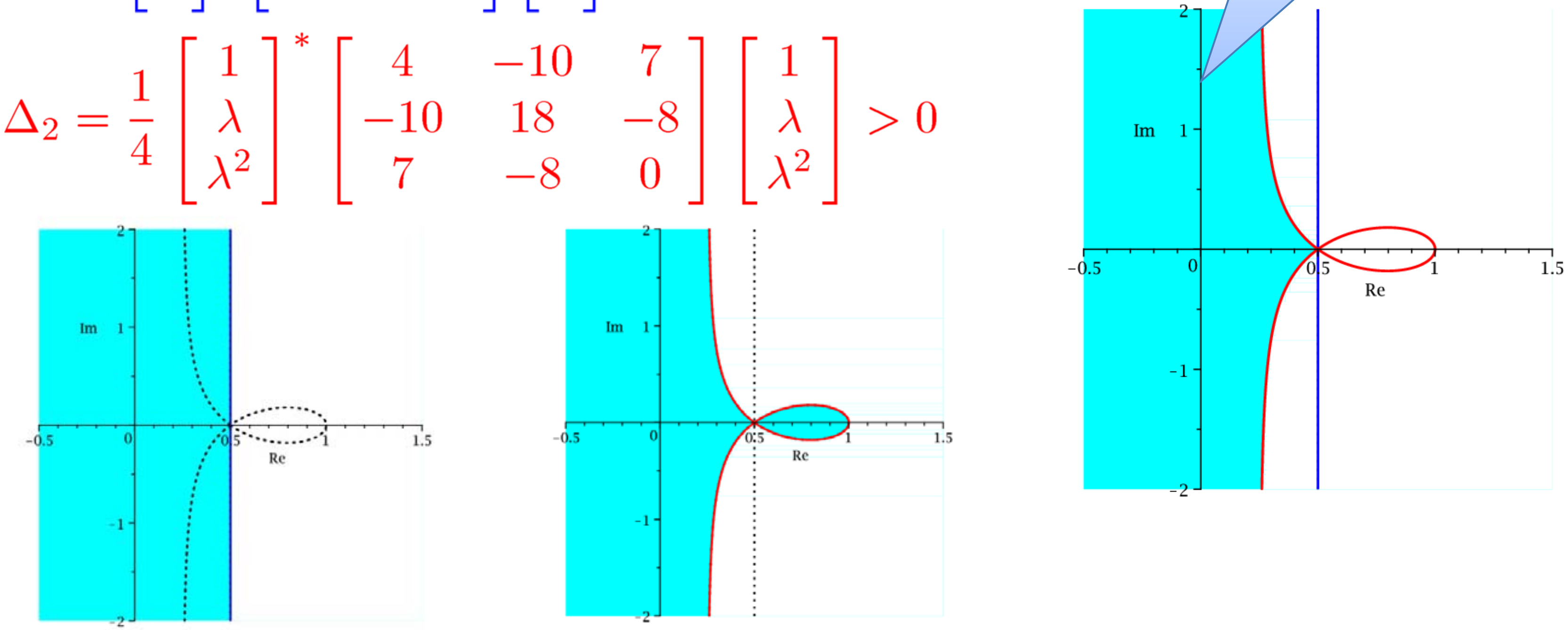
Extended Routh-Hurwitz Criterion (Frank, 1948)

$$\sigma(A) \subset \Sigma := \{\ \lambda \in \mathbb{C} \mid \Delta_1 > 0 \text{ and } \Delta_2 > 0\ \}$$

$$\Delta_1 = \begin{bmatrix} 1 \\ \lambda \end{bmatrix}^* \begin{bmatrix} 1 & -1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ \lambda \end{bmatrix} > 0$$

$$\Delta_2 = \frac{1}{4} \begin{bmatrix} 1 \\ \lambda \\ \lambda^2 \end{bmatrix}^* \begin{bmatrix} 4 & -10 & 7 \\ -10 & 18 & -8 \\ 7 & -8 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ \lambda \\ \lambda^2 \end{bmatrix} > 0$$

Stability region



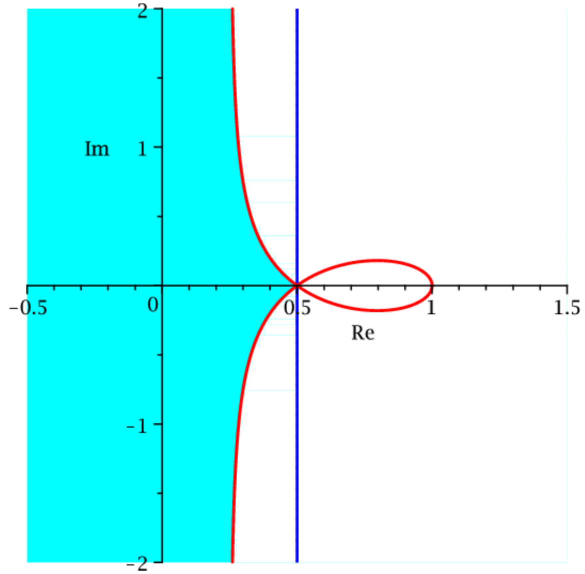
Numerical Example : 2nd order (2/2)

