Collision Checks for Air- and Spacecrafts

Background

When checking whether an air- or spacecraft trajectory is safe, one has to check that the trajectory in question does not hit a dangerous object, such as another aircraft, a satellite, or any other object that may interfere with the planned flight path. To that end, a common strategy is to model the vehicle and the (dangerous) object as sets, compute the trajectory of such sets, and then check for a given point in time whether the two sets intersect. If the dangerous object is significantly smaller than the vehicle (e.g., a bird or a meteorite), it suffices to check whether the object lies inside the trajectory of the vehicle.

These kind of problems are called containment problems. Although these problems can not always be solved algorithmically, they are often solvable for most usual set representations, e.g., polytopes, zonotopes, ellipsoids, ...

More generally, containment problems are used for reachability analysis, set-based observers, fault detection, robust control, controller synthesis, and conformance checking. Currently, containment checks are performed mostly on zonotopes or polytopes, as they are convex and can thus easily represent an autonomous car or a robot arm. However, air- and spacecrafts may be non-convex due to their wings, and thus other set representations need to be used, for example polynomial zonotopes [1]. Due to the non-convexity of such set representations, many properties that were easy to check for zonotopes or polytopes become more intricate and difficult.

This is also the case for containment problems, and one way to remedy this issue is to split a polynomial zonotope, over-approximate each part with a zonotope, and then check containment. Due to this over-approximation, significant errors can be made, and it is therefore of interest whether a better method can be found.

Description

The focus of this thesis is the containment problem for air- and spacecrafts trajectories, represented using polynomial zonotopes. Concretely, the goal is twofold: First, the complexity of the problem will be determined; if you want to learn how to show that a problem can only have exponential runtime (i.e., that it is (co-)NP-hard), this is the perfect opportunity to learn how to prove such results. In a second step, an algorithm to approximately solve the containment problem will be developed. All programming will be done in Matlab, and the final implementation of the approaches should be integrated into the CORA toolbox so that it can be made publicly available in the next CORA release.

Tasks

- Literature review on the topic of containment problems
- Analysis of the complexity of the containment problem for polynomial zonotopes
- Implementation of an approximate algorithm
- Evaluation of the performance by comparing the result to the currently implemented method in CORA
- Integration of the final implementation into the CORA toolbox
- Testing the implementation on a functional air- or spacecraft model

References

[1] Niklas Kochdumper and Matthias Althoff. Constrained polynomial zonotopes, 2020.

[2] A. Kulmburg and M. Althoff. On the co-np-completeness of the zonotope containment problem. *European Journal of Control*, To appear.

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Research project:
justITSELF

Type:
Bachelor Thesis

Research area:
Containment Problems, Reachability Analysis

Programming language:
MATLAB

Required skills:
Good mathematical background. A basic understanding of complexity theory may be useful, but can be learned along the way.

Language:
English

Date of submission:
25. Januar 2022

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