# Optimization-based Verification of Cyber-Physical Systems

## Background

A typical problem for many cyber-physical applications is to check whether a given system (e.g., an autonomous car, a robot,...) reaches a specified goal area (e.g., performed a correct lane change, placed an object in the correct spot,...). For example, a welding robot may need to verify whether it soldered the correct parts, and to do so needs to check whether its arm was in the right place at the right time.

One common way of performing such a check is by representing the system (e.g., the robot arm), as well as the goal area (e.g. the area around the parts to be soldered), as sets, and then check whether the system is contained within the goal area.

This sort of problems are called containment problems. Although in general they can not be solved algorithmically, they can often be solved in practice, e.g., if the sets are zonotopes. Zonotopes are sets that can be represented numerically very easily, while describing complex shapes. Owing to these favorable properties, zonotopes are also used for reachability analysis, set-based observers, fault detection, robust control, controller synthesis, and conformance checking. The aforementioned applications often require solving containment problems for zonotopes as well.

Recently, the containment problem for zonotopes was shown to be co-NP-complete [1]. That means that, unless P=NP, no polynomial-time algorithm solving this problem can exist. Algorithms that solve the problem approximately exist [2], but have a fixed accuracy that can not be tuned.

### Description

The goal of this thesis is to find new ways to solve the zonotope containment problem approximately. While some approximative methods already exist, for most of them one can not tune the accuracy of the method by requiring the solution to be within a given error bound. One possible remedy to that would be optimization- and/or learning-based approaches, that can be given a specific amount of time to compute an approximation that is within a certain error-bound.

All programming will be done in Matlab, and the final implementation of the approaches should be integrated into the CORA toolbox so that it can be made publicly available in the next CORA release.

#### Tasks

- Literature review on the topic of containment problems
- Development and implementation of one or several new algorithms for the zonotope containment problem, based on optimization
- Evaluation of the performance by comparing the result to the currently implemented method in CORA
- Integration of the final implementation into the CORA toolbox
- Testing of the implemented methods on various models for cyber-physical systems

#### References

- [1] A. Kulmburg and M. Althoff. On the co-np-completeness of the zonotope containment problem. *European Journal of Control*, To appear.
- [2] S. Sadraddini and R. Tedrake. Linear encodings for polytope containment problems. In IEEE 58th Conference on Decision and Control, pages 4367–4372, 2019.



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Research project: justITSELF

**Type:** Bachelor Thesis

**Research area:** Containment Problems, Reachability Analysis

**Programming language:** MATLAB

**Required skills:** Good mathematical background. A basic understanding of statistics may be useful.

Language: English

Date of submission: 22. September 2022

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