Given $h(s) = \dfrac{2s+1}{s^2+s+1}, \quad A \in \mathbb{R}^{n\times n}$

$$\sigma(A) \subset \Sigma := \{ \lambda \in \mathbb{C} \mid \Delta_1 > 0 \text{ and } \Delta_2 > 0 \}$$

$$\Delta_1 = \begin{bmatrix} 1 \\ \lambda \end{bmatrix}^* \begin{bmatrix} 1 & -1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ \lambda \end{bmatrix} > 0$$

$$\Delta_2 = \frac{1}{4} \begin{bmatrix} 1 \\ \lambda \\ \lambda^2 \end{bmatrix}^* \begin{bmatrix} 4 & -10 & 7 \\ -10 & 18 & -8 \\ 7 & -8 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ \lambda \\ \lambda^2 \end{bmatrix} > 0$$



Generalized Lyapunov inequality

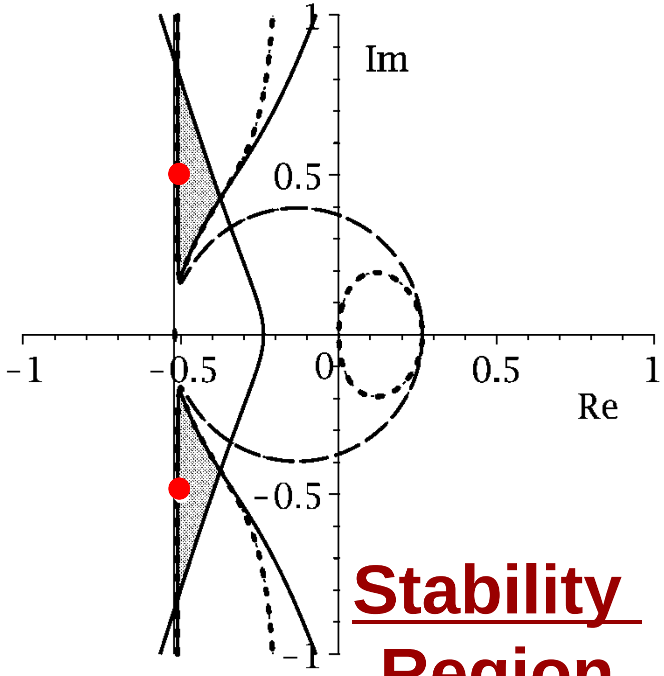
$$X_1 = X_1^T > 0 \quad \text{s.t.} \quad \begin{bmatrix} I \\ A \end{bmatrix}^T \left(\begin{bmatrix} 1 & -1 \\ -1 & 0 \end{bmatrix} \otimes X_1 \right) \begin{bmatrix} I \\ A \end{bmatrix} > 0$$

$$X_2 = X_2^T > 0 \quad \text{s.t.} \quad \begin{bmatrix} I \\ A \\ A^2 \end{bmatrix}^T \left(\begin{bmatrix} 4 & -10 & 7 \\ -10 & 18 & -8 \\ 7 & -8 & 0 \end{bmatrix} \otimes X_2 \right) \begin{bmatrix} I \\ A \\ A^2 \end{bmatrix} > 0$$

An example : Cope. Stab. ≠ Soley Stab.

$$h(s) = \frac{100(s+2)(\frac{19}{10}s^2 - \frac{1}{500000}s + \frac{21}{10})}{(s-1)^2(s+1)(s+100)}$$

$$\lambda = (-1 \pm j)/2$$



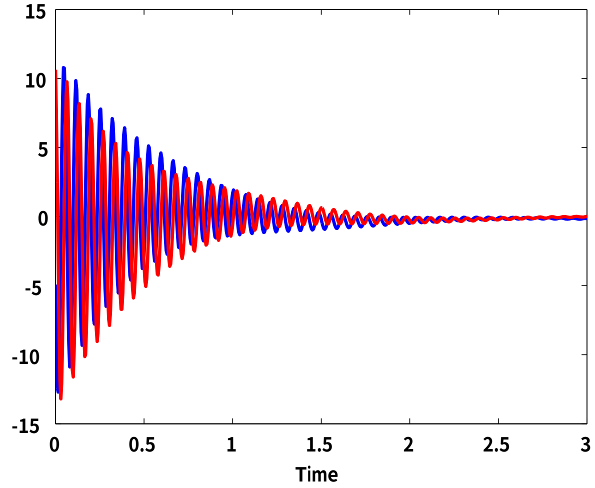
Stability Region

Follower

Control Law

$$\begin{cases} u_1 &= -(y_1 - y_2)/2 \\ u_2 &= -(y_1 + y_2)/2 \end{cases}$$

Leader



OUTLINE : Part 2

2. A Unified Framework for Networked Dynamical Systems with Stability Analysis

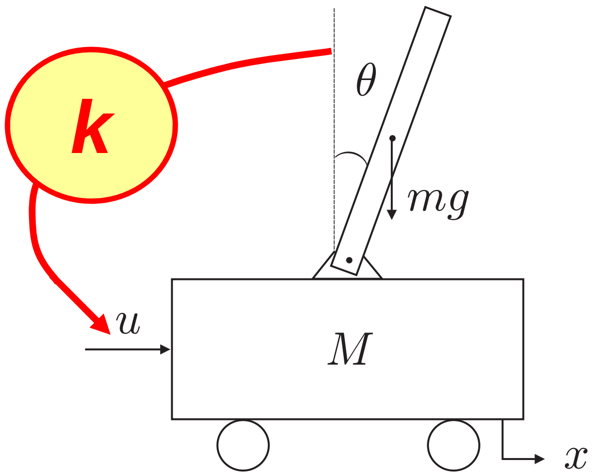
- LTI System with Generalized Frequency Variables:

