## Robust Control of Linear Systems with Uncertain Parameters

## Background

Model Predictive Control (MPC) has been widely adopted for motion planning and control of systems such as autonomous vehicles and drones [7, 8]. In contrast to classical control algorithms, a model of the controlled system is employed to predict its behavior and an optimization problem is formulated to compute the control inputs. Thereby, constraints on the state and the control inputs (e.g. actuator limitations) can be considered seamlessly.

To ensure safety despite modeling errors and (external) disturbances such as wind in autonomous driving, robust MPC schemes have been proposed, see e.g. [4]. Most of the works in the literature have been devoted to additive disturbances that are well suited for modeling external disturbances. However, modeling uncertainties like unknown friction coefficients in autonomous driving is more straightforward using models with parametric uncertainties.



Using robust control algorithms enables us to ensure safety despite uncertainties like unknown friction coefficients in autonomous driving. (Image from kroschke.com).

## Description

Therefore, the goal of this thesis is to extend the MPC algorithm proposed in [4] to linear systems with parametric uncertainties. The approach in [4] leverages reachability analysis [2], which is a tool for formal verification, to compute the control inputs while ensuring safety for every reachable state of the system. In contrast to existing works, the approach in [4] scales better with the dimension of the state space, which makes it appealing for real-world applications. By combining the MPC algorithm in [4] with the reachability algorithm for linear systems with uncertain parameters in [3], a novel MPC algorithm should be derived.

All programming will be done in MATLAB and the final implementation should be integrated into our toolbox AROC [6].

This thesis provides you with the opportunity to get to know a state of the art algorithm for motion planning and control, which provides formal guarantees. Moreover, you gain/deepen your knowledge in control theory, numerical optimization, and reachability analysis, which is a tool for formal verification.

## Tasks

- Familiarization with our open-source MATLAB tool CORA [1].
- Familiarization with the MPC algorithm in [4] and the reachability algorithm in [3].
- Formulation of the optimization problem and derivation of suitable constraints by extending the optimal control problem in [4] to account for parametric uncertainties based on [3].
- Computation of a suitable terminal region, see e.g. [5].



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Research project: justITSELF

**Type:** Master's thesis

Research area: Safety-preserving control, robust control, formal methods

**Programming language:** MATLAB

#### Required skills:

Background in control theory and dynamical systems. Background in numerical optimization beneficial.

Language: English, German

Date of submission: 6. Februar 2023

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- Analysis of the developed MPC scheme to guarantee safety.
- Evaluation of your algorithm.
- Documentation of your results.

### References

- Matthias Althoff. An introduction to cora 2015. In ARCH@ CPSWeek, pages 120–151, 2015.
- [2] Matthias Althoff, Goran Frehse, and Antoine Girard. Set propagation techniques for reachability analysis. Annual Review of Control, Robotics, and Autonomous Systems, 4(1):369–395, 2021.
- [3] Matthias Althoff, Colas Le Guernic, and Bruce H. Krogh. Reachable set computation for uncertain time-varying linear systems. In *Proceedings of the 14th International Conference* on Hybrid Systems: Computation and Control, HSCC '11, page 93–102, New York, NY, USA, 2011. Association for Computing Machinery.
- [4] Felix Gruber and Matthias Althoff. Scalable robust model predictive control for linear sampled-data systems. In *IEEE Conference on Decision and Control*, pages 438–444, 2019.
- [5] Felix Gruber and Matthias Althoff. Computing safe sets of linear sampled-data systems. IEEE Control Systems Letters, 5(2):385–390, 2021.
- [6] Niklas Kochdumper, Felix Gruber, Bastian Schürmann, Victor Gaßmann, Moritz Klischat, and Matthias Althoff. Aroc: A toolbox for automated reachset optimal controller synthesis. In Proceedings of the 24th International Conference on Hybrid Systems: Computation and Control, HSCC '21, New York, NY, USA, 2021. Association for Computing Machinery.
- [7] Jason Kong, Mark Pfeiffer, Georg Schildbach, and Francesco Borrelli. Kinematic and dynamic vehicle models for autonomous driving control design. In *IEEE Intelligent Vehicles* Symposium, pages 1094–1099, 2015.
- [8] Huan Nguyen, Mina Kamel, Kostas Alexis, and Roland Siegwart. Model predictive control for micro aerial vehicles: A survey. In *European Control Conference*, pages 1556–1563, 2021.



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