Exploiting Mixed-Monotonicity in Reachability Analysis

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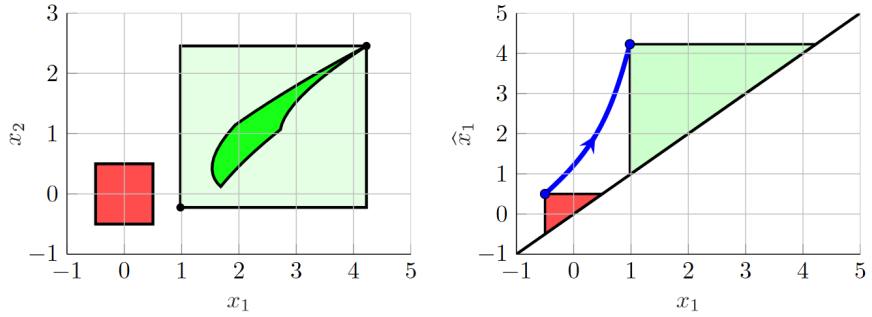
Background

The application of cyber-physical systems in safety-critical environments requires formal verification techniques in order to ensure correct functionality. Reachability analysis is one of the main techniques to provide safety guarantees: In general, only tight over-approximations of the reachable sets of states are computable, which are subsequently checked against a set of unsafe states given by unwanted system behavior. If the reachable set and the unsafe set do not intersect, safety is formally guaranteed. General model-based reachability algorithms are still cost-intensive, which has encouraged the development of more efficient algorithms exploiting special dynamical structures such as mixed-monotonicity.



Original system (left) and embedding system (right): Starting from the initial set (red), the exact reachble set (green) is over-approximated by a hyperrectangular over-approximation (light green) using a single trajectory (blue) of the embedding system. Image taken from [1].

Description

In certain cases, the dynamics of a continuous-time system $\dot{x} = f(x)$ can be separated into increasing and decreasing components [1] using a decomposition function $d(x, \hat{x})$ whose application to the dynamics yielding an embedding mixed-monotone system. The rewritten dynamics allow for great simplifications in the computation of reachable sets as a single trajectory suffices to construct hyperrectangular over-approximations, see Figure above. In this thesis, we are interested in the performance of reachability algorithms for mixed-monotone systems from the literature compared to standard model-based algorithms in terms of computational efficiency and accuracy. Furthermore, we aim for a two-fold extension of existing algorithms: (1) adaptive parameter tuning of the reachability algorithm, (2) an estimate for the committed over-approximation error with respect to the exact reachable set. All programming will be done in MATLAB, and the final implementation should be integrated into the CORA toolbox [2].

Tasks

- Implementation of reachability analysis for mixed-monotone systems CORA
- Adaptive parameter tuning for the reachability algorithm of mixed-monotone systems
- Estimation of the over-approximation with respect to the exact reachable set
- Evaluation of the performance and comparison to more general approaches

References

[1] Matthew Abate and Samuel Coogan. Computing robustly forward invariant sets for mixed-monotone systems. In *2020 59th IEEE Conference on Decision and Control (CDC)*, pages 4553–4559. IEEE, 2020.

[2] M. Althoff. An introduction to CORA 2015. In *Proc. of the Workshop on Applied Verification for Continuous and Hybrid Systems*, pages 120–151, 2015.

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Research project:
ConVeY

Type:
MA

Research area:
Reachability Analysis

Programming language:
MATLAB

Required skills:
Good mathematical background (calculus), programming in MATLAB

Language:
English

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