  System representation & stability tests

- **Co-operative Stabilization**
- D-stability Analysis

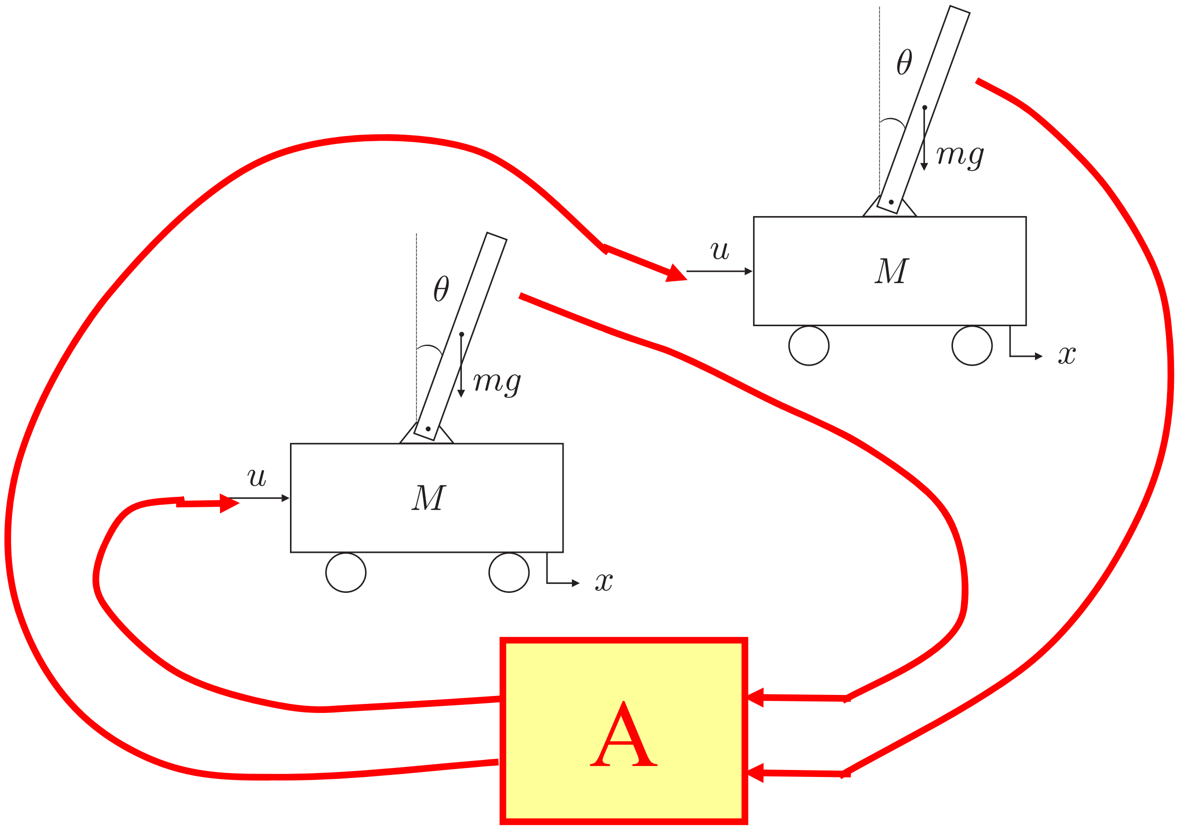
(Hara et al.: CDC-CCC2009)

An Application: Inverted Pendulum



Not stabilizable !

Cooperatively stabilizable ?



<u>Remarks</u> : No physical interactions
memory-less feedback

Property 1

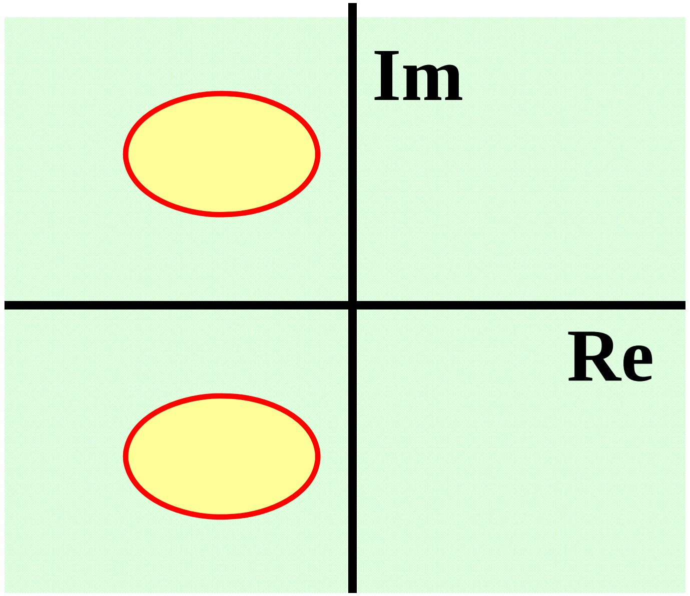
(i) $\sigma(A) \subset \Omega_+^c$

Cooperatively stabilizable：

Ω_+^c is non-empty.

Solely stabilizable：

Ω_+^c interacts the real axis.



N **: odd：Coop. Stab. = Solely Stab.**

N **: even：Coop Stab. (*N*=*2*) → any *N*=*2m***

Property 2

$$\mathcal{H}_A(s) := \left(\frac{d(s)}{n(s)} I - A\right)^{-1} \text{ is stable.} \quad ; \; h(s) = \frac{n(s)}{d(s)}$$

⇕

$$p(\lambda, s) := d(s) - \lambda\, n(s)$$

(ii) $\sigma(A) \subset \mathbf{\Lambda} \;:=\; \{\, \lambda \in \mathbb{C} \mid p(\lambda, s)$ is Hurwitz stable.$\}$

Solely Stabilizable： ⇔

Stabilizable by a **real gain** output feedback

Cooperatively Stablizable： ⇔

Stabilizable by a **complex gain** output feedback

Theorem: Coop. Stabliz. = Soley Stabiliz.

2nd order systems:

$$h_2(s) = \frac{cs + d}{s^2 + as + b}$$

Higher order systems:

$$\mathcal{H}_0(s) \triangleq \{\ h(s) = \frac{k}{d(s)} \mid k \neq 0\ \}$$

$$\mathcal{H}_1(s) \triangleq \{\ h(s) = \frac{ks}{d(s)} \mid k \neq 0,\ d(0) \neq 0\ \}$$

$$\mathcal{H}_2(s) \triangleq \{\ h(s) = \frac{k(s^2 - b^2)}{d(s)} \mid k \neq 0,\ d(\pm b) \neq 0\}$$

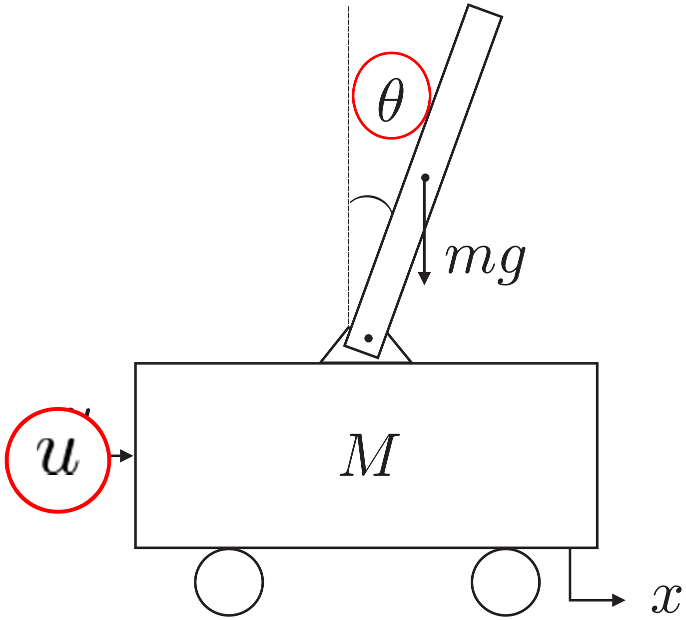
$$d(s) = s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0$$

Example：Inverted Pendulum

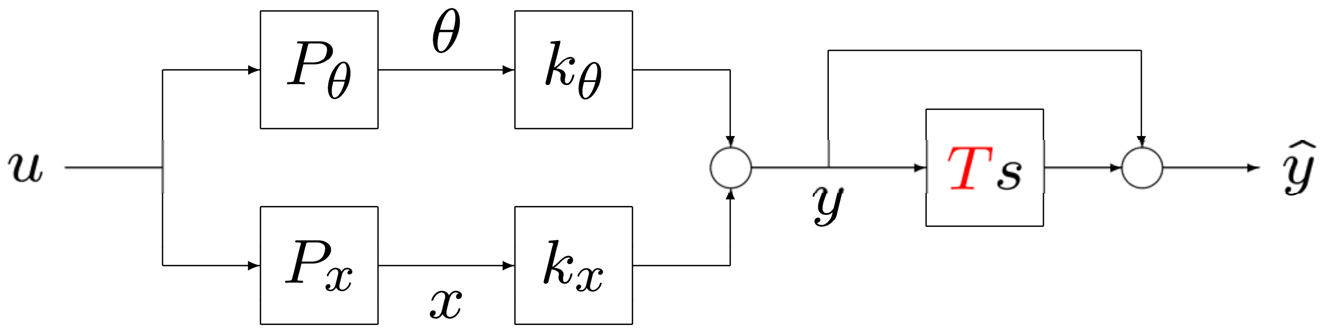
$$P_\theta(s) = \frac{-m\ell s}{D(s)} \quad \in \ \mathcal{H}_1(s)$$

$$D(s) = a_3 s^3 + a_2 s^2 + a_1 s + a_0 \ ,$$

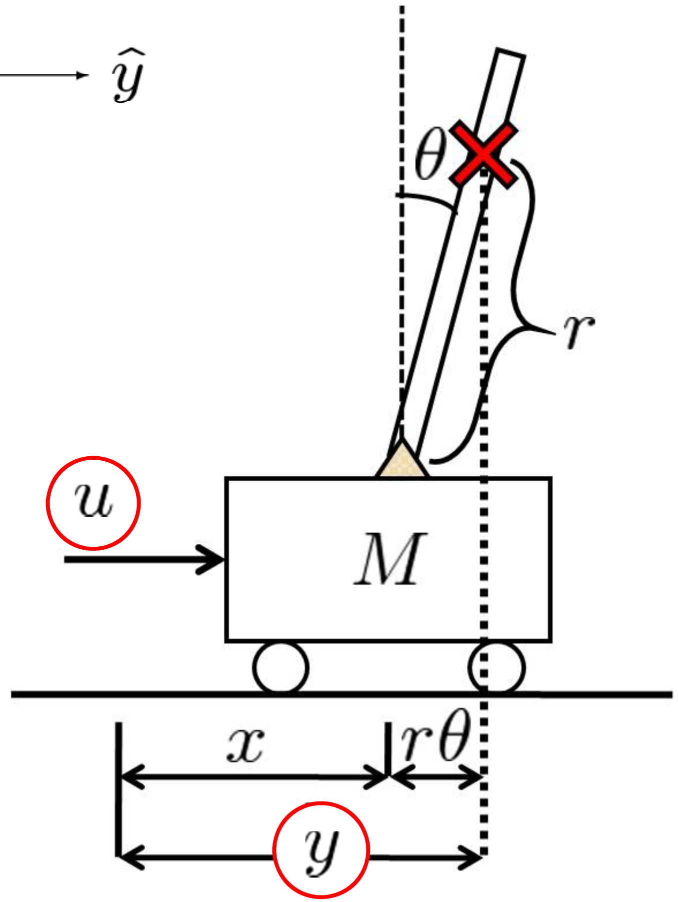
$$\begin{aligned}
a_3 \ &:= \ \frac{1}{3}(4M + m)m\ell^2 \ , \\
a_2 \ &:= \ (M + m)\mu_\mathrm{p} + \frac{4}{3}\mu_\mathrm{t} m\ell^2 \ , \\
a_1 \ &:= \ -(M + m)mg\ell + \mu_\mathrm{p}\mu_\mathrm{t} \ , \\
a_0 \ &:= \ -\mu_\mathrm{t} mg\ell \ .
\end{aligned}$$



Inverted Pendulum : PD control (1/2)



$$h(s) = \frac{(Ts+1)(\frac{19}{10}s^2 - \frac{1}{500}s + \frac{21}{10})}{s(s-2)(s+1)(s+5)}$$

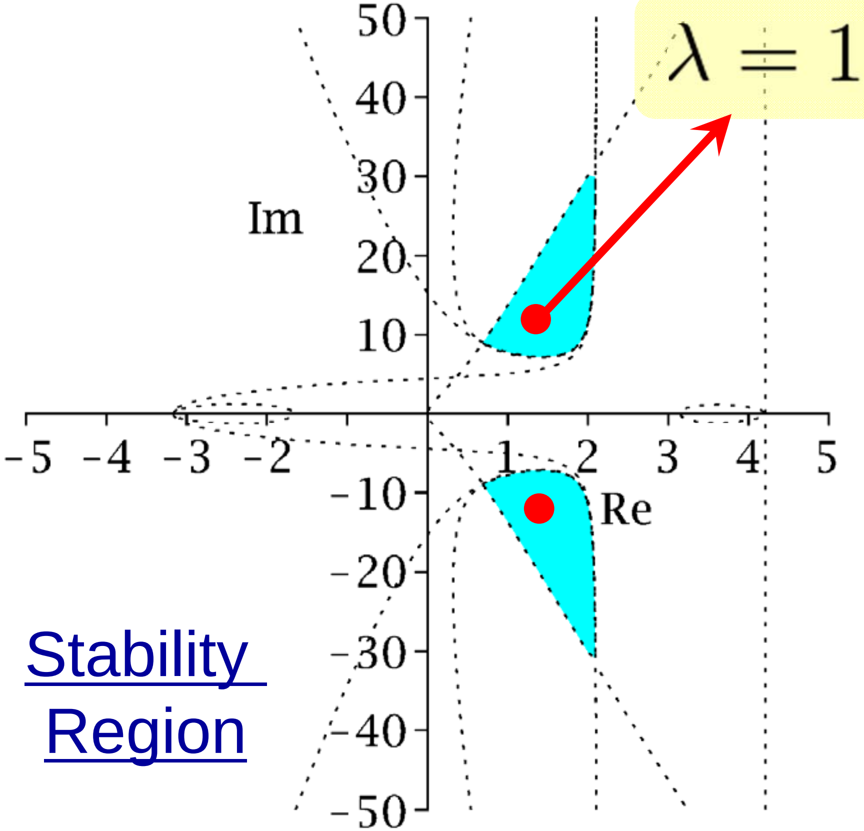


We can prove by a symbolic computation (QE) that the system can not be stabilized alone no matter how we choose *T>0* .

Inverted Pendulum : PD control (2/2)

T=1/2 :

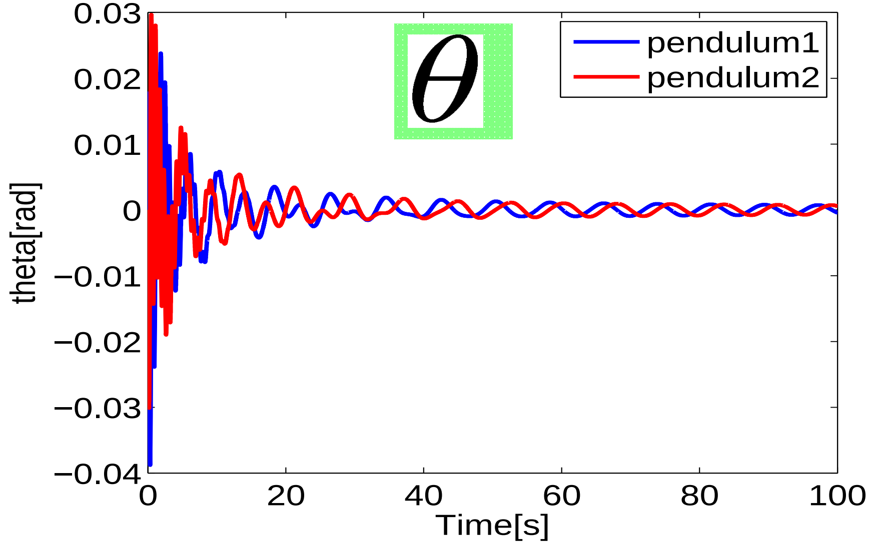
$$h(s) = \frac{(\frac{1}{2}s+1)(\frac{19}{10}s^2 - \frac{1}{500}s + \frac{21}{10})}{s(s-2)(s+1)(s+5)}$$



Stability Region

$$\lambda = 1.5 \pm 12j$$

$$A = \begin{bmatrix} 1.5 & -12 \\ 12 & 1.5 \end{bmatrix}$$



Robust Stability Analysis for LTI Systems with GFV

Fundamental Questions in Control

From Stability Analysis to

- *D-Stability Analysis ?*

  Stability Margins

- *Robust Stability Analysis ?*

  Homogeneous → Heterogeneous

OUTLINE Part 2

2. A Unified Framework for Networked Dynamical Systems with Stability Analysis

- **LTI System with Generalized Frequency Variables:**

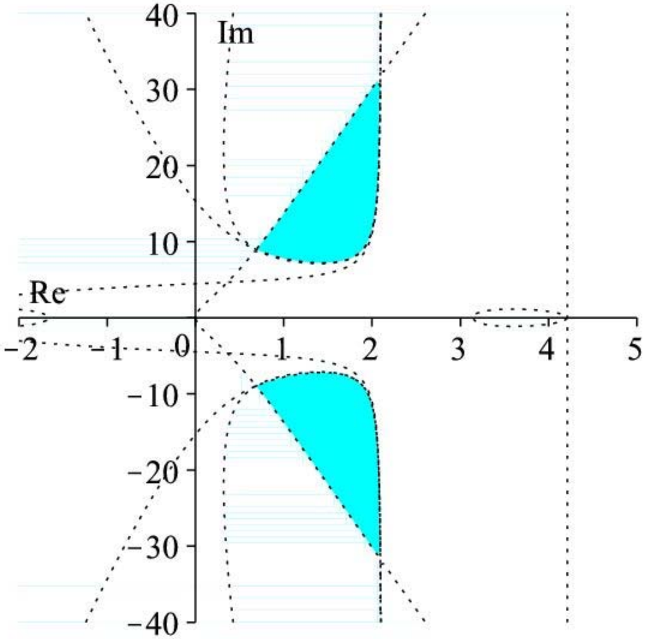
  System representation & stability tests

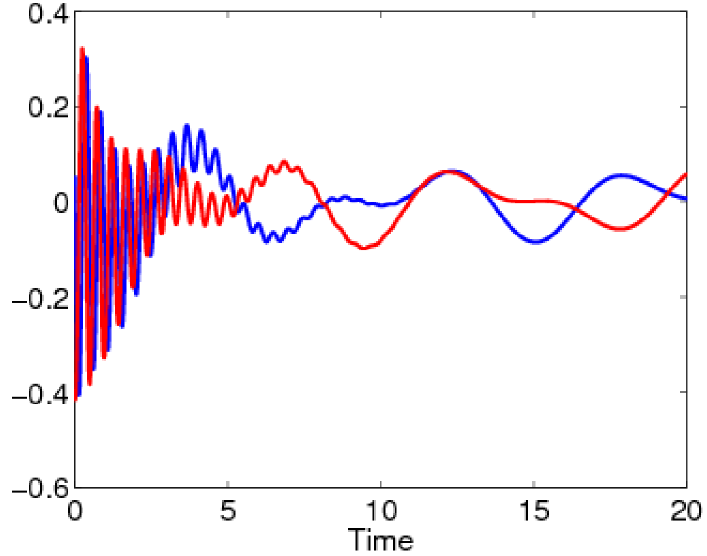
- **Co-operative Stabilization**
- **D-stability Analysis**

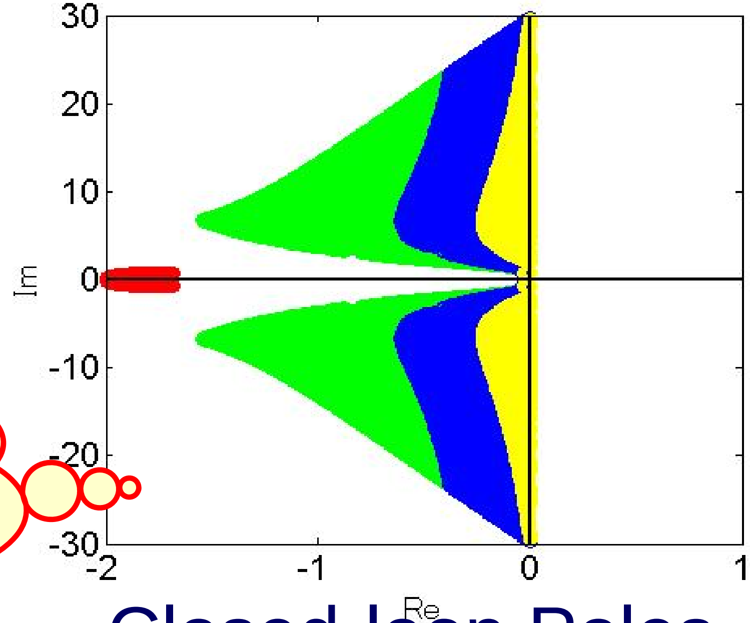
(Hara, Tanaka: CDC2010)

Why D-Stability Analysis ?

$$h(s) = \frac{(\frac{1}{2}s+1)(\frac{19}{10}s^2 - \frac{1}{500}s + \frac{21}{10})}{s(s-2)(s+1)(s+5)}$$



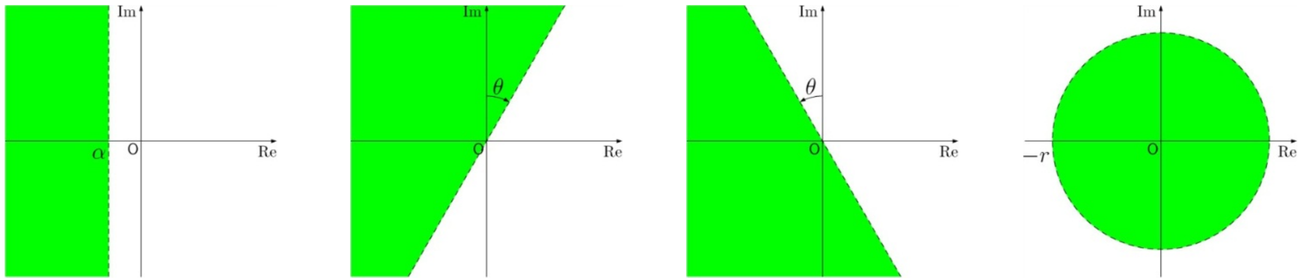


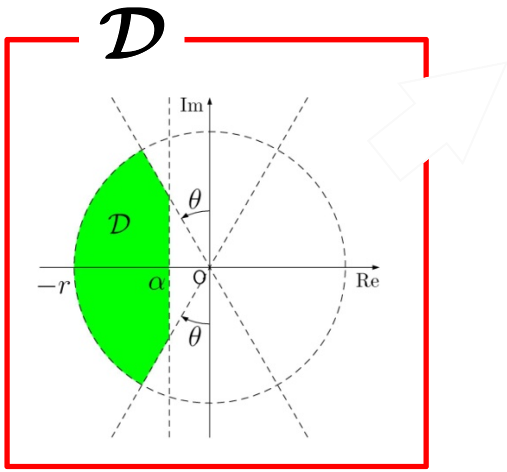


Closed-loop Poles

We can not assign all the closed-loop poles as you want.

Unified Approach to D-Stability Analysis





Unified Stability Analysis for Disks and Half Planes

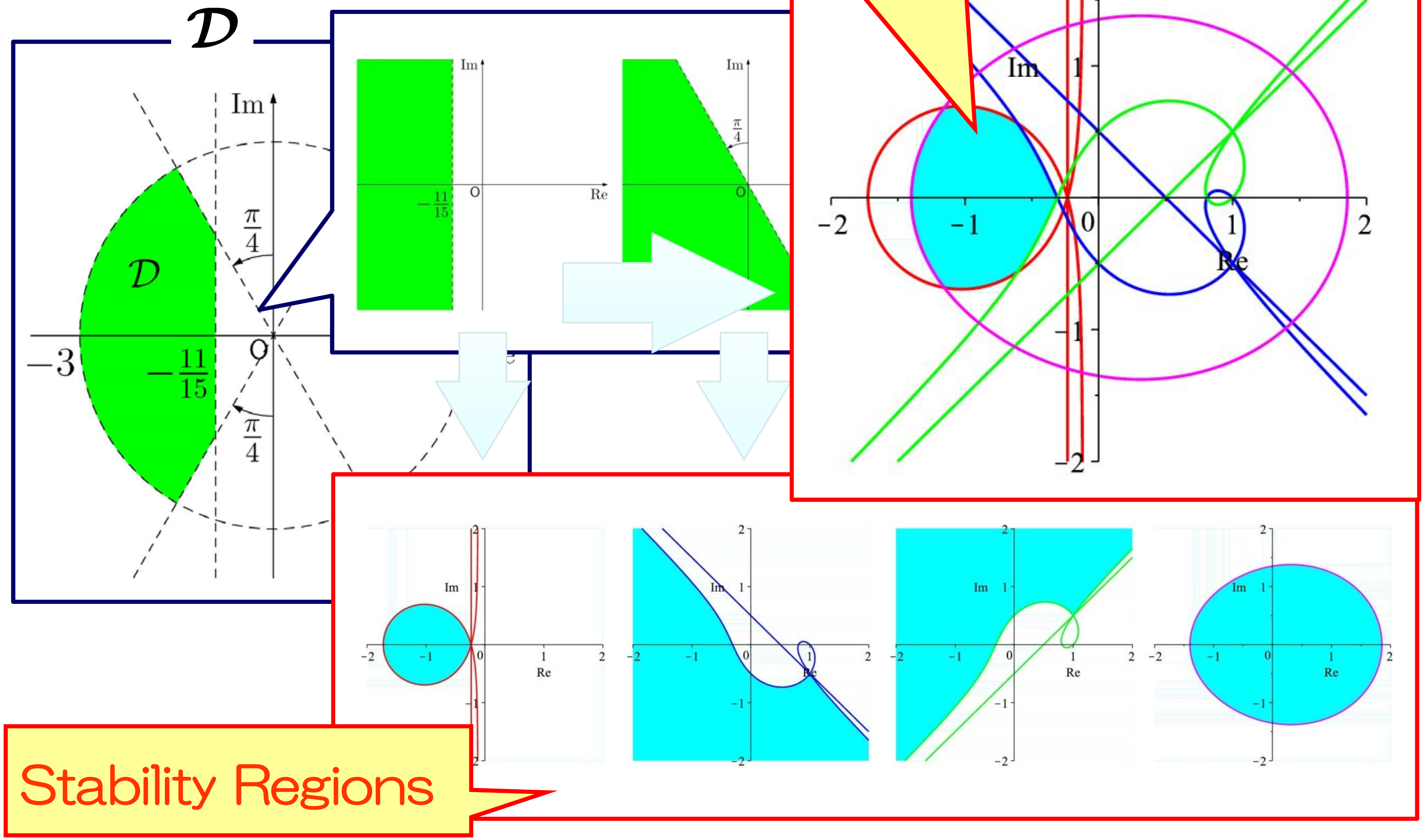
Derive the stability conditions

Intersection

D-stability Condition

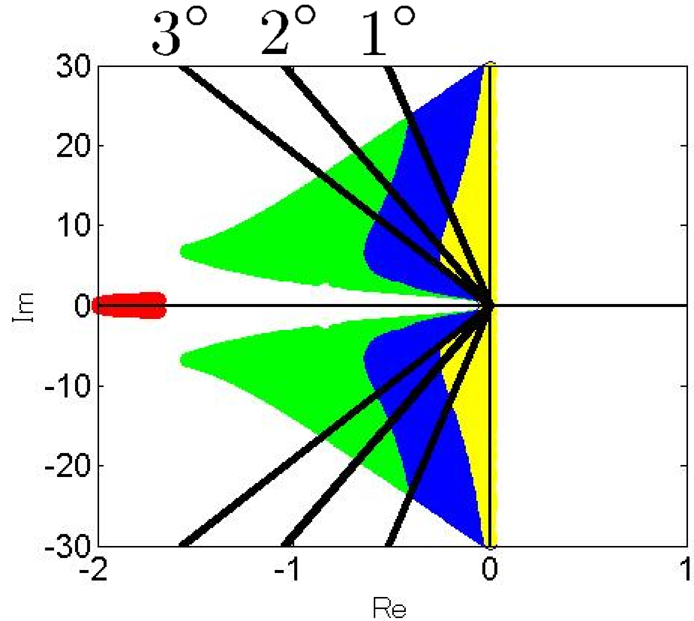
A Numerical Example

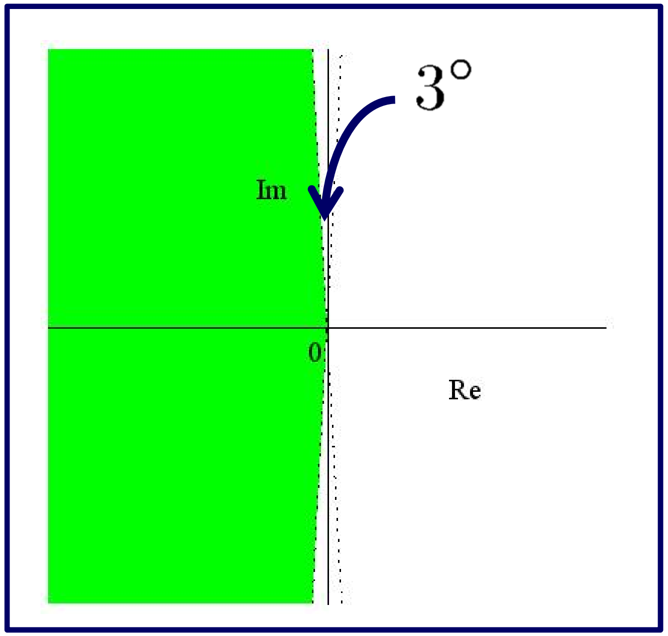
$$h(s) = \frac{2s+1}{s^2+s+1}$$



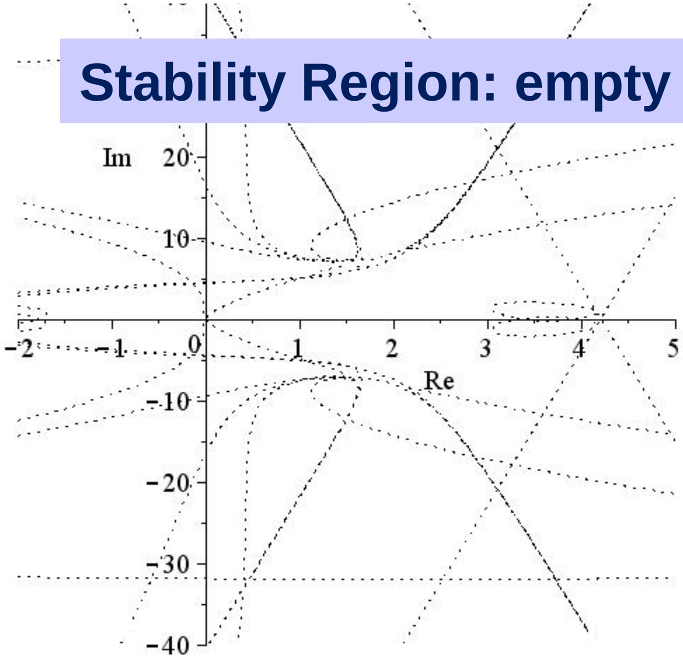
Motivating Example

$$h(s) = \frac{(\frac{1}{2}s+1)(\frac{19}{10}s^2 - \frac{1}{500}s + \frac{21}{10})}{s(s-2)(s+1)(s+5)}$$





Stability Region: empty



Messages : A New Framework

① **LTI system with generalized freq. variable**

a proper class of homogeneous multi-agent dynamical systems

② **Three types of stability tests, namely graphical, algebraic, and numeric (LMI)**

powerful tools for analysis

Q3: from **Homogeneous**
to **Heterogeneous** ***?***

Q4: from **Flat Structure**
to **Hierarchical Structure** ***?***

New Framework for System Theory